Towards improved hadron femtography with hard exclusive reactions, edition IV, Jefferson Lab, 2025

QCD First Inverse problem using Maximum Likelihood Analysis via Exclusive Reactions

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Outline

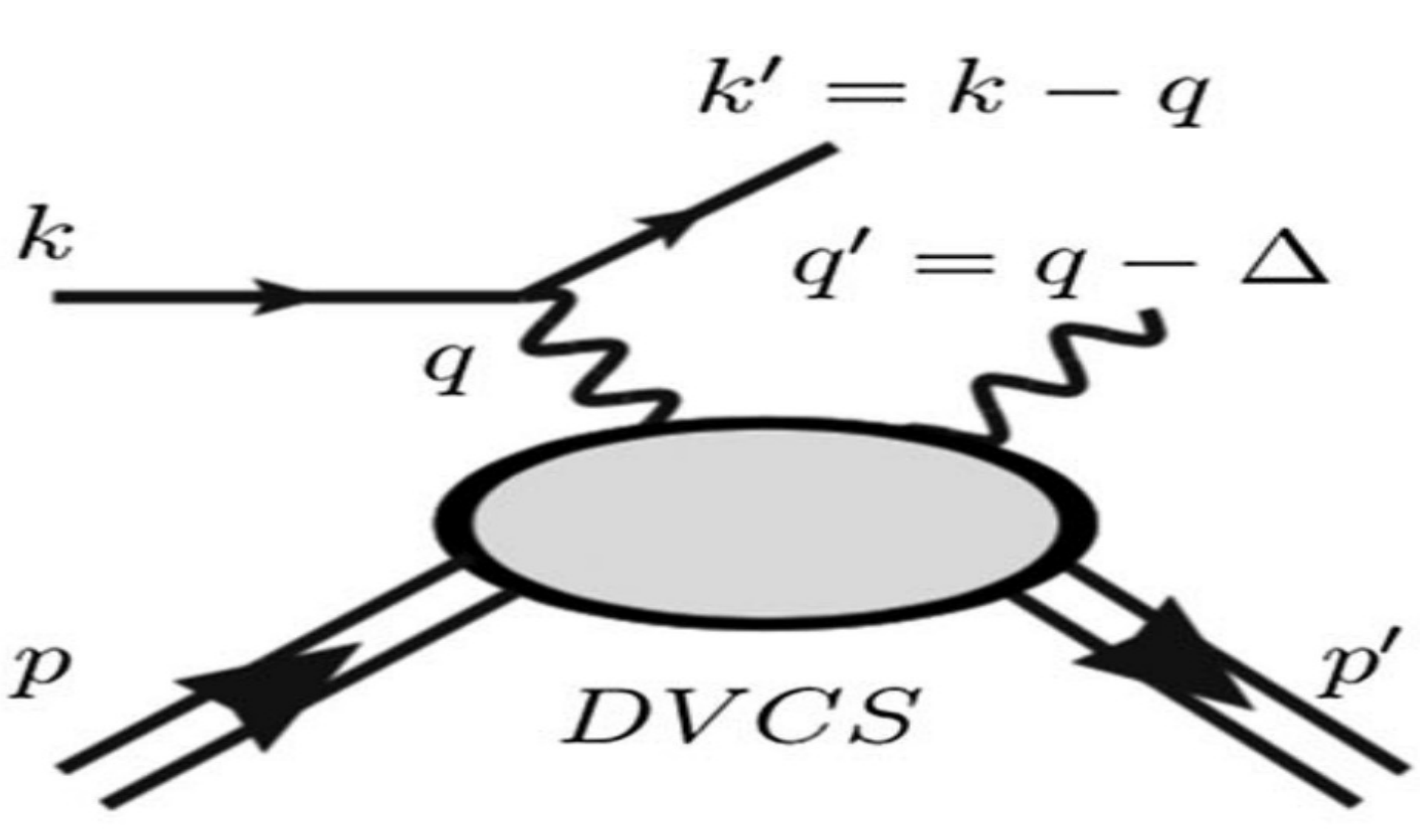
- Motivation
  - Inverse problems in QCD
- Formalism
- Extraction CFFs from exclusive meson (π^0) data
  - Approach
    - Likelihood Analysis
      - Canonical Method
  - Obtained Results
- Conclusion and Next Steps

Motivation

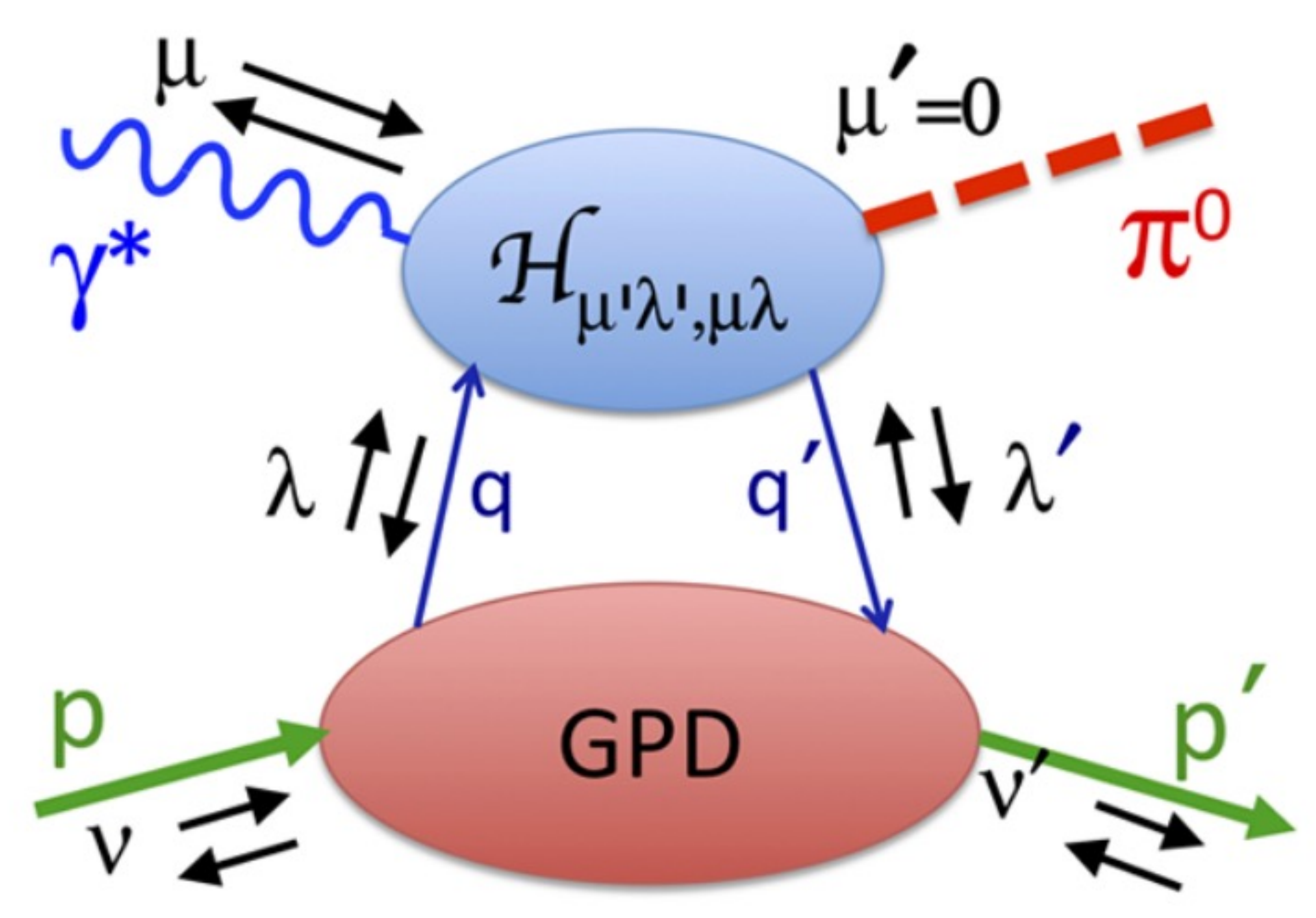
Imaging transverse spatial distribution of quarks and gluons

Exclusive reactions

DVCS



DVMP



Inverse problems in QCD

- First inverse problems

QCD Theory

Observables
(fragmentation functions, Compton form factors, etc)

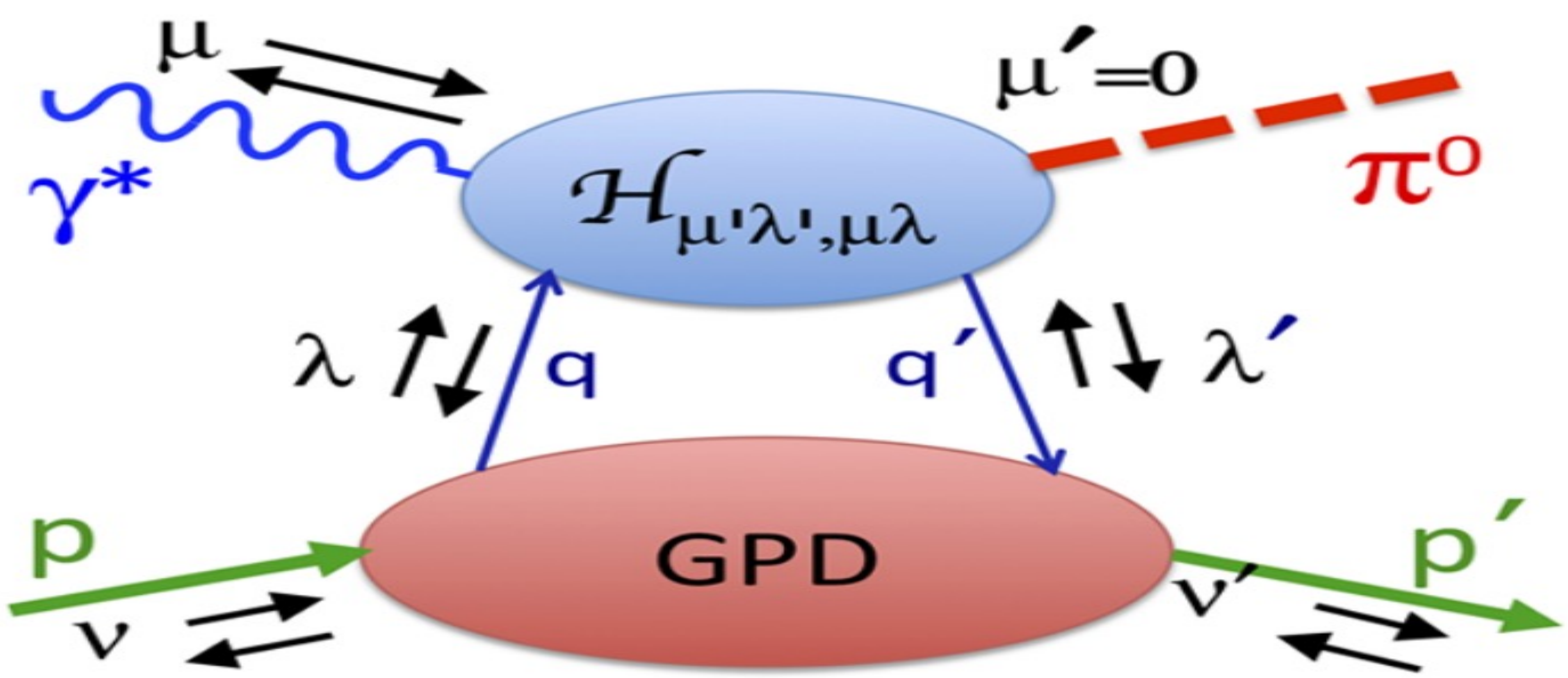
Inverse problem

Experiments
(JLAB, EIC)

- Second inverse problem

$$\mathrm{CFF} = \int (\mathrm{QCDKernel}) \times \mathrm{GPD}$$

Formalism



e p ⟶ e' p' π⁰

Liuti et. al, Phys. Rev. D 91, 114013 (2015)
Liuti et. al, Phys. Rev. D 79, 054014 (2009)
arxiv : 1401.0438

$$\begin{aligned}\frac{d^4\sigma}{dx_{Bj}dyd\phi dt} &= \Gamma\Big\{\Big[F_{UU,T}+\epsilon F_{UU,L}+\epsilon\cos 2\phi F_{UU}^{\cos 2\phi}+\sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi}+h\sqrt{\epsilon(1-\epsilon)}\,\sin\phi F_{LU}^{\sin\phi}\Big]\\
&+ S_{||}\Big[\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}+\epsilon\sin 2\phi F_{UL}^{\sin 2\phi}+h\left(\sqrt{1-\epsilon^2}\,F_{LL}+\sqrt{\epsilon(1-\epsilon)}\,\cos\phi\, F_{LL}^{\cos\phi}\right)\Big]\\
&- S_{\perp}\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)}+\epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right)+\frac{\epsilon}{2}\left(\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)}+\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}\right)\\
&+ \sqrt{\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S}+\sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big]\\
&+ S_{\perp}h\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)}+\sqrt{\epsilon(1-\epsilon)}\left(\cos\phi_S F_{LT}^{\cos\phi_S}+\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\right)\Big]\Big\}\end{aligned} \quad (74)$$

$$A_{UL}=\frac{N_{s_z=+}-N_{s_z=-}}{N_{s_z=+}+N_{s_z=-}}=\frac{\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T}+\epsilon F_{UU,L}}+\frac{\epsilon\sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T}+\epsilon F_{UU,L}}=A_{UL}^{\sin\phi}\sin\phi+A_{UL}^{\sin 2\phi}\sin 2\phi$$

$$A_{LL}=\frac{N^{\rightarrow}_{s_z=+}-N^{\rightarrow}_{s_z=-}+N^{\leftarrow}_{s_z=+}-N^{\leftarrow}_{s_z=-}}{N_{s_z=+}+N_{s_z=-}}=\frac{\sqrt{1-\epsilon^2}\;F_{LL}}{F_{UU,T}+\epsilon F_{UU,L}}+\frac{\sqrt{\epsilon(1-\epsilon)}\;\cos\phi\,F_{LL}^{\cos\phi}}{F_{UU,T}+\epsilon F_{UU,L}}=A_{LL}+A_{LL}^{\cos\phi}\cos\phi$$

Theoretical background

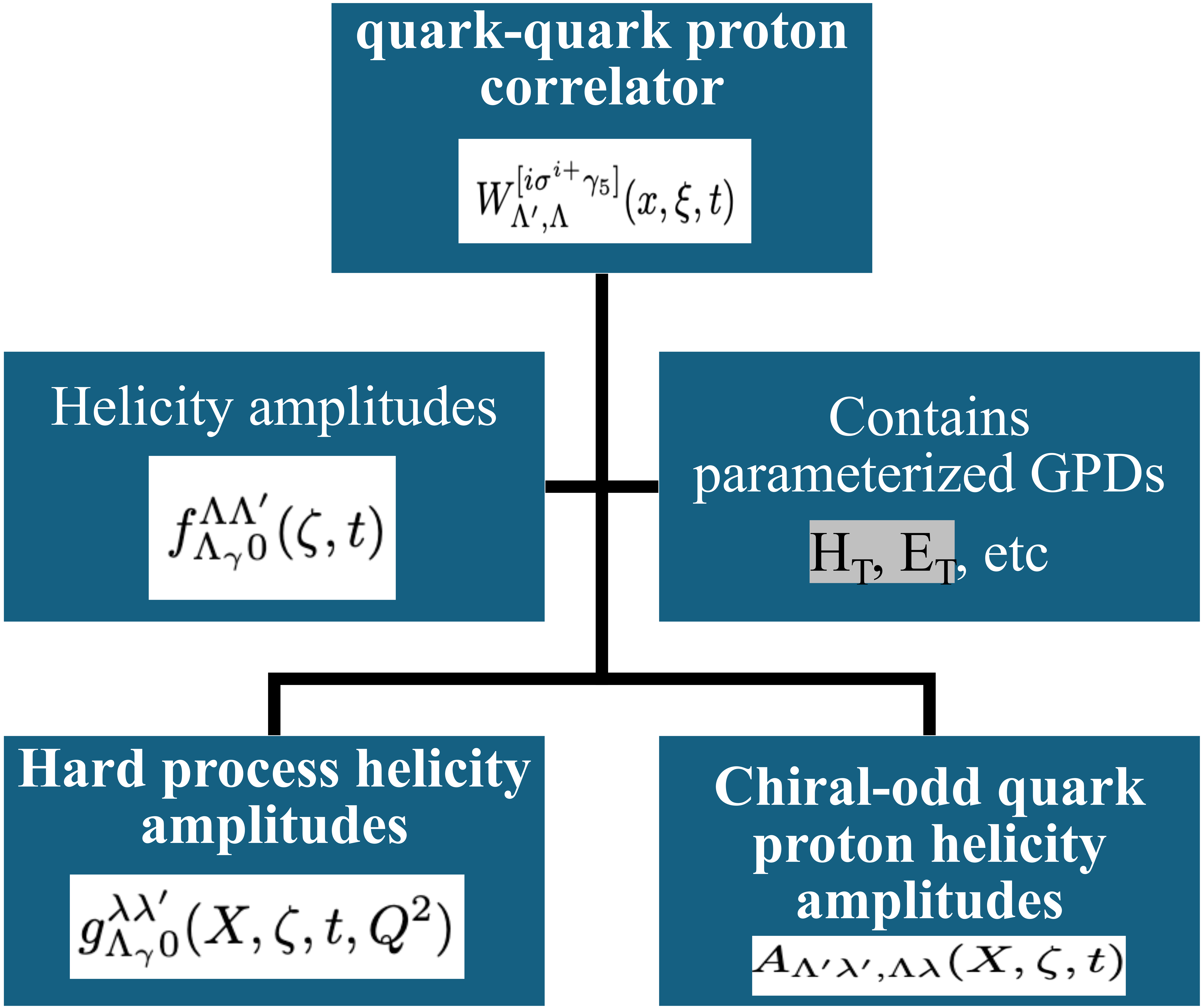
$$W_{\Lambda',\Lambda}^{[i\sigma^{i+}\gamma_5]}(x,\xi,t) = \frac{1}{2\overline{P}^+}\overline{U}(P',\Lambda')\left(i\sigma^{+i}H_T(x,\xi,t) + \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2M}E_T(x,\xi,t)\right.$$
$$\left. + \frac{P^+\Delta^i - \Delta^+P^i}{M^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^+P^i - P^+\gamma^i}{2M}\widetilde{E}_T(x,\xi,t)\right)U(P,\Lambda)$$

$$f_{\Lambda_\gamma 0}^{\Lambda\Lambda'}(\zeta,t) = \sum_{\lambda,\lambda'} g_{\Lambda_\gamma 0}^{\lambda\lambda'}(X,\zeta,t,Q^2) \otimes A_{\Lambda'\lambda',\Lambda\lambda}(X,\zeta,t),$$

hard process helicity amplitude

$$f_{10}^{++} = g_{10}^{+-} \otimes A_{+-,++}$$
$$f_{10}^{+-} = g_{10}^{+-} \otimes A_{--,++}$$
$$f_{10}^{-+} = g_{10}^{+-} \otimes A_{+-,-+}$$
$$f_{10}^{--} = g_{10}^{+-} \otimes A_{++,+-}$$
$$f_{00}^{+-} = g_{00}^{+-} \otimes (A_{--,++} - A_{+-,-+})$$
$$f_{00}^{++} = g_{00}^{+-} \otimes (A_{++,+-} - A_{+-,++}),$$

chiral-odd quark proton helicity amplitude


quark-quark proton correlator
$W^{[i\sigma^{i+}\gamma_5]}_{\Lambda',\Lambda}(x,\xi,t)$
Helicity amplitudes
$f^{\Lambda\Lambda'}_{\Lambda_\gamma 0}(\zeta,t)$
Contains parameterized GPDs
H_T, E_T, etc
Hard process helicity amplitudes
$g^{\lambda\lambda'}_{\Lambda_\gamma 0}(X,\zeta,t,Q^2)$
Chiral-odd quark proton helicity amplitudes
$A_{\Lambda'\lambda',\Lambda\lambda}(X,\zeta,t)$

$$
\begin{aligned}
F_{UU,T} &= \frac{1}{2}(F_{11}^{++} + F_{11}^{--}) = \frac{1}{2}\sum_{\Lambda'}(f_{10}^{+\Lambda'*} f_{10}^{+\Lambda'} + f_{10}^{-\Lambda'*} f_{10}^{-\Lambda'}) \\
&= \frac{1}{2}\left(| f_{10}^{++} |^2 + | f_{10}^{+-} |^2 + | f_{10}^{-+} |^2 + | f_{10}^{--} |^2\right) \\
F_{UU,L} &= F_{00}^{++} = \sum_{\Lambda'} f_{00}^{+\Lambda'*} f_{00}^{+\Lambda'} =| f_{00}^{++} |^2 + | f_{00}^{+-} |^2 \\
F_{UU}^{\cos 2\phi} &= -\Re e\, F_{1-1}^{++} = -\Re e \sum_{\Lambda'} f_{10}^{+\Lambda'*} f_{-10}^{+\Lambda'} \\
&= -\,\Re e\left[(f_{10}^{++})^*(f_{10}^{--}) - (f_{10}^{+-})^*(f_{10}^{-+})\right] \\
F_{UU}^{\cos\phi} &= \Re e(F_{10}^{++} + F_{10}^{--}) = \Re e \sum_{\Lambda'}(f_{00}^{+\Lambda'*} f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'*} f_{10}^{-\Lambda'}) \\
&= \Re e\big[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})\big] \\
F_{LU}^{\sin\phi} &= -\Im m(F_{10}^{++} + F_{10}^{--}) = -\Im m \sum_{\Lambda'}(f_{00}^{+\Lambda'*} f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'*} f_{10}^{-\Lambda'}) \\
&= -\Im m\big[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})\big]
\end{aligned}
$$

Experimental data

Cross-section

Q2	W	xB	tmin-t	epsilon	sig0	dsig0 stat	dsig0 sys	sigLT	dsigLT stat	dsigLT sys	sigTT	dsigTT stat	dsigTT sys	sigLT'	dsigLT' stat	dsigLT' sys
3.11	2.53	0.36	0.03	0.61	195.46	3.66	11.93	7.05	3.19	0.25	-4.66	7.62	0.16	14.52	6.91	0.51
3.11	2.53	0.36	0.1	0.61	217.29	4.22	13.26	-5.97	3.81	0.21	-67.45	9.03	2.36	18.91	8.05	0.66
3.11	2.53	0.36	0.18	0.61	195.76	4.15	11.95	-11.55	4.01	0.4	-67.67	8.97	2.37	27.63	7.17	0.97
3.11	2.53	0.36	0.29	0.61	183.18	4.54	11.18	-28.08	4.63	0.98	-87.12	10.32	3.05	9.05	6.38	0.32
3.57	2.69	0.36	0.03	0.62	115.04	2.53	4.64	-2.37	2.18	0.08	-16.48	5.25	0.58	-5.08	4.97	0.18
3.57	2.69	0.36	0.1	0.62	117.51	277	4.74	-9.7	2.54	0.34	-46.96	5.85	1.64	18.11	5.31	0.63
3.57	2.69	0.36	0.17	0.62	119.61	3.35	4.82	-5.32	3.52	0.19	-35.39	6.99	1.24	14.58	5.31	0.51
3.57	2.69	0.36	0.28	0.62	105.37	4.06	4.25	-5.32	3.52	0.19	-35.39	6.99	1.24	14.58	5.31	0.51
4.44	2.96	0.36	0.03	0.63	57.04	1.88	2.08	-1.84	1.44	0.06	-2.42	3.43	0.08	4.92	3.24	0.17
4.44	2.96	0.36	0.09	0.63	62.86	2.16	2.29	-0.23	1.83	0.01	-13.17	4.03	0.46	5.04	3.63	0.18
4.44	2.96	0.36	0.17	0.63	64.53	2.47	2.35	0.62	2.35	0.02	-13.49	4.66	0.47	6.39	3.65	0.22
4.44	2.96	0.36	0.28	0.63	51.63	2.56	1.88	-5.66	2.61	0.2	-28.8	4.76	1.01	5.79	3.13	0.2
2.67	1.94	0.48	0.01	0.51	525.95	14.48	41.16	25.07	16.53	0.88	-29.14	43.88	1.02	7.6	30.21	0.27
2.67	1.94	0.48	0.04	0.51	520.4	16.36	40.73	-38.25	19.21	1.34	-7.88	45.79	0.28	-5.32	31.83	0.19
2.67	1.94	0.48	0.08	0.51	488.33	17.33	38.22	-31.6	21.72	1.11	-55.44	47.02	1.94	16.7	28.69	0.5
2.67	1.94	0.48	0.14	0.51	480.77	23.45	37.63	-60.2	30.66	2.11	-116.12	57.67	4.06	14.05	27.05	0.49
4.06	2.3	0.45	0.02	0.71	126.23	3.84	6.71	-3.93	3.36	0.14	-17.69	7.8	0.6	16.81	8.43	0.59
4.06	2.3	0.45	0.07	0.71	128.7	4.65	6.84	-9.18	4.45	0.32	-13.9	8.66	0.49	26.38	8.65	0.92
4.06	2.3	0.45	0.12	0.71	115.22	6.01	6.12	-16.42	6.24	0.57	-23.1	10.78	0.81	30.12	8.41	1.05

Jefferson Lab Hall A

Phys. Rev. Lett. 127, 152301

Phys. Rev. C 83, 025201

Phys. Lett. B 768 (2017) 168–173

Q2	W	xB	epsilon	-t	AUL^sinphi	dAUL^sinphi stat	dAUL^sinphi sys	AUL^sin2phi	dAUL^sin2phi stat	dAUL^sin2phi sys
1.94	2.59	0.25	0.51	0.15	0.272	0.037	0.03	-0.033	0.036	0.044
1.94	2.59	0.25	0.51	0.26	0.223	0.033	0.06	-0.047	0.032	0.03
1.94	2.59	0.25	0.51	0.49	0.292	0.036	0.021	-0.073	0.036	0.031
1.94	2.59	0.25	0.51	0.92	0.269	0.036	0.019	-0.12	0.036	0.024
1.94	2.59	0.25	0.51	1.46	0.188	0.047	0.032	-0.116	0.043	0.013
2.83	2.26	0.4	0.58	0.27	0.165	0.046	0.036	-0.006	0.042	0.029
2.83	2.26	0.4	0.58	0.52	0.208	0.03	0.022	-0.021	0.025	0.021
2.83	2.26	0.4	0.58	0.88	0.135	0.035	0.022	-0.043	0.026	0.012
2.83	2.26	0.4	0.58	1.24	0.164	0.049	0.051	-0.09	0.042	0.036
2.83	2.26	0.4	0.58	1.59	0.26	0.055	0.029	-0.155	0.047	0.029

Q2	W	xB	epsilon	-t	ALL^const	dALL^const stat	dALL^const sys	ALL^cosphi	dALL^cosphi stat	dALL^cosphi sys
1.94	2.59	0.25	0.51	0.15	0.594	0.029	0.019	0.163	0.041	0.064
1.94	2.59	0.25	0.51	0.26	0.593	0.028	0.03	0.162	0.042	0.081
1.94	2.59	0.25	0.51	0.49	0.564	0.036	0.031	0.2	0.064	0.078
1.94	2.59	0.25	0.51	0.92	0.395	0.03	0.025	0.158	0.049	0.05
1.94	2.59	0.25	0.51	1.46	0.214	0.036	0.037	-0.023	0.042	0.029
2.83	2.26	0.4	0.58	0.27	0.705	0.039	0.059	0.19	0.064	0.089
2.83	2.26	0.4	0.58	0.52	0.667	0.022	0.032	0.156	0.029	0.089
2.83	2.26	0.4	0.58	0.88	0.49	0.029	0.014	0.193	0.027	0.076
2.83	2.26	0.4	0.58	1.24	0.504	0.038	0.017	0.158	0.052	0.089
2.83	2.26	0.4	0.58	1.59	0.449	0.046	0.034	0.114	0.056	0.083

Thanks to Prof. Garth & Alicia

Jefferson Lab Hall B

Asymmetry

best overlapping kinematic bins

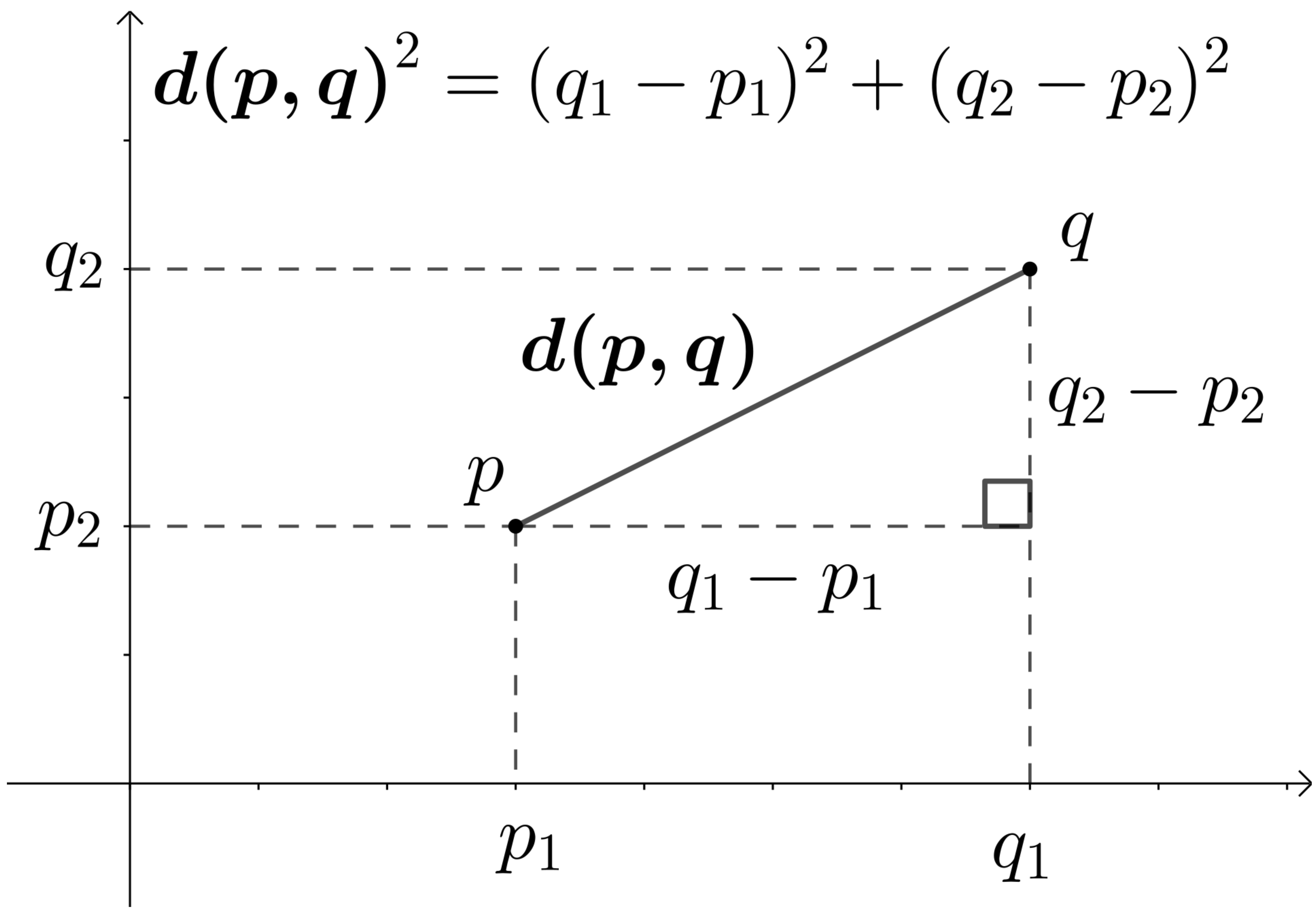
Datatype / kinematic	Q^2	W	x_{Bj}	|t|	ε
σ's	2.67	1.94	0.48	0.470	0.510
A	2.83	2.26	0.40	0.520	0.580
Joint (σ, A)	2.75	2.1	0.44	0.495	0.545

Finding the best kinematic bin for analysis

- Find the closest set of kinematic bins from all the data

- Pythagorean theorem

- Find the Euclidean distance



https://commons.wikimedia.org/w/index.php?curid=67617313

Likelihood Analysis

- We use a "canonical" approach to perform a Bayesian likelihood analysis.

- The joint likelihood of the parameters of interest is calculated as a simple product of Gaussians.

$$\mathcal{L} = \prod_{i=1}^{N} Gaussian(x, \mu, \sigma)$$

$$Gaussian(x, \mu, \sigma) \propto exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

- Markov Chain Monte Carlo (MCMC) algorithms are used to take multidimensional probability density functions and generate set of representative samples.

- These samples are used to create easy visualizations of the samples in the form corner plots.

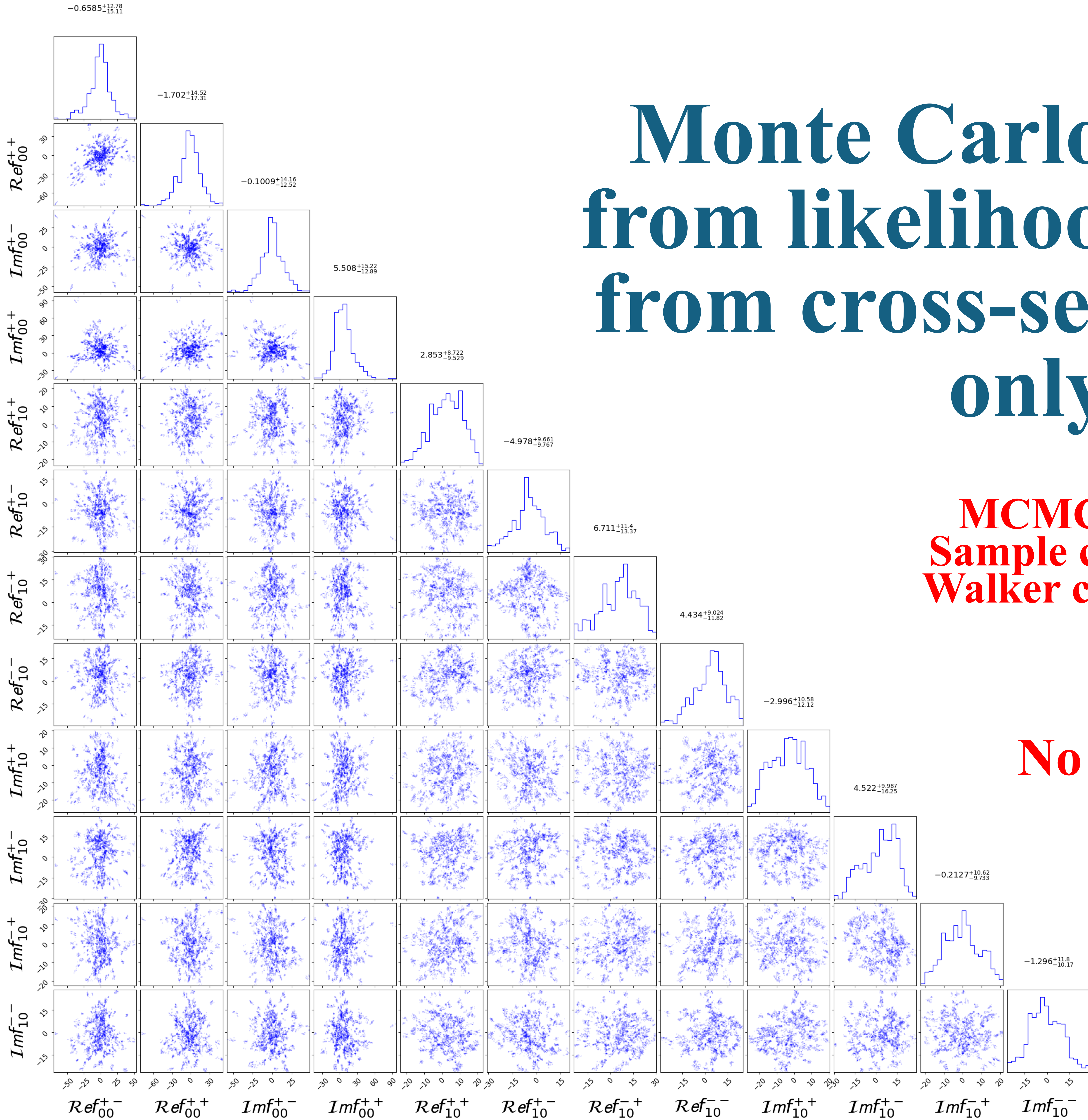
https://arxiv.org/abs/2410.23469

What is more important for CFFs fundamentally?

$$
\begin{aligned}
f_{10}^{++} &= g_\pi^{V,odd}(Q)e^{i\phi}\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T+(1+\xi)\left(\mathcal{E}_T-\widetilde{\mathcal{E}}_T\right)\right] \\
&= g_\pi^{V,odd}(Q)e^{i\phi}\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T+\frac{1}{1-\zeta/2}\left(\mathcal{E}_T-\widetilde{\mathcal{E}}_T\right)\right], \\
f_{10}^{+-} &= \frac{g_\pi^{V,odd}(Q)+g_\pi^{A,odd}(Q)}{2}\sqrt{1-\xi^2}\left[\mathcal{H}_T+\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T-\frac{\xi^2}{1-\xi^2}\mathcal{E}_T+\frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T\right] \\
&= \frac{g_\pi^{V,odd}(Q)+g_\pi^{A,odd}(Q)}{2}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\left[\mathcal{H}_T+\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T-\frac{\zeta^2/4}{1-\zeta}\mathcal{E}_T+\frac{\zeta/2}{1-\zeta}\widetilde{\mathcal{E}}_T\right] \\
f_{10}^{-+} &= -\frac{g_\pi^{A,odd}(Q)-g_\pi^{V,odd}(Q)}{2}e^{-i2\phi}\sqrt{1-\xi^2}\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T \\
&= -\frac{g_\pi^{A,odd}(Q)-g_\pi^{V,odd}(Q)}{2}e^{-i2\phi}\frac{\sqrt{1-\zeta}}{1-\zeta/2}\frac{t_0-t}{4M^2}\widetilde{\mathcal{H}}_T \\
f_{10}^{--} &= g_\pi^{V,odd}(Q)e^{i\phi}\frac{\sqrt{t_0-t}}{4M}\left[2\widetilde{\mathcal{H}}_T+(1-\xi)\left(\mathcal{E}_T+\widetilde{\mathcal{E}}_T\right)\right] \\
&= g_\pi^{V,odd}(Q)\,e^{i\phi}\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{\mathcal{H}}_T+\frac{1-\zeta}{1-\zeta/2}\left(\mathcal{E}_T+\widetilde{\mathcal{E}}_T\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
F_{UU,T} &= \frac{1}{2}(F_{11}^{++}+F_{11}^{--})=\frac{1}{2}\sum_{\Lambda'}(f_{10}^{+\Lambda'*}f_{10}^{+\Lambda'}+f_{10}^{-\Lambda'*}f_{10}^{-\Lambda'}) \\
&= \frac{1}{2}\left(|\,f_{10}^{++}\,|^2+|\,f_{10}^{+-}\,|^2+|\,f_{10}^{-+}\,|^2+|\,f_{10}^{--}\,|^2\right) \\
F_{UU,L} &= F_{00}^{++}=\sum_{\Lambda'}f_{00}^{+\Lambda'*}f_{00}^{+\Lambda'}=|\,f_{00}^{++}\,|^2+|\,f_{00}^{+-}\,|^2 \\
F_{UU}^{\cos 2\phi} &= -\Re e\,F_{1-1}^{++}=-\Re e\sum_{\Lambda'}f_{10}^{+\Lambda'*}f_{-10}^{+\Lambda'} \\
&= -\Re e\left[(f_{10}^{++})^*(f_{10}^{--})-(f_{10}^{+-})^*(f_{10}^{-+})\right] \\
F_{UU}^{\cos\phi} &= \Re e(F_{10}^{++}+F_{10}^{--})=\Re e\sum_{\Lambda'}(f_{00}^{+\Lambda'*}f_{10}^{+\Lambda'}+f_{00}^{-\Lambda'*}f_{10}^{-\Lambda'}) \\
&= \Re e\left[(f_{00}^{+-})^*(f_{10}^{+-}+f_{10}^{-+})+(f_{00}^{++})^*(f_{10}^{++}-f_{10}^{--})\right] \\
F_{LU}^{\sin\phi} &= -\Im m(F_{10}^{++}+F_{10}^{--})=-\Im m\sum_{\Lambda'}(f_{00}^{+\Lambda'*}f_{10}^{+\Lambda'}+f_{00}^{-\Lambda'*}f_{10}^{-\Lambda'}) \\
&= -\Im m\left[(f_{00}^{+-})^*(f_{10}^{+-}+f_{10}^{-+})+(f_{00}^{++})^*(f_{10}^{++}-f_{10}^{--})\right]
\end{aligned}
$$

In a QCD factorized scenario

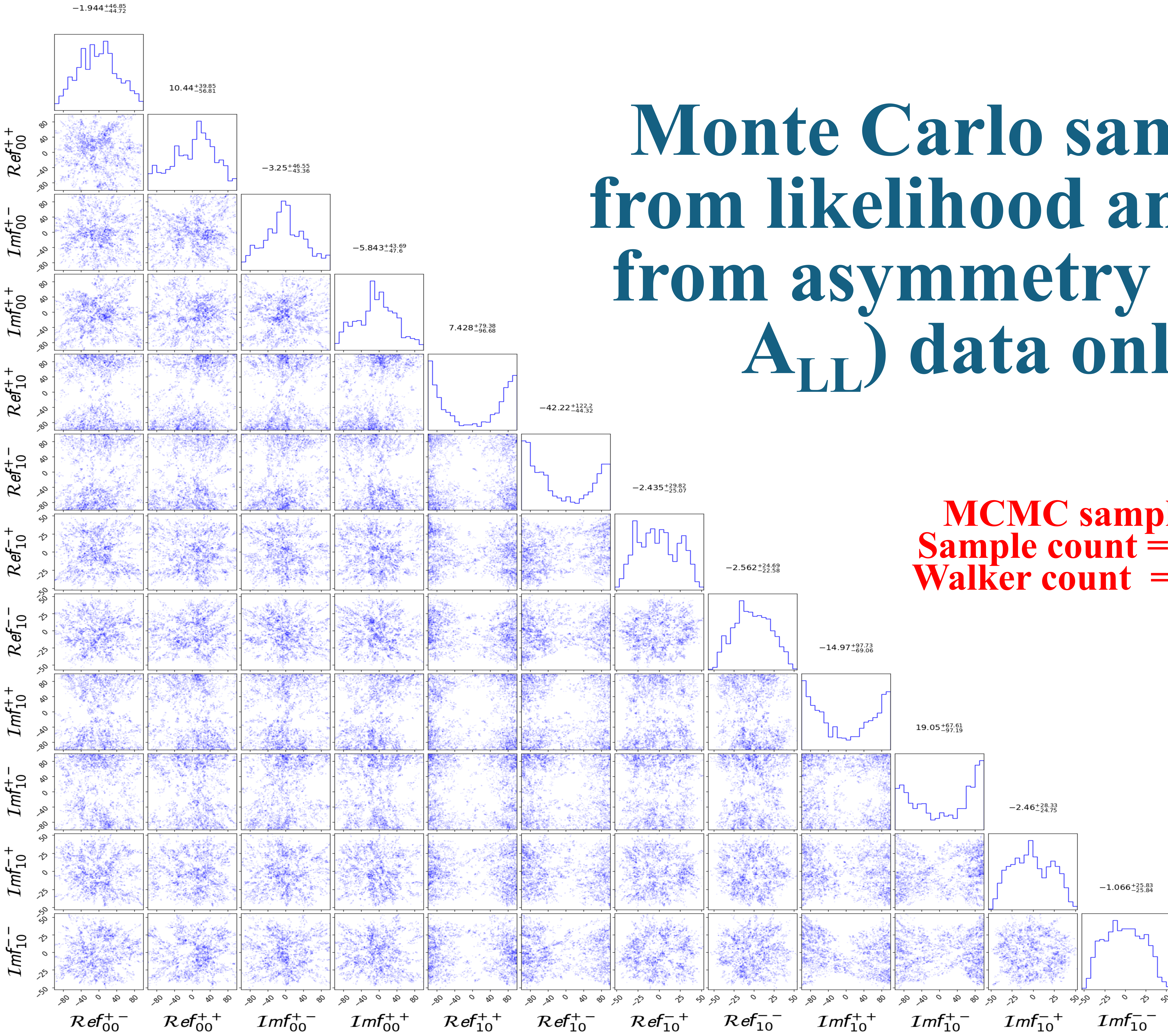


Monte Carlo samples from likelihood analysis from cross-section data only

MCMC samples
Sample count = 1e6
Walker count = 144

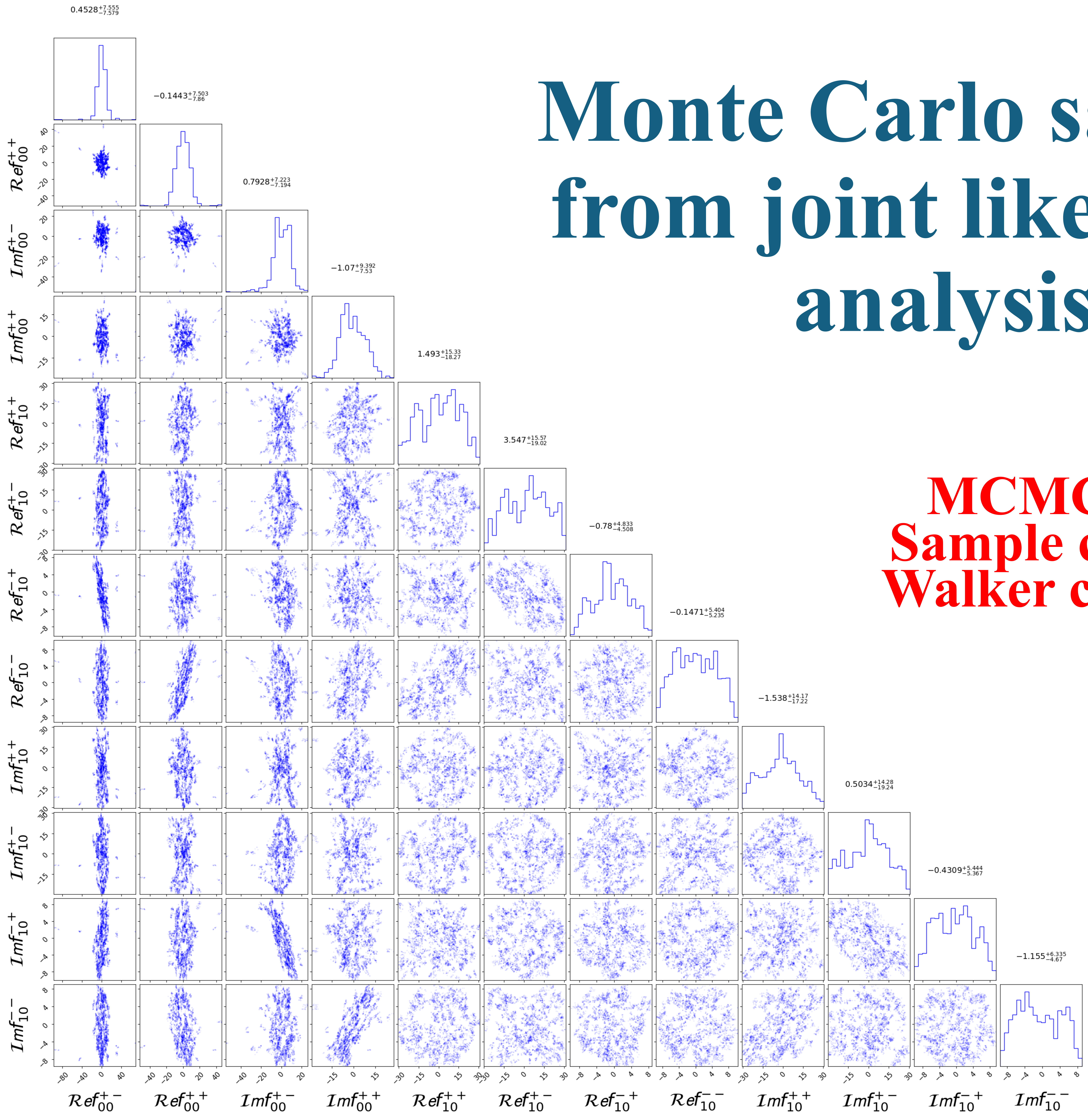
No information prior (flat sampling)

Monte Carlo samples from likelihood analysis from asymmetry (A_{UL}, A_{LL}) data only



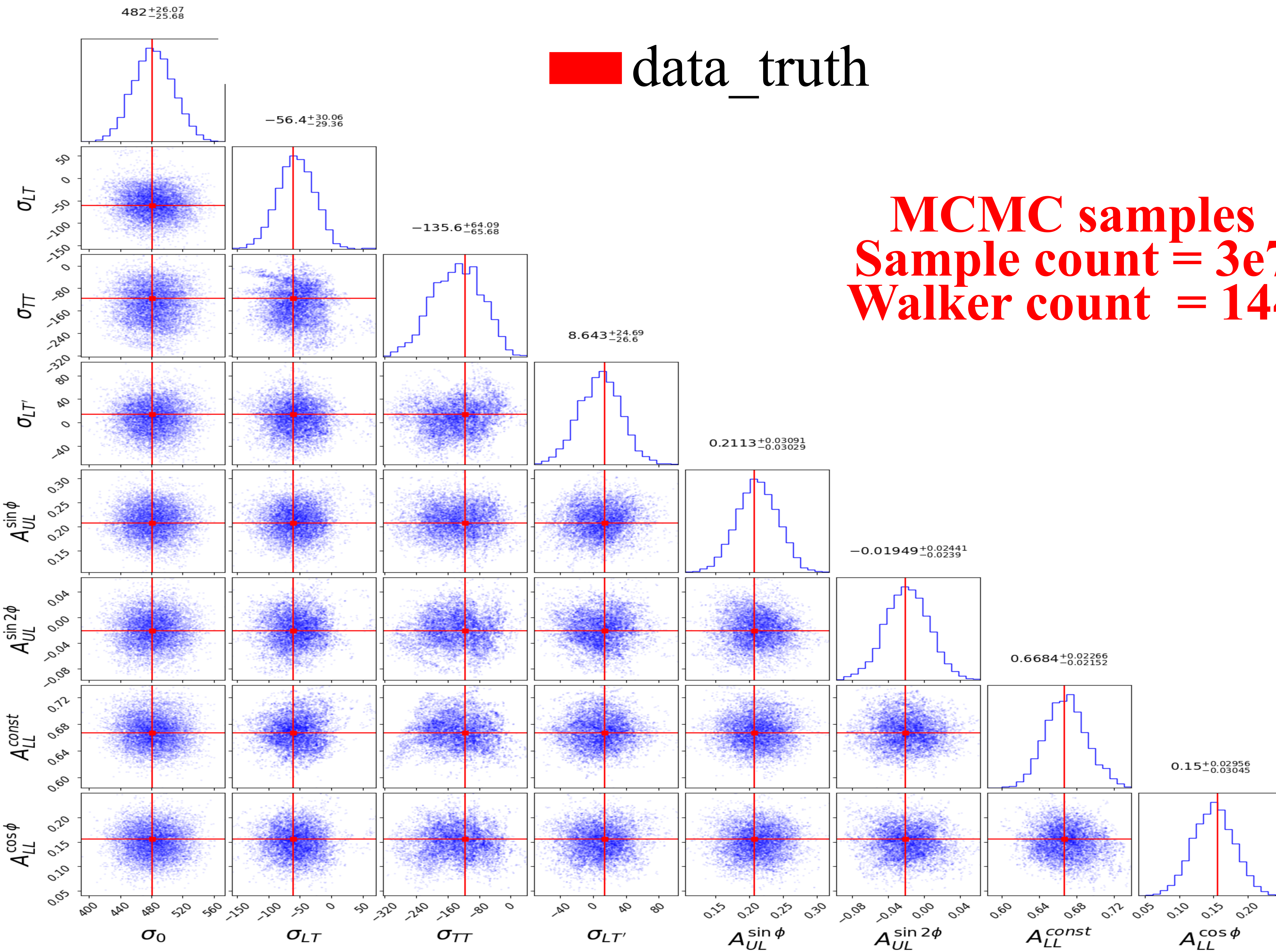
MCMC samples
Sample count = 1e6
Walker count = 144

Monte Carlo samples from joint likelihood analysis



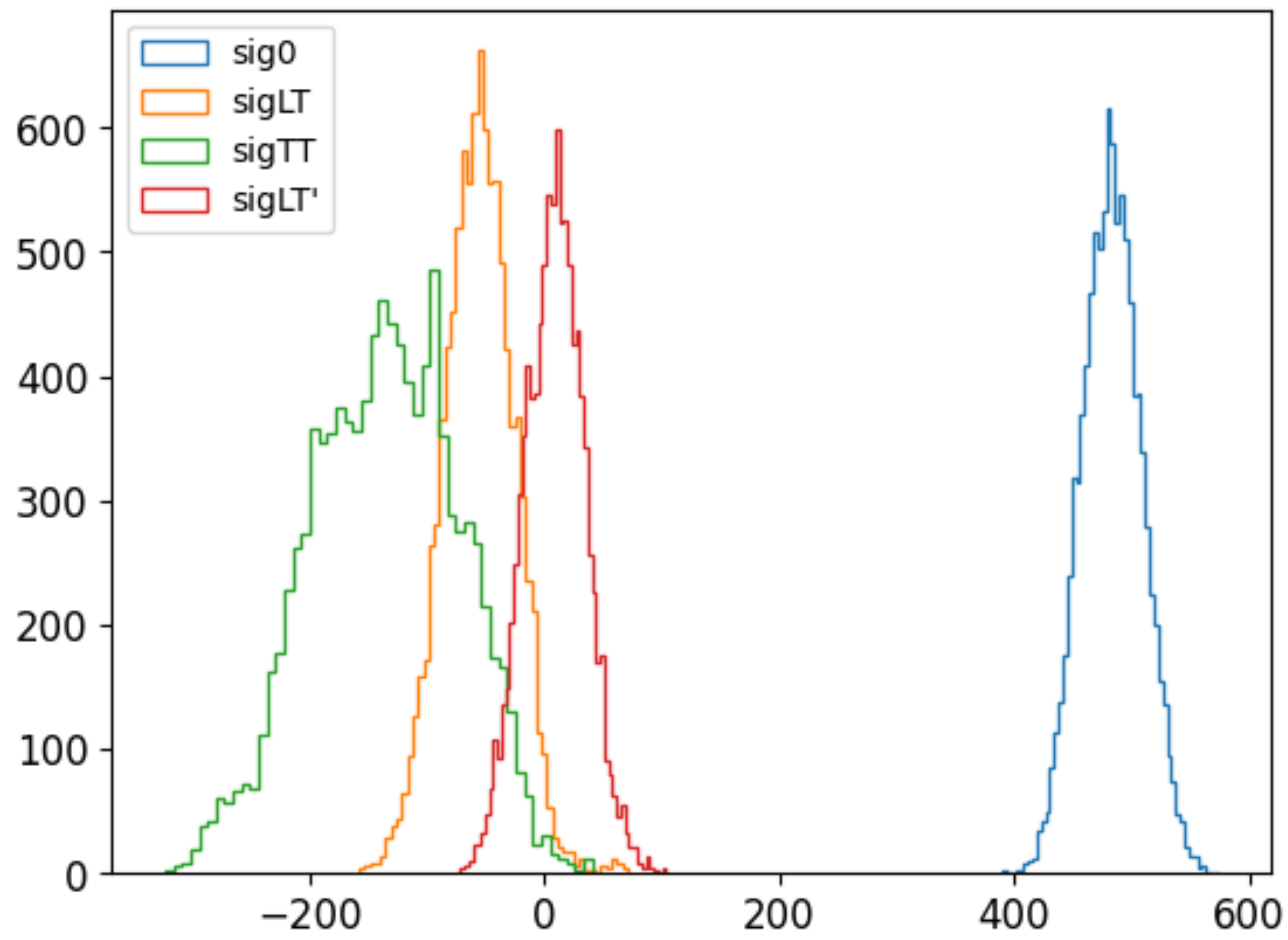
MCMC samples
Sample count = 3e7
Walker count = 144

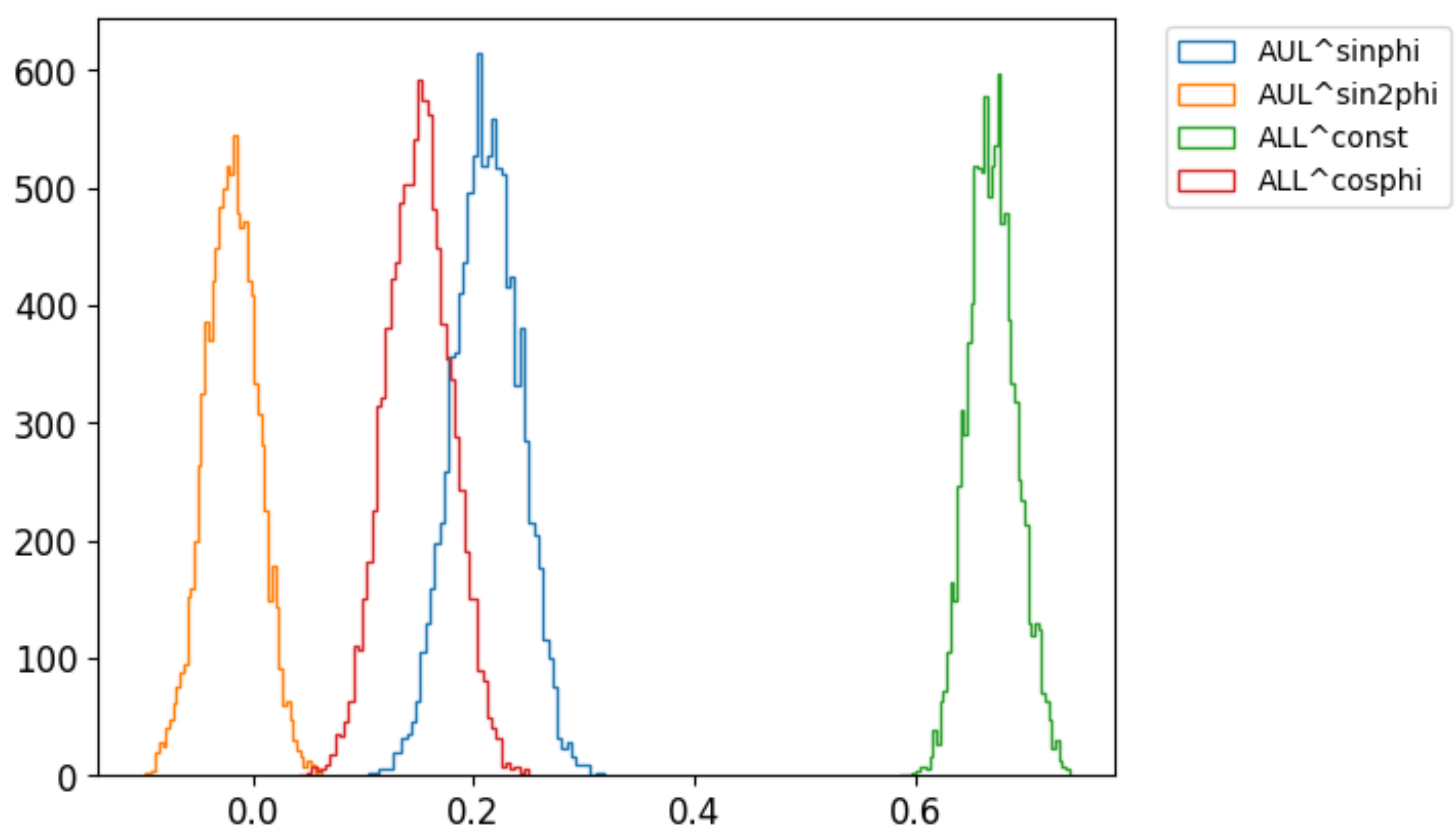
Consistency check



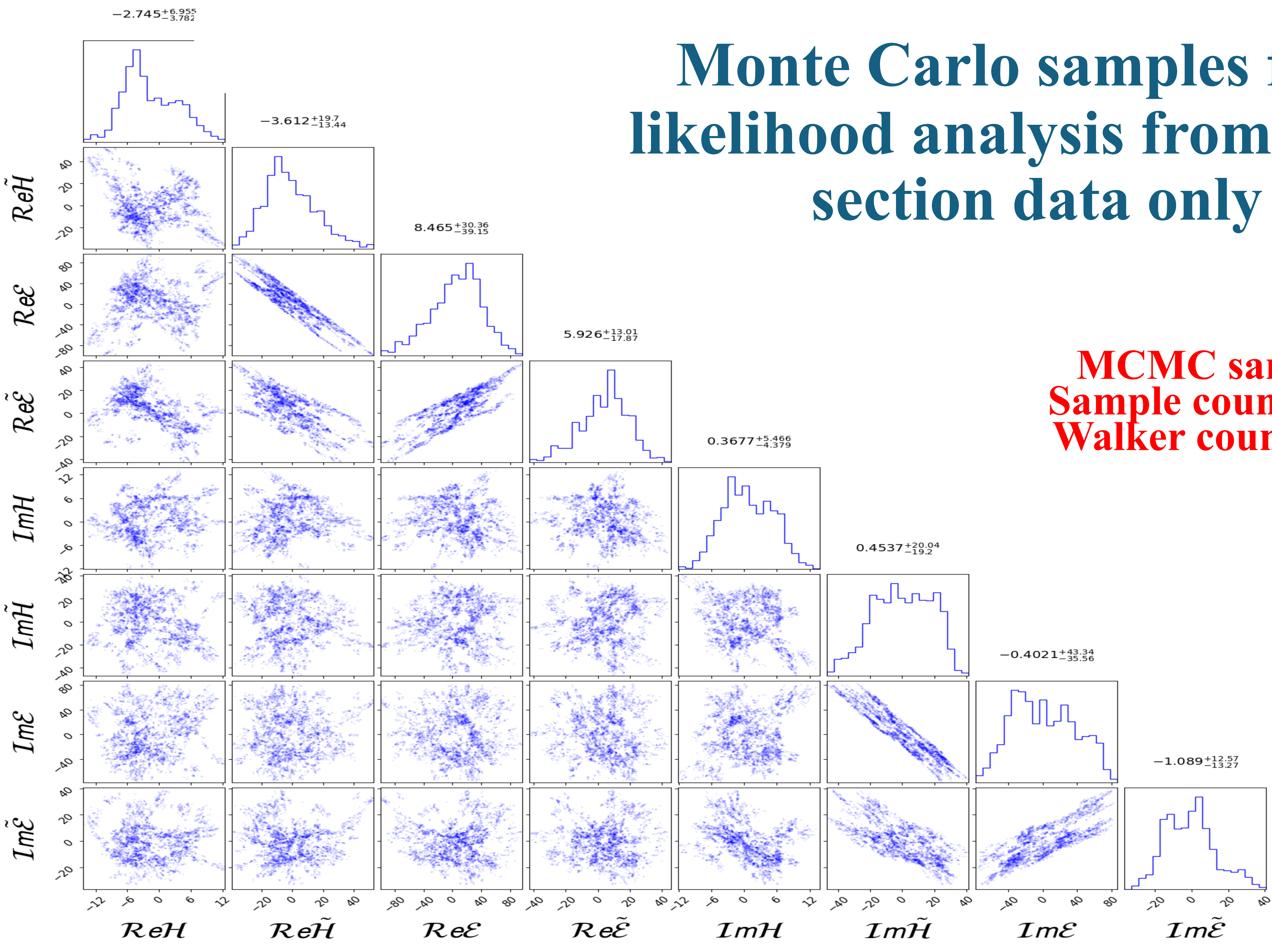
MCMC samples
Sample count = 3e7
Walker count = 144

Consistency check





Similar extraction of CFFs



Monte Carlo samples from likelihood analysis from cross-section data only

MCMC samples
Sample count = 1e6
Walker count = 64

Monte Carlo samples from likelihood analysis from asymmetry (A_{UL}, A_{LL}) data only

MCMC samples
Sample count = 1e6
Walker count = 64

Monte Carlo samples from joint likelihood analysis

MCMC samples
Sample count = 5e6
Walker count = 64

$$
\begin{aligned}
F_{UU,T} &= \frac{1}{2}(F_{11}^{++} + F_{11}^{--}) = \frac{1}{2}\sum_{\Lambda'}(f_{10}^{+\Lambda'*} f_{10}^{+\Lambda'} + f_{10}^{-\Lambda'*} f_{10}^{-\Lambda'}) \\
&= \frac{1}{2}\left(| f_{10}^{++} |^2 + | f_{10}^{+-} |^2 + | f_{10}^{-+} |^2 + | f_{10}^{--} |^2\right)
\end{aligned}
$$

$$
F_{UU,L} = F_{00}^{++} = \sum_{\Lambda'} f_{00}^{+\Lambda'*} f_{00}^{+\Lambda'} =| f_{00}^{++} |^2 + | f_{00}^{+-} |^2
$$

$$
\begin{aligned}
F_{UU}^{\cos 2\phi} &= -\Re e\, F_{1-1}^{++} = -\Re e \sum_{\Lambda'} f_{10}^{+\Lambda'*} f_{-10}^{+\Lambda'} \\
&= -\,\Re e\left[(f_{10}^{++})^*(f_{10}^{--}) - (f_{10}^{+-})^*(f_{10}^{-+})\right]
\end{aligned}
$$

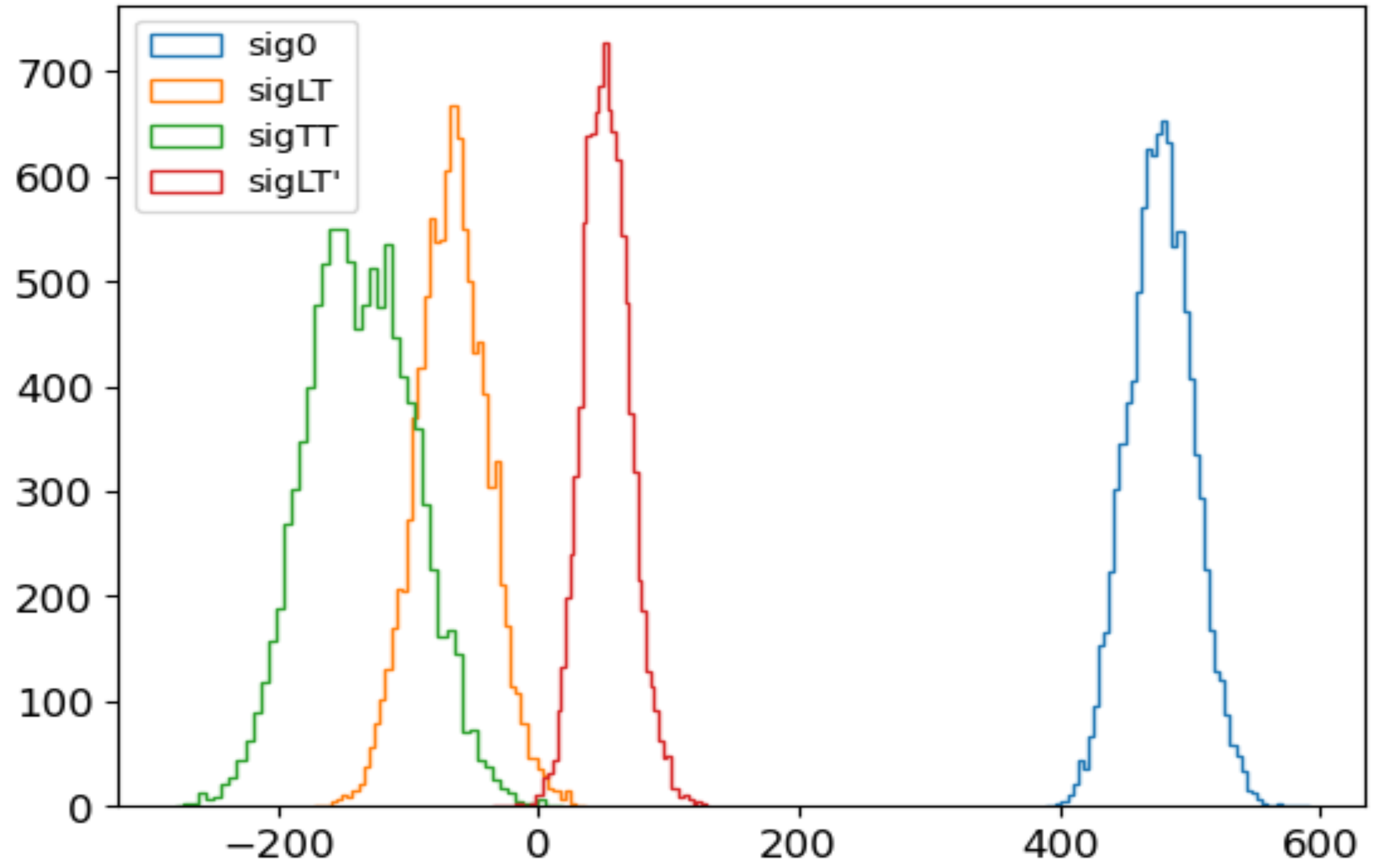
$$
\begin{aligned}
F_{UU}^{\cos\phi} &= \Re e(F_{10}^{++} + F_{10}^{--}) = \Re e \sum_{\Lambda'}(f_{00}^{+\Lambda'*} f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'*} f_{10}^{-\Lambda'}) \\
&= \Re e\left[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})\right]
\end{aligned}
$$

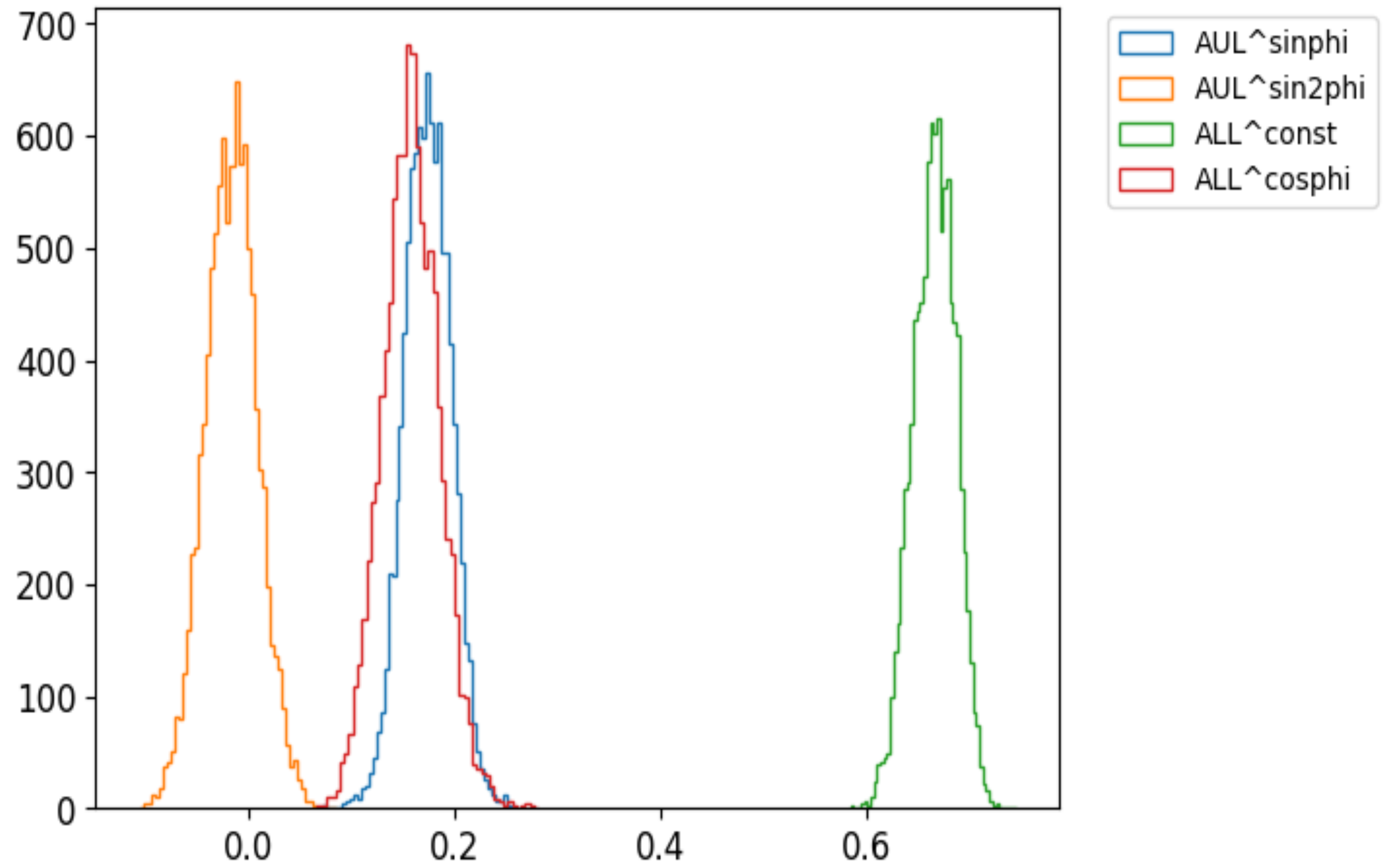
$$
\begin{aligned}
F_{LU}^{\sin\phi} &= -\Im m(F_{10}^{++} + F_{10}^{--}) = -\Im m \sum_{\Lambda'}(f_{00}^{+\Lambda'*} f_{10}^{+\Lambda'} + f_{00}^{-\Lambda'*} f_{10}^{-\Lambda'}) \\
&= -\Im m\left[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})\right]
\end{aligned}
$$

Consistency check

MCMC samples
Sample count = 5e6
Walker count = 64

Consistency check





Conclusion

- Attempt to have a possible extraction of Compton form factors from $\boldsymbol{\pi^0}$ cross-section and asymmetry data.

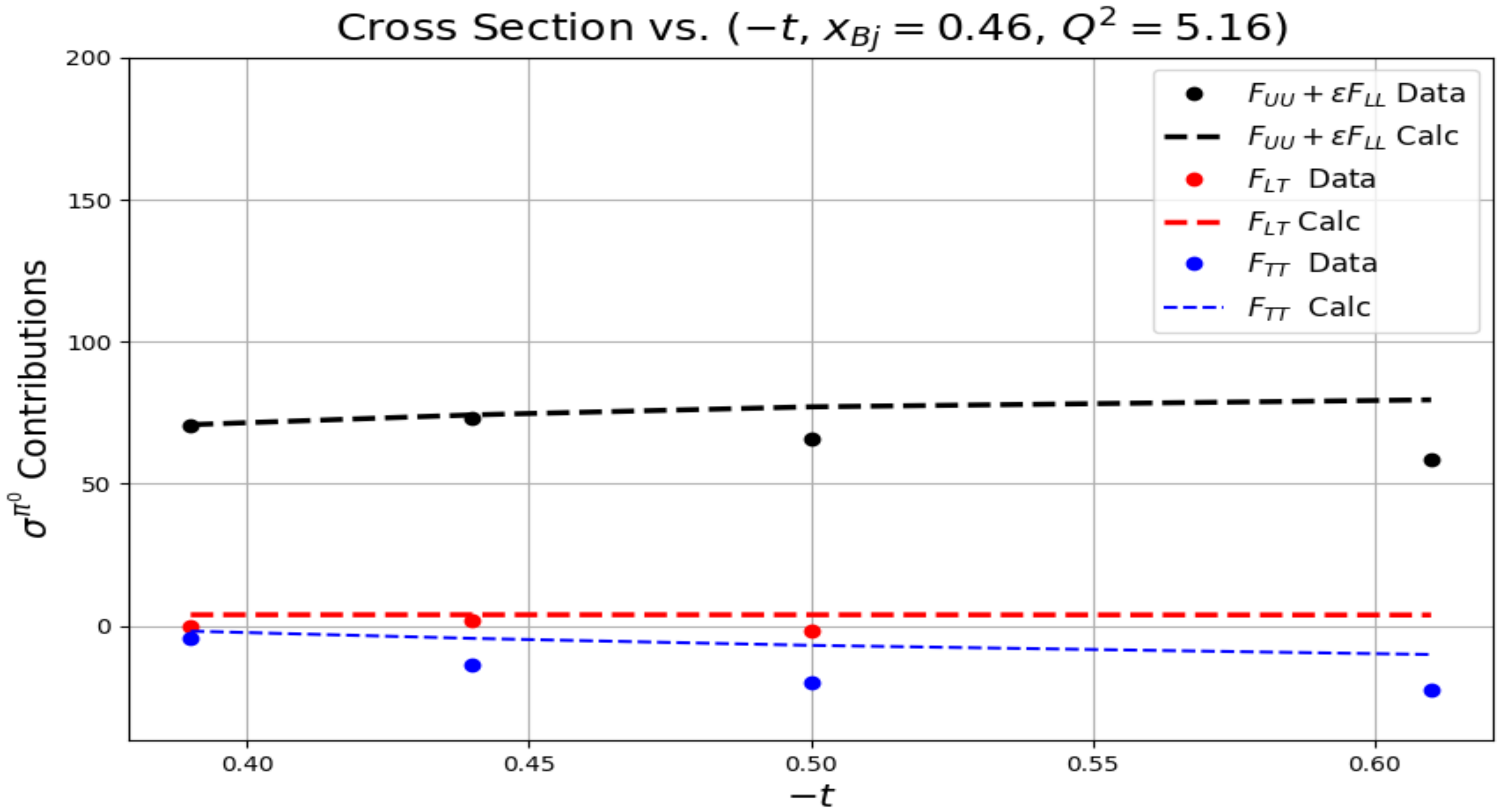
- Need of more different kinds data for any particular kinematic bin!
  - Like more Φ dependent observables
  - More kind of polarization observables.

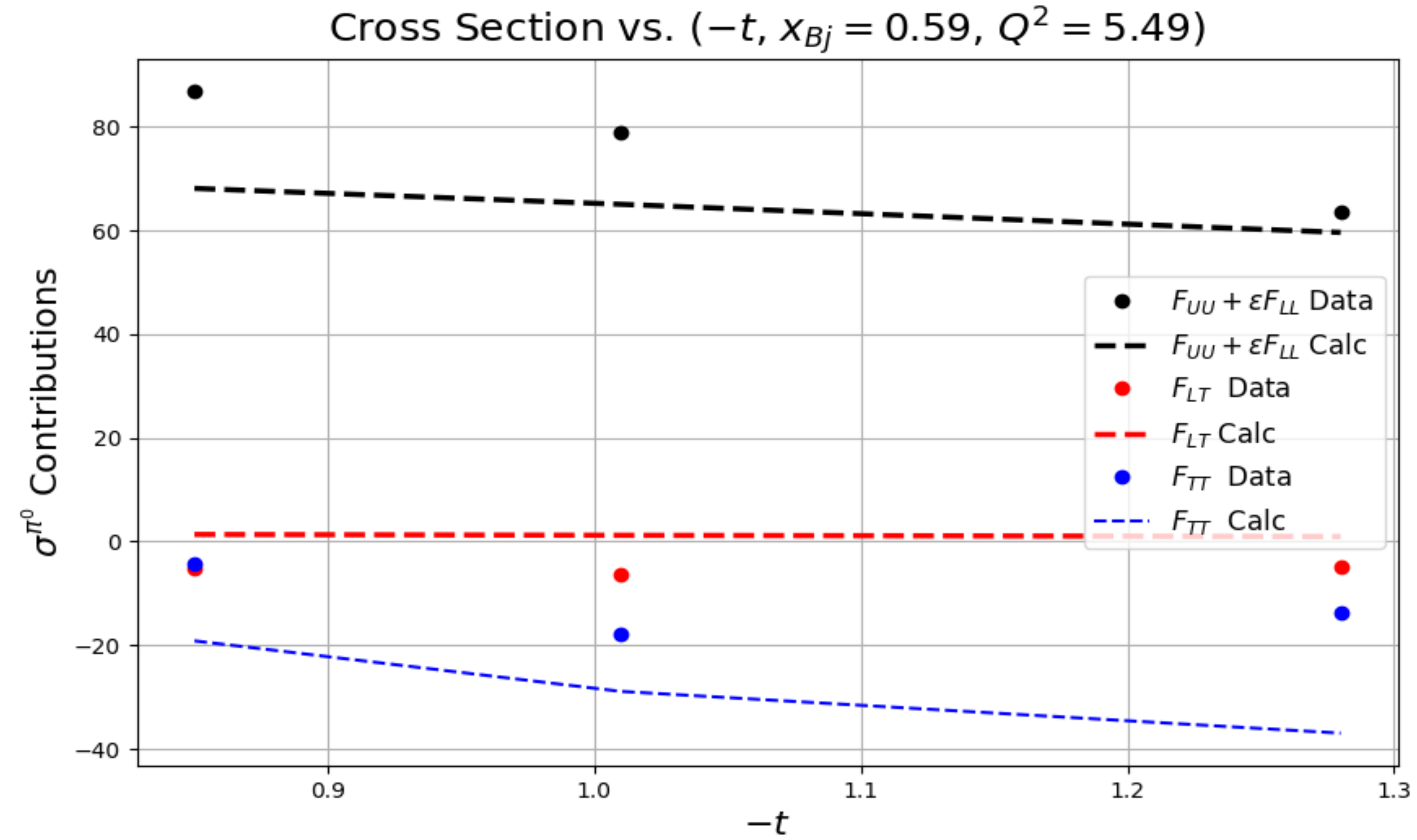
- Broader range of x and Q^2

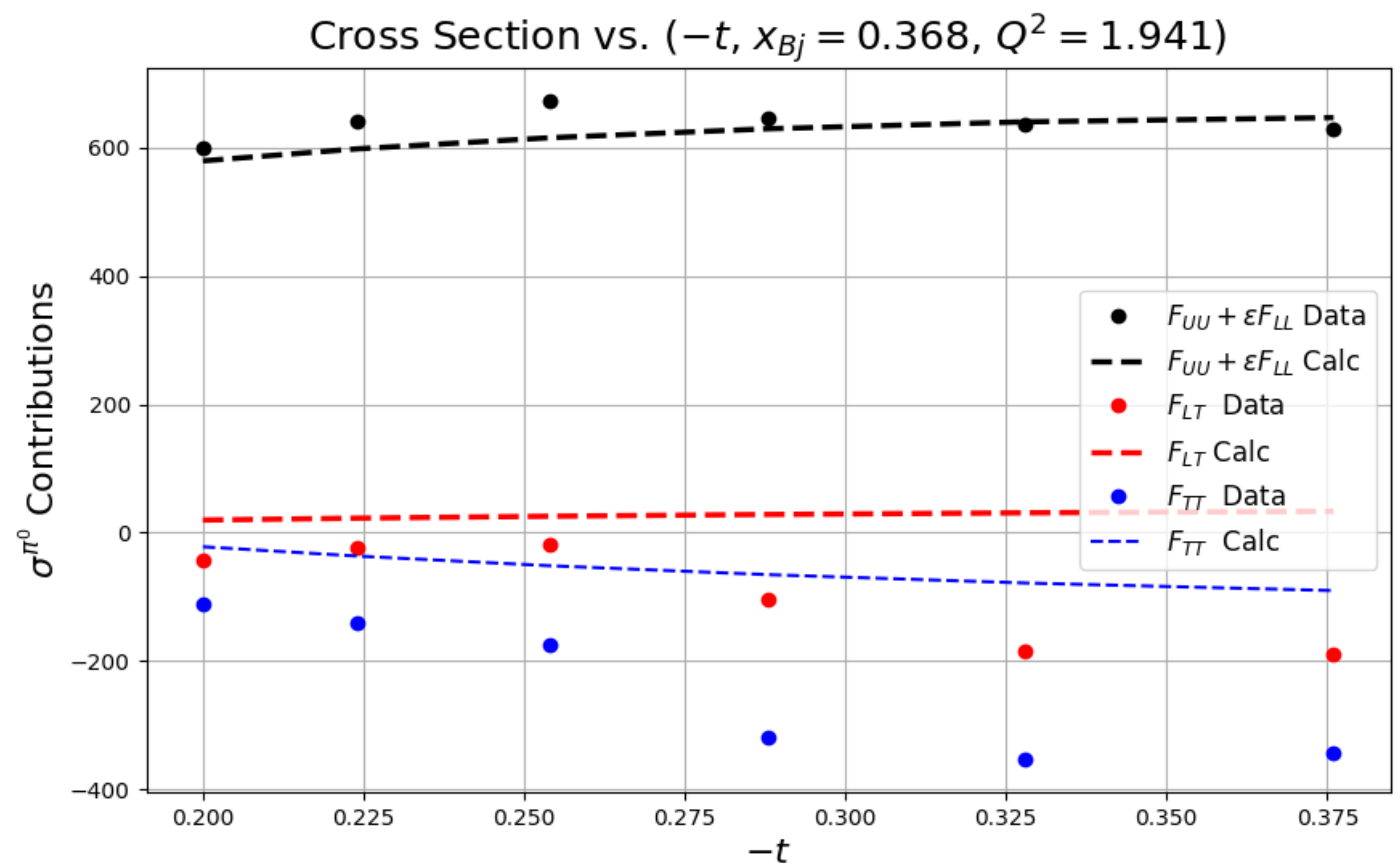
What next can be done!

- ❑Analysis on more kinematic bins
- ❑An analysis in t

Model comparison with data at HALL A

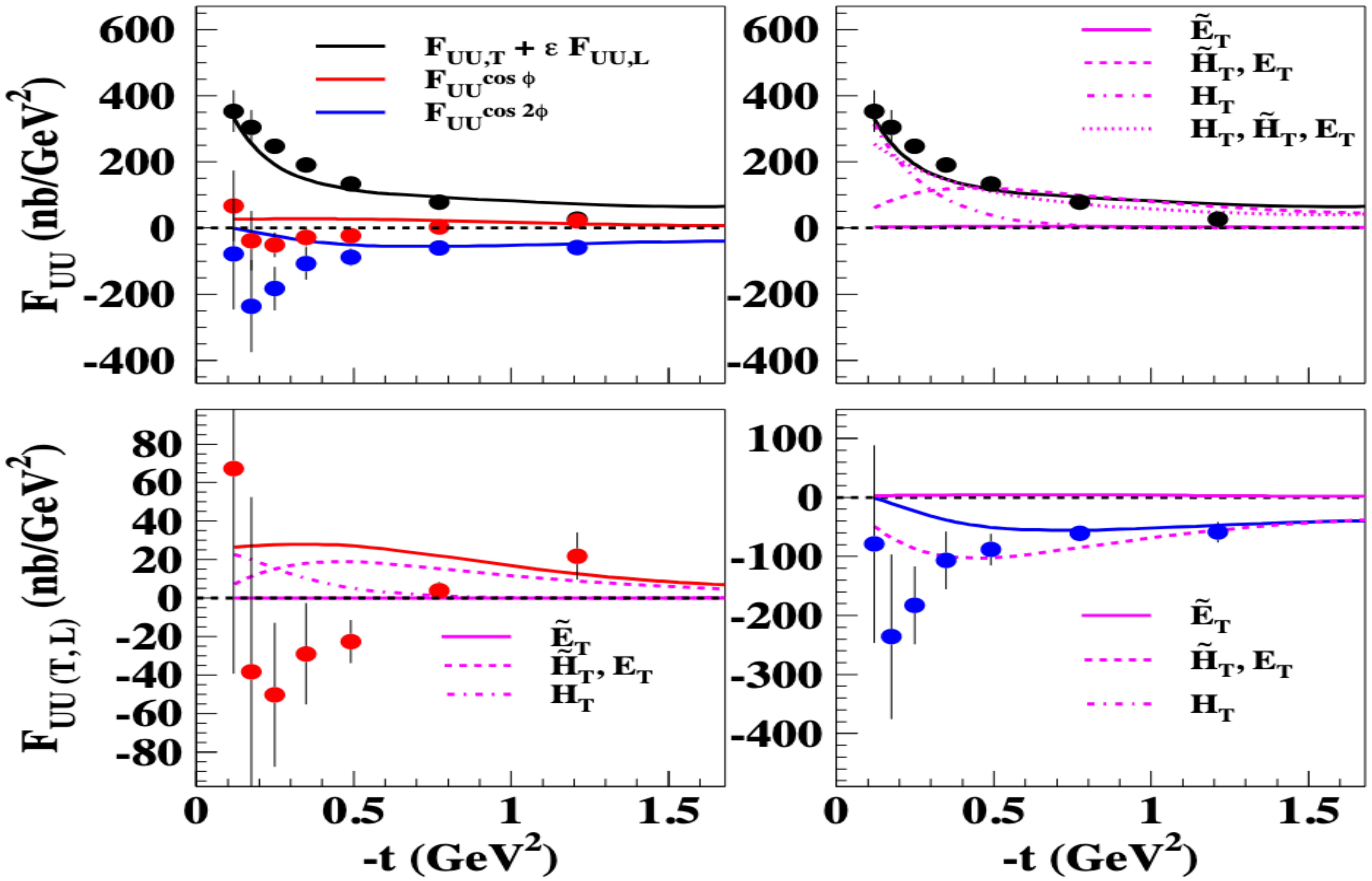






Phys. Rev. Lett. 127, 152301
GGL, Phys. Rev. D 91, 114013 (2015)

$F_{UU,T} + \varepsilon F_{UU,L}$



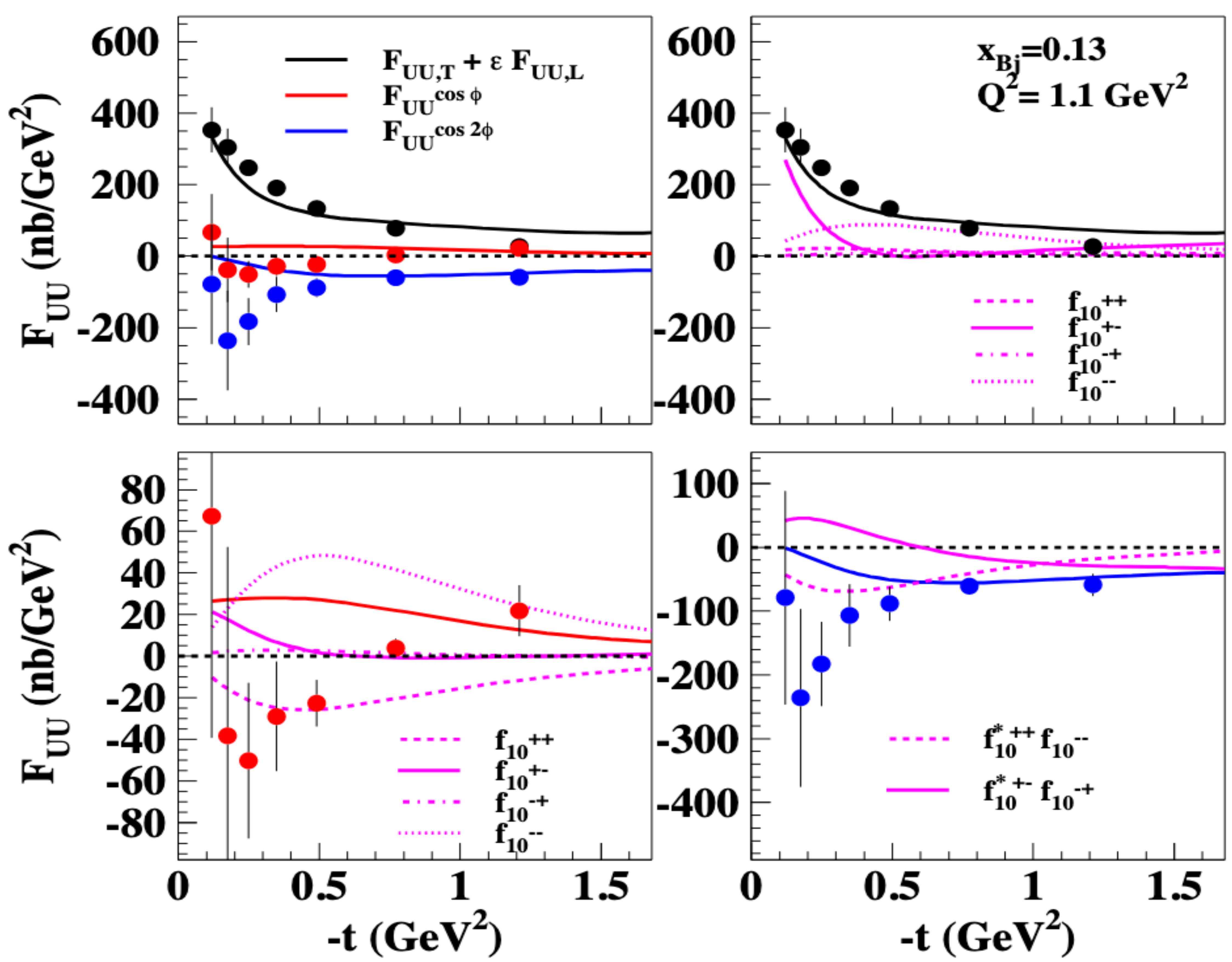
$F_{UU}^{\cos\phi}$ $F_{UU}^{\cos 2\phi}$

Second Inverse problem

GPDs??

GGL, Phys. Rev. D 91, 114013 (2015)
GGL, Phys. Rev. D 79, 054014 (2009)
arxiv : 1401.0438

$F_{UU,T} + \varepsilon F_{UU,L}$



$F_{UU}^{\cos\phi}$ $F_{UU}^{\cos 2\phi}$

Thank you

Any Questions?

Backup slides

E_{beam} (GeV)	x_{Bj}	Q^2 (GeV^2)	t (GeV^2)	ϕ (deg)	σ_{total}	$\Delta\sigma$
10.591	0.369	4.53	-0.2094	7.5	0.01394	0.00058
10.591	0.369	4.53	-0.2094	22.5	0.01292	0.00056
10.591	0.369	4.53	-0.2094	37.5	0.01305	0.00056
10.591	0.369	4.53	-0.2094	52.5	0.01216	0.00054
10.591	0.369	4.53	-0.2094	67.5	0.01147	0.00052
10.591	0.369	4.53	-0.2094	82.5	0.01128	0.00051
10.591	0.369	4.53	-0.2094	97.5	0.00875	0.00046
10.591	0.369	4.53	-0.2094	112.5	0.00915	0.00046
10.591	0.369	4.53	-0.2094	127.5	0.00904	0.00045
10.591	0.369	4.53	-0.2094	142.5	0.00838	0.00044
10.591	0.369	4.53	-0.2094	157.5	0.00828	0.00044
10.591	0.369	4.53	-0.2094	172.5	0.00798	0.00043
10.591	0.369	4.53	-0.2094	187.5	0.00774	0.00043
10.591	0.369	4.53	-0.2094	202.5	0.00841	0.00045
10.591	0.369	4.53	-0.2094	217.5	0.00853	0.00045
10.591	0.369	4.53	-0.2094	232.5	0.00991	0.00049
10.591	0.369	4.53	-0.2094	247.5	0.00969	0.00049
10.591	0.369	4.53	-0.2094	262.5	0.01021	0.00049
10.591	0.369	4.53	-0.2094	277.5	0.01093	0.00051
10.591	0.369	4.53	-0.2094	292.5	0.01223	0.00054
10.591	0.369	4.53	-0.2094	307.5	0.01236	0.00054
10.591	0.369	4.53	-0.2094	322.5	0.01382	0.00057
10.591	0.369	4.53	-0.2094	337.5	0.01543	0.00061
10.591	0.369	4.53	-0.2094	352.5	0.01376	0.00058