A Framework Between Quantum and Classical: An Illustration With Neutrinos

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What this talk is about

- This talk is about a genereralization of a particular aspect of quantum mechanics.
- Generalization: A framework with extra parameters that quantify the deviation from QM.
- Reduces to QM when these parameters are set to zero and interpolates between QM and CM at some other values.
- Can be seen as a way to study the foundations of QM.

Overview

Necessity of quantum foundations

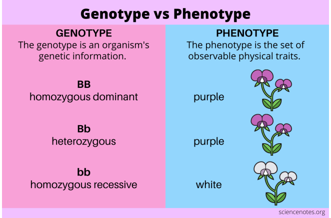
- Lorentz Transformations ⇒ Principle of relativity
- Mathematical ‘axioms’ of QM ⇒?

Necessity of quantum foundations

- Lorentz Transformations $\Rightarrow$ Principle of relativity
- Mathematical 'axioms' of QM $\Rightarrow$?

- SR + Equivalence principle $\Rightarrow$ GR
- Physical QM + Additional insight $\Rightarrow$ A bigger theory (QG?)

Layers of explanation



Mathematical 'genotype'	**Physical 'phenotype'**
Linearity of SE	Superposition, Interference
Complex Hilbert space	Tensor product, Entanglement
...	...

Wait! Isn't quantum foundations just philosophy?

- Early debates on quantum foundations mostly concerned interpretations.
- Bell (1964) devised an 'operational' inequality that separates classical and quantum regimes by quantifying the amount of non-local correlation.
- Rigorously defined and provided a test for seemingly 'philosophical' issues like the viability of hidden-variable theories.
- Experimental confirmation of the violation by CHSH revitalized the field and led to applications in information processing.
- Many researchers follow this example to rigorously formulate other issues and design experiments to test them (Ex. macrorealism, non-contextuality).

Why tamper with quantum mechanics?

- Better understanding: Relaxing the mathematical structure or generalizing QM can give insights into the very aspects that were generalized.
- New phenomenology: Potentially describe phenomena not present in canonical QM but present in Nature.
- More parameters $\Longrightarrow$ Wider testing: Could allow for a wider testing of certain aspects of QM. Ex. SM and GR.
- Environmental mutations: Considerations in quantum gravity *might* make the modification unavoidable.

Quantum correlations

- Quantum theory violates the Bell-CHSH inequality

$$S := |\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle| \leq 2,$$

where $A_0,\ B_0,\ A_1,\ B_1 \in \{\pm 1\}$

- The upper bound on S in quantum mechanics is $S = 2\sqrt{2}$ (Tsirelson bound).

- The algebraic maximum of $S = 4$ is consistent with relativity (Popescu & Rohrlich (1994)).

Higher-order interference

- Classically, for n available paths for a system in state α to end up in state β,

$$P(A, B, C, \cdots) = P(A) + P(B) + P(C) + \cdots$$

- In a double-slit experiment, quantum-mechanically

$$P(A, B) \ = \ |\psi_A + \psi_B|^2 \ = \ \underbrace{|\psi_A|^2}_{P(A)} + \underbrace{|\psi_B|^2}_{P(B)} + \underbrace{(\psi_A^* \psi_B + \psi_B^* \psi_A)}_{I_2(A,B)} .$$

- For three slits/paths,

$$\begin{aligned} P(A, B, C) \ &= \ |\psi_A + \psi_B + \psi_C|^2 \\ &= \ P(A) + P(B) + P(C) + I_2(A, B) + I_2(B, C) + I_2(C, A) \end{aligned}$$

- Define (Sorkin, 1994)

$$\begin{aligned} I_3(A, B, C) := P(A, B, C) - P(A, B) - P(B, C) - P(C, A) \\ + P(A) + P(B) + P(C). \end{aligned}$$

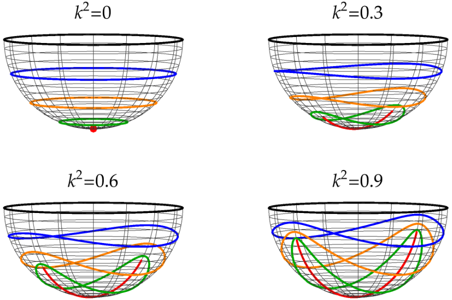
Hierarchy of “quantumness”

Quantumness	Correlations	Interference
Classical theory	$S \leq 2$	$I_2 = 0,\ I_3 = 0$
Quantum theory	$2 < S \leq 2\sqrt{2}$	$I_2 \neq 0,\ I_3 = 0$
“Super-quantum” theory	$2\sqrt{2} < S < 4?$	$I_2 \neq 0,\ I_3 \neq 0?$

A minimal* generalization of QM

Our work only changes the "phases" $U(1) = \{e^{-iEt}\}$ of energy eigenstates using two deformation parameters (k, ξ) that quantify the deviation from QM.

Parameters $k \propto$ eccentricity and $\xi \propto$ size. Can be thought of as a 'mutation' of the phase.



Neutrino oscillation probability

- Flavor eigenstates of neutrinos, $|\alpha\rangle$ and $|\beta\rangle$, are superpositions of their mass eigenstates, $|1\rangle$ and $|2\rangle$.

$$\begin{aligned} |\alpha\rangle &= \quad \cos\theta\,|1\rangle + \sin\theta\,|2\rangle \\ |\beta\rangle &= -\sin\theta\,|1\rangle + \cos\theta\,|2\rangle \end{aligned}$$

- This causes the phenomena of interference and oscillation.

$$P(\alpha \to \beta) = \sin^2 2\theta \sin^2\left(\frac{\delta m^2 L}{4E}\right) = \sin^2 2\theta \sin^2 (t_2 - t_1),$$

$$\left(\frac{\delta m^2 c}{4E}\right) = 1,\ L \approx c(t_2 - t_1).$$

Observable phenotype: Modified oscillation formula

- In our framework, QM is deformed using two parameters $0 \leq k^2 < 1$ and $0 \leq \xi \leq \dfrac{\pi}{2}$.

- The neutrino oscillation probability is now[1]

$$P_G(\alpha \to \beta) = \left(\cos^2\xi + \frac{k^2}{2}\sin^2\xi\right)\sin^2 2\theta\,\sin^2(t_2 - t_1) + \mathcal{O}(k^4)\,.$$

- $\left(c_\xi^2 + \frac{k^2}{2}s_\xi^2\right) \leq 1$, so k and ξ can be bound by considering bounds on $\sin^2 2\theta$.

[1] NB, D. Minic, & T. Takeuchi, JHEP 2024, 31 (2024)

The "pseudoclassical" limit

- For $k = 0$,

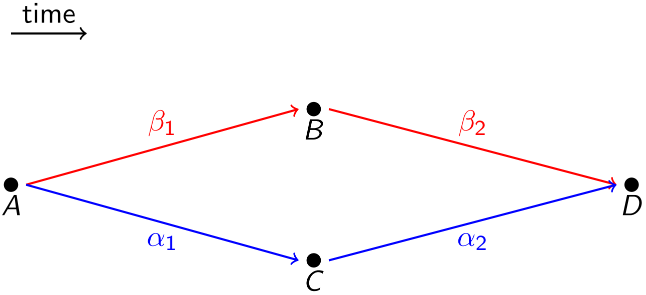
$$P_G(\alpha \to \beta) = \cos^2 \xi \sin^2 2\theta \sin^2 (t_2 - t_1) \,.$$

- Note that when $\xi = 0$, the deformation is turned off

$$P_G(\alpha \to \beta) = P(\alpha \to \beta).$$

- For $\xi = \frac{\pi}{2}$, $P_G(\alpha \to \beta) = 0$, a classical behaviour!

Two flavor oscillation/Double slit experiment



$$P_{\alpha\alpha}(A \to D) = |\alpha_1\alpha_2 + \beta_1\beta_2|^2$$

$$= \underbrace{|\alpha_1|^2|\alpha_2|^2}_{P_{ACD}} + \underbrace{|\beta_1|^2|\beta_2|^2}_{P_{ABD}} + \underbrace{2\,\mathrm{Re}(\alpha_1^*\alpha_2^*\beta_1\beta_2)}_{I_2(\alpha,\beta)}.$$

How to quantify interference?

- $I_2(\alpha, \beta)$ is just the difference between the survival probabilities with and without intermediate measurement![2]

$$I_2 := \underbrace{P_{\alpha\alpha}(0,2t)}_{P_{\alpha\alpha}(A\to D)} - \left\{ \underbrace{P_{\alpha\alpha}(0,t)P_{\alpha\alpha}(t,2t)}_{P_{ACD}} + \underbrace{P_{\alpha\beta}(0,t)P_{\beta\alpha}(t,2t)}_{P_{ABD}} \right\}$$

- For canonical QM,

$$I_2(\alpha,\beta) = -\frac{1}{2}\sin^2(2t).$$

- In the current framework,

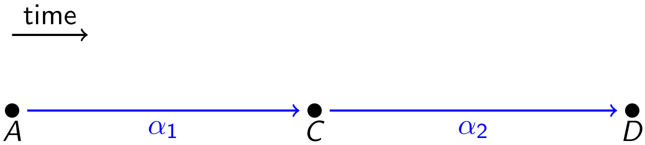
$$I_2(\alpha,\beta) = -2\,c_\xi^2\,\sin^2 t\,\left(2\cos^2 t + c_\xi^2\,\sin^2 t - 1\right).$$

[2] $I_2 = 0$ called No Signaling in Time in literature. Phys. Rev. A 87, 052115

Covering one of the slits

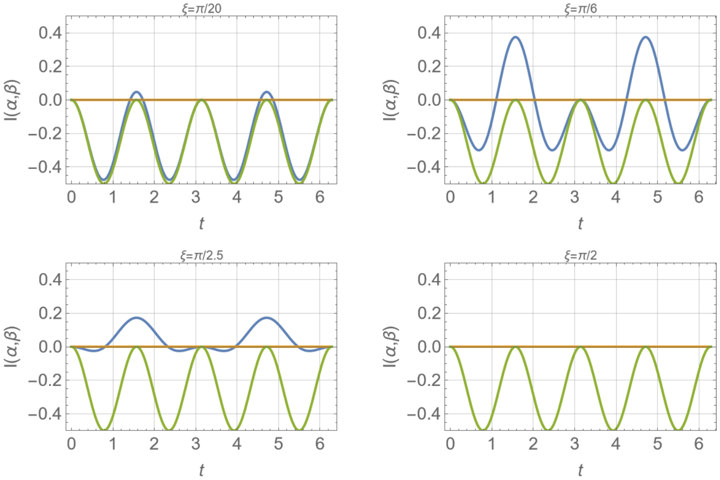
- For $\xi = \frac{\pi}{2}$,

$$I_2(\alpha, \beta) = 0 \text{ identically!}$$



- Superposition is lost.
- Taking the limit $\xi \to \frac{\pi}{2}$ is mathematically trivial but continuously interpolates between quantum-like and classical-like behavior.
- Could provide insights into quantum to classical transition.

Between quantum and classical



Summary

- The foundations of QM can be studied rigorously and confronted with experiments.
- Modifications to QM not meant as empirical competitors but serve to clarify why QM is the way it is.
- Having alternative formulations *might* be useful in surviving environment-induced ‘mutations’, ex. quantum gravity.
- Particle physics processes like neutrino oscillations can be exploited for experimental tests.