Relativistic spatial distributions:

energy-momentum tensor in polarized nucleons

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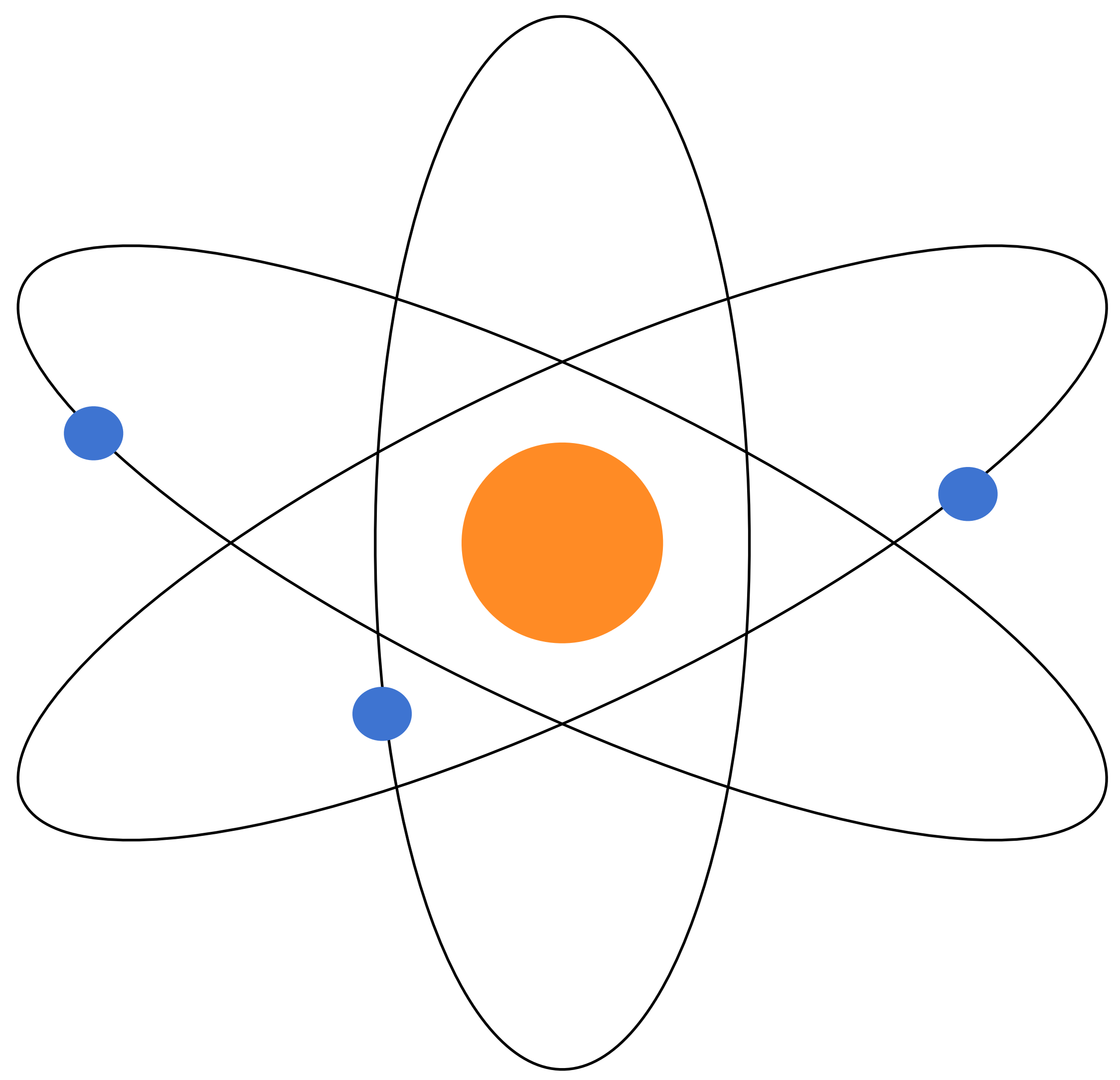
Based on:
Won, and Lorcé, PRD 111 (2025)
Lorcé, Mukherjee, Singh, and Won, arXiv 2606.20468 (accepted in PLB)

Contents

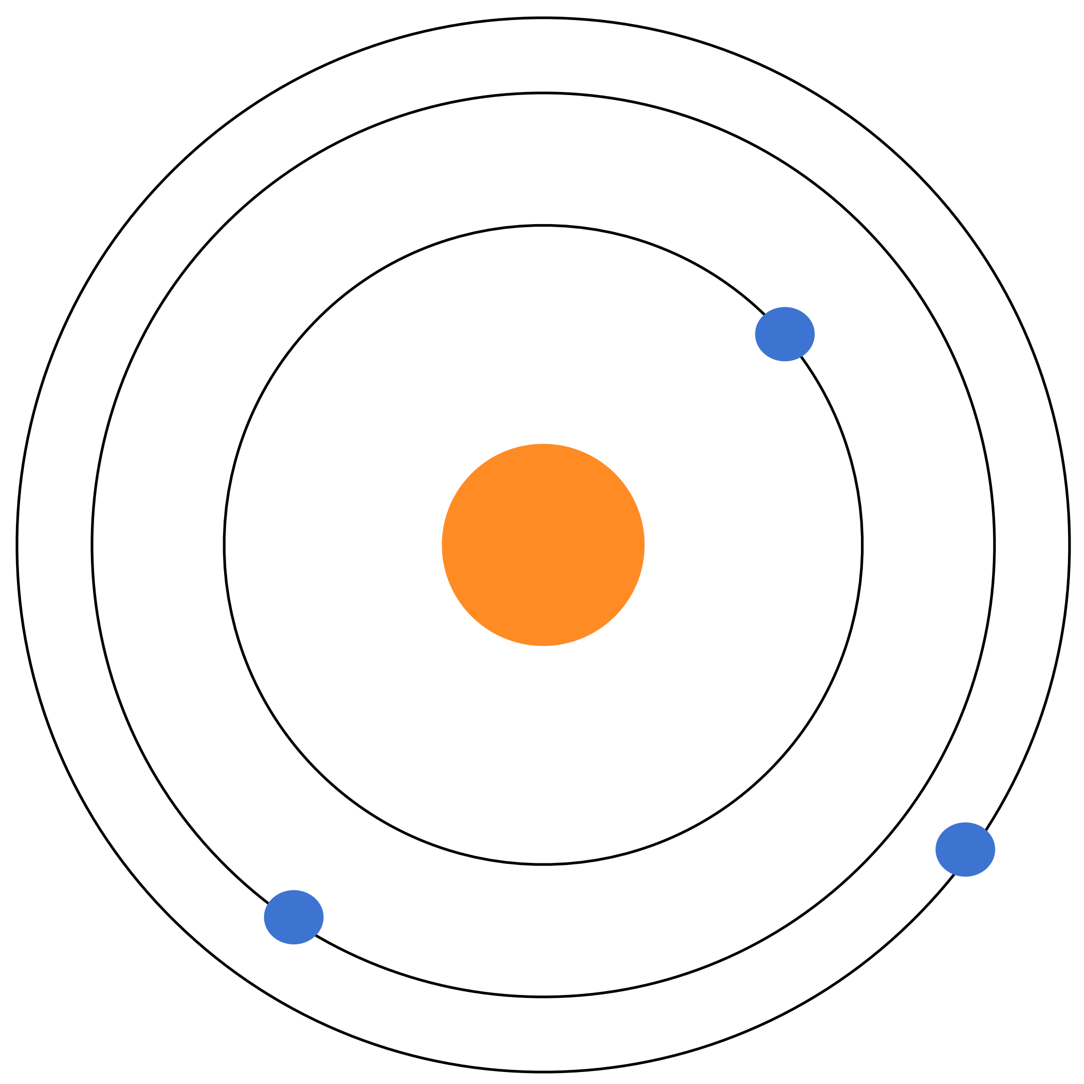
Introduction

Atom models

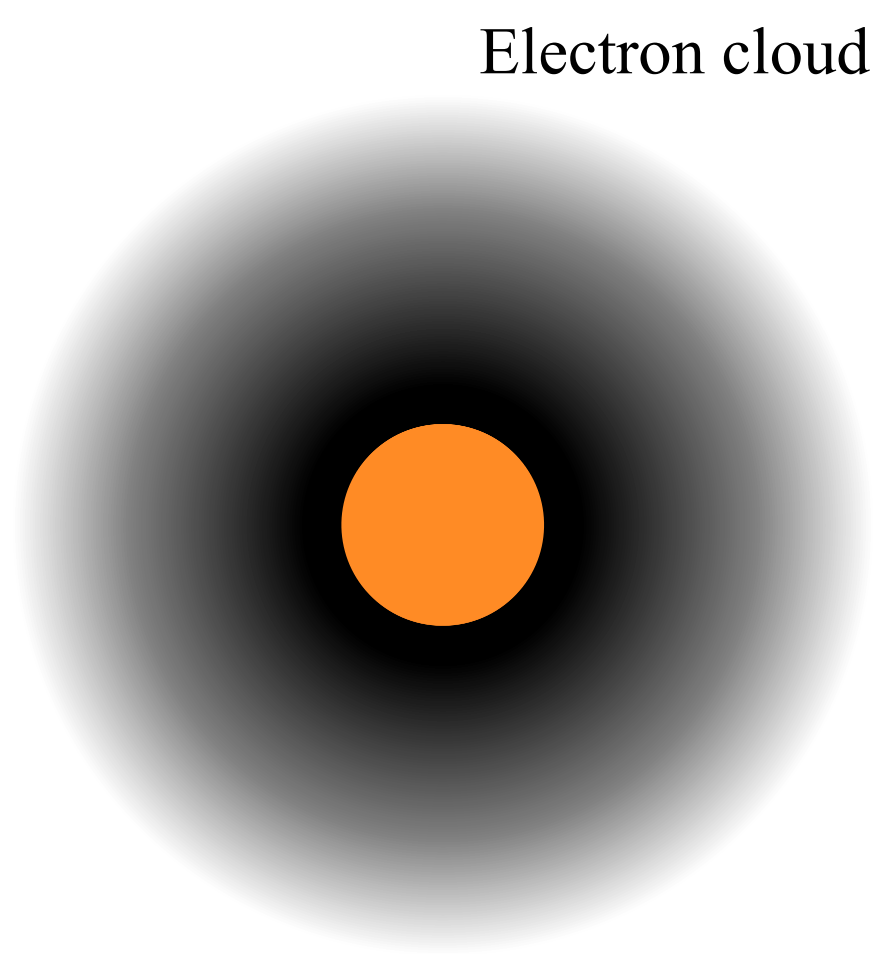
In 1909, E. Rutherford's model



In 1921, N. Bohr's model



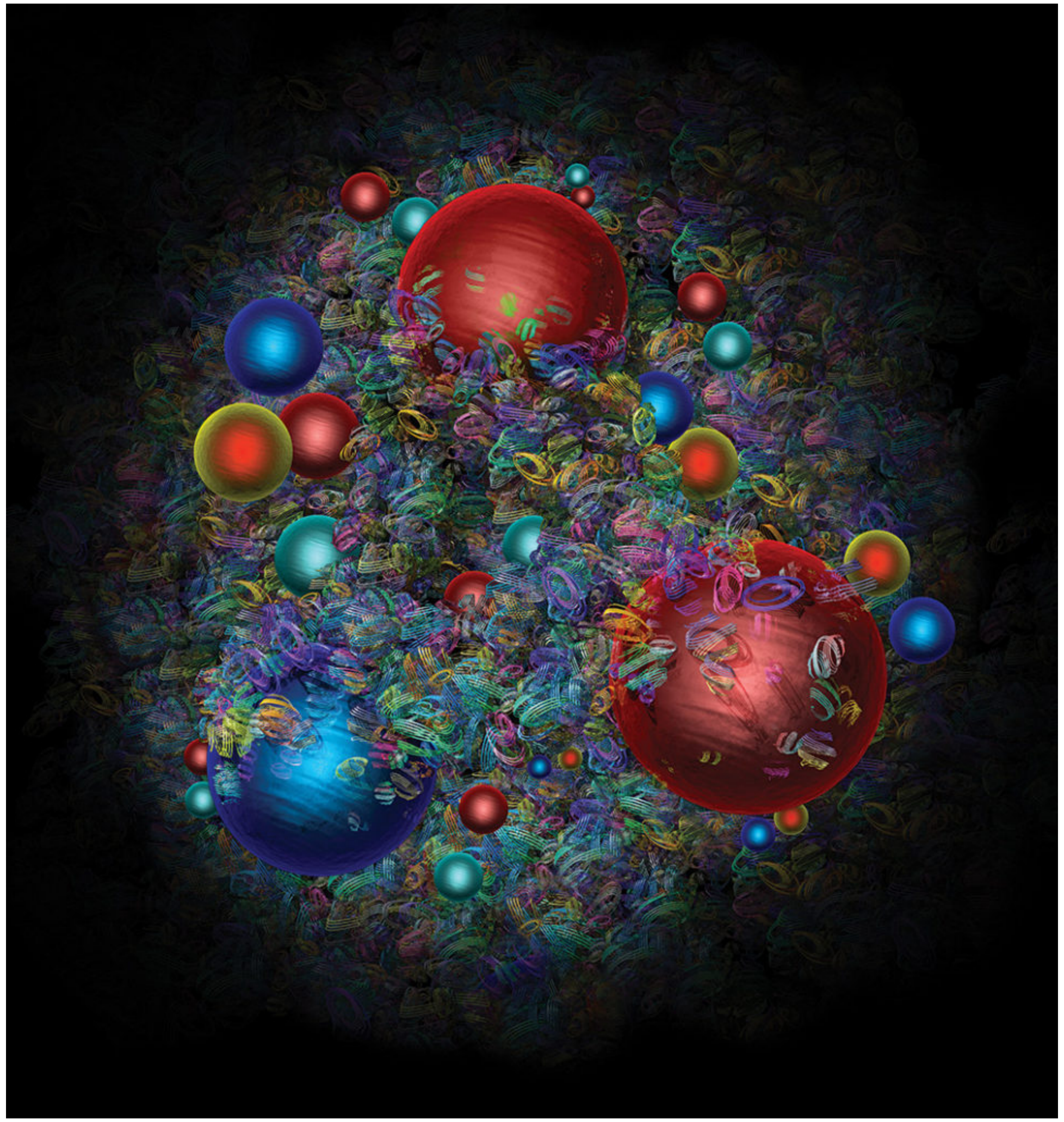
In 1932, E. Schrödinger model



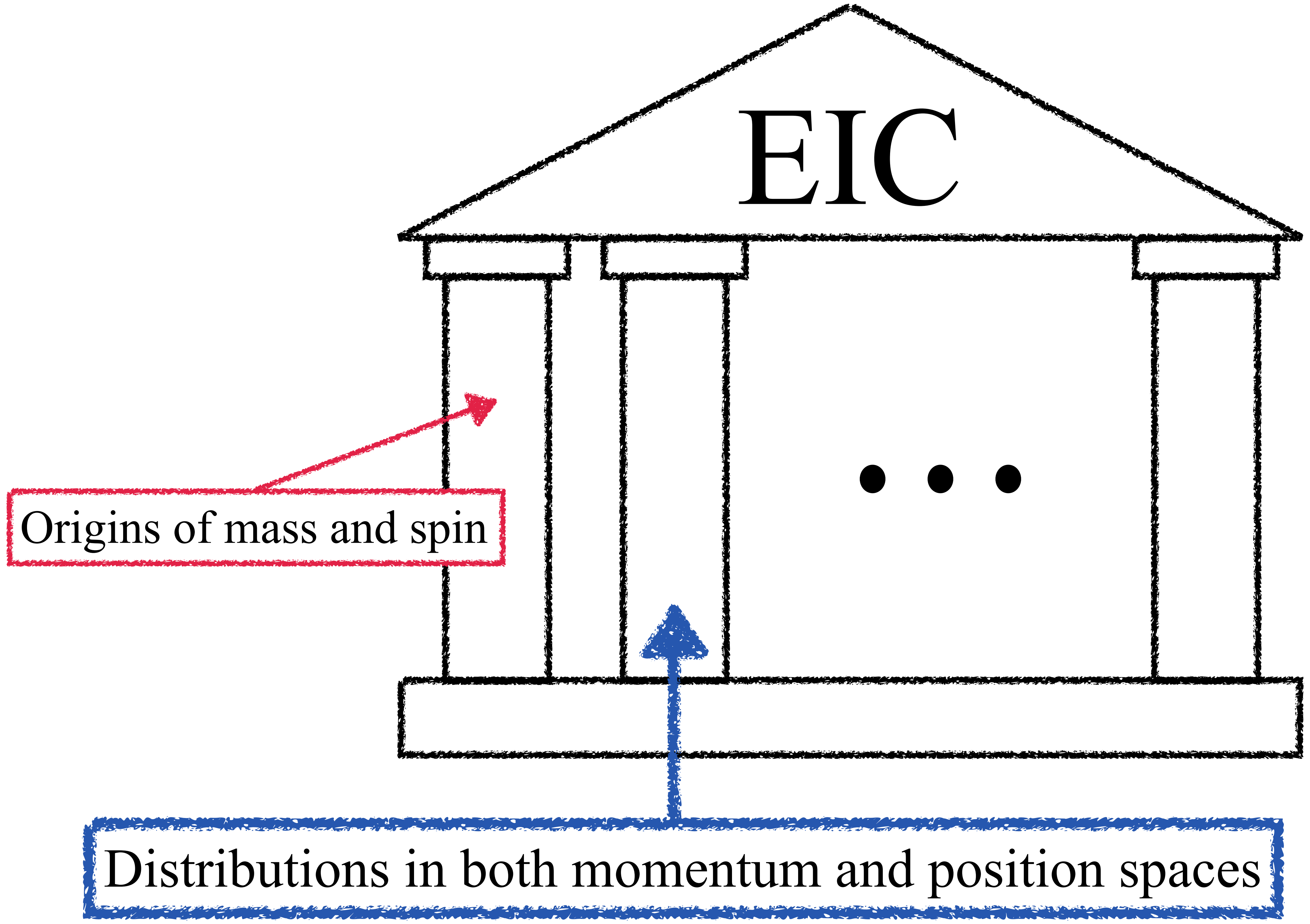
A core of atoms: the nucleon

Abdul Khalek et al., NPA 1026 (2022)

Internal structure of the nucleon



Electron-Ion Collider (EIC) project



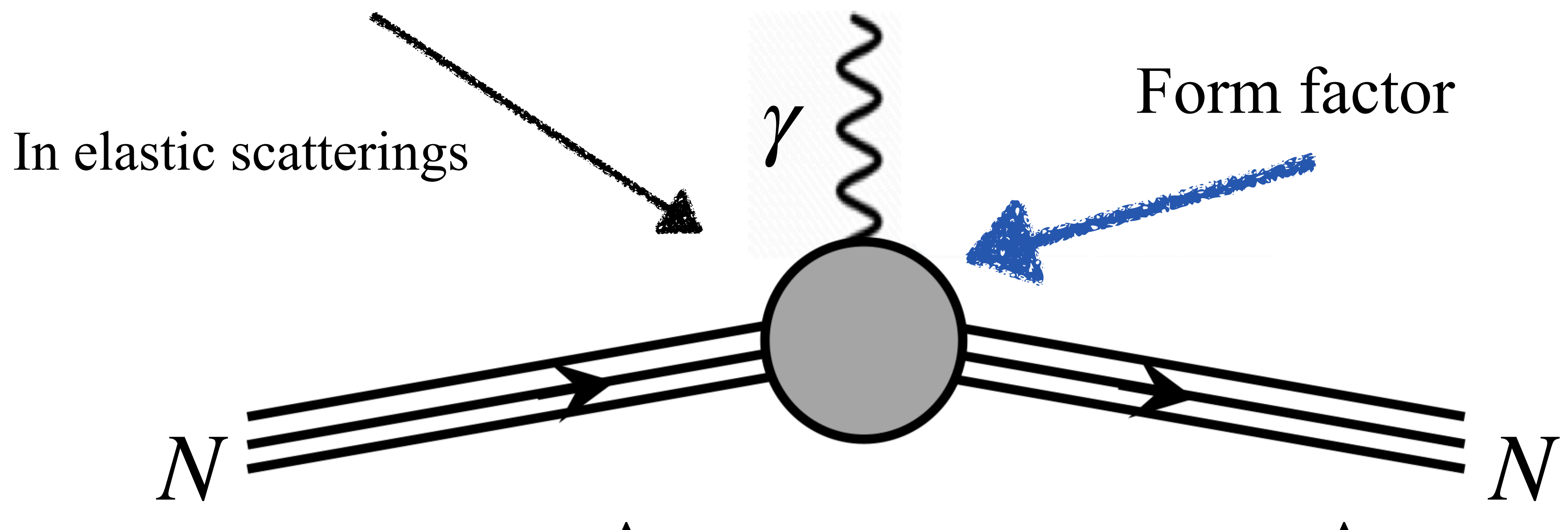
Images: CERN

Relativistic spatial distribution

Form factor

Electromagnetic form factors

Internal structure, e.g., $\mu_N \neq 1$



$$p = P - \frac{\Delta}{2} \qquad p' = P + \frac{\Delta}{2}$$

$$\tau = \frac{Q^2}{4M^2} \quad t = -\Delta^2 = -Q^2$$

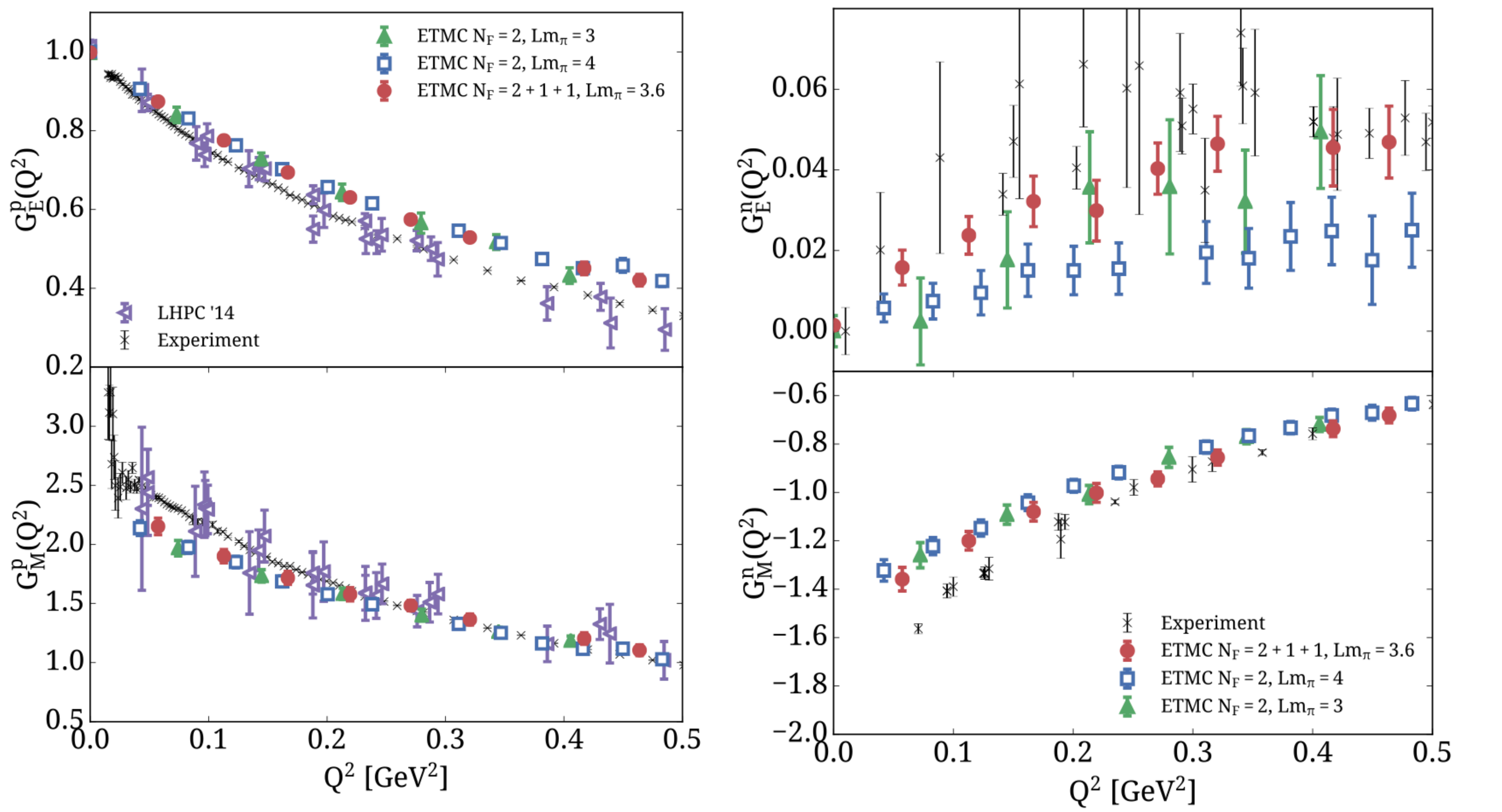
$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M} F_2(Q^2) \right] u(p, s)$$

Dirac form factor   Pauli form factor

Electric : $G_E = F_1 + \tau F_2,$
Magnetic: $G_M = F_1 + F_2,$

Lorentz invariant quantities

Alexandrou, et. al., PRD 100 (2019)



Spatial distribution depending on Lorentz frame

Image: Lorcé’s talk

Rest frame distribution

Non-relativistic charge distribution

Sachs, PR 126 (1962)
Jaffe, RRD 103 (2021)

$$\rho|_{\text{Sachs}}(\boldsymbol{r}) := \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} G_E(t)$$

Regime: $R \gg \bar{\lambda}$ (Particle size R; Reduced Compton wavelength $\bar{\lambda}$)

However,

I. Comparable charge radius

$$\sqrt{\langle r^2\rangle^Q_{\text{exp}}} = 0.84 \text{ fm v.s. } \bar{\lambda} = \frac{\hbar}{Mc} \sim 0.2 \text{ fm}$$

PDG, PTEP 2022 (2022)

II. Recoil correction

$$P^0_{\text{BF}} = M\sqrt{1+\tau},$$
$$\text{not } P^0_{\text{BF}} \approx M,\ Q \ll 2M$$

→ Motivation of relativistic spatial distribution

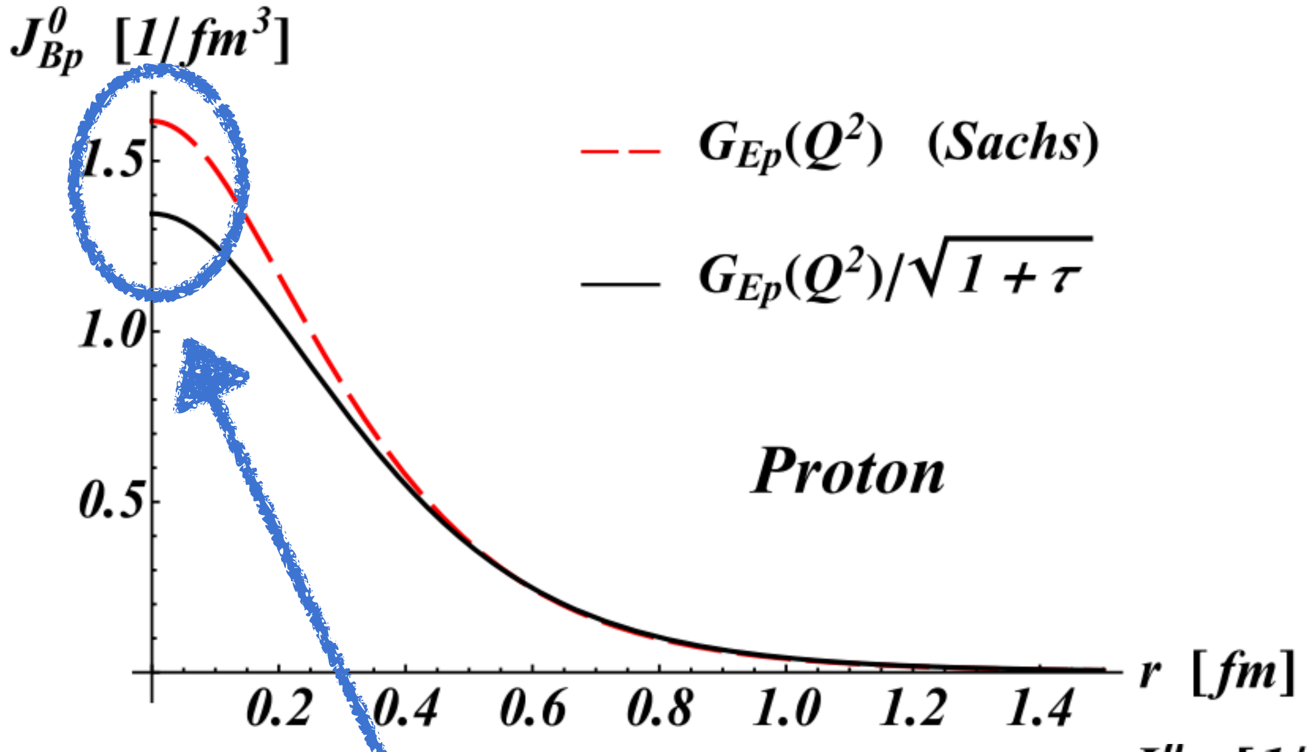
$$\rho_{\text{BF}}(\boldsymbol{r}) := \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \frac{\langle p'|\hat{j}^0(0)|p\rangle|_{\text{BF}}}{2P^0_{\text{BF}}},$$
$$= \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \frac{M}{P^0_{\text{BF}}} G_E(t)$$

Friar, Negele, ANP 8 (1975)
Lorcé, PRL 125 (2020)

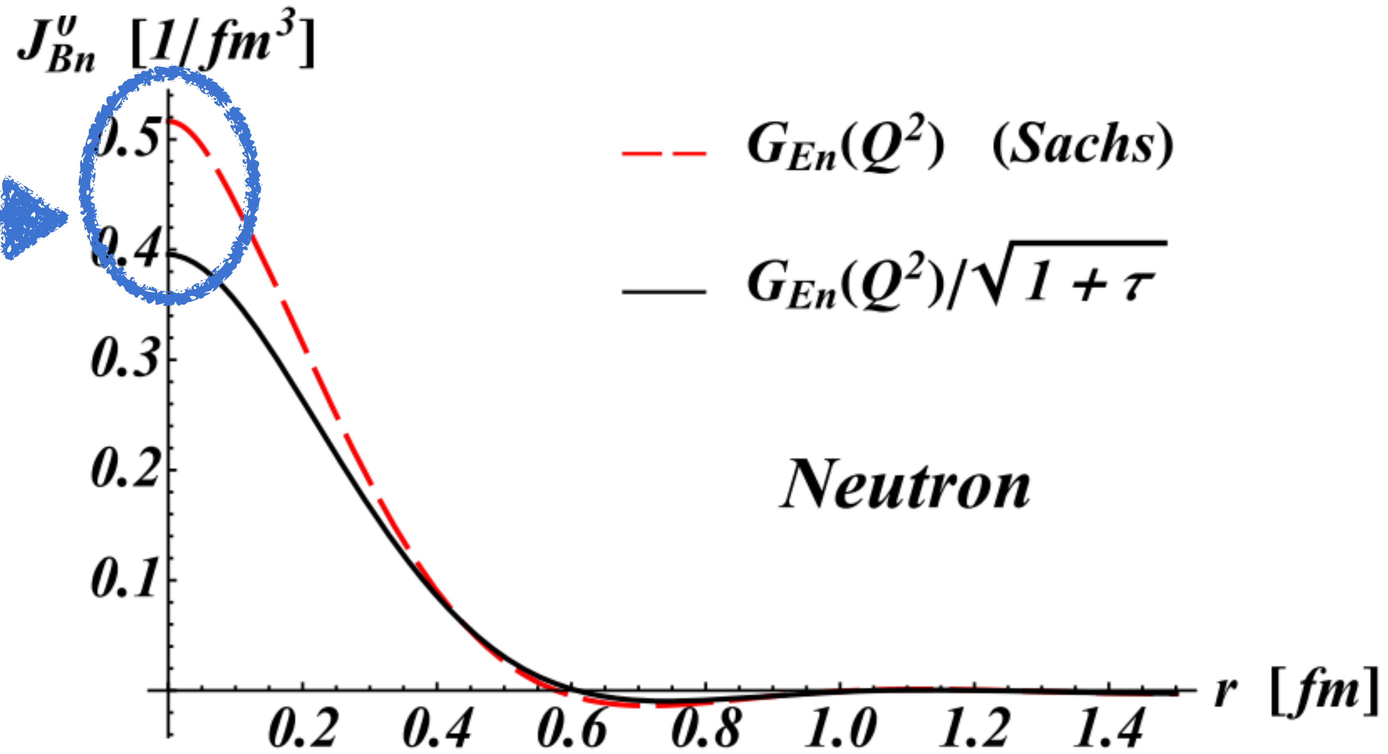
Non-relativistic limit

$$\frac{M}{P^0_{\text{BF}}} \approx 1 \ \rightarrow\ \rho_{\text{BF}} \to \rho|_{\text{Sachs}}$$

Lorcé, PRL 125 (2020)

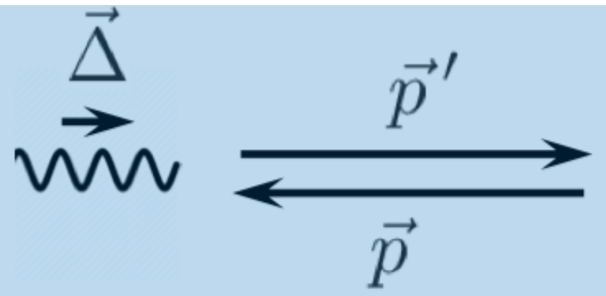


Recoil correction effect



Relativistic spatial distribution in Breit frame

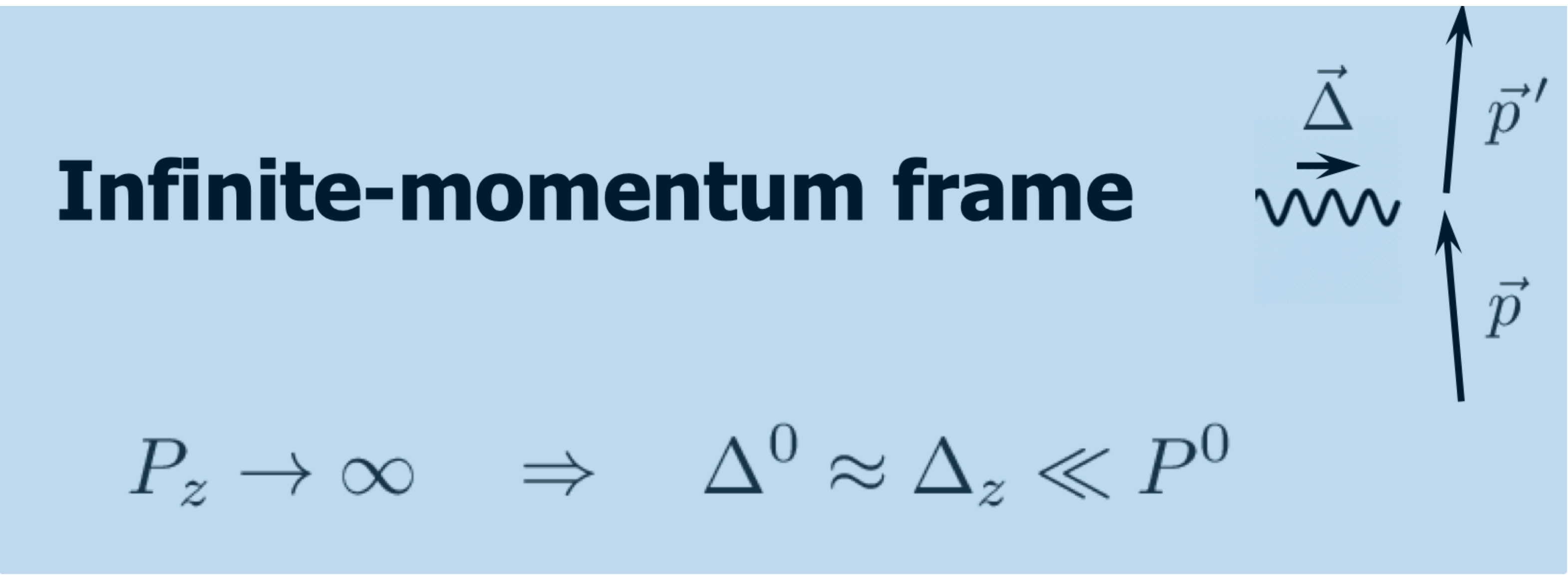
Breit (aka brick-wall) **frame**



$$\vec{P} = \vec{0} \quad \Rightarrow \quad \Delta^0 = \frac{\vec{P}\cdot\vec{\Delta}}{P^0} = 0$$

Infinite momentum frame distribution

Infinite momentum frame distribution

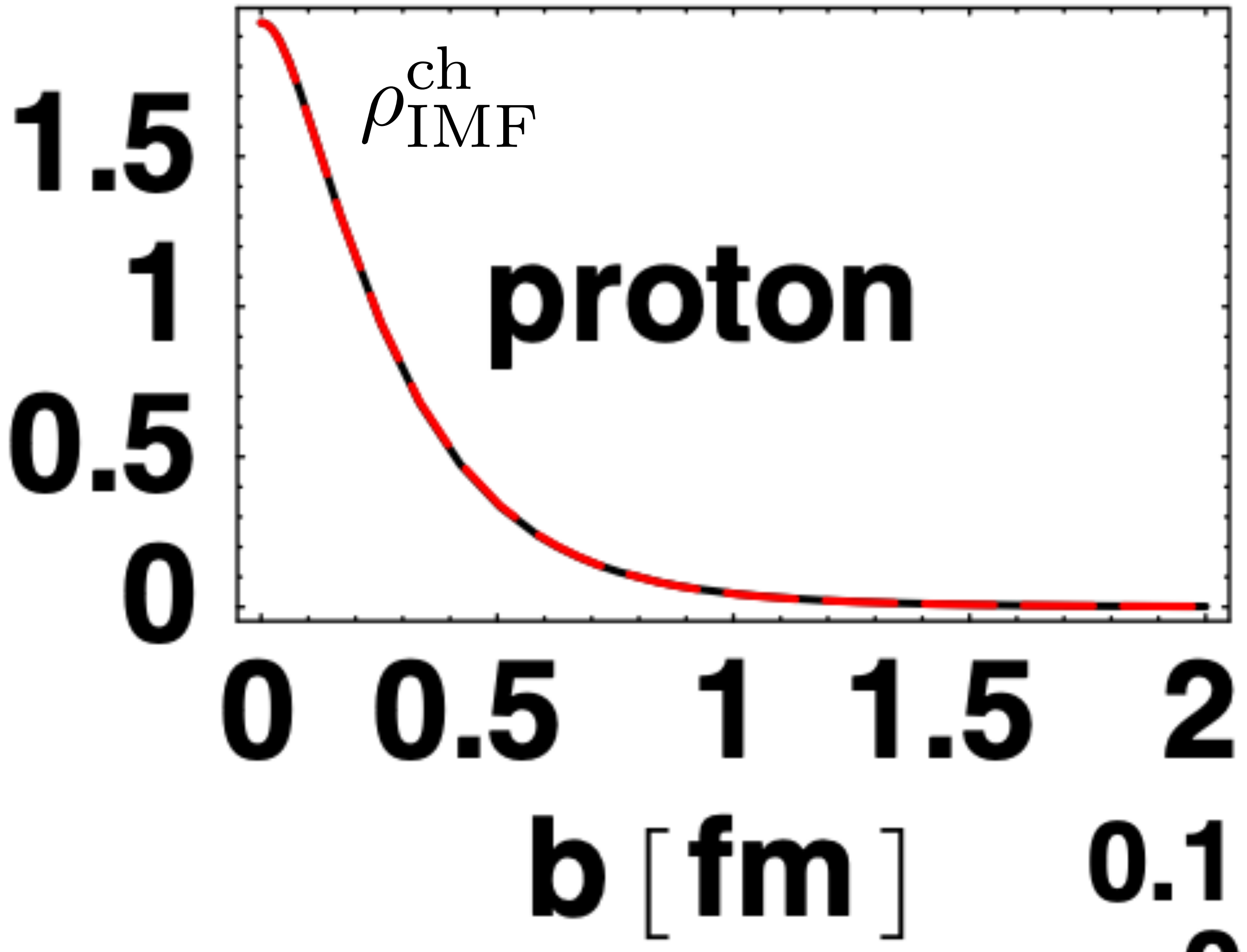


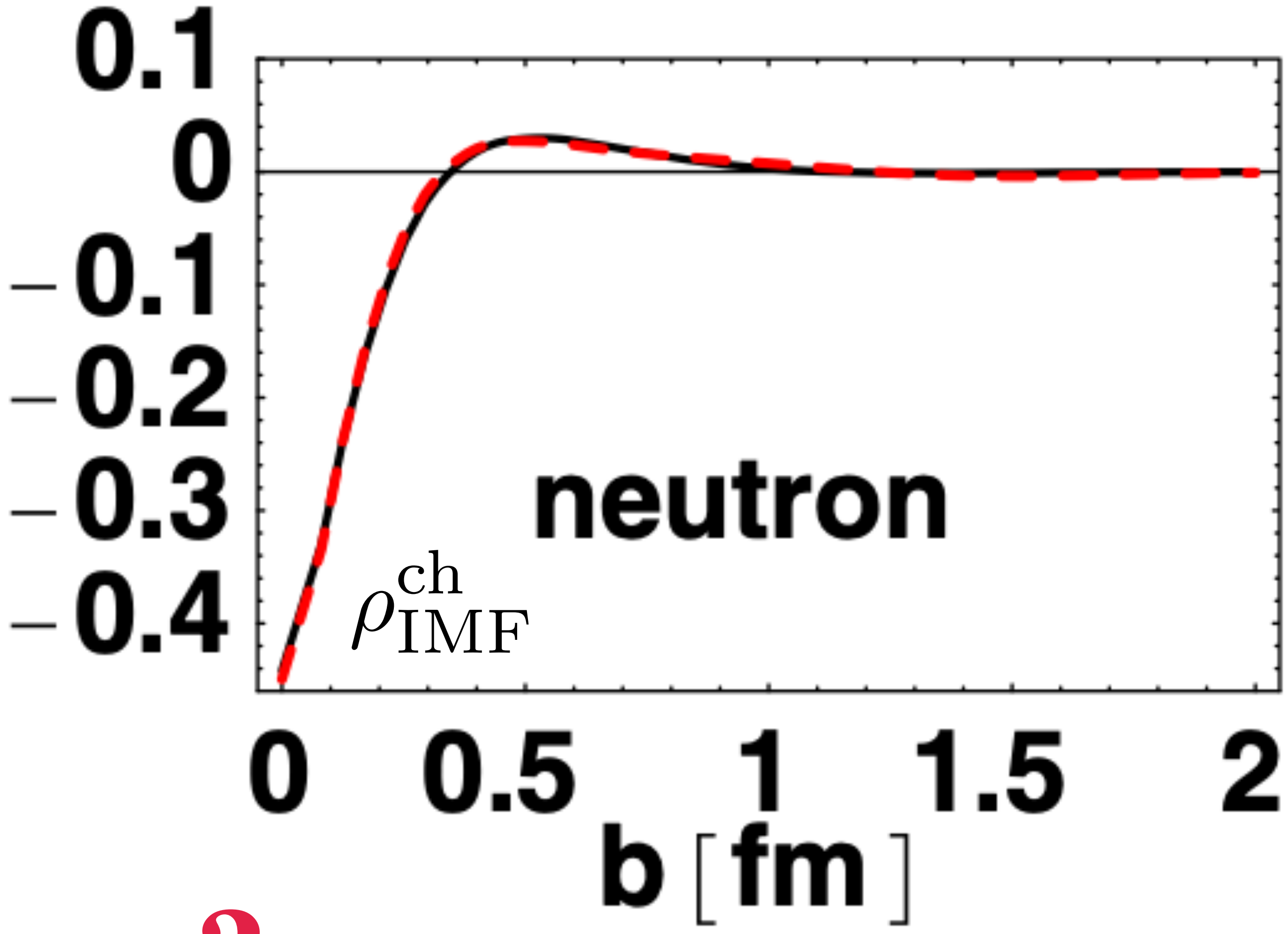
$$\rho^{\text{ch}}_{\text{IMF}}(\boldsymbol{b}_\perp) := \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left.\frac{\langle p'|\hat{j}^0(0)|p\rangle}{2P^0}\right|_{\text{IMF}},$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[F_1(t) + \frac{(\boldsymbol{\sigma}\times i\boldsymbol{\Delta}_\perp)_z}{2M}F_2(t)\right]$$

$\boldsymbol{b}_\perp$: impact parameter vector

Lorcé, Wang, PRD 105 (2022)
Chen, Lorcé, PRD 106 (2022)

Miller, PRL 99 (2007)





No recoil correction! $\because$ Galilean symmetry in the transverse plane

Consistency with the light-front distribution

How to interpolate two different distributions?

Quantum phase-space formalism

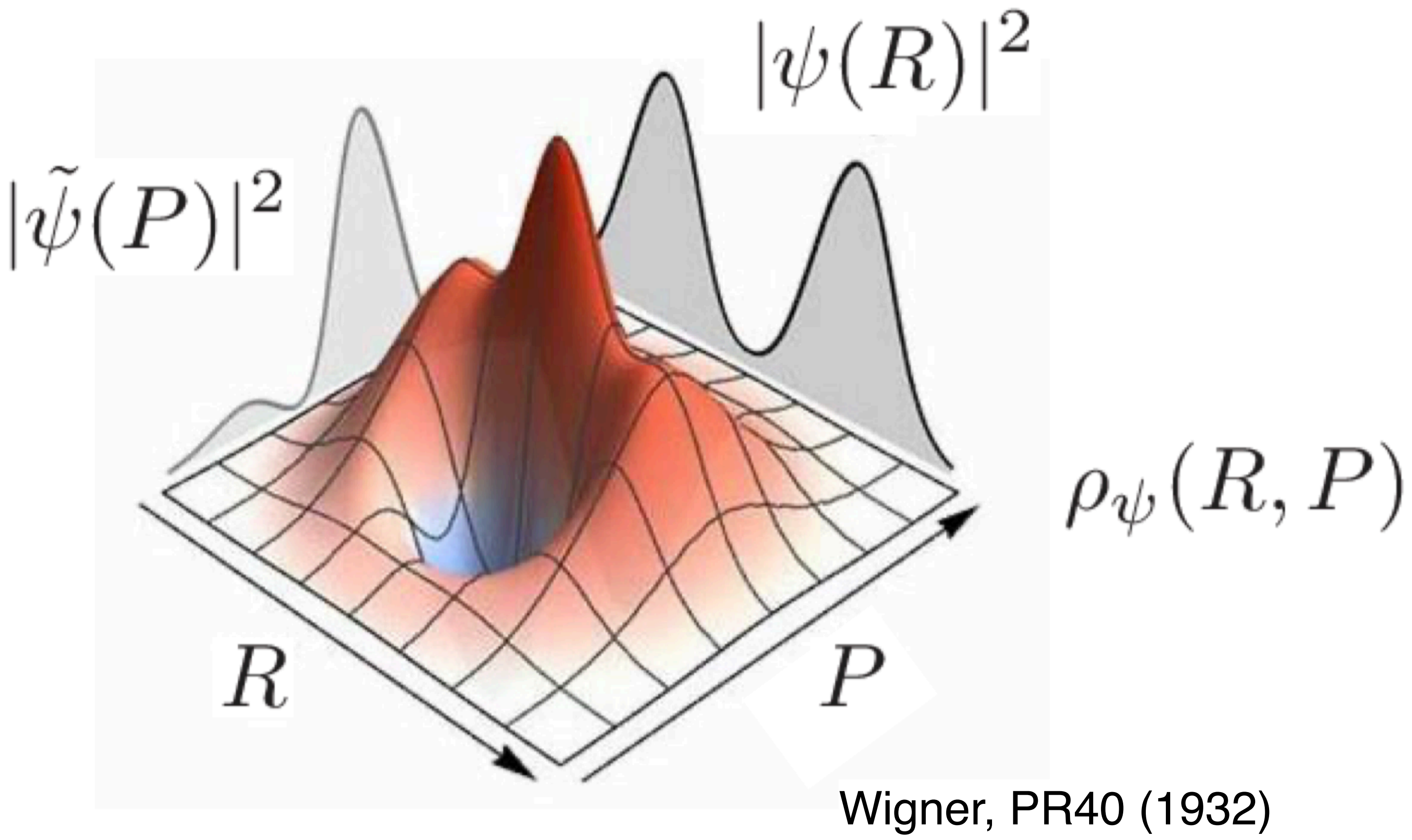
Hadronic matrix element

In quantum phase-space formalism

$$\langle\Psi|\hat{O}(\boldsymbol{r})|\Psi\rangle = \int \frac{d^3P}{(2\pi)^3}\int d^3R\, \rho_\Psi(\boldsymbol{R},\boldsymbol{P})\left\langle \hat{O}(\boldsymbol{r})\right\rangle_{\boldsymbol{R},\boldsymbol{P}}$$

I. Wigner distribution (hadronic wave packet)

$$\rho_\Psi(\boldsymbol{R},\boldsymbol{P}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\boldsymbol{q}\cdot\boldsymbol{R}}\widetilde{\psi}^\dagger\left(\boldsymbol{P}+\frac{\boldsymbol{q}}{2}\right)\widetilde{\psi}\left(\boldsymbol{P}-\frac{\boldsymbol{q}}{2}\right)$$
$$= \int d^3Y\, e^{i\boldsymbol{P}\cdot\boldsymbol{Y}}\psi^\dagger\left(\boldsymbol{R}+\frac{\boldsymbol{Y}}{2}\right)\psi\left(\boldsymbol{R}-\frac{\boldsymbol{Y}}{2}\right)$$



Wavepacket

$$\psi(\boldsymbol{r}) = \langle \boldsymbol{r}|\Psi\rangle$$
$$\widetilde{\psi}(\boldsymbol{p}) = \frac{1}{\sqrt{2p^0}}\langle \boldsymbol{p}|\Psi\rangle$$

Quasi-probablistic interpretation

$$\int d^3R\, \rho_\Psi(\boldsymbol{R},\boldsymbol{P}) = \left|\widetilde{\psi}(\boldsymbol{P})\right|^2$$
$$\int \frac{d^3P}{(2\pi)^3}\rho_\Psi(\boldsymbol{R},\boldsymbol{P}) = |\psi(\boldsymbol{R})|^2$$

Lorcé, Moutarde, and Trawiński, EPJC 79 (2019)
Lorcé, EPJC 78 (2018)
Lorcé, PRL 125 (2020)

II. Phase-space amplitude (internal distribution)

$$\left\langle \hat{O}(\boldsymbol{r})\right\rangle_{\boldsymbol{R},\boldsymbol{P}} = \int \frac{d^3\Delta}{(2\pi)^3} e^{i\boldsymbol{\Delta}\cdot\boldsymbol{R}}\frac{1}{\sqrt{2p_i^0}\sqrt{2p_f^0}}\left\langle P+\frac{\Delta}{2}\middle|\hat{O}(\boldsymbol{r})\middle|P-\frac{\Delta}{2}\right\rangle$$

Projection onto the 2D space

$$\int dr_z\left\langle \hat{O}(\boldsymbol{r})\right\rangle_{\boldsymbol{R},\boldsymbol{P}} = \int \frac{d^2\Delta}{(2\pi)^2} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{R}_\perp}\frac{1}{\sqrt{2p_i^0}\sqrt{2p_f^0}}\left\langle P+\frac{\Delta}{2}\middle|\hat{O}(\boldsymbol{r}_\perp)\middle|P-\frac{\Delta}{2}\right\rangle\Bigg|_{\Delta_z=0}$$

Transverse Lorentz frame with $\Delta_z = 0$

Wigner, PR40 (1932)
Hillery, O'Connell, Scully, and Wigner, PR106 (1984)
Bialynicki-Birula, Gornicki, and Rafelski, PRD 44 (1991)

Image: Lorcé's presentation

Elastic frame

I. A generic frame in the 2D space

Connection between the rest (Breit) frame and the moving frame.

$$P = \frac{p' + p}{2} = (P^0, \mathbf{0}_\perp, P_z) \qquad \Delta = p' - p = (0, \mathbf{\Delta}_\perp, 0)$$

$P_z = 0$ $\quad$ $P_z \to \infty$

Breit frame (BF)

$P = (P^0, \mathbf{0}_\perp, 0),$

$\Delta = (0, \mathbf{\Delta}_\perp, 0)$

Infinite momentum frame (IMF)

$P \approx (P_z, \mathbf{0}_\perp, P_z),$

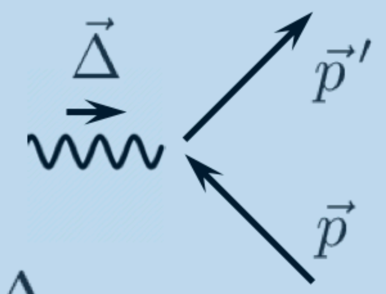
$\Delta = (0, \mathbf{\Delta}_\perp, 0)$

cf. light-front frame

$P^+ \sim P_z, \quad P^- \approx 0$

Lorcé, Mantovani, Pasquini, PLB 776 (2018)

Elastic frame



$$\vec{P} = P_z \vec{e}_z \quad \Rightarrow \quad \Delta^0 = \frac{P_z \Delta_z}{P^0}$$

$$\Delta^0 = 0 \quad \Rightarrow \quad \Delta_z = 0 \quad \Leftrightarrow \quad \int \mathrm{d}z$$

II. Relativistic spin dynamics

For spin-j particle $\quad$ Non-trivial structure $\quad$ $\because [K^i, K^j] = -i\varepsilon^{ijk} J^k$

$$\langle p', s' | \hat{O}^{\mu_1 \cdots \mu_n} | p, s \rangle = \sum_{s'_{\rm BF}, s_{\rm BF}} D^{(j)}_{s_{\rm BF} s}(p_{\rm BF}, \Lambda)\, D^{*(j')}_{s'_{\rm BF} s'}(p'_{\rm BF}, \Lambda)$$

Wigner rotation

$$\times \Lambda^{\mu_1}{}_{\alpha_1} \cdots \Lambda^{\mu_n}{}_{\alpha_n} \langle p'_{\rm BF}, s'_{\rm BF} | \hat{O}^{\alpha_1 \cdots \alpha_n} | p_{\rm BF}, s_{\rm BF} \rangle$$

Lorentz transformation of BF amplitudes

Durand, DeCelles, and Marr, PR 126 (1962)
Polyzou, Glökle, and Witala, FBS 54 (2013)

* Wigner rotation and Melosh rotation

$$\lim_{p_z \to \infty} D(p, \Lambda) = \mathcal{M}$$

Melosh rotation

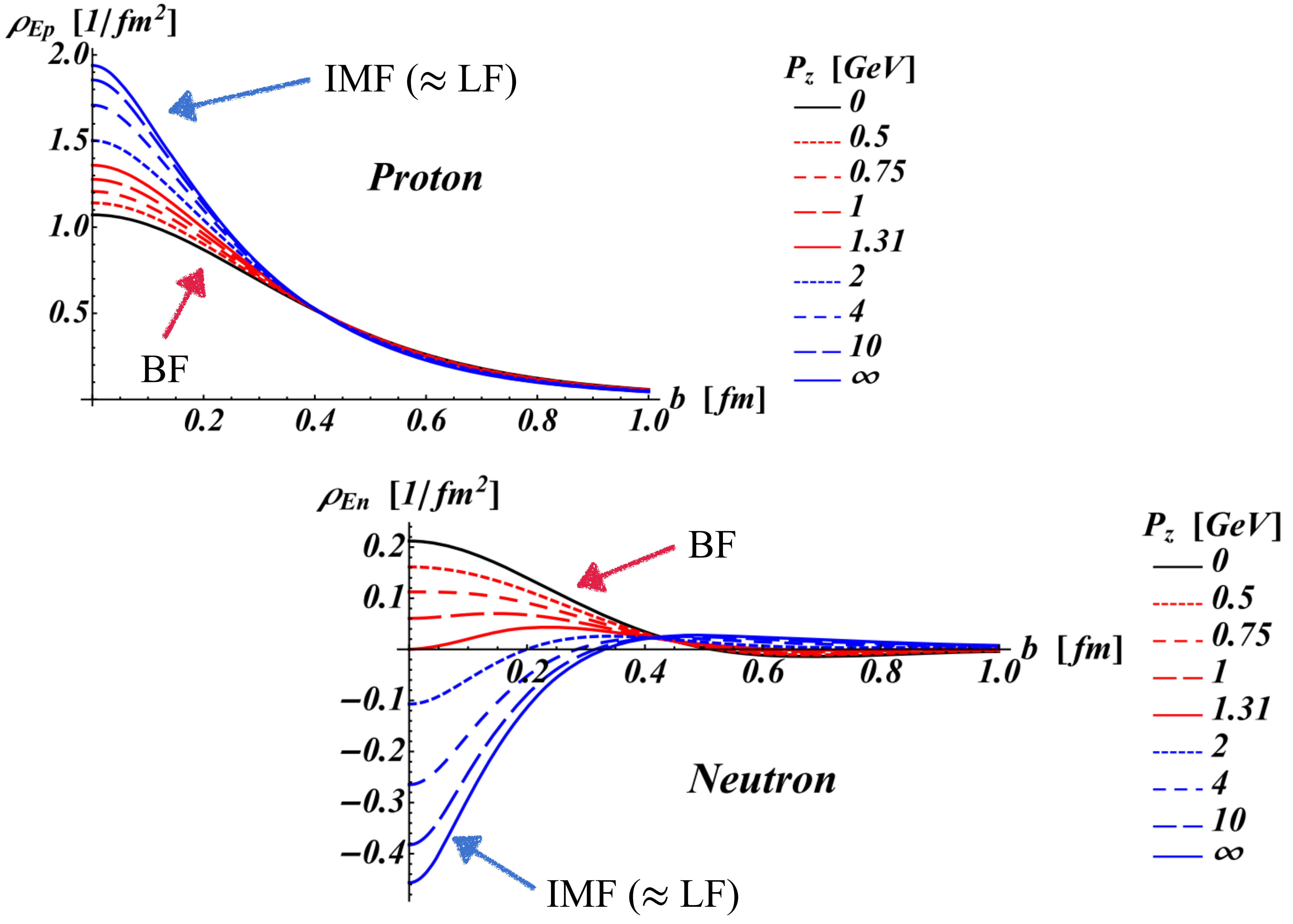
$$|p, \lambda\rangle = \sum_s \mathcal{M}_{s\lambda} |p, s\rangle$$

Chen, and Lorcé, PRD 106 (2022)

Charge distribution in the elastic frame

Lorcé, PRL 125 (2020)

Electric charge distribution in the elastic frame



What about the EMT?

Energy-momentum tensor

Energy-momentum tensor

Energy-momentum tensor (EMT)

Conserved current under space-time translations

Energy — Momentum

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

Energy flux — Stress tensor

Conservation law

$$\partial_\mu T^{\mu\nu} = 0$$

Asymmetric!

Angular momentum

Orbital angular momentum

$$L^i = \int d^3r\ \epsilon^{ijk} r^j T^{0k}(r)$$

c.f. intrinsic spin

$$S^i = \frac{1}{2}\int d^3r\ \bar{\psi}(r)\gamma^i\gamma_5\psi(r)$$

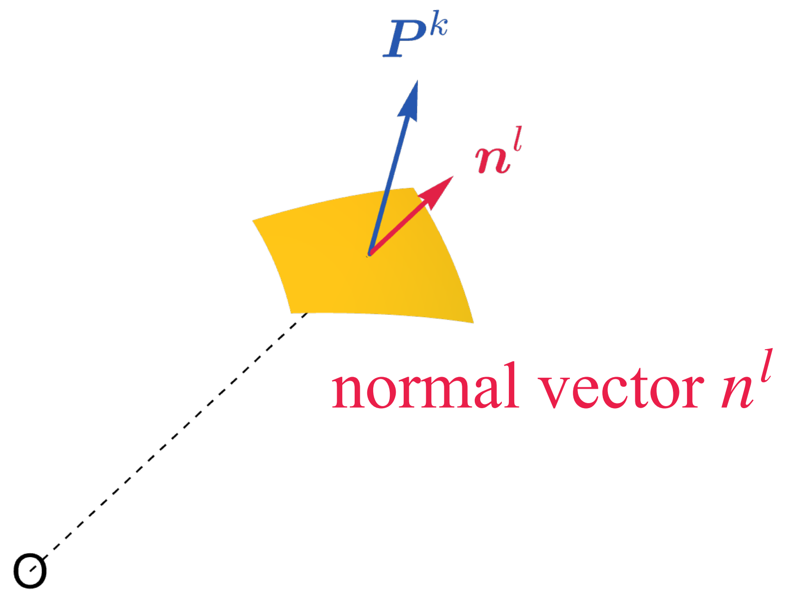
$$\longrightarrow J^i = L^i + S^i$$

Spin sum rule & individual contribution

Lorcé, Mantovani, and Pasquini, PLB 776 (2018)
Lorcé, Mukherjee, Singh, and Won, arXiv 2606.20468

Mechanical force and torque

Stress tensor T^{lk}



projection $\longrightarrow$

Mechanical forces

$$F^k_{(r,\theta,\phi)} = T^{lk} n^l_{(r,\theta,\phi)}$$

Generalized torque

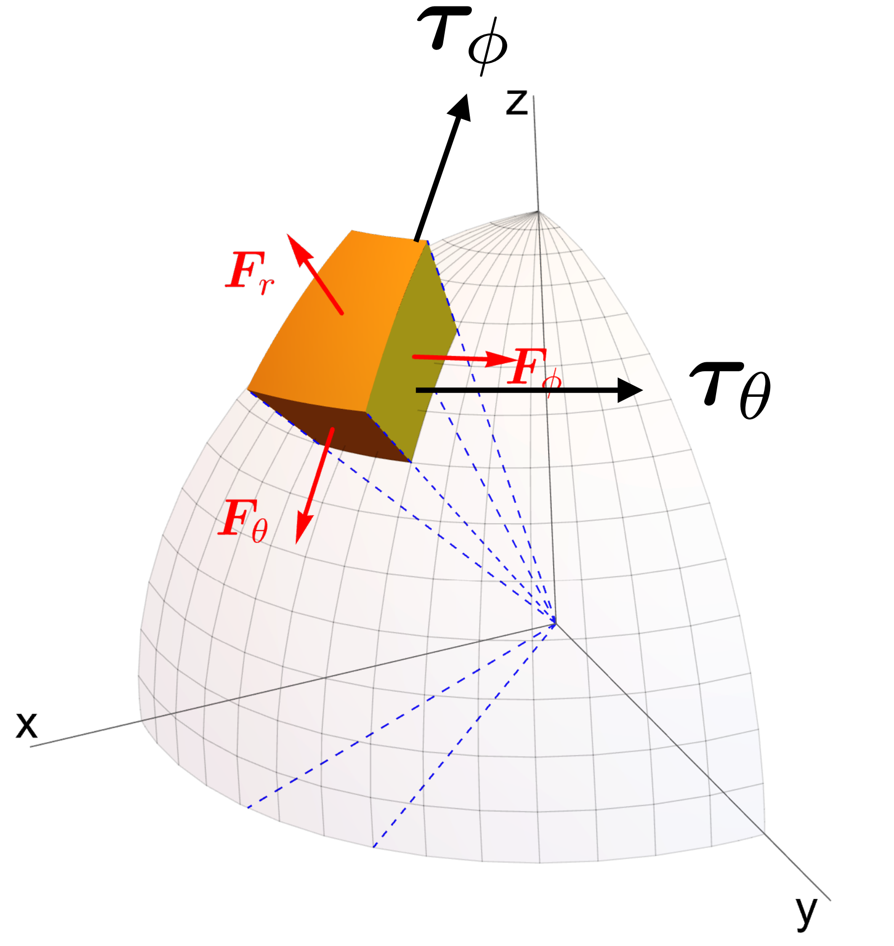
$$\mathcal{C}^{il} = \epsilon^{ijk} r^j T^{lk}$$

$\longrightarrow$ projection

$$\mathcal{C}^{il} n^l_{(\theta,\phi)} = \epsilon^{ijk} r^j F^k_{(\theta,\phi)} = \tau^i_{(\theta,\phi)}$$

Mechanical torque

Lorcé, and Song, PLB 864 (2025)



Kim, Kim, Polyakov, and Son, PRD 103 (2021)

Relativistic spatial distributions of the EMT

V.D. Burkert et al., RMP 95 (2023)
Won, and Lorcé, PRD 111 (2025)

Matrix elements of the EMT

Kinetic EMT operator; gauge-invariant and asymmetric

$$\langle p', s'|\hat{T}_a^{\mu\nu}(0)|p, s\rangle$$
$$= \bar{u}(p', s')\left[\frac{P^\mu P^\nu}{M}A_a(Q^2) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M}D_a(Q^2) + Mg^{\mu\nu}\bar{C}_a(Q^2) + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M}J_a(Q^2) - \frac{iP^{[\mu}\sigma^{\nu]\rho}\Delta_\rho}{2M}S_a(Q^2)\right]u(p, s)$$

$a = q,\ G$

Multi-pole ansatz

$$F_a(t) = \frac{F_a(0)}{(1 - t/\Lambda_{F_a}^2)^{n_F}}$$

$\mu = 2$ GeV

F_a	n_F	$F_q(0)$	Λ_{F_q} (GeV)	$F_G(0)$	Λ_{F_G} (GeV)
A_a	2	0.55	0.91	0.45	0.91
B_a	3	−0.07	0.80	0.07	0.80
D_a	3	−1.28	0.80	−2.24	0.80
$\bar{C}_a$	2	−0.11	0.91	0.11	0.91
S_a	2	0.33	1.00	–	–

C. Lorcé, Moutarde, and Trawiński, EPJC 79 (2019)

Relativistic spatial distributions

$$T_a^{\mu\nu}(\boldsymbol{b}_\perp, P_z; s', s) = \int \frac{d^2\Delta}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\, \frac{\langle p', s'|\hat{T}_a^{\mu\nu}(0)|p, s\rangle}{2P^0}\bigg|_{\text{EF}}$$

$$\longrightarrow \int d^2b_\perp\, T_a^{\mu\nu}(\boldsymbol{b}_\perp, P_z; s', s) = M\frac{u^\mu u^\nu}{u^0}\delta_{s's},$$

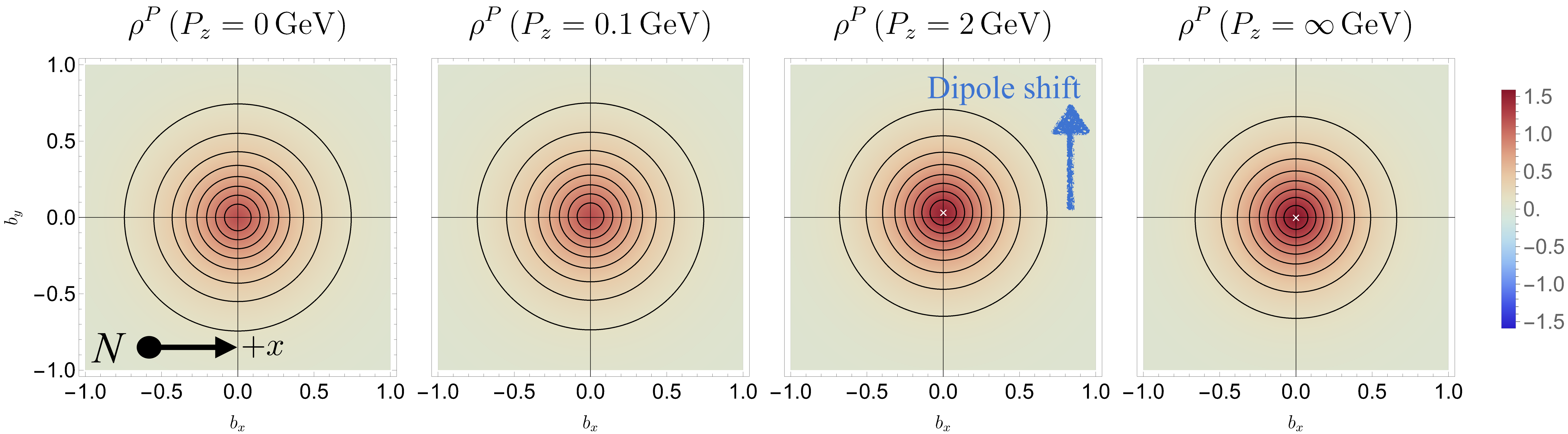
Four-velocity of the system $u = \gamma_P(1, \mathbf{0}_\perp, \beta_P)$

$$\gamma_P = \frac{\sqrt{P_z^2 + M^2}}{M}, \qquad \beta_P = \frac{P_z}{\sqrt{P_z^2 + M^2}},$$

EMT distributions in transversely polarized nucleon

$P_z \to \infty$

Energy, T^{00}/γ_P Won, and Lorcé, PRD 111 (2025)

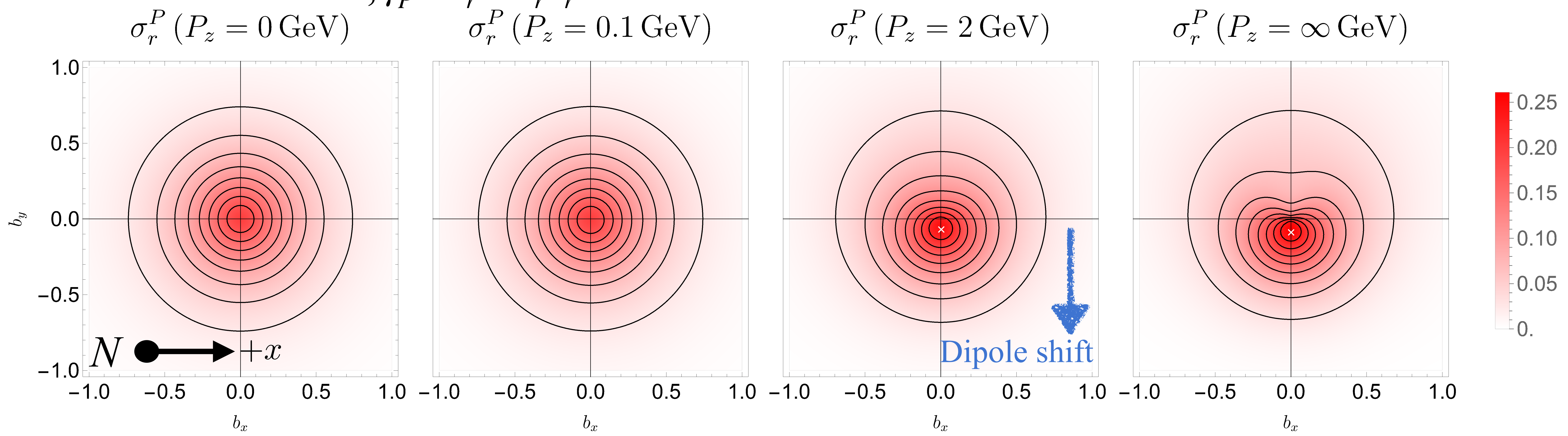


No distortion in the IMF

$$\rho_a(\boldsymbol{b}_\perp, \infty; s', s) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[A_a(t)\delta_{s's} + \frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta}_\perp)_z}{2M}B_a(t)\right]$$

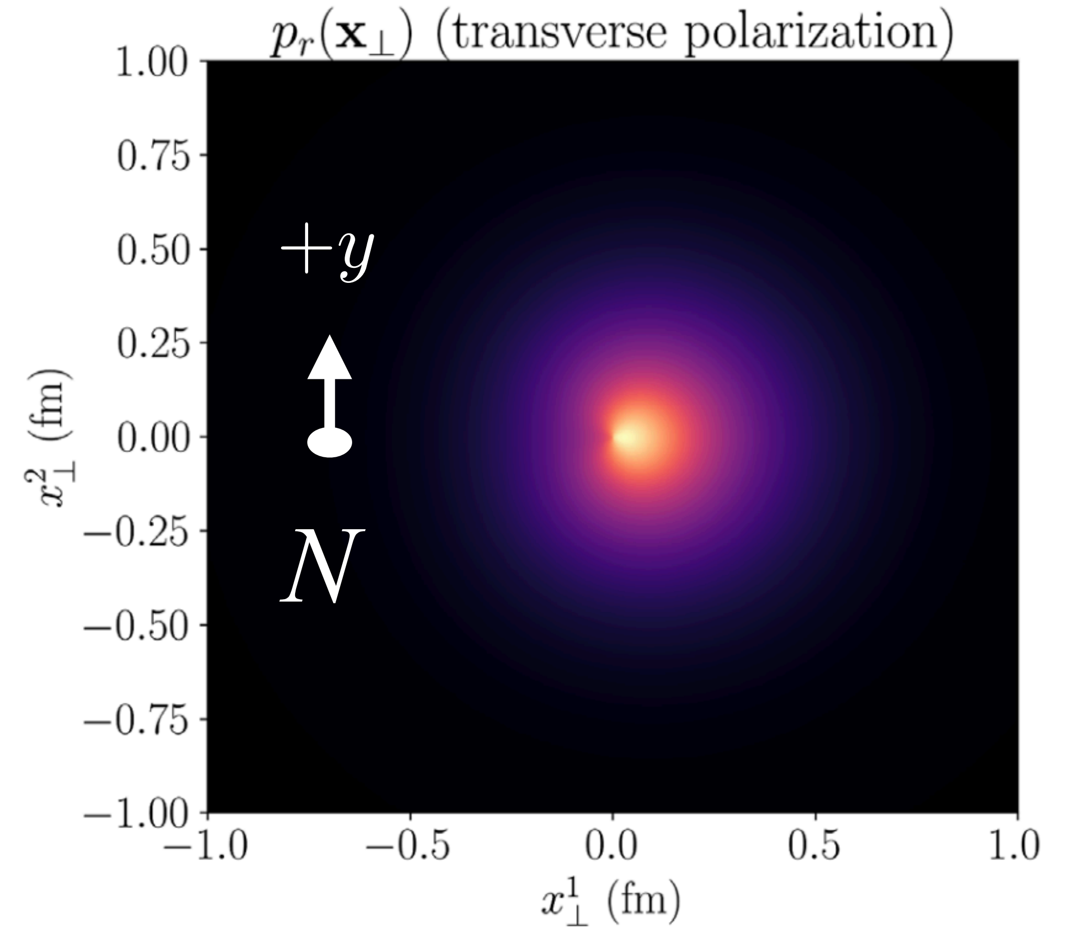
$\because B(t) = 0$ in our multi-pole ansatz

Transverse radial force, $\gamma_P T^{ij}\hat{n}^i_r = \sigma_r \hat{n}^j_r$ Won, and Lorcé, in preparation



Freese, and Miller, PRD 104 (2021)

consistency with LF work
(but polarized in the +y direction)



Angular momentum

Spin sum rules

For spin-1/2 target,

Longitudinal spin sum rule

$$\langle S^z \rangle = \frac{\sigma^z}{2}$$ Frame-independent

Clearly, only one

Transverse spin sum rule

$$\langle S_\perp^i \rangle = \frac{\sigma_\perp^i}{2}$$ Leader, PRD 85 (2012) Frame-independent

vs

$$\langle S_\perp^i \rangle = \gamma \frac{\sigma_\perp^i}{2}$$ Ji, and Yuan, PLB 810 (2020) Frame-dependent

with $\gamma = \sqrt{P_z^2 + M^2}/M$

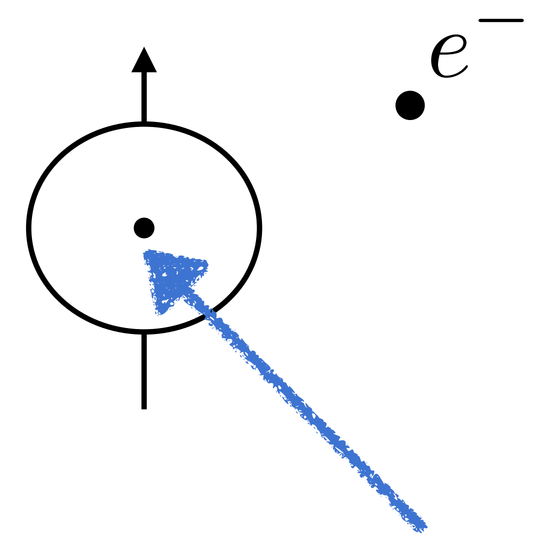
Why two sum rules?

Angular momentum operator

An AM operator is defined as a generator of rotations about the pivot

The origin of the coordinate system is “conventionally” choosen to coincide with the pivot.

Hydrogen atom:

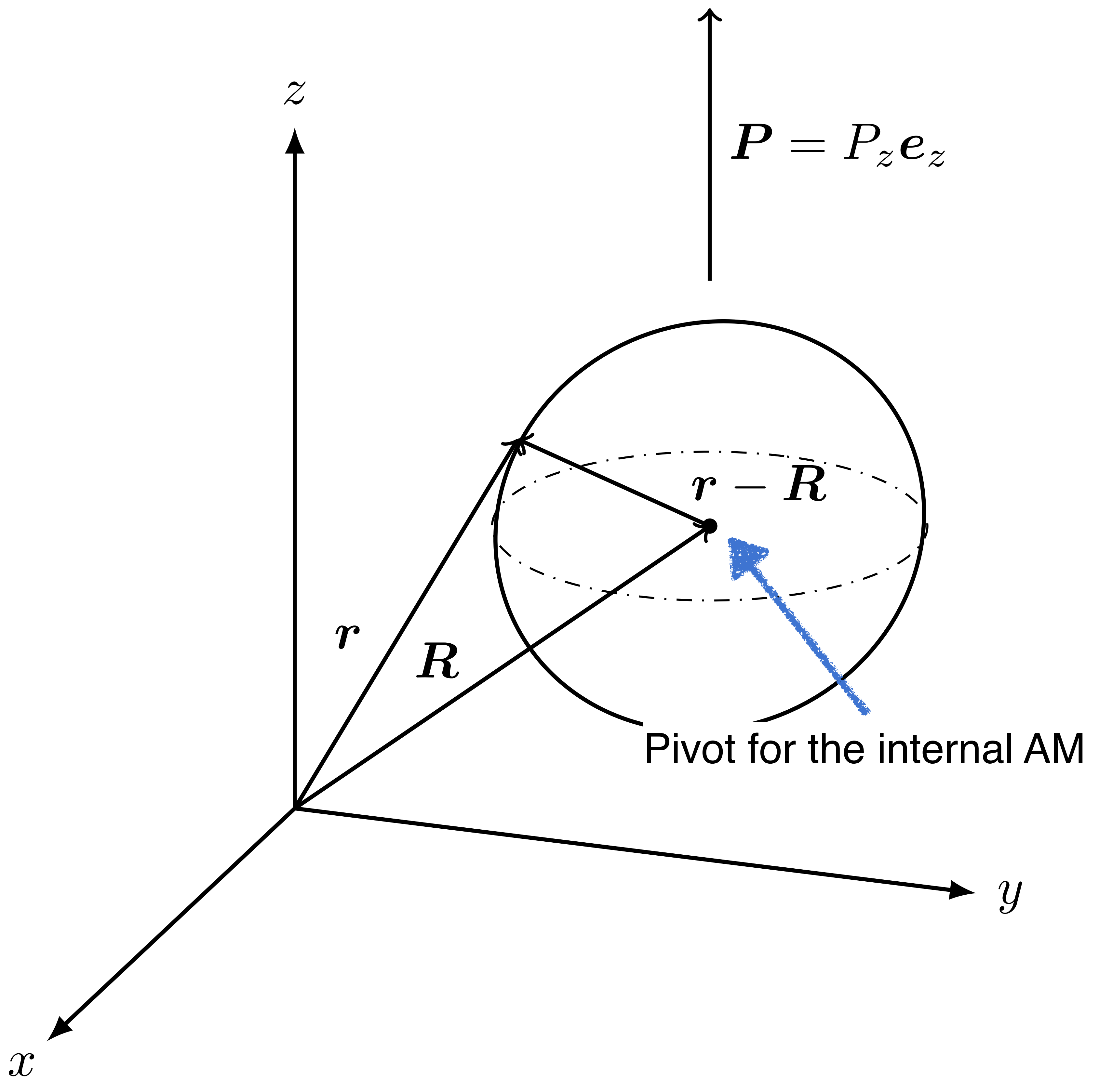


Pivot; the center of the rotation

Angular momentum operator for composite systems

Composite system:

Lorcé, EPJC 81 (2021)



Internal AM operator to composite systems in QFT

Definition: $\boldsymbol{S} = \boldsymbol{J} - \boldsymbol{R} \times \boldsymbol{P}$

total AM

Internal AM

External AM

$$[S^i, S^j] = i\epsilon^{ijk} S^k, \qquad [R^i, S^j] = 0, \qquad [P^i, S^j] = 0$$

su(2) spin algebra

Position operator R; from the origin to the pivot

$$[R^i, P^j] = i\delta^{ij}, \qquad [R^i, R^j] = 0,$$

Canonical relation

The center of mass is typically used as the pivot, but **the choice of the pivot is arbitrary**

Relativistic centers of composite systems in QFT

Qualifications of the position operator | Qualification of the internal AM operator

Relativistic center	Position operator	Canonical relation $[R^i, P^j] = i\delta^{ij}$	Vector under rotation $[R^i, J^j] = i\epsilon^{ijk}R^k$	Compatibility of components $[R^i, R^j] = 0$	Four-vector transformation $R'^\mu = \Lambda^\mu{}_\nu R^\nu$	su(2) spin algebra $[S^i, S^j] = i\epsilon^{ijk}S^k$	Transverse spin sum rule
Energy	$r^0 = 0$ $R^i_E = \frac{1}{P^0}\int d^3r\, r^i T^{00} = -\frac{K^i}{P^0}$	✓	✓	✗	✗	✗	$\langle S^i_{E,\perp}\rangle = \gamma^{-1}\frac{\sigma^i_\perp}{2}$ New
Mass	$R^\mu_M = \Lambda^\mu{}_\nu R^\nu_E\vert_{\text{rest}}$	✓	✓	✗	✓	✗	$\langle S^i_{M,\perp}\rangle = \gamma\frac{\sigma^i_\perp}{2}$
Spin	$R^i_c = \frac{P^0 R^i_E + M R^i_M}{P^0 + M}$	✓	✓	✓	✗	✓	$\langle S^i_{c,\perp}\rangle = \frac{\sigma^i_\perp}{2}$ Today's topic

Pauli-Lubanski pseudo-vector: $S^i_E = W^i/P^0$

Qualification of the localized state
$\boldsymbol{R}_c\,|\boldsymbol{r}\rangle = \boldsymbol{r}\,|\boldsymbol{r}\rangle$

$$\langle S^i_\perp\rangle = \frac{\sigma^i_\perp}{2} \quad \& \quad \langle S^i_\perp\rangle = \gamma\frac{\sigma^i_\perp}{2}$$

Leader, PRD 85 (2012) | Ji, and Yuan, PLB 810 (2020)

Measured relative to different relativistic centers

Pryce, PRSLA 195 (1948)
Newton, and Wigner, RMP 21 (1949)
Fleming, PR 137 (1965)
Murkardt, PRD 72 (2005)
Lorcé, EPJC 81 (2021)

Relativistic spatial distributions of OAM and intrinsic spin

Relativistic spatial distributions about the relativsitic center of spin

$$L^i(\boldsymbol{b}_\perp; s', s) = \epsilon^{ijk} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}_\perp} \left[-i\frac{\partial}{\partial \Delta^j} \frac{\langle p', s'|\hat{T}^{0k}(0)|p, s\rangle}{2P^0} \right]_{\Delta_z=0},$$

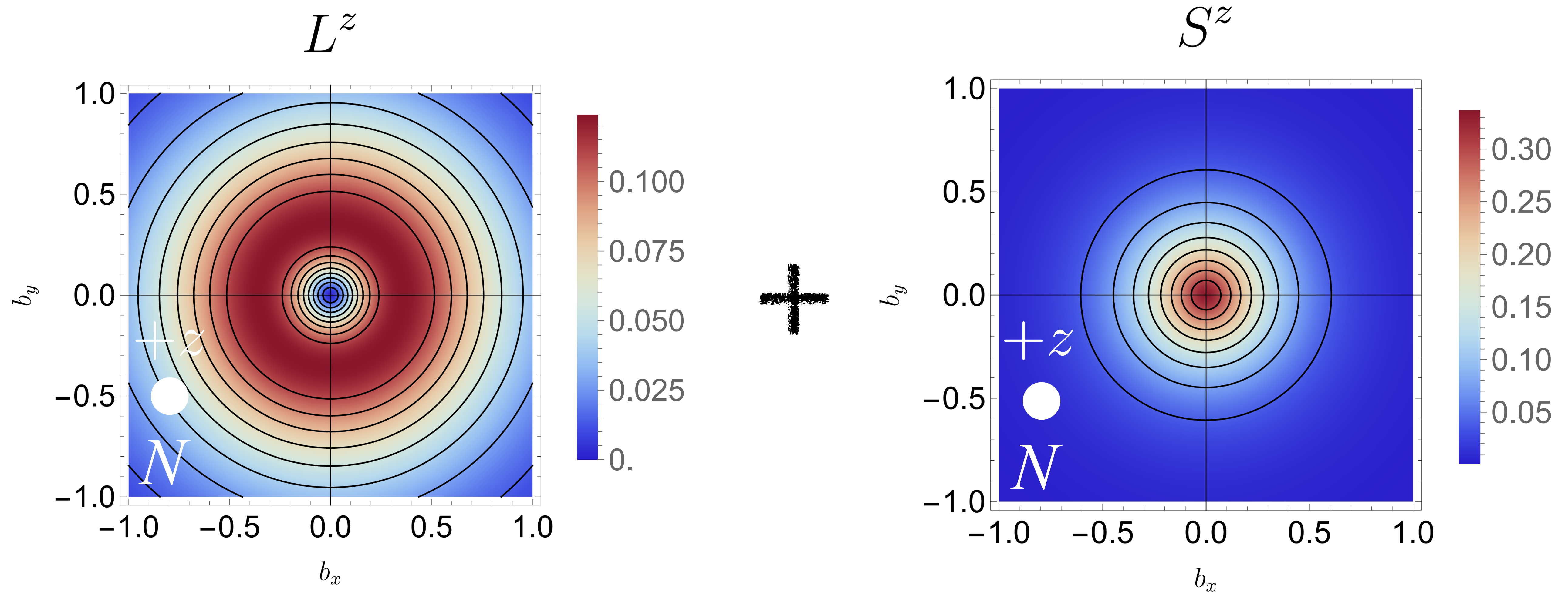
$$S^i(\boldsymbol{b}_\perp; s', s) = \frac{1}{2}\epsilon^{ijk} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}_\perp} \left. \frac{\langle p', s'|\hat{S}^{0jk}(0)|p, s\rangle}{2P^0} \right|_{\Delta_z=0},$$

Integrating spin sum rule →

$$\langle S_c^i \rangle := \langle J^i \rangle = \int d^2b_\perp\, L^i + \int d^2b_\perp\, S^i$$

Internal distribution condition $\because \boldsymbol{R}_c = \boldsymbol{0}$

Longitudinal component in longitudinally polarized nucleon



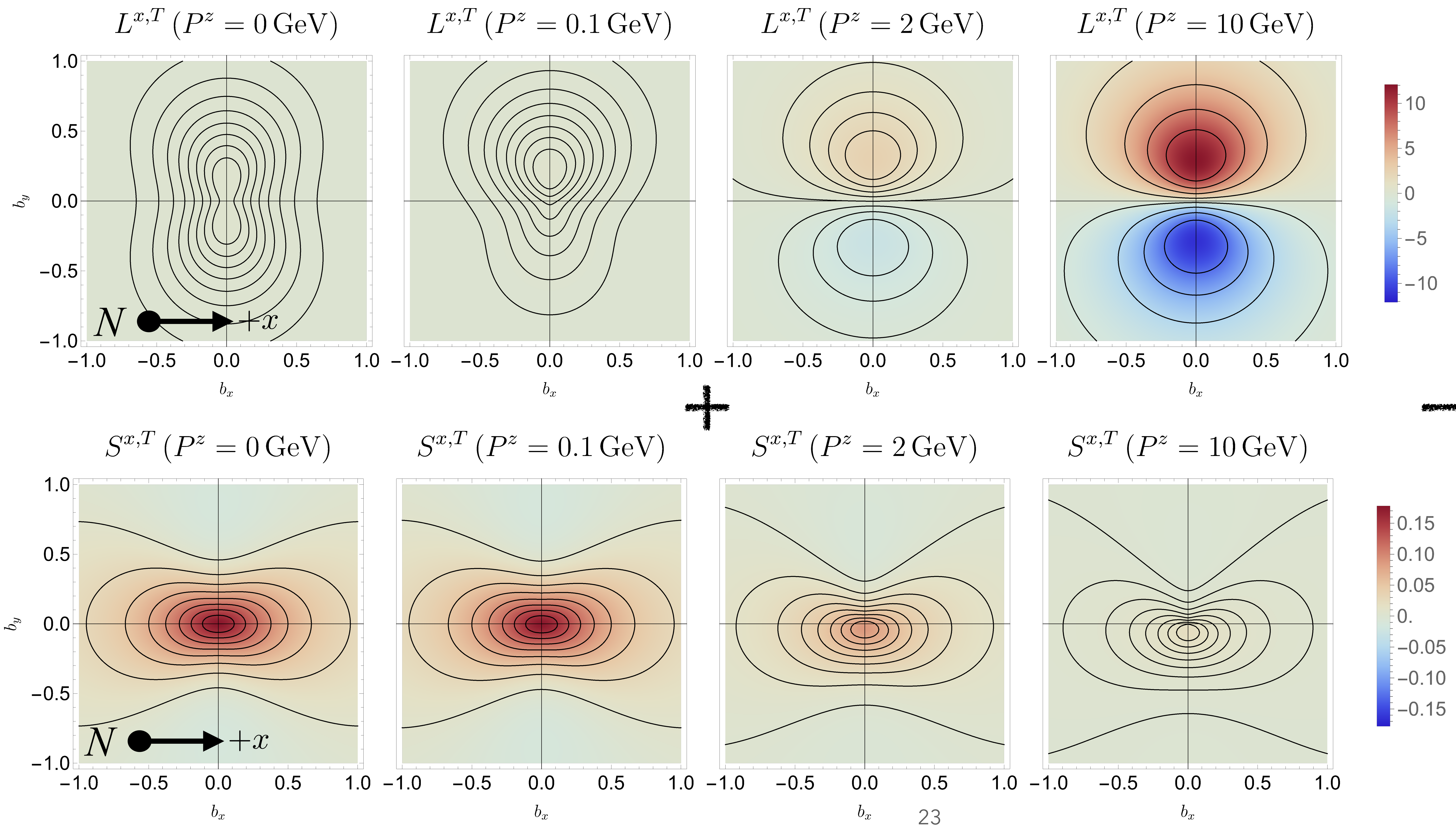
→ $\langle J^z \rangle = \frac{\sigma_z}{2}$

Longitudinal spin sum rule
about **the relativistic center of spin**

Frame-independent $\because [J^z, K^z] = 0$

Relativistic spatial distributions of OAM and intrinsic spin

Transverse components in transversely polarized nucleon



Frame-dependent

$$\because [J_\perp^i, K^j] = i\epsilon^{ijk}K^k$$

$$\longrightarrow \langle J_\perp^i \rangle = \frac{\sigma_\perp^i}{2}$$

Transverse spin sum rule about **the relativistic center of spin**

Lorcé, Mukherjee, Singh, and Won, arXiv 2606.20468

Conclusion

Conclusion

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Won, and Lorcé, PRD 111 (2025)
Lorcé, Mukherjee, Singh, and Won, arXiv 2606.20468 (accepted in PLB)

- We review the **energy-momentum tensor** and related physical observables.

- Quantum phase-space formalism developed the relativistic spatial distribution in the **elastic frame**, which is a generic frame to interpolate both **rest** and **infinite momentum frames**, as well as to include the relativistic spin effect emerged as **Wigner rotation**.

- We show the **relativistic spatial distributions** of the **EMT**, and longitudinal and transverse **angular momentum** for polarized nucleons in the transverse plane.

- This work can be extended to study the **parity-odd EMT**, including longitudinal spin-orbit correlation and unknown quantities.

Parity-odd EMT operator in QCD

$$\hat{T}_5^{\mu\nu} = \bar{\psi}\gamma^\mu\gamma_5 \frac{i}{2}\overleftrightarrow{D}^\nu\psi - 2i\mathrm{Tr}\,\tilde{F}^{\mu\lambda}F^\nu{}_\lambda + \frac{i}{2}g^{\mu\nu}\mathrm{Tr}\,\tilde{F}^{\lambda\rho}F_{\lambda\rho}$$

Thank you for listening

Back up

The spatial distributions of the EMT

In the EF, the longitudinal EMT distributions, i.e., T^{00}, T^{03}, T^{30}, and T^{33}

$$g_a\left(\boldsymbol{b}_\perp, P_z; s', s\right) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's}\tilde{g}_a^U\left(Q^2, P_z\right) + \frac{\left(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta}_\perp\right)_z}{2M}\tilde{g}_a^T\left(Q^2, P_z\right)\right],$$

$g = \rho, \mathcal{P}^i, \mathcal{I}^i, \Pi^{zz}, \sigma$

Monopole; spin-independent — Dipole; spin-dependent → Rank: spin=position in multipole order

For example, the amplitudes in the energy distribution

$$\tilde{\rho}_a^U\left(Q^2, P_z\right) = M\frac{\gamma}{\gamma_P}\left[\frac{P^0 + M(1+\tau)}{(P^0+M)\sqrt{1+\tau}}\left\{E_a\left(Q^2\right) + \beta^2 F_a\left(Q^2\right)\right\} + \frac{\sqrt{\tau}P_z}{(P^0+M)\sqrt{1+\tau}}2\sqrt{\tau}\beta J_a\left(Q^2\right)\right],$$

$$\tilde{\rho}_a^T\left(Q^2, P_z\right) = M\frac{\gamma}{\gamma_P}\left[\frac{\sqrt{\tau}P_z}{(P^0+M)\sqrt{1+\tau}}\frac{1}{\sqrt{\tau}}\left\{-E_a\left(Q^2\right) - \beta^2 F_a\left(Q^2\right)\right\} + \frac{P^0 + M(1+\tau)}{(P^0+M)\sqrt{1+\tau}}\frac{1}{\sqrt{\tau}}2\sqrt{\tau}\beta J_a\left(Q^2\right)\right],$$

$E = A + \bar{C} + \tau(A - 2J + D)$,
$J = \frac{A+B}{2}$,
$F = -\tau D - \bar{C}$,

Normalized by γ_P, so that $\int d^2 b_\perp\, \rho = M$

→ Wigner rotation effects × mixing of currents under a longitudinal Lorentz boost

$$\hat{T}^{00} \to \gamma^2\hat{T}^{00} + \gamma^2\beta\hat{T}^{03} + \gamma^2\beta\hat{T}^{30} + \gamma^2\beta^2\hat{T}^{33}$$

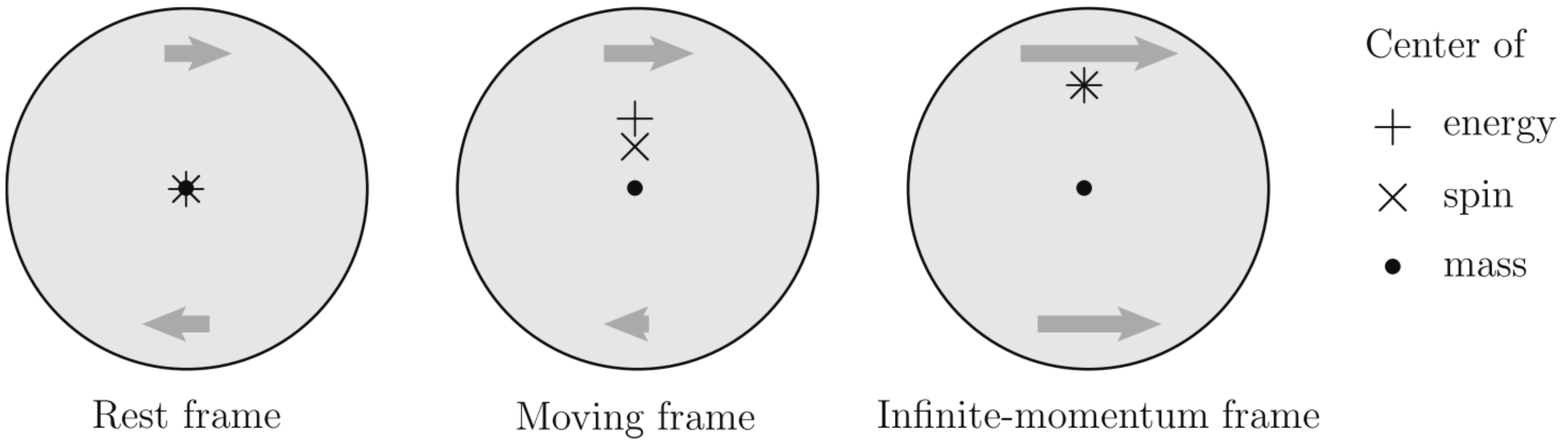
The transverse shear force

$$\Pi_a\left(\boldsymbol{b}_\perp, P_z; s', s\right) = \frac{1}{M^2 b^2}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\tilde{\Pi}_a^U\left(Q^2, P_z\right)\delta_{s's} + \frac{\left(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta}_\perp\right)_z}{2M}\tilde{\Pi}_a^T\left(Q^2, P_z\right)\right]$$

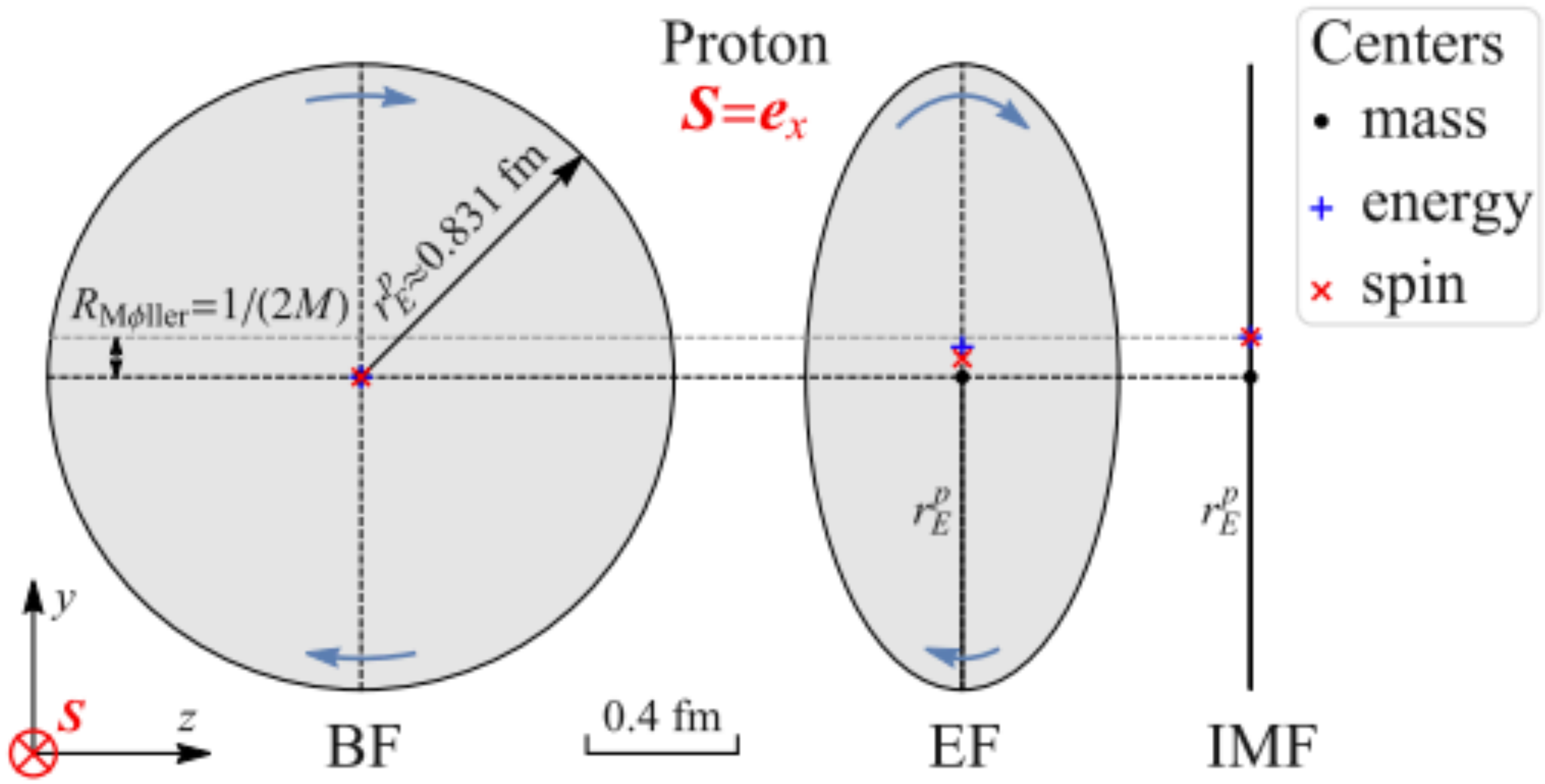
Position operator in QFT

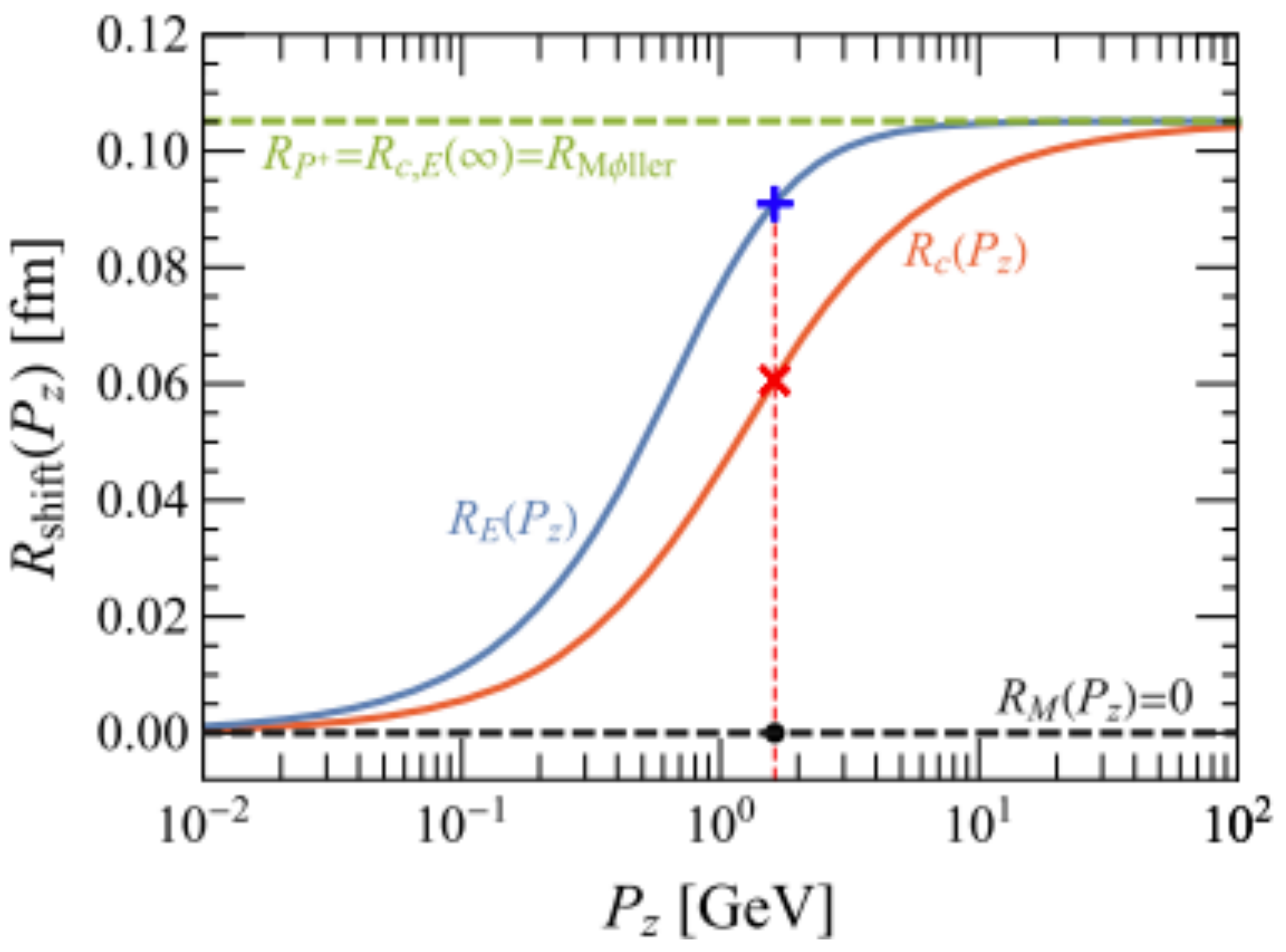
$$[J^i, K^j] = i\epsilon^{ijk}K^k$$

Lorcé, EPJC 81 (2023)



Chen, Lorcé, PRD 107 (2023)





Parity-odd EMT operator

Parity-odd EMT operator

$$\hat{T}_5^{\mu\nu} = \bar{\psi}\gamma^\mu\gamma_5 \frac{i}{2}\overleftrightarrow{D}^\nu \psi - 2i\mathrm{Tr}\ \tilde{F}^{\mu\lambda}F^\nu{}_\lambda + \frac{i}{2}g^{\mu\nu}\mathrm{Tr}\ \tilde{F}^{\lambda\rho}F_{\lambda\rho}.$$

Dual field tensor

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

Chiral decomposition

$$\psi_{R/L} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

Helicity decomposition in the field strength tensor

$$F_\pm^{\mu\nu} = \frac{1}{2}\left(F^{\mu\nu} \pm i\tilde{F}^{\mu\nu}\right)$$

Matrix elements of P-odd EMT operator

$$\langle p', s'|\hat{T}_{5a}^{\mu\nu}(0)|p, s\rangle = \bar{u}(p', s')\left[\frac{P^{\{\mu}\gamma^{\nu\}}\gamma_5}{2}\tilde{A}_a(Q^2) + \frac{P^{\{\mu}\Delta^{\nu\}}\gamma_5}{2}\tilde{B}_a(Q^2)\right.$$
$$\left. + \frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{2}\tilde{C}_a(Q^2) + \frac{P^{[\mu}\Delta^{\nu]}\gamma_5}{2}\tilde{D}_a(Q^2) + Mi\sigma^{\mu\nu}\gamma_5\tilde{F}_a(Q^2)\right]u(p, s)$$