Small-x asymptotics of GPDs

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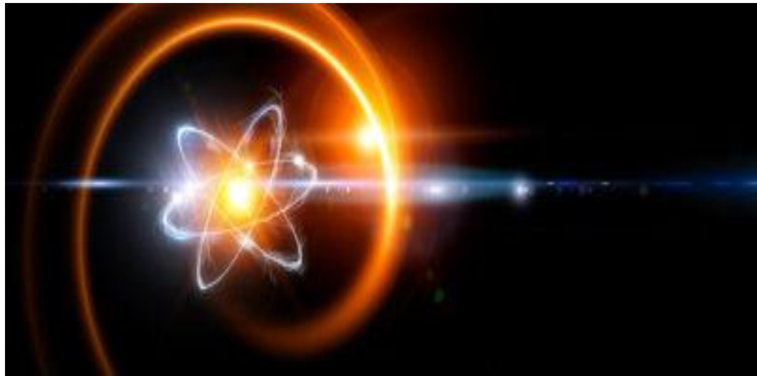
in collaboration with *Yuri Kovchegov and M. Gabriel Santiago*



Things to address:

- Main goal: understand small-x asymptotics of GPDs at nonzero skewness ξ.

- Account for real part of scattering amplitude. Real part of the scattering amplitude is either discarded in small-x calculations of exclusive processes or is included using a model calculation. We would like to improve on that.

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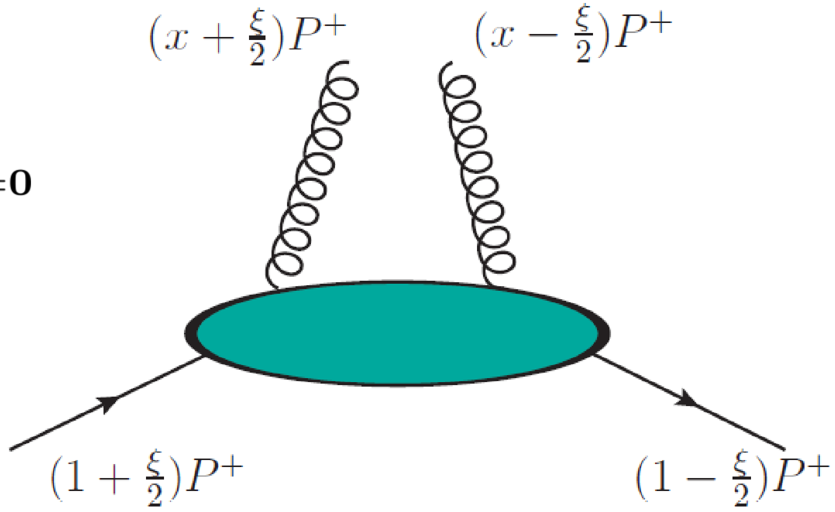
Gluon generalized parton distribution

$$F^g = \frac{1}{2P^+}\int_{-\infty}^{\infty}\frac{dz^-}{2\pi}e^{ixP^+z^-}\left\langle p'\left|F^{+\mu}\left(-\frac{1}{2}z\right)F_\mu^+\left(\frac{1}{2}z\right)\right|p\right\rangle\Big|_{z^+=0,\mathbf{z}=\mathbf{0}}$$
$$= \frac{1}{2P^+}\left[H^g(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E^g(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right].$$



- Skewness $\xi = \frac{p'-p}{p}$.
- Momentum transferred squared $t = \Delta^2$

Our goal is to understand $x \ll 1$ limit of F^g with $\xi \neq 0$.

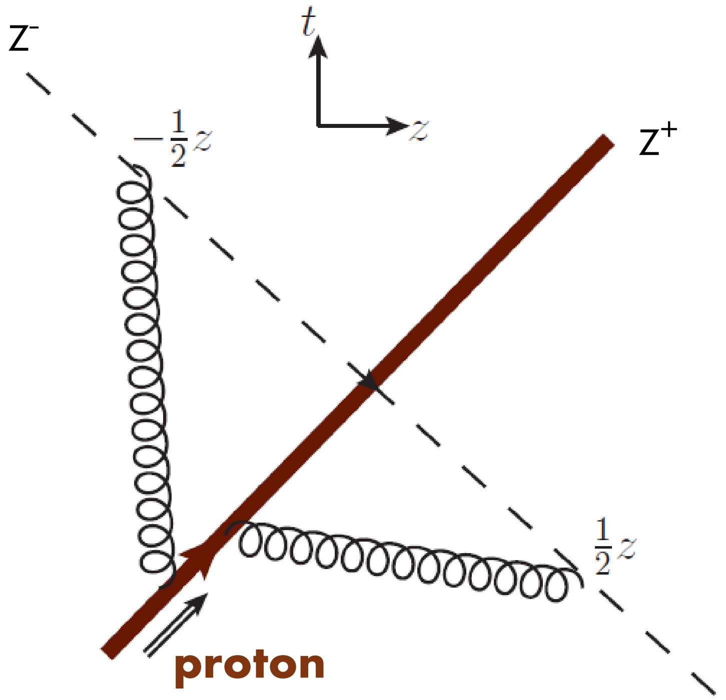
Small-x evolution of E^g was studied in the $\xi \to 0$ limit.
Y. Hatta and J. Zhou, Phys. Rev. Lett. 129, 252002 (2022), arXiv:2207.03378 [hep-ph]

GPD at small-x

$$F^g = \frac{1}{2P^+}\int_{-\infty}^{\infty}\frac{dz^-}{2\pi}e^{ixP^+z^-}\left\langle p'\left|F^{+\mu}\left(-\frac{1}{2}z\right)F_\mu^{\ +}\left(\frac{1}{2}z\right)\right|p\right\rangle$$

$$z^- \sim \frac{1}{xP^+},$$

Small x → large z^- separation



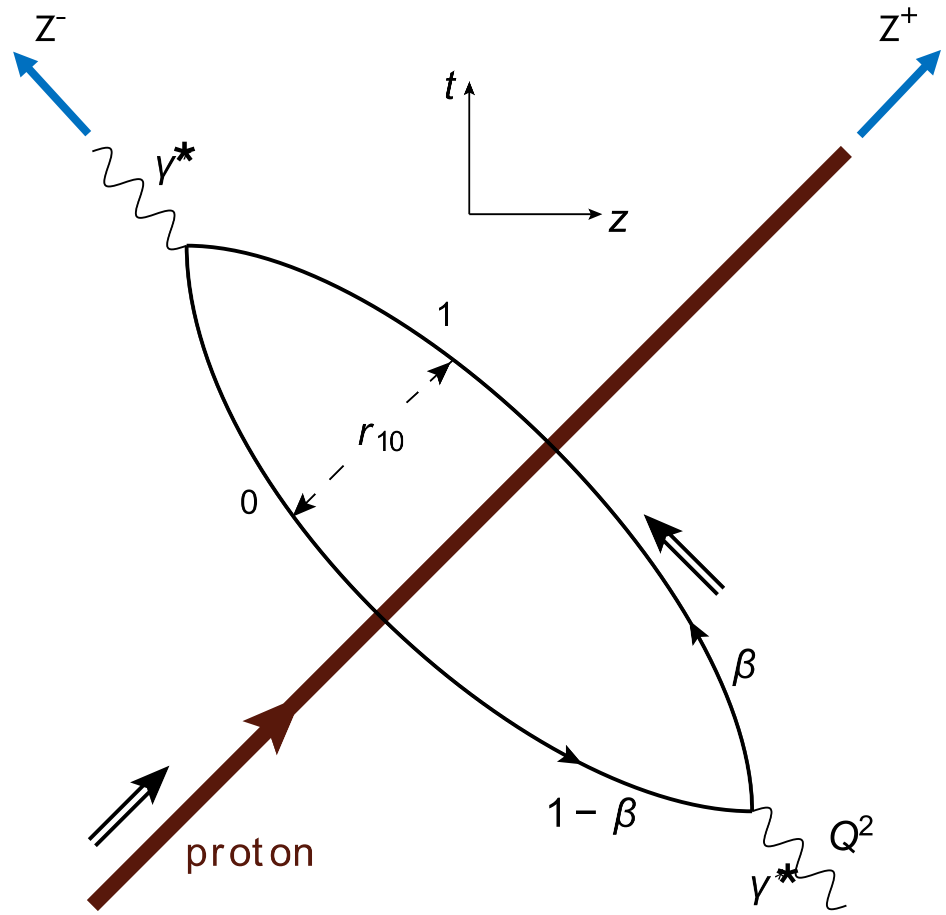
Small-x dipole picture

$$q^\mu = \left(\frac{Q^2}{2q^-}, q^-, 0_\perp\right)$$

$$z^- \sim \frac{2q^-}{Q^2} >> \frac{1}{P^+},$$

Lifetime of $q\bar{q}$ pair

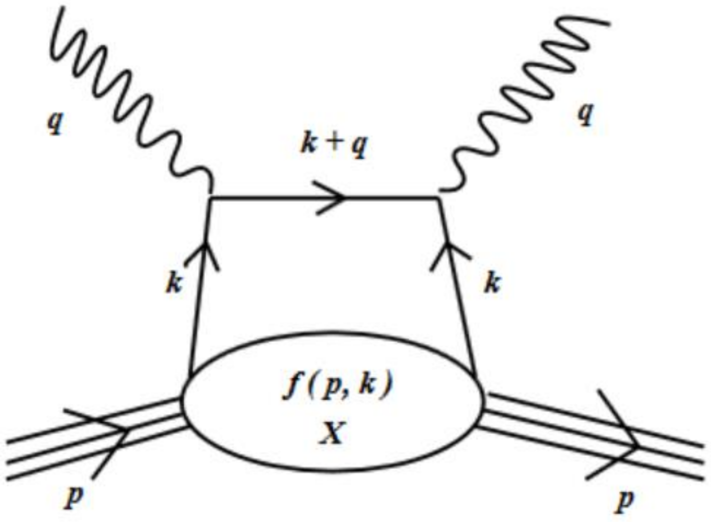
Large q^- → large z^- separation



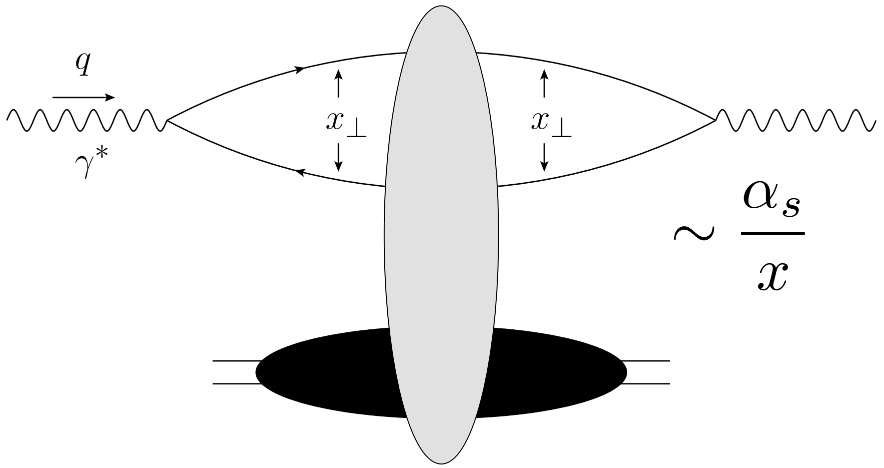
aka the "shock wave"

Dipole picture of GPD

- At small x, the dominant contribution to the matrix element F^g does not come from the handbag diagram.



- Instead, the dominant term comes from the dipole picture, where the virtual photon splits into a quark-antiquark pair, which then interacts with the target.

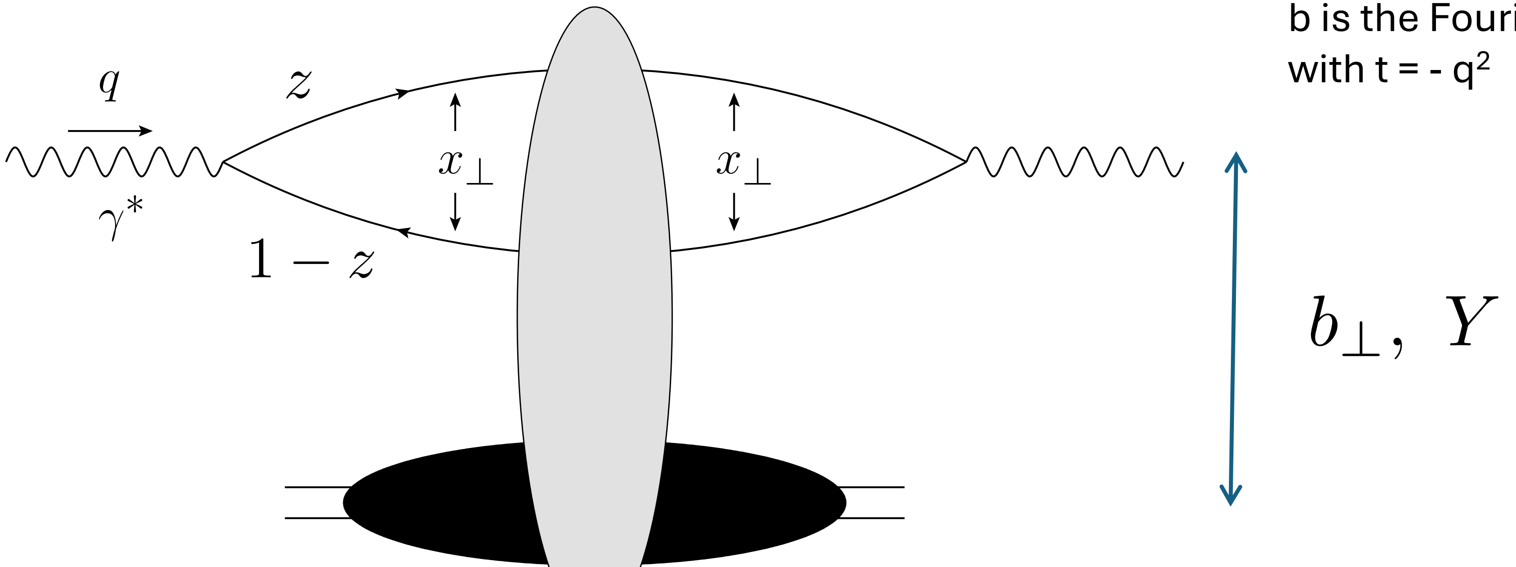


Dipole Amplitude

- The total DVCS cross section is expressed in terms of the (Im part of the) forward quark dipole amplitude N:

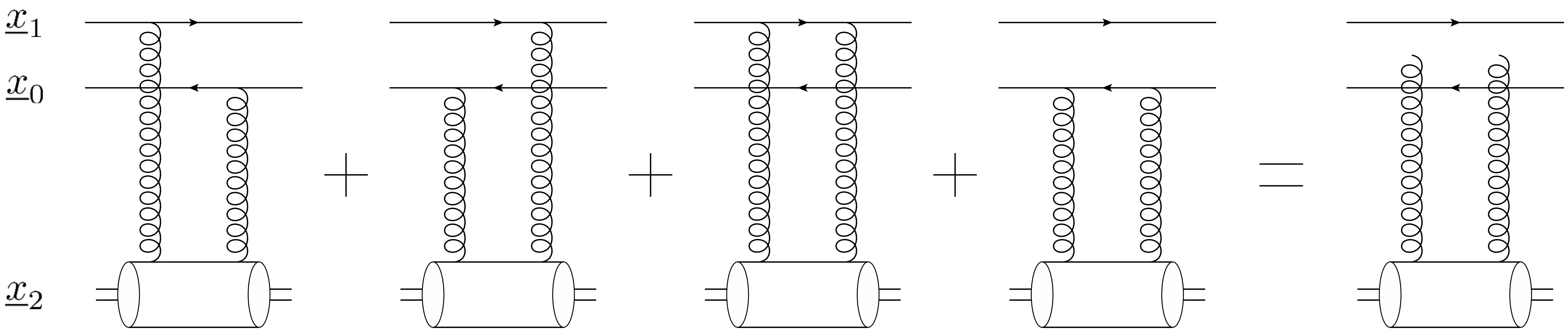
$$\sigma^{\gamma^* A \to \gamma A} = \int d^2 b_\perp \left| \int \frac{d^2 r_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} \Psi^{\gamma^*}(\underline{r}, z) N(\underline{r}, \underline{b}, Y) \Psi^{\gamma}(\underline{r}, z)^* \right|^2$$

b is the Fourier conjugate to q with t = - q^2



with rapidity Y=ln(1/x)

Quasi-classical dipole amplitude (i.c. for evolution)



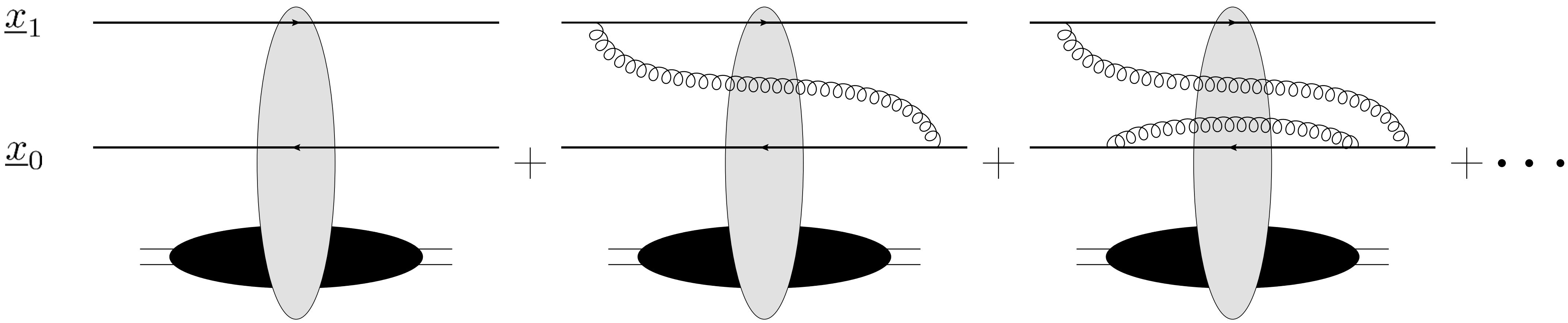
$$N(\underline{x}_{12}, \underline{b}, Y = 0) = 2\alpha_s^2 C_F \ln^2 \frac{x_{12}}{x_{02}}$$

- The dipole amplitude N is independent of energy at the lowest order.

Small-x Evolution

- Energy dependence comes in through the long-lived *s*-channel gluon corrections (higher Fock states):

$$\alpha_s \ \ln s \sim \alpha_s \ \ln \frac{1}{x} \sim 1$$



These extra gluons bring in powers of α_S ln s, such that when α_S << 1 and ln s >>1 this parameter is α_S ln s ~ 1 (leading logarithmic approximation, LLA).

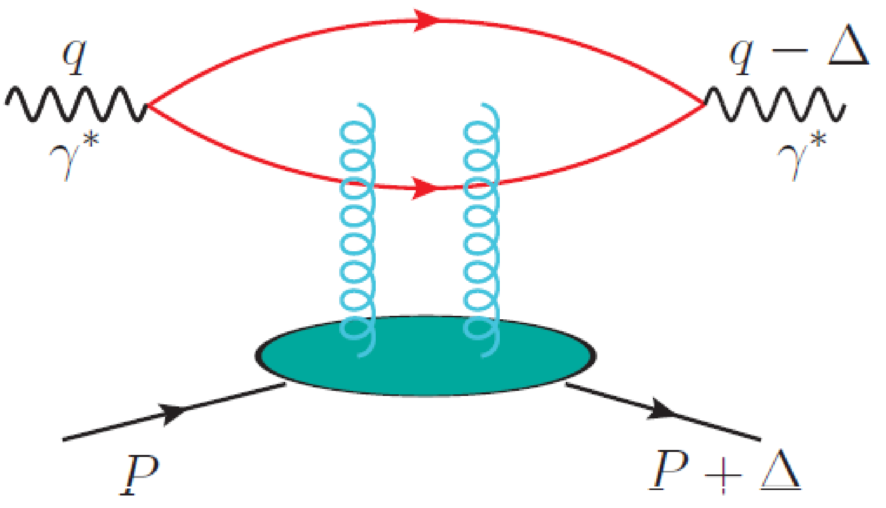
$$Y = \ln \frac{1}{x}$$

$$\partial_Y N(\underline{x}_0, \underline{x}_1, Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} [N(\underline{x}_0, \underline{x}_2, Y) + N(\underline{x}_2, \underline{x}_1, Y) - N(\underline{x}_0, \underline{x}_1, Y) - N(\underline{x}_0, \underline{x}_2, Y) N(\underline{x}_2, \underline{x}_1, Y)]$$

Balitsky '96, Kovchegov '99

- Scattering amplitude for DVCS/DVMP

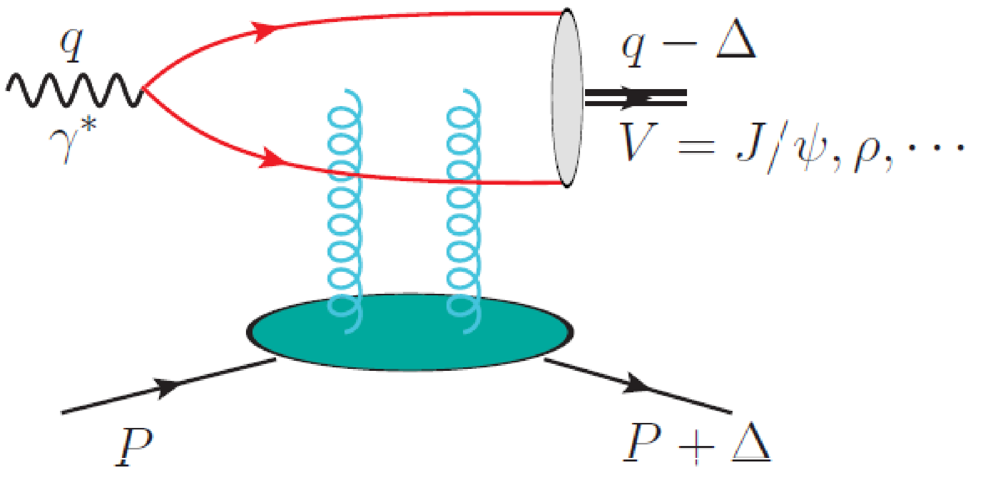
$$\sigma^{\gamma^* A\to\gamma A} = \int d^2b_\perp \left| \int \frac{d^2r_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} \Psi^{\gamma^*}(\underline{r},z) N(\underline{r},\underline{b},Y) \Psi^{\gamma}(\underline{r},z)^* \right|^2$$



- Y. Hatta and J. Zhou, Phys. Rev. Lett. 129, 252002 (2022).

$$\int \frac{d^2x_{10}}{(2\pi)^2} e^{i\underline{k}\cdot\underline{x}_{10}} \mathcal{N}\left(\underline{x}_{10}, \underline{\Delta}, \ln\frac{1}{x}\right) \approx (2\pi)^2 \delta^2(\underline{\Delta})\delta^2(\underline{k}) - \frac{\pi g^2}{2N_c k_\perp^2} f_{1,1}(x,\xi,\underline{k},\underline{\Delta})$$

$$xH_g(x,\xi,-\underline{\Delta}^2) = \int d^2k_\perp f_{1,1}(x,\xi,\underline{k},\underline{\Delta}).$$



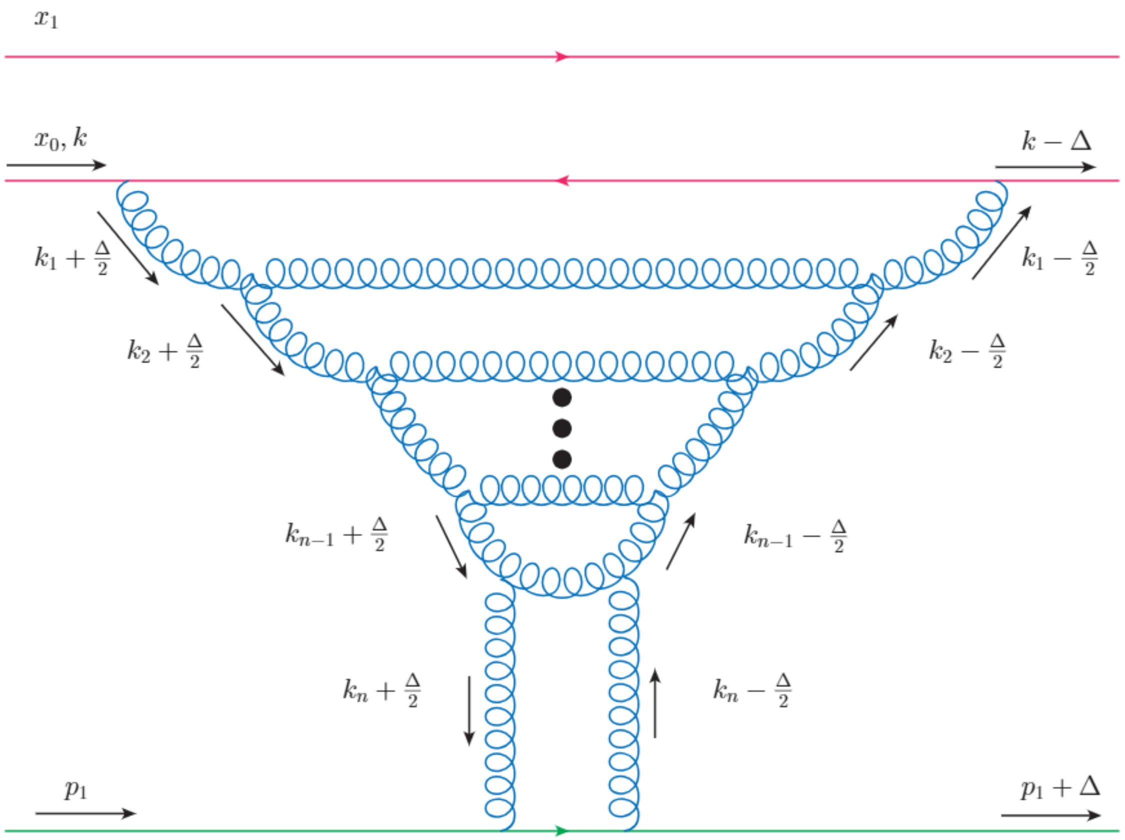
Small-x evolution in the dipole picture

Dipole emits soft gluons undergoes a gluon cascade before interacting with the target shockwave.

In the leading logarithmic approximation (LLA), evolution is governed by lifetime ordering of the soft gluons,

$$x_1^- \gg x_2^- \gg \cdots \gg x_n^- .$$

- How does skewness affect small-x evolution?



Effect of skewness

Lifetime ordering of gluons leads to the strong ordering of the longitudinal momenta to the left of the shockwave,

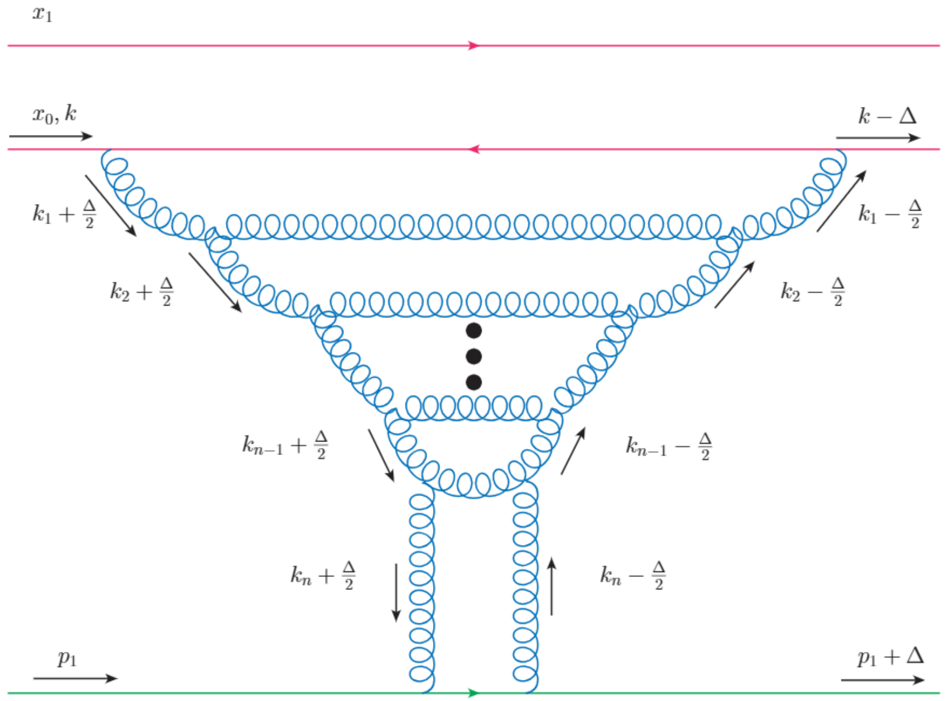
$$q^{+} \ll \frac{\left(\underline{k}_1 + \frac{\underline{\Delta}}{2}\right)^2}{2k_1^{-}} \ll \frac{\left(\underline{k}_2 + \frac{\underline{\Delta}}{2}\right)^2}{2k_2^{-}} \ll \cdots \ll \frac{\left(\underline{k}_n + \frac{\underline{\Delta}}{2}\right)^2}{2k_n^{-}}.$$

To the right of the shockwave, LLA enforces similar momenta ordering

.

$$q^{+}, \Delta^{+} \ll \frac{\left(\underline{k}_1 - \frac{\underline{\Delta}}{2}\right)^2}{2(k_1^{-} - \Delta^{-})} \ll \frac{\left(\underline{k}_2 - \frac{\underline{\Delta}}{2}\right)^2}{2(k_2^{-} - \Delta^{-})} \ll \cdots \ll \frac{\left(\underline{k}_n - \frac{\underline{\Delta}}{2}\right)^2}{2(k_n^{-} - \Delta^{-})}.$$

$k_i^{-} \gg \Delta^{-}$ is also required.



Effect of skewness

The plus momenta of the soft gluons are larger than the longitudinal fraction of momentum transferred:

$$\frac{\left(\underline{k}_i - \frac{\underline{\Delta}}{2}\right)^2}{z_i q^-} \gg \xi P^+.$$

This is constrained by the “longest living” gluon (gluon 1). The transverse momenta are of the same order

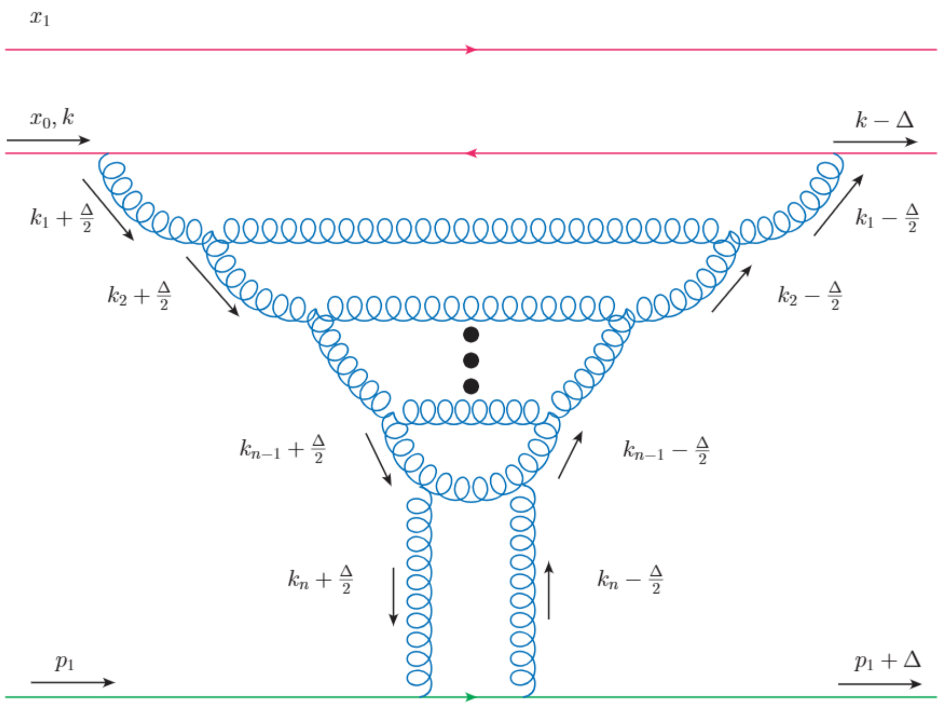
$$k_{1\perp}^2 \sim k_{2\perp}^2 \sim \cdots \sim k_{n\perp}^2 \sim Q^2 ,$$

substituting $x = \frac{Q^2}{s}$ leads to

$$z_1 \ll \min\left(1, \frac{x}{\xi}\right).$$

The integral for the longitudinal momentum of this gluon gives

$$\int_{\frac{1}{s x_{10}^2}}^{\min\{1,\frac{x}{\xi}\}} \frac{dz_1}{z_1} \approx \ln \min\left\{\frac{1}{x}, \frac{1}{\xi}\right\}.$$



Main result for GPDs with non-zero skewness

In the presence of skewness ξ, dipole amplitude N evolves with the known Balitsky-Kovchegov equation, but with skewness modifying the rapidity argument for the evolution to $\ln \min\left\{\frac{1}{x}, \frac{1}{\xi}\right\}$.

$$N\left(\underline{x}_{10}, \underline{b}, \ln\frac{1}{x}\right) \to N\left(\underline{x}_{10}, \underline{b}, \ln\min\left\{\frac{1}{x}, \frac{1}{\xi}\right\}\right).$$

In the $\xi \to 0$ limit, we get back the usual rapidity argument $\ln\frac{1}{x}$.

H_g is given through the relation:

$$xH_g(x,\xi,-\underline{\Delta}^2) = -\frac{N_c}{2\pi^3 g^2}\int d^2k_\perp e^{i\underline{k}\cdot\underline{x}_{10}} k_\perp^2 N\left(\underline{x}_{10}, \underline{\Delta}, \ln\min\left\{\frac{1}{x}, \frac{1}{\xi}\right\}\right).$$

Things to address:

- Main goal: understand small-x asymptotics of GPDs at nonzero skewness ξ.

- Account for real part of scattering amplitude. Real part of the scattering amplitude is either discarded in small-x calculations of exclusive processes or is included using a model calculation. We would like to improve on that.

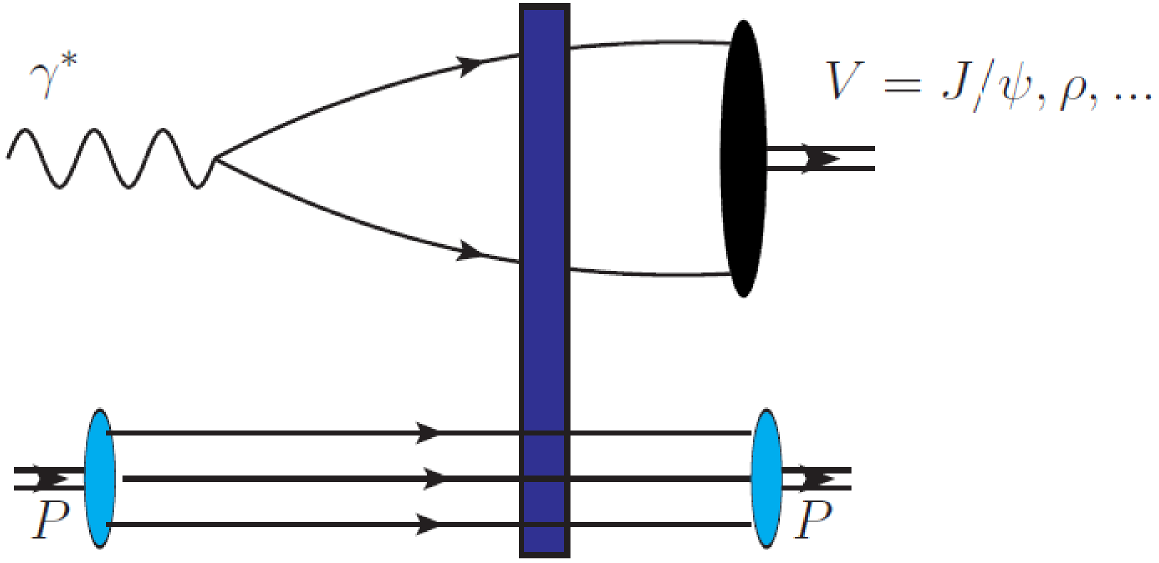
Vector meson production

$$\sigma^{\gamma^* A \to VA} = \int d^2\mathbf{b} \left| \int \frac{d^2\mathbf{r}}{4\pi} \int_0^1 \frac{dz}{z(1-z)} \Psi^{\gamma^* \to q\bar{q}}(\mathbf{r}, z) N(\mathbf{r}, \mathbf{b}) \Psi^V(\mathbf{r}, z)^* \right|^2$$

- The usual initial condition for dipole amplitude

$$N(\underline{x}_{10}, \underline{b}) = 1 - \exp\left\{ -\frac{x_{10}^2 Q_s^2(\underline{b})}{4} \ln \frac{1}{x_{10}\Lambda} \right\}.$$

is real.



- In phenomenology, people account for the imaginary part of the dipole amplitude by multiplying the cross section by an R-factor.

Quark-quark scattering amplitude

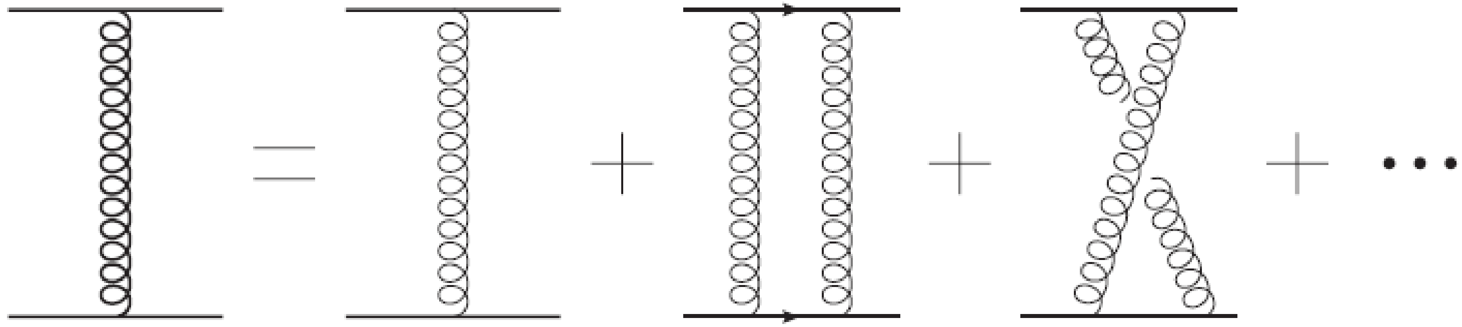


Fig. 3.10. Reggeized gluon (bold corkscrew line) represented as the sum of all leading-ln s corrections to the single-gluon exchange amplitude for $qq \to qq$ scattering.

The scattering amplitude for an octet exchange is

$$A^{(8)}_{qq\to qq} = 4\pi\alpha_s \frac{\mathrm{s}}{\mathrm{t}}\left[\left(\frac{\mathrm{s}}{t}\right)^{\alpha_{\mathrm{G}}(\mathrm{t})-1} + \left(\frac{-\mathrm{s}}{\mathrm{t}}\right)^{\alpha_{\mathrm{G}}(\mathrm{t})-1}\right] = 8\pi\alpha_s \left(\frac{s}{t}\right)^{\alpha_{\mathrm{G}}(\mathrm{t})} \frac{1-\mathrm{e}^{\mathrm{i}\pi\alpha_G(t)}}{2}.$$

Summing s-channel and u-channel contributions leads to the odd signature factor. The signature factor encapsulates the imaginary part of the amplitude.

What about pomeron (color singlet)?

Pomeron has even signature, so

$$A^{(1)}_{qq\to qq} \propto \frac{1 + e^{i\pi\alpha_P(t)}}{2}.$$

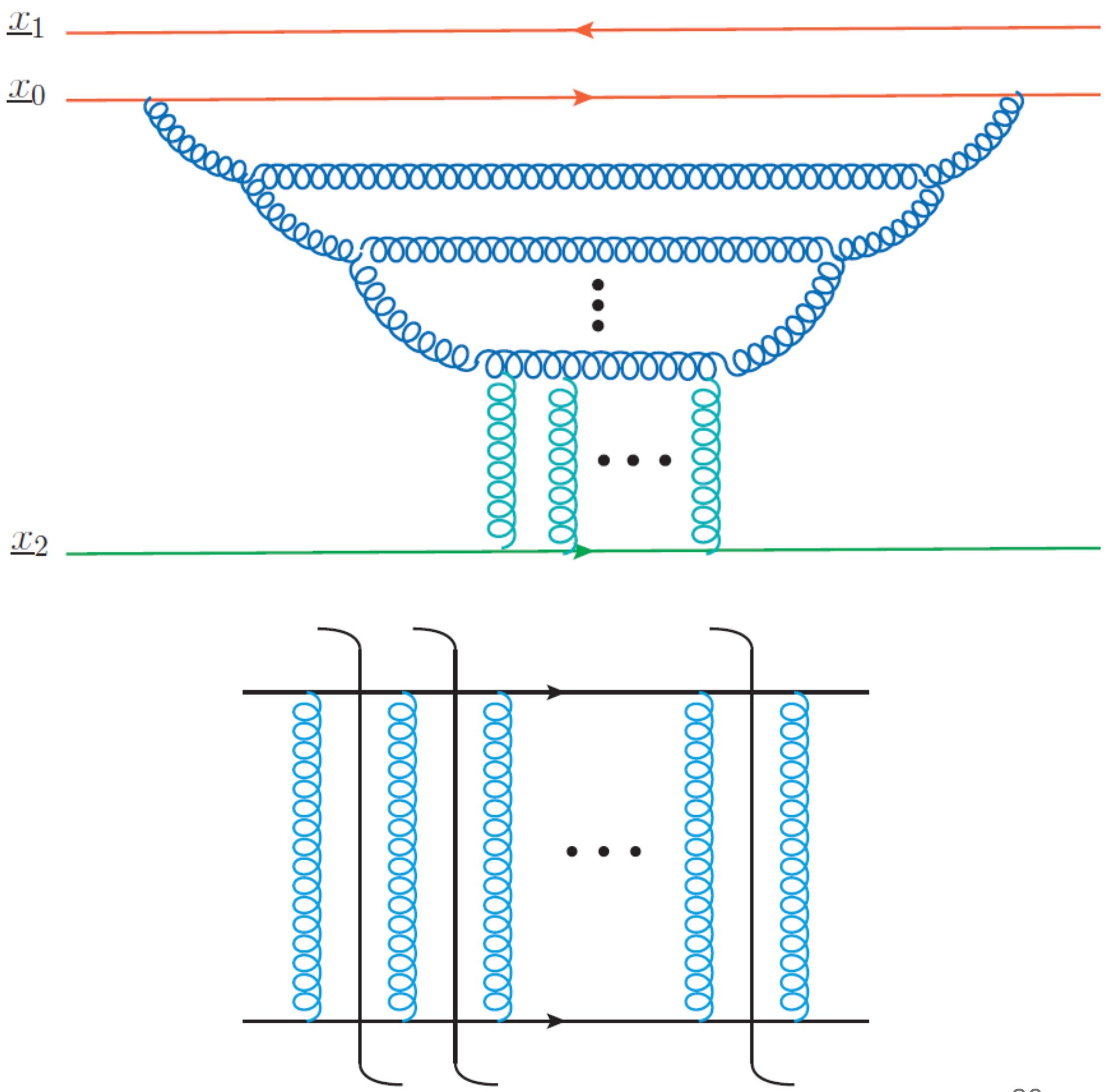
The real part of the amplitude cancels in LLA, so the leading contribution is purely imaginary.

The dipole amplitude is usually defined to be the imaginary part of the scattering amplitude:

$$N \equiv \frac{1}{2s} \Im A^{(1)}_{qq\to qq}$$

Where does the imaginary parts come from?

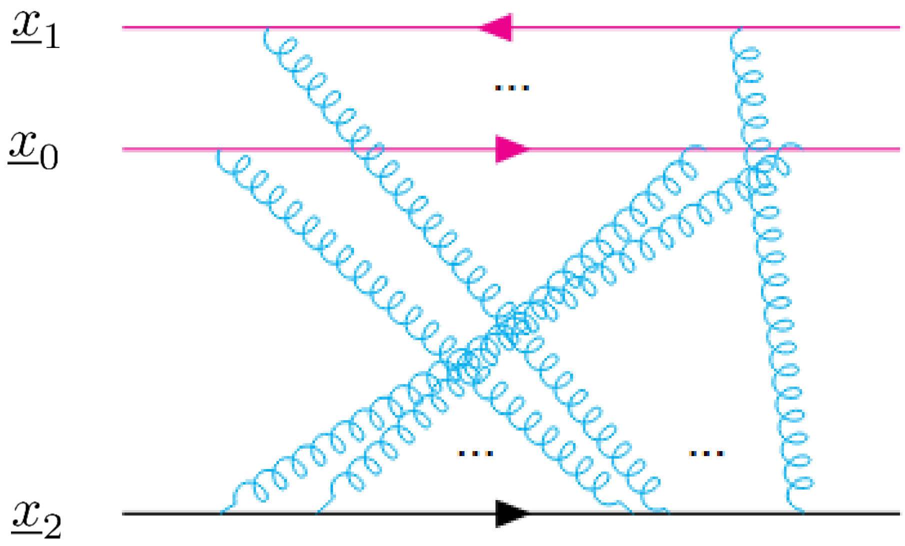
- Cutkosky rules tell us that discontinuities of scattering amplitudes can be obtained by cutting diagrams.
- One cannot cut through the cascade because that would correspond to a space-like photon decaying into real quarks and gluons. The cut has to go between t-channel gluons of the initial condition.
- Dipole evolution resums $\ln s$. By considering multiple t-channel gluon exchanges, we sum the $i\pi$'s to higher orders.



Dipole amplitude as correlators of Wilson lines

It can be shown that the dipole amplitude in the impact parameter space $N(\underline{x}_{10}, \underline{b}) = \frac{1}{\mathrm{N_c}} \mathrm{tr}\left[\mathrm{V}_{\underline{x}_0} \mathrm{V}_{\underline{x}_1}^{\dagger}\right] \mathrm{tr}[\mathrm{V}_{\underline{x}_2}]$.

Diagrams with multiple t-channel gluon exchanges can be calculated by expanding the Wilson lines and contracting the gluon fields.



- Consider the case where t-channel exchanged particles are photons, expand the Wilson line to all orders

$$V_{\underline{x}} = \sum_{n=0}^{\infty} \prod_{i=1}^{n} \int_{x_{i-1}^-}^{\infty} dx_i^- A^+(x_i^-, \underline{x}_i) = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{-\infty}^{\infty} dx^- A^+(x^-, \underline{x}) \,.$$

- We get $1/n!$ from each Wilson line, there are $n!$ ways to contract the photon fields:

$$\left(\frac{1}{n!}\right)^2 \cdot n! = \frac{1}{n!}$$

The quark-quark scattering amplitude for t-channel photon exchanges to all orders is

$$\left\langle V_{\underline{x}_0} V_{\underline{x}_1} \right\rangle = \exp(-2i\alpha_{EM} \ln \frac{1}{x_{10}\Lambda}).$$

The imaginary parts exponentiate!

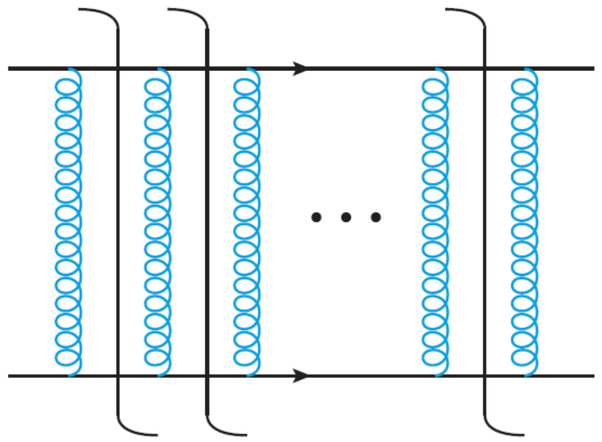
What about gluon exchanges?

- Expand the Wilson lines:

$$V_{\underline{x}} = \sum_{n=0}^{\infty} \prod_{i=1}^{n} \int_{x_{i-1}^-}^{\infty} dx_i^- A^+(x_i^-, \underline{x}_i),$$

but this time the nested integral cannot be easily converted to integral from $-\infty$ to ∞ because of the color factor.

- Alternative: notice that the leading $i\pi$ contribution comes from diagrams with the maximum number of cuts, so these terms give the same contribution as the case with photon exchanges apart from a color factor. Hence we should again get an exponentiation which is characteristic of the signature factor.



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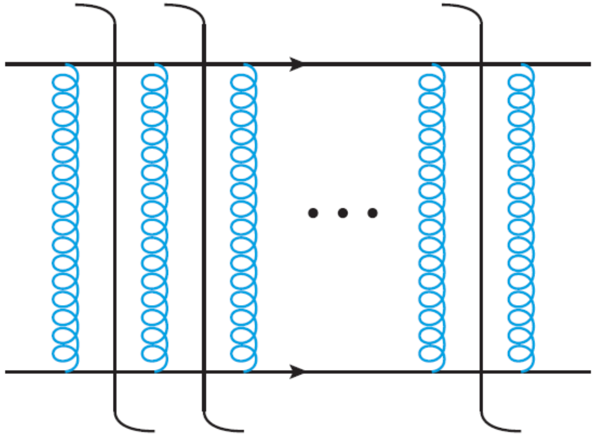
Exact calculation with nested integrals is needed for cross check.



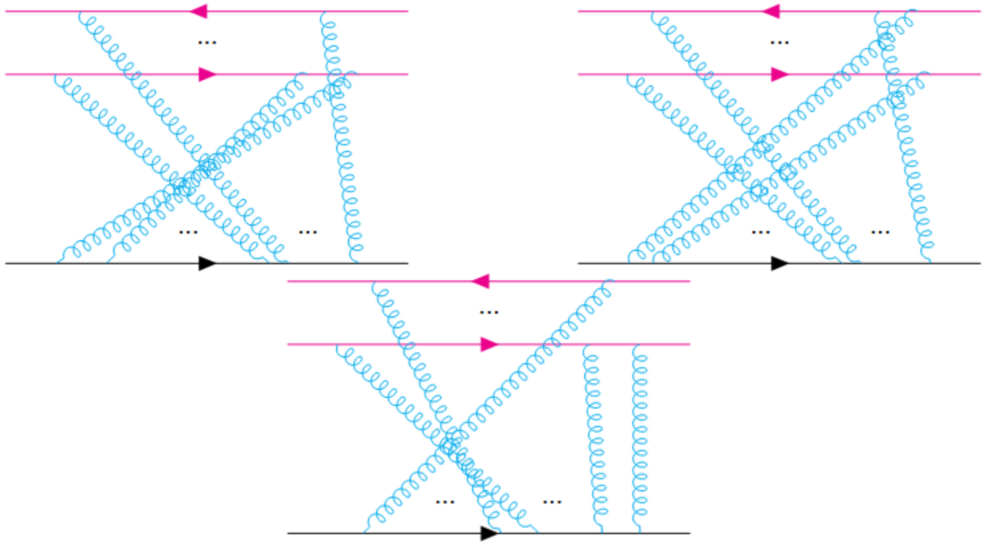
Summary

- We study evolution of H_g in the small-x dipole picture, and show that skewness effect modifies the rapidity argument in dipole amplitude N from $\ln 1/x$ to $\ln \min\{1/x\,, 1/\xi\}$.

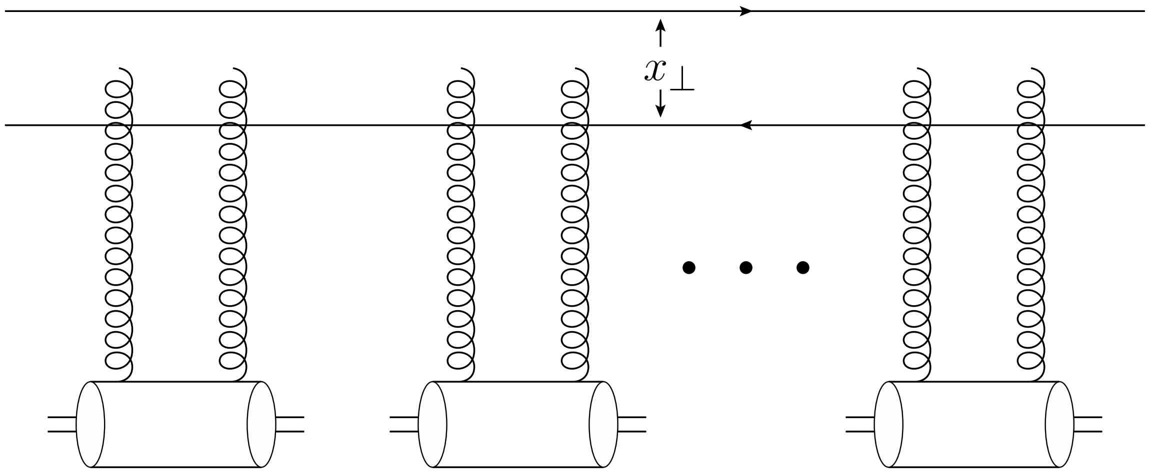
- We account for the imaginary part of N by including multiple t-channel gluon exchanges in the initial condition. This leads to exponentiation of $i\pi$'s which is characteristic of signature factor. The signature factor needs to be included into the initial condition. Evolution for N stays the same. Since N now has an imaginary part in the initial conditions, it will have an imaginary part after evolution, giving the real part to the scattering amplitude.

Large-N_c diagrams (backup)

- For multiple t-channel gluon exchanges, we encounter color factors such as $\frac{1}{\mathrm{N_c}}\mathrm{tr}\left[t_{a_1}t_{a_2}\cdots t_{a_n}\right]\mathrm{tr}\left[t_{a_{i_1}}t_{a_{i_2}}\cdots t_{a_{i_n}}\right]$. Only n of the diagrams contribute at large N_c.



Multiple rescattering (backup)

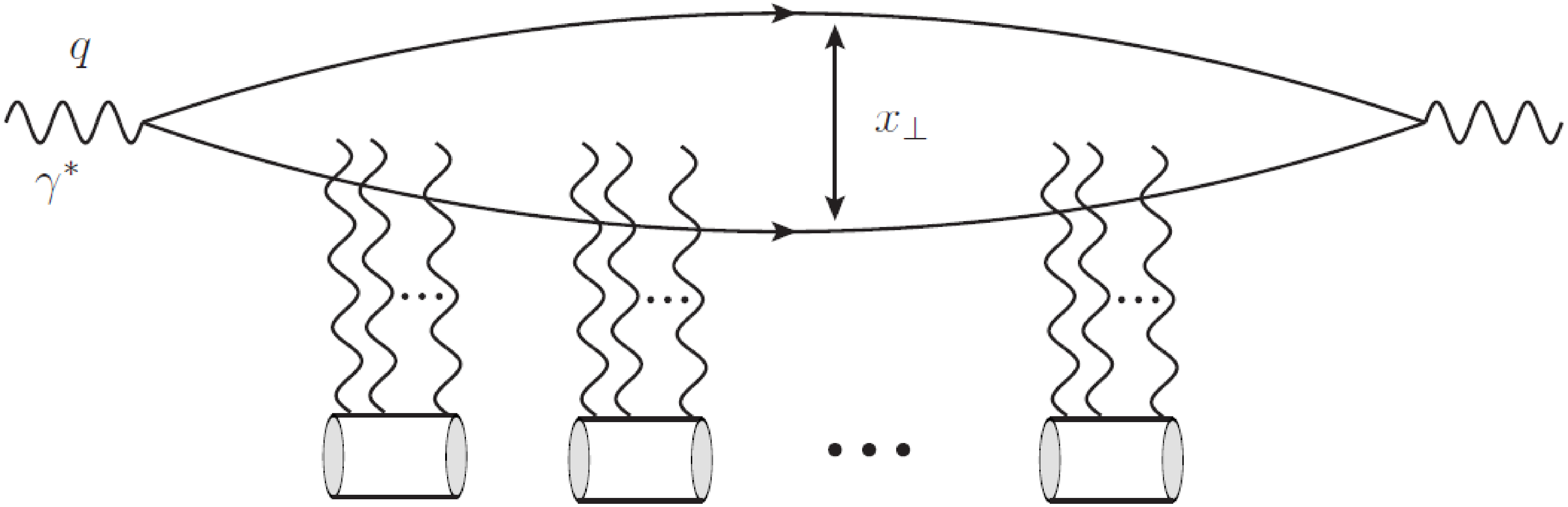


A.H. Mueller, ‘90

Lowest-order interaction with each nucleon – two gluon exchange – lead to the following resummation parameter:

$$\alpha_s^2\, A^{1/3}$$

Including multiple rescattering (backup)



$$N(\underline{x}_{10}, \underline{b}, Y = 0) = 1 - \exp(-\frac{x_{10}^2 Q_s^2(\underline{b})}{4} \ln \frac{1}{x_{10}\Lambda})$$

$$N(\underline{x}_{10}, \underline{b}, Y = 0) \to 1 - \exp\left[-\int db^- d^2 b'_\perp \rho(b'^+, \underline{b}') N(\underline{x}_{10}, \underline{b} - \underline{b}')\right].$$