A Unified Software for Factorization-Based and HQCD Models

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Towards Improved Hadron Femtography with Hard Exclusive Reactions

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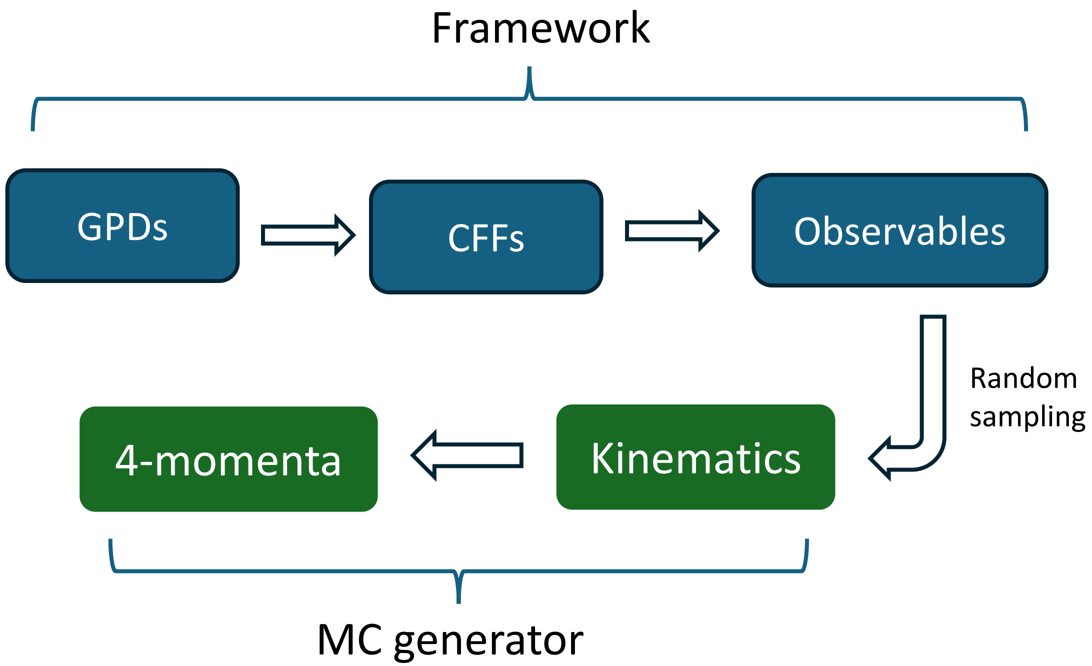
Outline

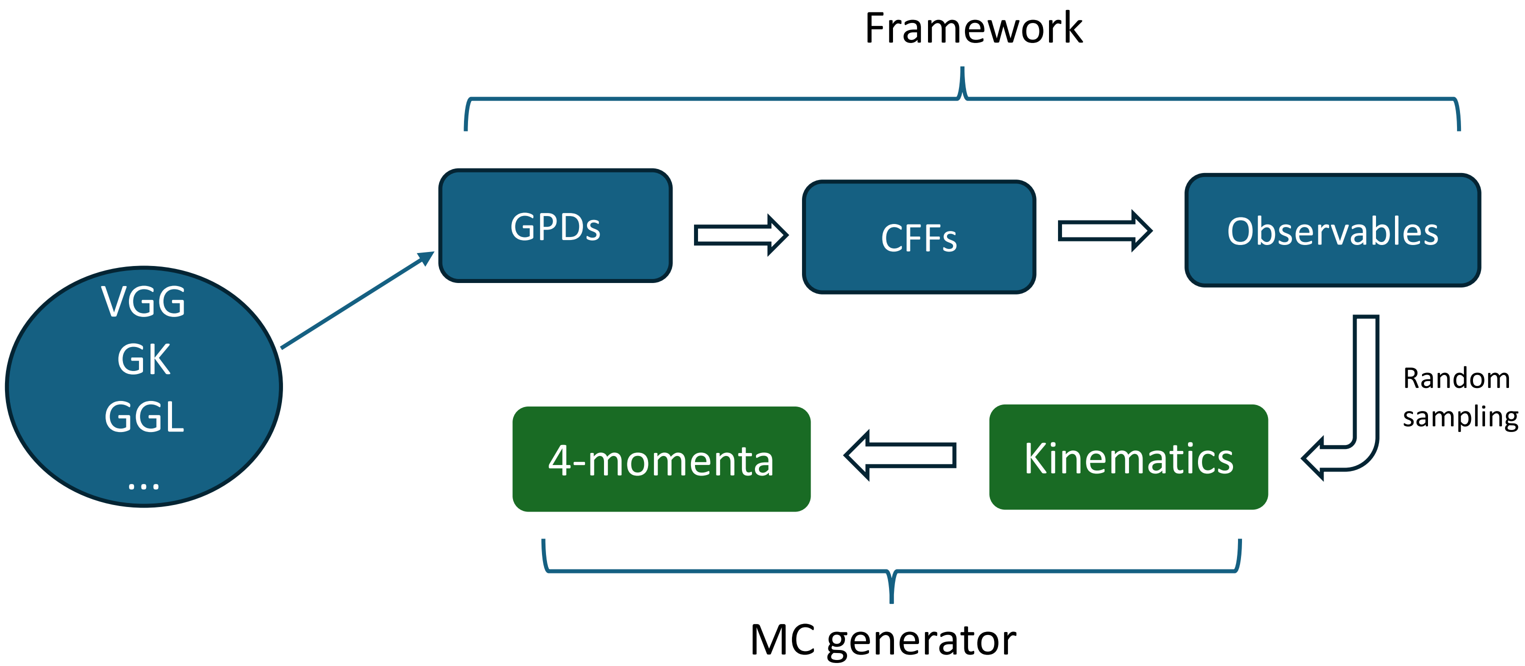
- A C++ program to analyze exclusive processes
- Different phenomenological models: VGG, GK, GGL, HQCD
- Compute various observables for DVCS and DVMP
- Enable multi-channel analysis
- Produce tables of cross-section for the DEEPGen Event Gen

Architecture



- **Modular structure:** each component has an isolated role and can easily be extended
- **Processes:** DVCS, π^+ and π^0 (handbag-based), J/ψ, Υ, and ϕ (HQCD)
- **Concept:** use building blocks of the hadrons, such as GPDs and/or CFFs, to describe observables

GPDs



- Available GPD models: VGG, GK; In progress: GGL
- PDFs used: MRST1998, LSS1998 (for the VGG model)
- EM Form Factors used: Kelly's parametrization (for the VGG model)

Common Software Tools

PARTONS (Talk by Victor)
Gepard
...

GPDs

Chiral-even GPDs parametrize the off-forward nucleon bilinear matrix elements at a light-like separation:

$$P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}_q(-\frac{z}{2}) \mathcal{W}(-\frac{z}{2}, \frac{z}{2})\, \gamma^+ \psi_q(\frac{z}{2})\, |p, \lambda\rangle \Big|_{z^+=0, \vec{z}_T=0}$$
$$= \bar{u}(p', \lambda') \left[H^q \gamma^+ + E^q \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} \right] u(p, \lambda)\,,$$

Müller, Robaschik, Geyer, Dittes, Hořejši (1994)
Radyushkin (1996)
Ji (1997)

$$P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}_q(-\frac{z}{2}) \mathcal{W}(-\frac{z}{2}, \frac{z}{2})\, \gamma^+ \gamma_5 \psi_q(\frac{z}{2})\, |p, \lambda\rangle \Big|_{z^+=0, \vec{z}_T=0}$$
$$= \bar{u}(p', \lambda') \left[\tilde{H}^q \gamma^+ \gamma_5 + \tilde{E}^q \frac{\gamma_5 \Delta^+}{2m} \right] u(p, \lambda)\,,$$

parameters: $x, \quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+}, \quad t = \Delta^2, \quad \mu^2 \qquad \text{with} \qquad \Delta^\alpha = p'^\alpha - p^\alpha$

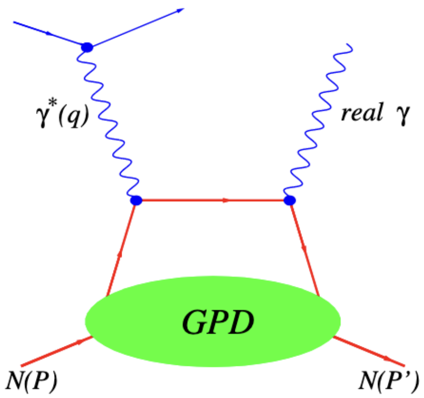
GPDs

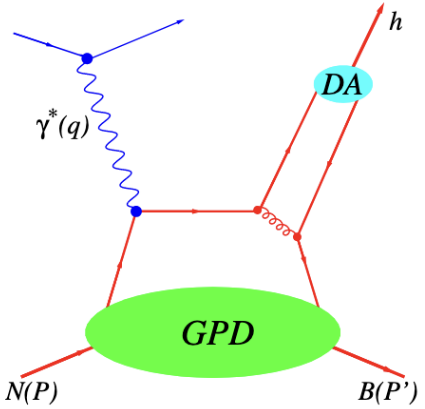
Chiral-odd GPDs: Diehl (2001)

$$P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}_q(-\frac{z}{2}) \mathcal{W}(-\frac{z}{2}, \frac{z}{2})\, i\sigma^{+i} \psi_q(\frac{z}{2})\, |p, \lambda\rangle \Big|_{z^+=0, \vec{z}_T=0}$$

$$= \bar{u}(p', \lambda') \Big[H_T^q i\sigma^{+i} + \tilde{H}_T^q \frac{P^+\Delta^i - \Delta^+ P^i}{m^2} + E_T^q \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m}$$

$$+ \tilde{E}_T^q \frac{\gamma^+ P^i - P^+\gamma^i}{m} \Big] u(p, \lambda)\,,$$





- Chiral-even GPDs: accessible through DVCS and DVMP with longitudinally polarized photons (leading twist)
- Chiral-odd GPDs: accessible through DVMP, if one assumes an effective handbag mechanism for transversely polarized photons Goloskokov, Kroll (2010)

GPD Models

The VGG model is based on double distributions

Vanderhaeghen, Guichon, Guidal (1999)

polynomiality
forward limit

$$H^q(x,\xi,t) = \frac{1}{1+\xi}\{f^q(X,\zeta,t)\} \quad \text{for} \quad \xi \leq x \leq 1,$$

$$H^q(x,\xi,t) = \frac{1}{1+\xi}\{f^q(X,\zeta,t) - f^{\bar{q}}(\zeta - X,\zeta,t)\}$$

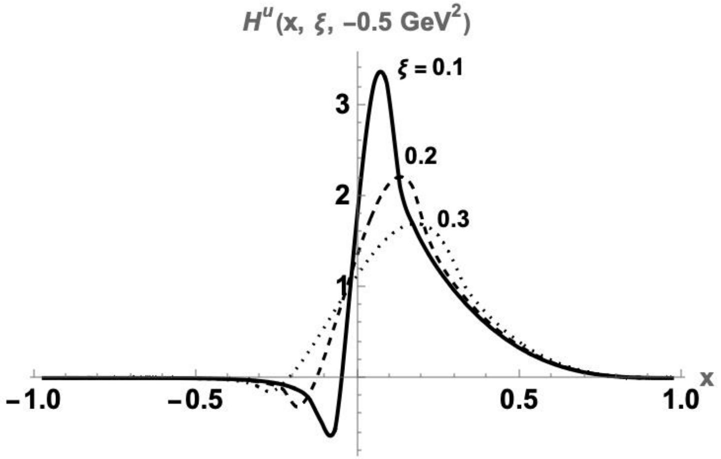
$$\text{for} \quad -\xi \leq x \leq \xi,$$

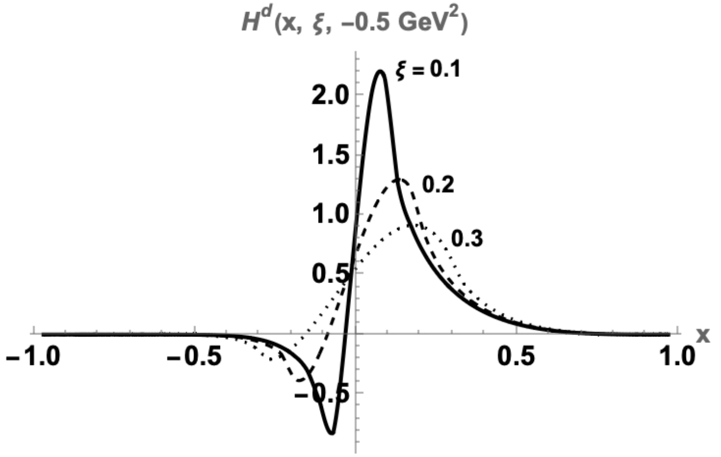
$$H^q(x,\xi,t) = \frac{1}{1+\xi}\{-f^{\bar{q}}(\zeta - X,\zeta,t)\}, \quad \text{for} \quad -1 \leq x \leq -\xi$$

$$f^q(X,\zeta,t) = \int_0^{\bar{X}/\bar{\zeta}} dy\ F^q(X - \zeta y, y, t), \quad \text{for} \quad X \geq \zeta$$

$$f^q(X,\zeta,t) = \int_0^{X/\zeta} dy\, F^q(X - \zeta y, y, t), \quad \text{for} \quad X \leq \zeta$$

$$F^q(\tilde{x},y,t) = F_1^q(t)/F_1^q(0)\, q(\tilde{x})\, 6\, \frac{y(1-\tilde{x}-y)}{(1-\tilde{x})^3}$$





GPD Models

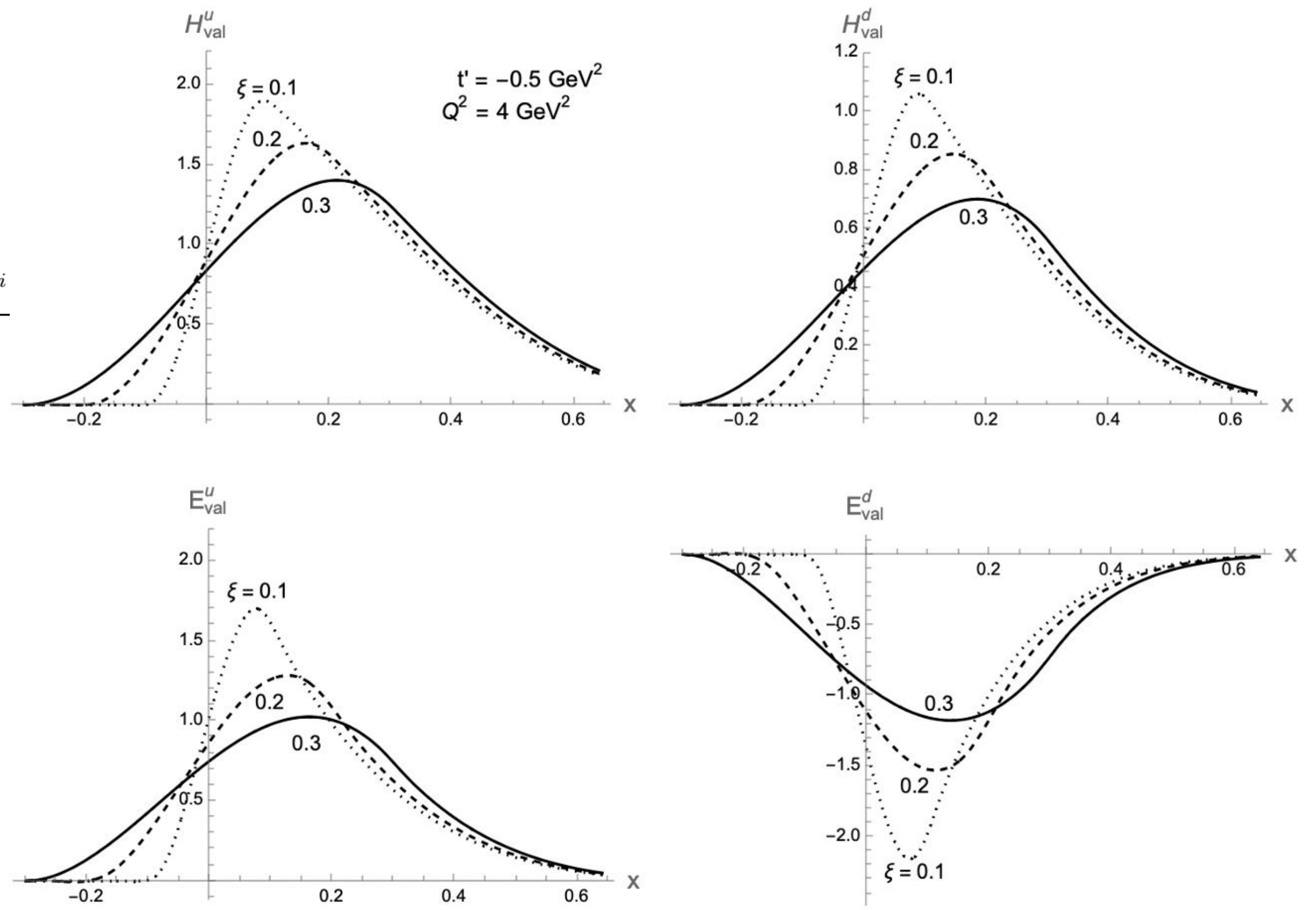
The GK model is also based on double distributions

Goloskokov, Kroll (2008)

$$H_i(\bar{x},\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-\bar{x})\, f_i(\beta,\alpha,t)$$

$$f_i(\beta,\alpha,t) = \mathrm{e}^{b_i t}\ |\,\beta\,|^{-\alpha_i' t}\ h_i(\beta)\, \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\,\Gamma^2(n_i+1)}\, \frac{[(1-|\beta|)^2-\alpha^2]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\begin{aligned} h_g(\beta) &= |\beta| g(|\beta|) & n_g &= 2\,, \\ h^q_{\rm sea}(\beta) &= q_{\rm sea}(|\beta|)\,\mathrm{sign}(\beta) & n_{\rm sea} &= 2\,, \\ h^q_{\rm val}(\beta) &= q_{\rm val}(\beta)\,\Theta(\beta) & n_{\rm val} &= 1\,. \end{aligned}$$



CFFs at LO

CFFs at LO can be computed as:

$$\mathcal{H}^q = \int_{-1}^{1} dx\ C_0 H^q(x,\xi,t)$$
$$= \left[-P.V \int_{-1}^{1} dx \left(\frac{1}{x+\xi} + \frac{1}{x-\xi} \right) H^q(x,\xi,t) \right] + i\pi (H^q(\xi,\xi,t) - H^q(-\xi,\xi,t))$$

Imaginary part can easily be computed analytically:

$$Im\mathcal{H} = \pi (H(\xi,\xi,t) - H(-\xi,\xi,t))$$
$$= \pi \left(\frac{H^+(\xi,\xi,t) + H^-(\xi,\xi,t)}{2} - \frac{H^+(-\xi,\xi,t) + H^-(-\xi,\xi,t)}{2} \right)$$
$$= \pi H^+(\xi,\xi,t)$$

$$Re\mathcal{H} = -\left(\int_0^1 dx \frac{H^+(x,\xi,t)}{x+\xi} + \int_0^1 dx \frac{H^+(x,\xi,t) - H^+(\xi,\xi,t)}{x-\xi} + H^+(\xi,\xi,t) \log\left(\frac{1-\xi}{\xi} \right) \right)$$

Goldstein, Gonzalez-Hernandez, Liuti (2011)

Cross section of the electroproduction process

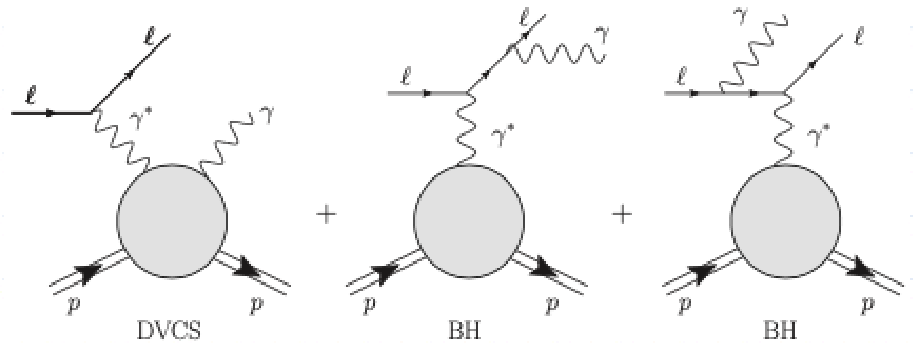
$$\frac{d\sigma}{dx_{\mathrm{B}}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{\mathrm{B}} y}{16\,\pi^2\,\mathcal{Q}^2\sqrt{1+\epsilon^2}} \left|\frac{\mathcal{T}}{e^3}\right|^2 .$$

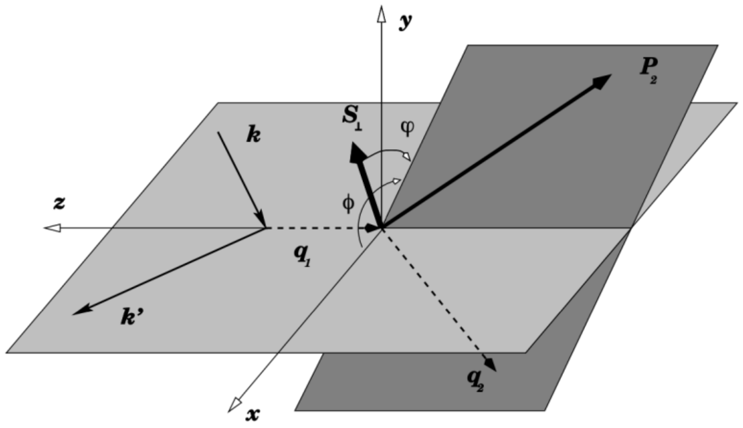
Belitsky, Müller, Kirchner (2002)
Kriesten, Liuti, Calero-Diaz, Keller, Meyer, Goldstein, Gonzalez-Hernandez (2020)

$$|\mathcal{T}_{\mathrm{BH}}|^2 = \frac{e^6}{x_{\mathrm{B}}^2 y^2 (1+\epsilon^2)^2 \Delta^2\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) + s_1^{\mathrm{BH}} \sin(\phi) \right\} ,$$

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} \left[c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi) \right] \right\} ,$$

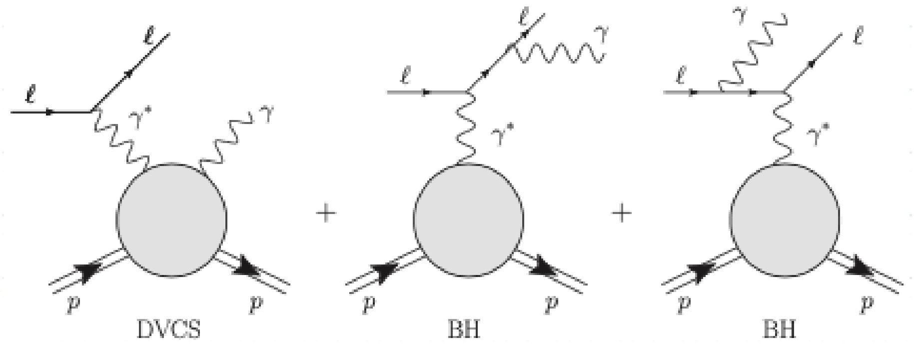
$$\mathcal{I} = \frac{\pm e^6}{x_{\mathrm{B}} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\} ,$$

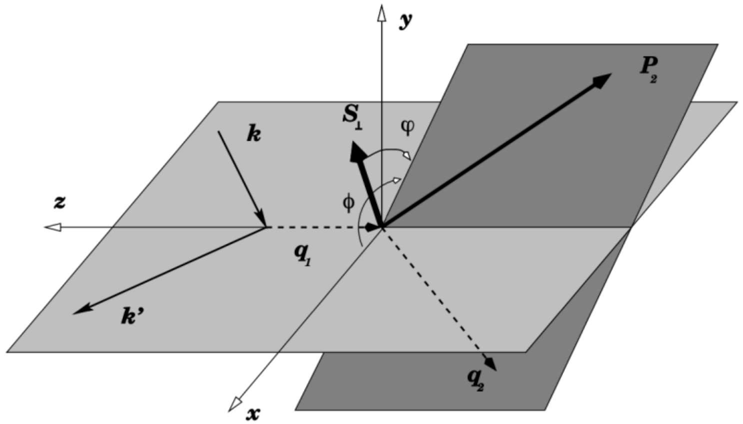




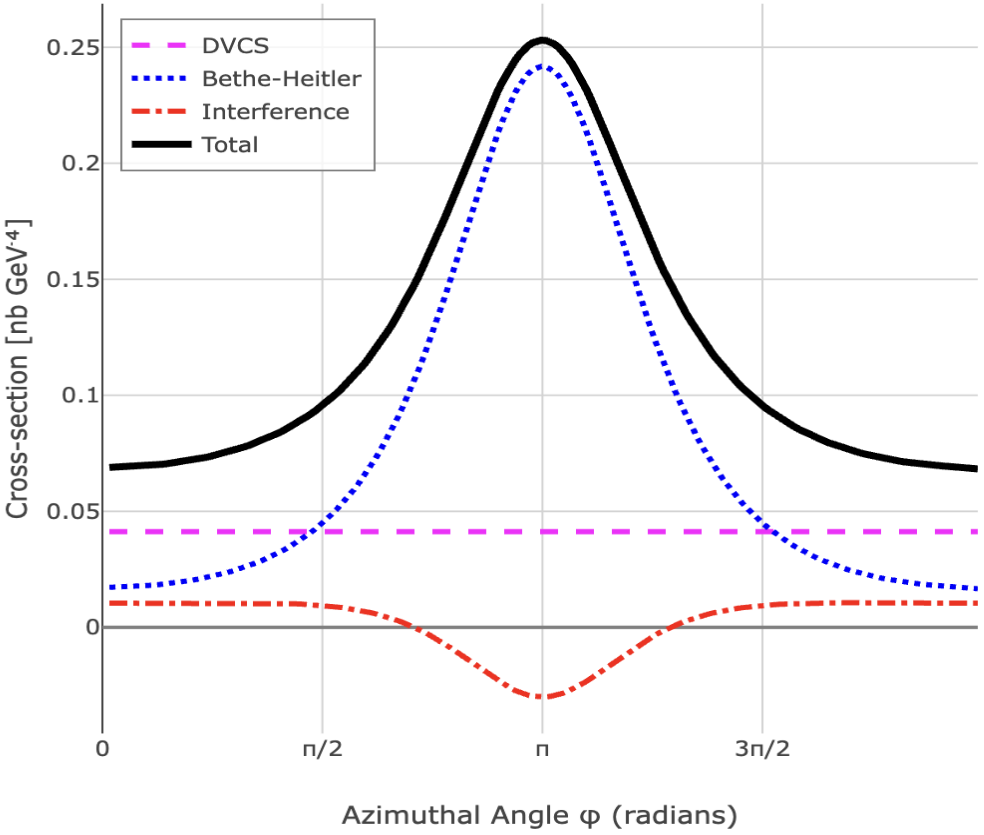
BMK cross section

- Unpolarized cross section of the electroproduction process with the VGG GPDs
- Leading twist and leading order



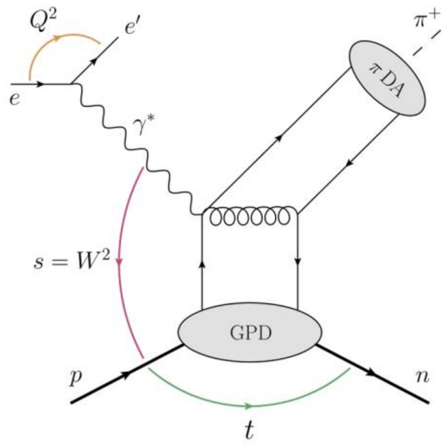


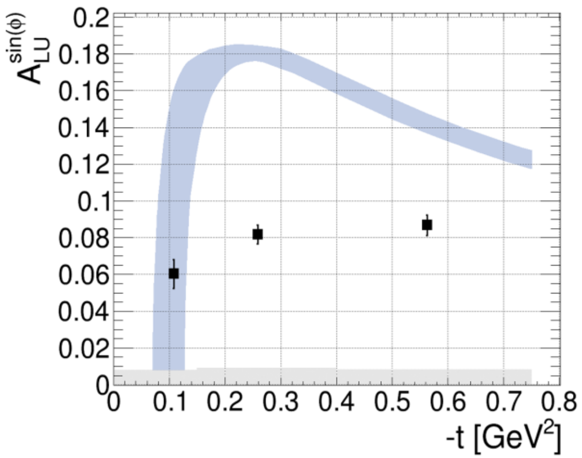
x_B=0.1, y=0.3, Q^2=2 GeV^2, t=-0.2 GeV^2



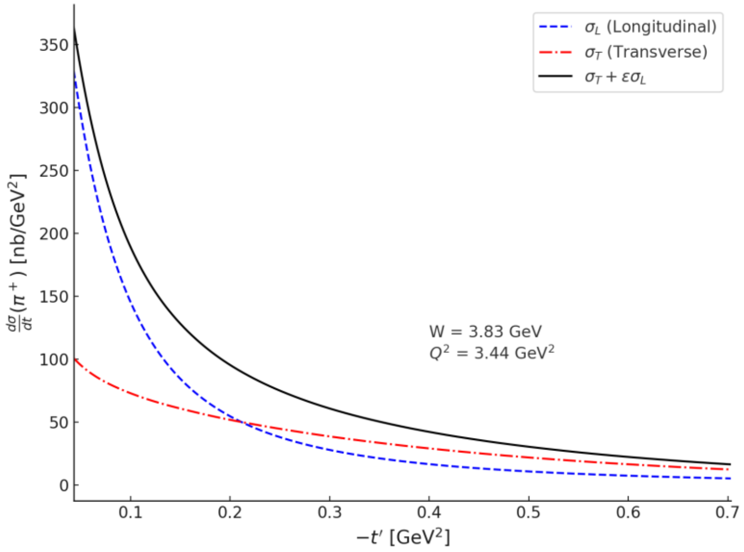
DVMP (π^+)

- Based on effective handbag approach in the GK model: factorizes the process into GPDs and subprocesses Goloskokov, Kroll (2010)
- Allow extraction and constrain of the chiral-even GPDs ($\widetilde{H}$, $\tilde{E}$) & the chiral-odd GPD H_T
- Higher twist effects: Helicity-flip GPD H_T coupled with twist-3 pion wave function
- Dominance of the pion-pole contribution at low momentum transfer
- $\sigma_L > \sigma_T$ at low |t|, $\sigma_T > \sigma_L$ at high |t|





S. Diehl et al., PRL 125 (2020)

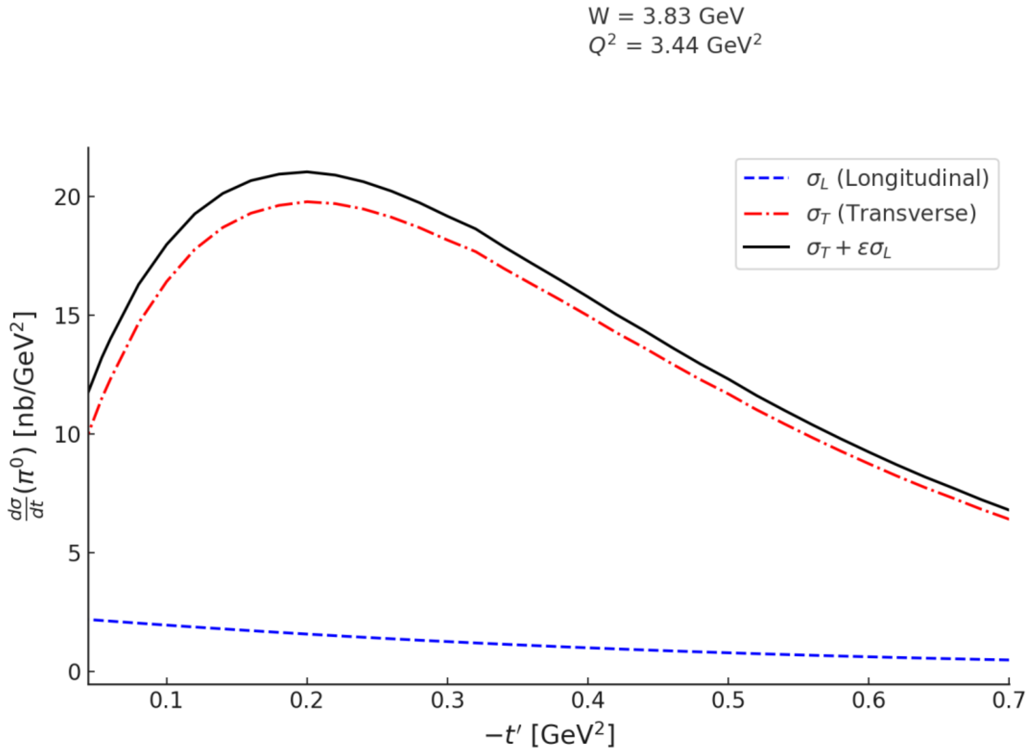


DVMP (π^0)

- Based on effective handbag approach in the GK model: factorizes the process into GPDs and subprocesses

Goloskokov, Kroll (2011)

- Particularly sensitive to the transversity GPDs H_T, $\widetilde{H}_T$ and E_T
- No pion-pole contribution
- Dominance of transversity GPDs
- Highlights the higher-twist effects



Pion production talks by Andrey, Avnish, Harut, Jihee, Karolina, Valery

Transversity GPDs talk by Jun-Young

Holographic QCD

- Holographic QCD is based on the AdS/CFT correspondence: certain strongly coupled gauge theories in the large-N_c limit are dual to weakly coupled gravitational theories in a higher dimensional AdS-space Maldacena (1998)

- QCD is neither supersymmetric nor conformal, but HQCD is a phenomenological extension that uses a 5D gravity dual to mimic QCD behavior

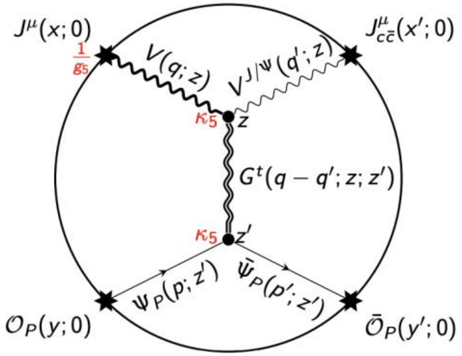
- Proton is treated as a bulk Dirac fermion, and vector mesons are normalizable modes (wave-like excitations) of a 5D gauge field

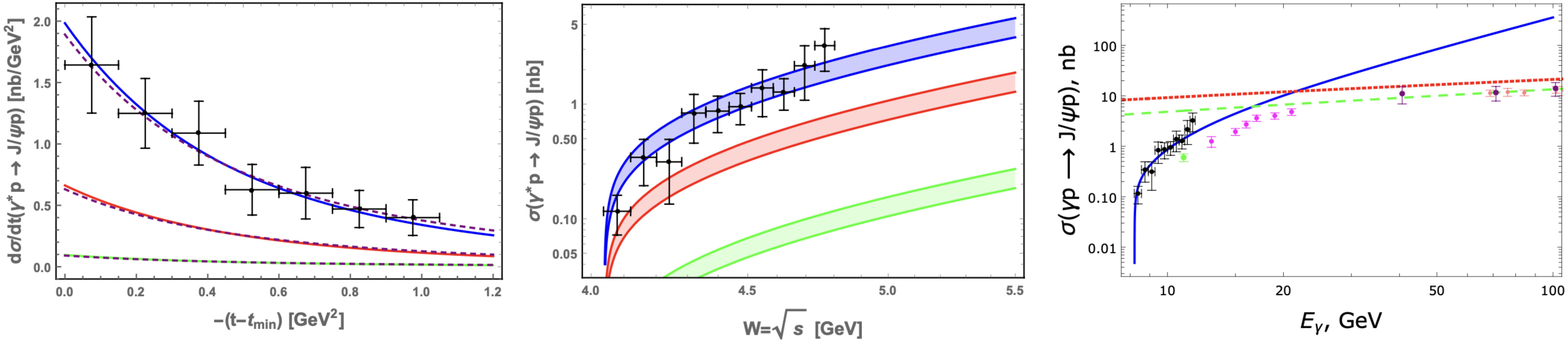
- Vector meson production probes the nucleon's gravitational structure via Witten diagrams in AdS, dominated by graviton (spin-2) exchange near threshold and higher-spin Reggeized exchanges far from threshold

HQCD talks by Ismael and Kiminad

Vector Mesons in HQCD (Near Threshold)

- Near threshold the leading contribution comes from spin-2 graviton exchange
- Sensitive to the Gravitational Form Factors



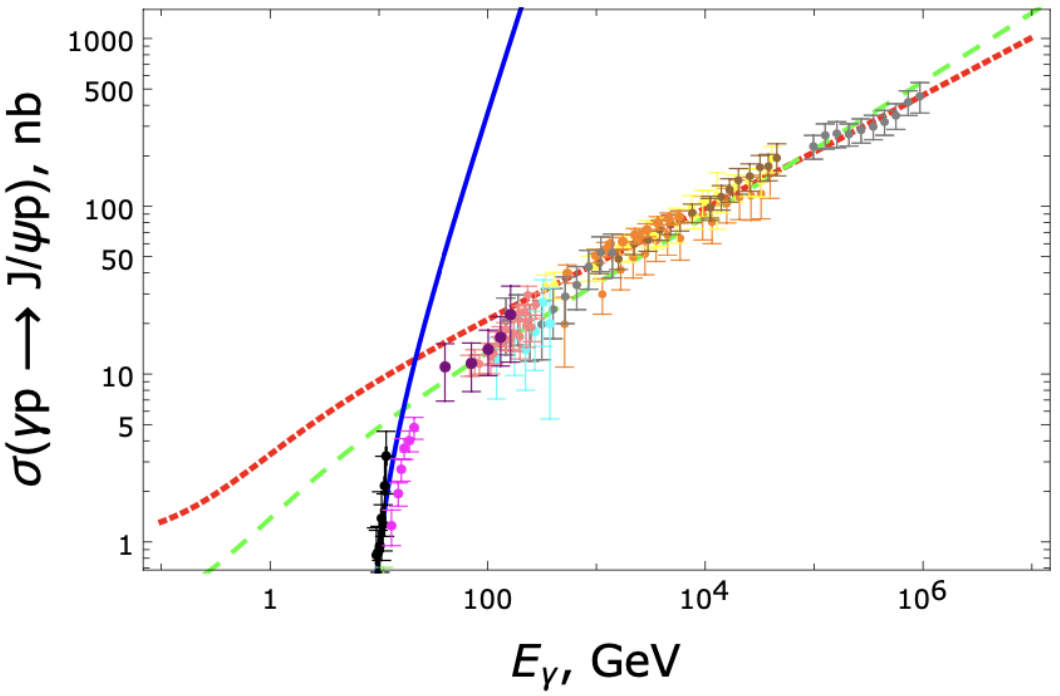


Blue $Q^2 = 0$
Red $Q^2 = 1.2\ \mathrm{GeV}^2$
Green $Q^2 = 2.2\ \mathrm{GeV}^2$

Mamo, Zahed, Phys. Rev. D 104 (2021)
Mamo, Zahed, Phys. Rev. D 101 (2020)

Vector Mesons in HQCD (Far from Threshold)

- Far from threshold the leading contribution comes from the Reggeized graviton, representing a tower of higher spin gluonic excitations



Mamo, Zahed, Phys. Rev. D 104 (2021)
Mamo, Zahed, Phys. Rev. D 101 (2020)

Vector Mesons in HQCD (Complete Picture)

- Summing over all spin exchanges

$$\frac{d\sigma(s,t,Q,\epsilon_T,\epsilon'_T)}{dt} = \frac{\mathcal{N}_T^2}{16\pi(s-(-Q^2+m_N^2))^2} \times \left|\mathcal{A}^{TT}_{\gamma^* p\to Vp}(s,t,Q,\epsilon_T,\epsilon'_T)\right|^2 ,$$

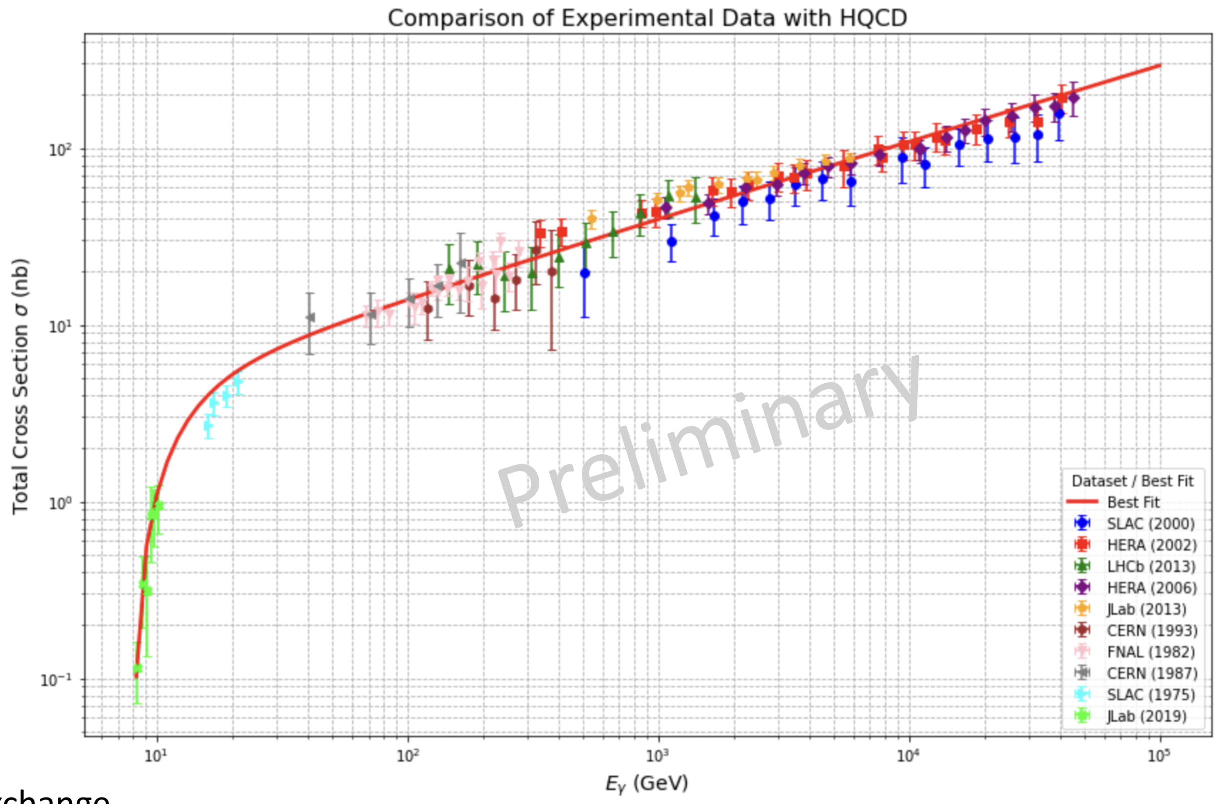
$$\frac{d\sigma(s,t,Q,\epsilon_L,\epsilon'_L)}{dt} = \frac{\mathcal{N}_T^2}{16\pi(s-(-Q^2+m_N^2))^2} \times \frac{Q^2}{M_V^2} \times \left|\mathcal{A}^{LL}_{\gamma^* p\to Vp}(s,t,Q,\epsilon_L,\epsilon'_L)\right|^2 ,$$

$$\mathcal{A}^{TT}_{\gamma^* p\to Vp}(s,t,Q,\epsilon_T,\epsilon'_T) = -\int_{\mathbb{C}} \frac{dj}{2\pi i} \frac{\left(\frac{s}{\kappa_N^2}\right)^j + \left(-\frac{s}{\kappa_N^2}\right)^j}{\sin \pi j} \times \mathcal{I}(Q,j) \times A(t,j) ,$$

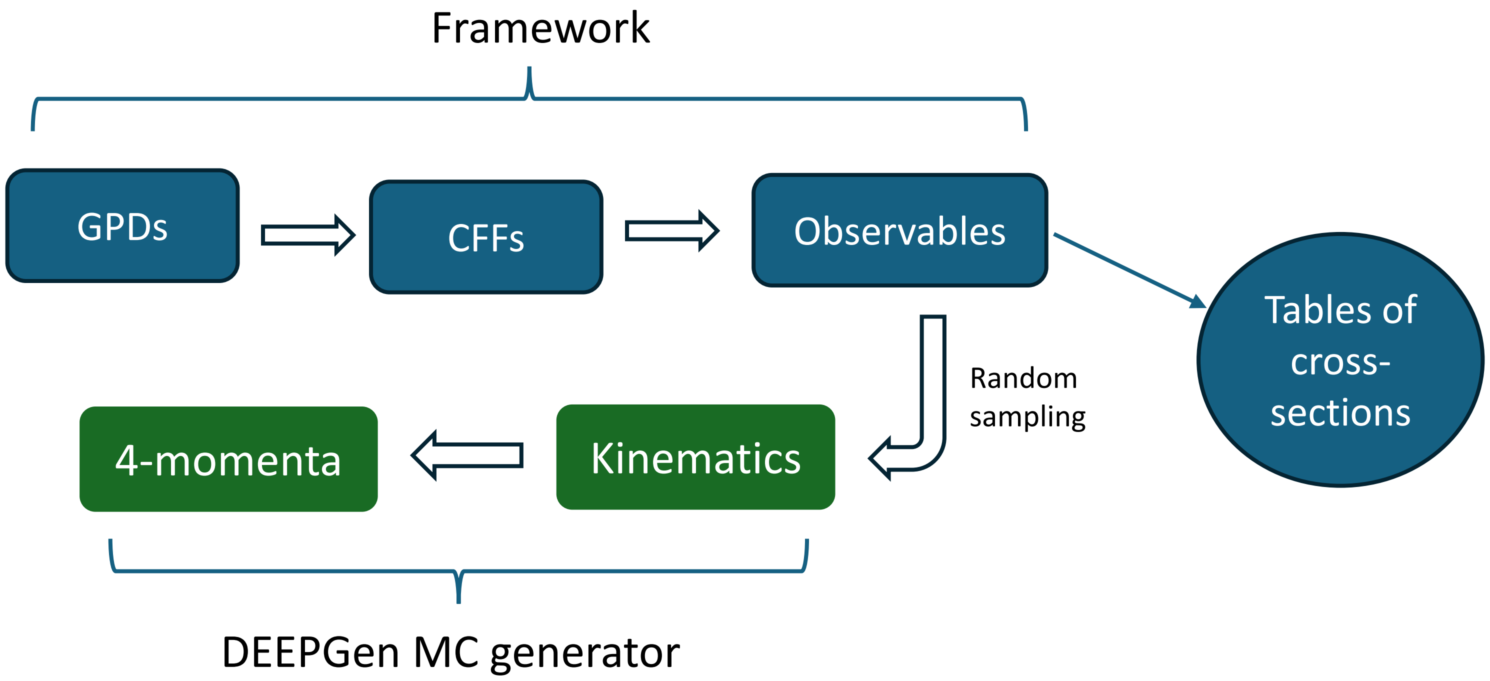
$$\mathcal{A}^{LL}_{\gamma^* p\to Vp}(s,t,Q,\epsilon_L,\epsilon'_L) = -\int_{\mathbb{C}} \frac{dj}{2\pi i} \frac{\left(\frac{s}{\kappa_N^2}\right)^j + \left(-\frac{s}{\kappa_N^2}\right)^j}{\sin \pi j} \times \frac{2}{j+\Delta(j)} \times \mathcal{I}(Q,j) \times A(t,j) ,$$

encodes the bulk overlap between the photon and meson

describes how the spin-j exchange couples to the nucleon



Next Steps



- Integration with the DEEPGen MC generator (by Marie Boër)
- Perform simulations and multi-channel analysis
- Compare factorization-based and HQCD-based approaches

Summary

- A new computer framework to accommodate factorization-based and HQCD-based approaches
- Multiple GPD models, processes (DVCS and DVMP), BMK formalism, and soon the UVA formalism
- Generation of pseudodata with the help of DEEPGen generator
- For the growing number of meson data, a tool for multi-channel analysis (DVCS + DVMP)

Thank you!