Towards improved hadron femtography with hard exclusive reactions, IV

How to extract GPDs from DVCS ?

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PRD 107 (2023) 014007

PRD 111 (2025) 094014

papers in preparation

Jul/30/2025

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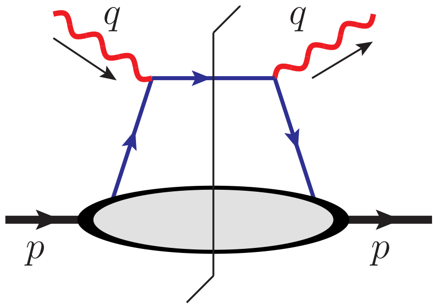
Deeply Virtual Compton Scattering (DVCS)

❑ DIS



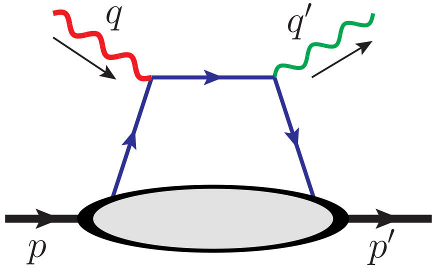
$$W^{\mu\nu}(p,q) = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\langle p|J^\nu(z/2)J^\mu(-z/2))|p\rangle$$
$$\simeq \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)\underbrace{\left[\frac{1}{2}\sum_q e_q^2 f_q(x_B)\right]}_{F_1(x_B,Q^2)} + \frac{1}{p\cdot q}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)\underbrace{\left[\sum_q e_q^2 x_B f_q(x_B)\right]}_{F_2(x_B,Q^2)}$$

$$\frac{d\sigma}{dx_B dQ^2} = \frac{4\pi\alpha_e^2}{x_B Q^4}\left[x_B y^2 F_1(x_B,Q^2) + \left(1 - y - x_B^2 y^2 \frac{m^2}{Q^2}\right) F_2(x_B,Q^2)\right]$$

Different y dependence $\longrightarrow$ **Separate F_1 and F_2**

Rosenbluth method

❑ DVCS



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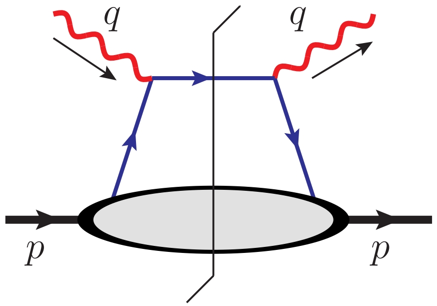
Deeply Virtual Compton Scattering (DVCS)

❑ DIS



$$W^{\mu\nu}(p,q) = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\langle p|J^\nu(z/2)J^\mu(-z/2))|p\rangle$$

$$\simeq \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\underbrace{\left[\frac{1}{2}\sum_q e_q^2 f_q(x_B)\right]}_{F_1(x_B,Q^2)} + \frac{1}{p\cdot q}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)\underbrace{\left[\sum_q e_q^2 x_B f_q(x_B)\right]}_{F_2(x_B,Q^2)}$$

$$\frac{d\sigma}{dx_B dQ^2} = \frac{4\pi\alpha_e^2}{x_B Q^4}\left[x_B y^2 F_1(x_B,Q^2) + \left(1 - y - x_B^2 y^2\frac{m^2}{Q^2}\right)F_2(x_B,Q^2)\right]$$

Different y dependence → **Separate F_1 and F_2**

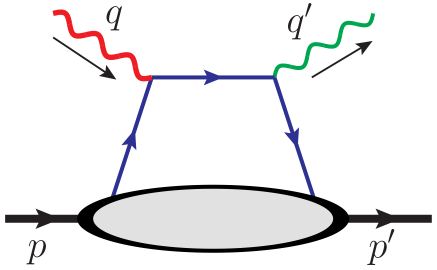
Rosenbluth method

❑ DVCS

$$T^{\mu\nu}(p,p',q) = i\int d^4z\, e^{i(q+q')\cdot z}\langle p'|\mathcal{T}\{J^\nu(z/2)J^\mu(-z/2))\}|p\rangle$$

(depends on the choice of n)

$$\simeq \frac{\bar{u}(p',s')}{2P\cdot n}\left\{\left[\mathcal{H}(\xi,t)\gamma\cdot n - \mathcal{E}(\xi,t)\frac{i\sigma^{n\Delta}}{2m}\right](g_\perp^{\mu\nu}) + \left[\widetilde{\mathcal{H}}(\xi,t)\gamma\cdot n\gamma_5 - \widetilde{\mathcal{E}}(\xi,t)\frac{\gamma_5\Delta\cdot n}{2m}\right](-i\,\epsilon_\perp^{\mu\nu})\right\}u(p,s)$$



$$\Delta = p - p'$$

$$P = (p+p')/2$$

$$\{\mathcal{H},\mathcal{E}\}(\xi,t) \equiv \sum_q e_q^2\int_{-1}^{1}dx\,\{H^q,E^q\}(x,\xi,t)\left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon}\right]$$

$$\{\widetilde{\mathcal{H}},\widetilde{\mathcal{E}}\}(\xi,t) \equiv \sum_q e_q^2\int_{-1}^{1}dx\,\{\widetilde{H}^q,\widetilde{E}^q\}(x,\xi,t)\left[\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon}\right]$$

← **GPD moments**

- **Both Re and Im parts**
- **8 real dof's in total!**

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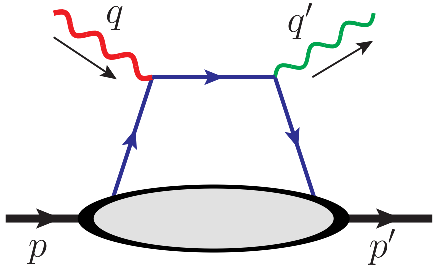
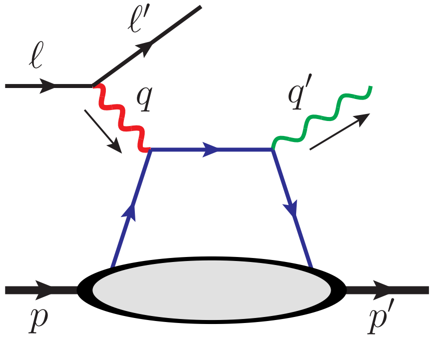
How to separate the GPD moments in DVCS? [Consider unpolarized target]

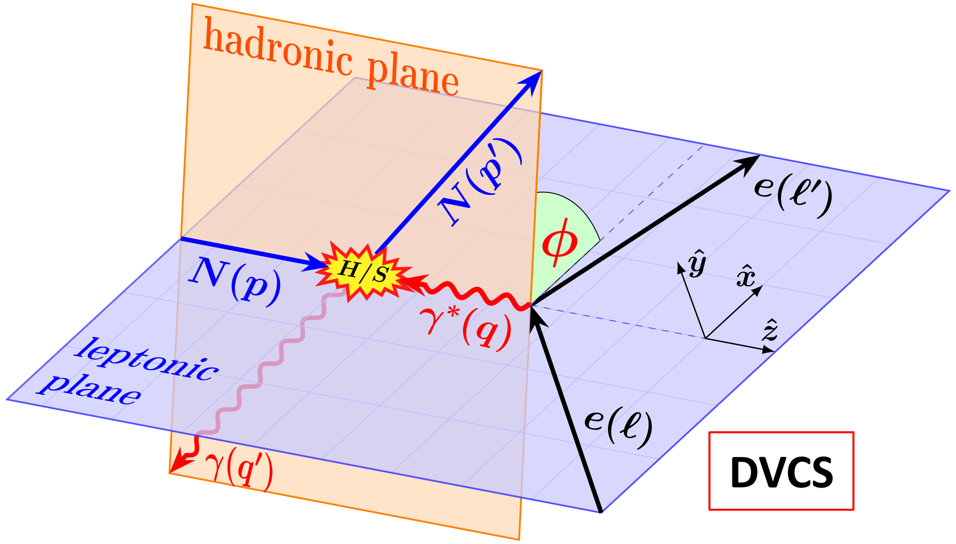
- Azimuthal modulation in the Breit frame



DVCS



Photon electroproduction



Breit frame observables: (x_B, Q^2, t, ϕ)

$$\Rightarrow \frac{d\sigma}{dx_B\, dQ^2\, dt\, d\phi}$$

[A.V. Belitsky et al., 2002]
[B. Kriesten et al., 2020, 2022]
[Y. Guo et al., 2021, 2022]
...

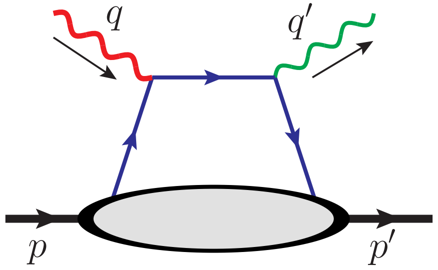
- (x_B, Q^2) determine: (1) electron scattering, (2) γ^* kinematics
- (t, ϕ) describe the $\gamma^* N$ scattering

ϕ dependence in the DVCS amplitude: $e^{i(\lambda_N - \lambda_{\gamma^*})\phi}$ + **polarization** ⇒ **separate GPDs (?)**

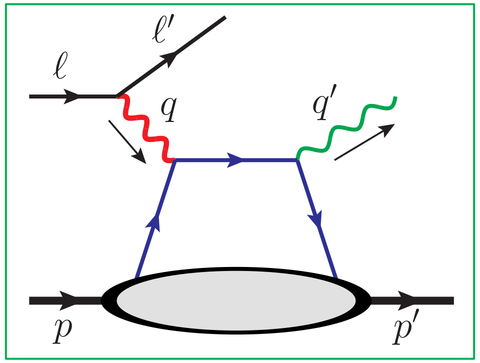
Bethe-Heitler subprocess!

[Consider unpolarized target]

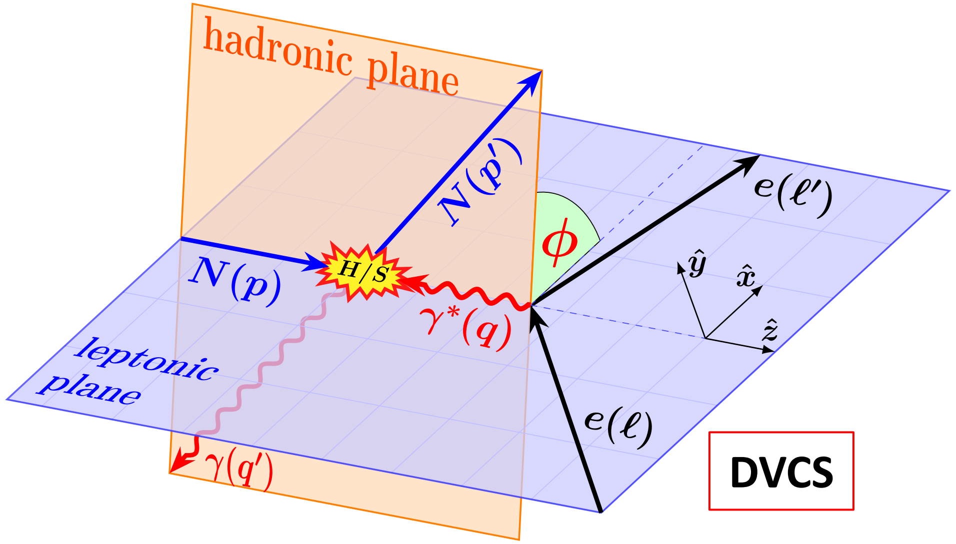
❑ Azimuthal modulation in the Breit frame



DVCS

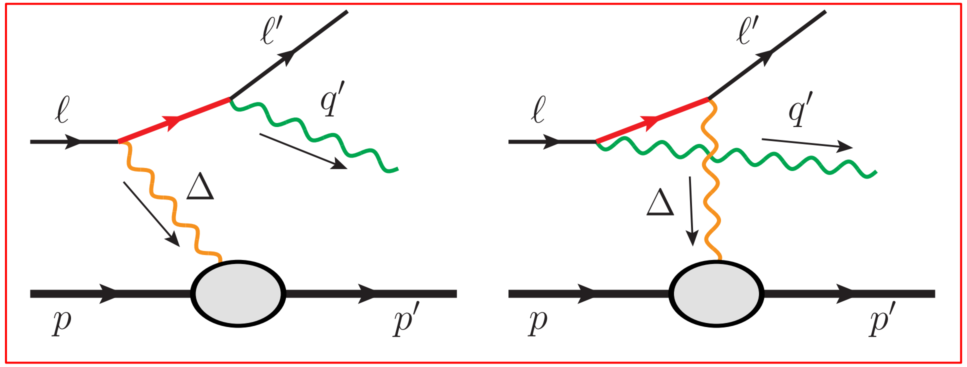


Photon electroproduction



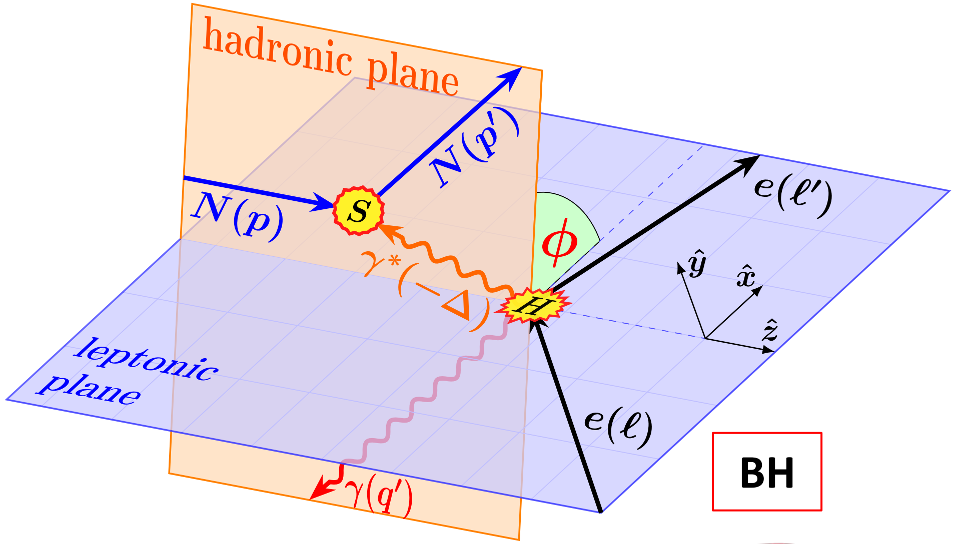
❑ DVCS is just a subprocess!

$$N(p) + e(\ell) \to N(p') + e(\ell') + \gamma(q')$$



$\Delta = p' - p$

Bethe-Heitler (BH) process



Measurement in Breit frame

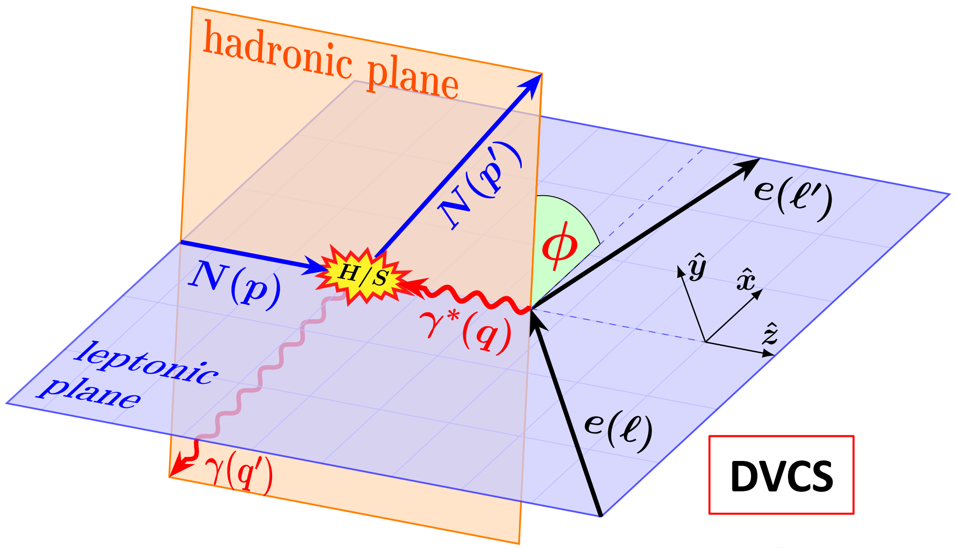
[Consider unpolarized target]

- **Experiments measure cross section** $N(p) + e(\ell) \to N(p') + e(\ell') + \gamma(q')$

$$\sigma \propto |\mathcal{M}_{\rm BH} + \mathcal{M}_{\rm DVCS}|^2 = \underbrace{|\mathcal{M}_{\rm BH}|^2 + 2\,{\rm Re}\{\mathcal{M}^*_{\rm BH}\mathcal{M}_{\rm DVCS}\}}_{\text{contains } \phi \text{ in denominator } \propto \frac{1}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \propto \frac{1}{(1+a\cos\phi)(1+b\cos\phi)}} + |\mathcal{M}_{\rm DVCS}|^2$$

- **Analysis procedure**
  - **For each event, boost to Breit frame and compute observables** $(x_B,\ Q^2,\ t,\ \phi)$
  - **Bin (x_B, Q^2, t).**
  - **For each (x_B, Q^2, t) bin, make histogram of ϕ.**
  - **Subtract $|M_{BH}|^2$ using known (F_1, F_2) measurements.**
  - **Fit ϕ histogram to some harmonic template.**
  - **Compare with theory to extract GPD moments.**

The binning introduces extra systematic uncertainties!



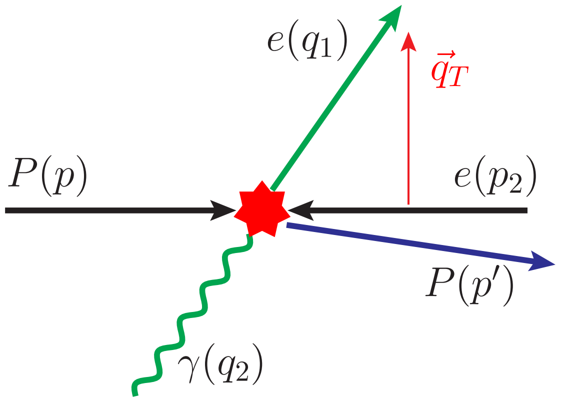
DVCS

A better way to think about DVCS

❑ Single diffractive hard exclusive process (SDHEP)

[Qiu & Yu, PRD 107 (2023) 014007]
[Qiu, Sato, Yu, PRD 111 (2025) 094014]
[in preparation]

View in lab frame $N(p) + e(p_2) \to N(p') + e(q_1) + \gamma(q_2)$



Two scales:

- **Hard q_T**
- **Soft t**

$t = (p - p')^2$

A better way to think about DVCS

❑ Single diffractive hard exclusive process (SDHEP)

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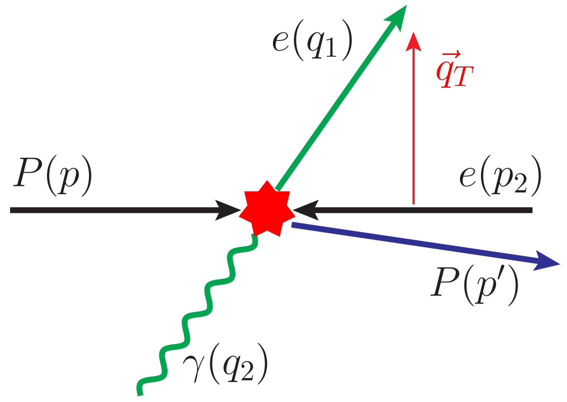
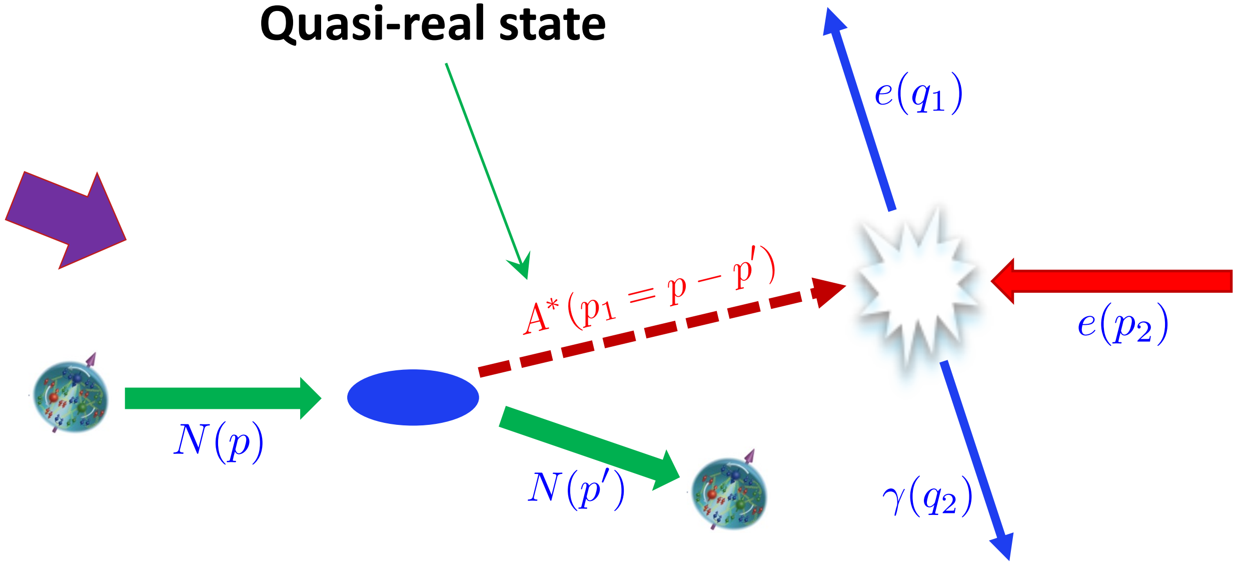


Two scales:

- **Hard q_T**
- **Soft t**

$t = (p - p')^2$



❑ Two-stage process paradigm

Single diffractive: $N(p) \to N(p') + A^*(p_1 = p - p')$

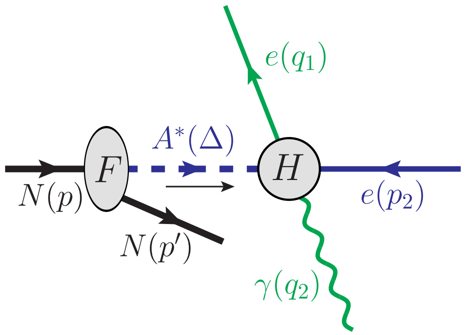
↓ **factorize**

Hard exclusive: $A^*(p_1) + e(p_2) \to e(q_1) + \gamma(q_2)$

***Necessary* condition for factorization:**

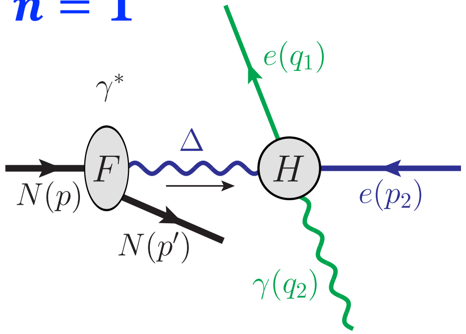
$q_T \gg \sqrt{-t} \simeq \Lambda_{\rm QCD}$

Channel expansion and power counting



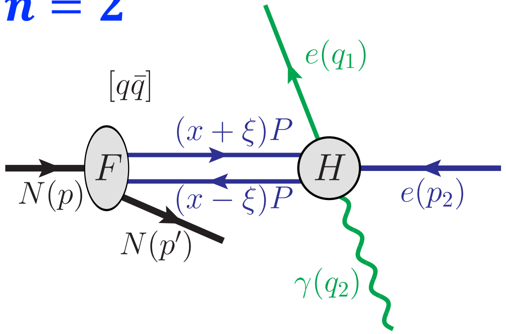
$=$

$\boldsymbol{n = 1}$



$+$

$\boldsymbol{n = 2}$



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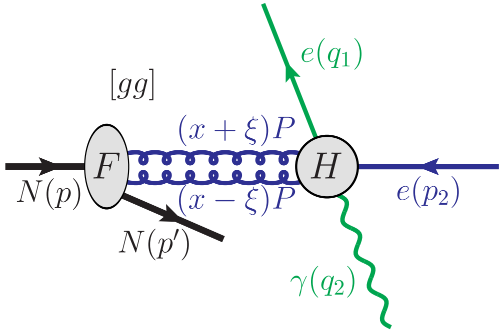
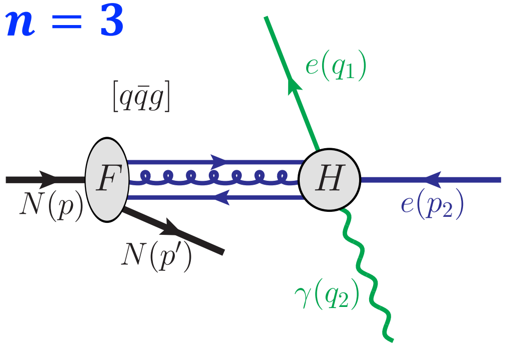


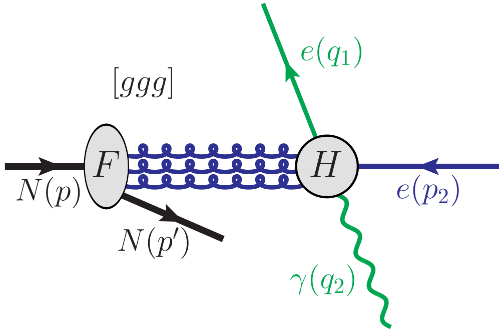
$$\Delta = p_1 = p - p'$$

$\boldsymbol{n = 3}$

$+$

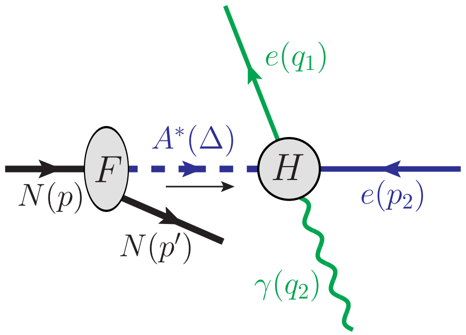


$+$



$+ \cdots \quad (\boldsymbol{n > 3})$

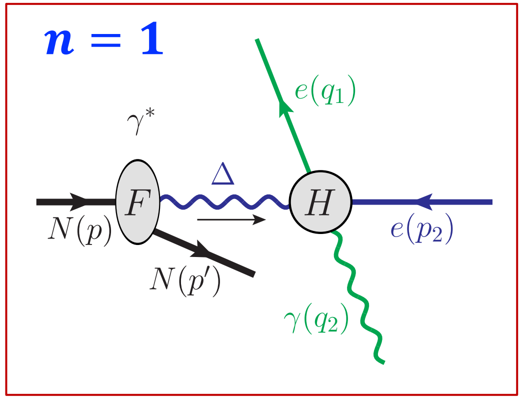
Channel expansion and power counting



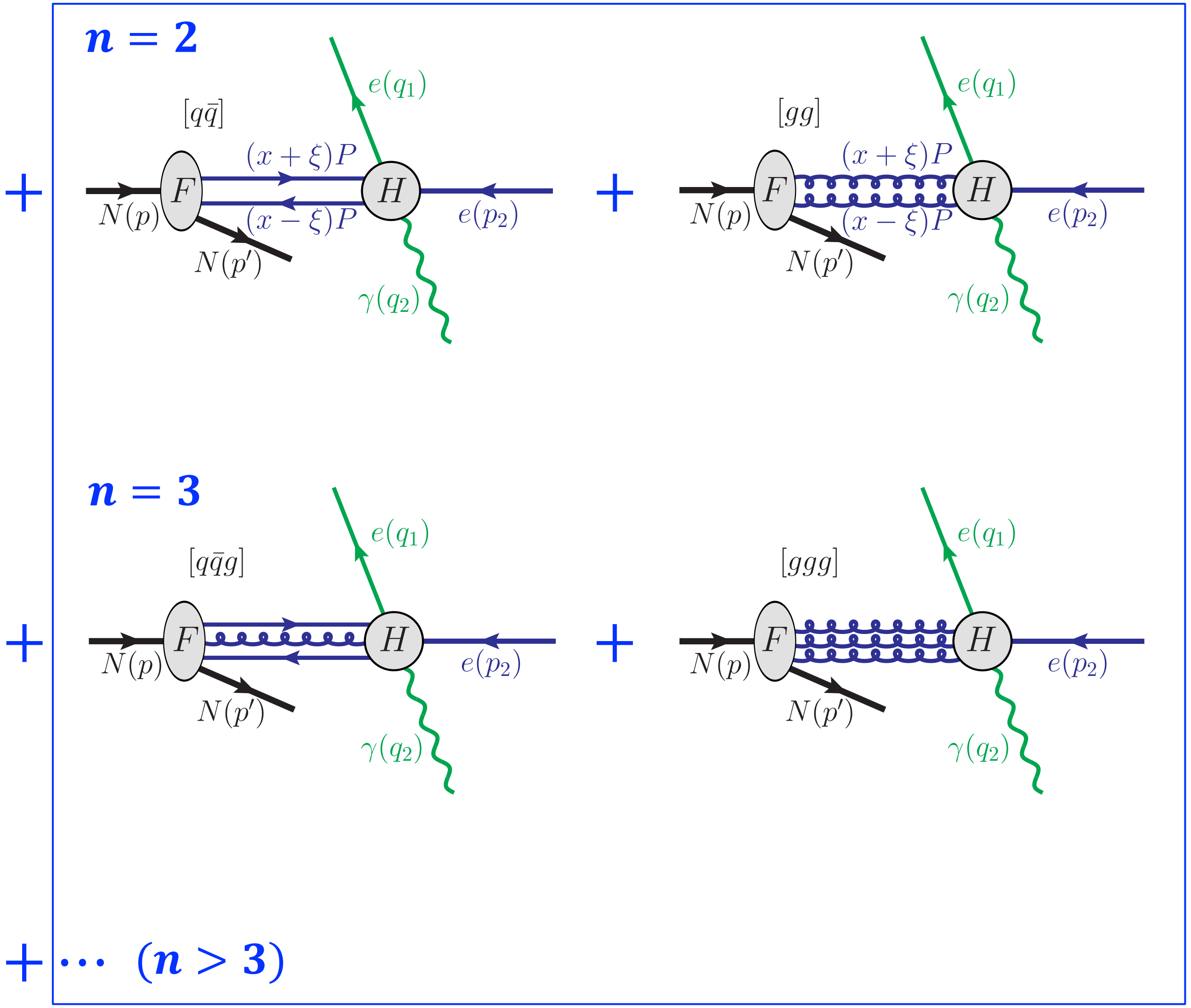
$$\Delta = p_1 = p - p'$$

=

BH

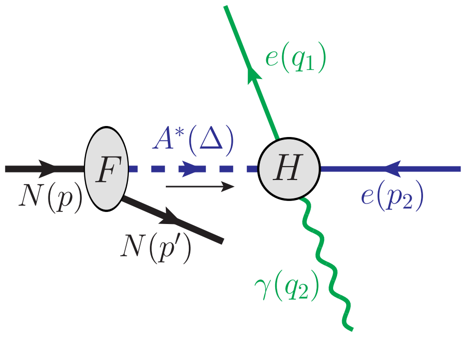


DVCS



BH and DVCS are treated on the *same* ground!

Channel expansion and power counting

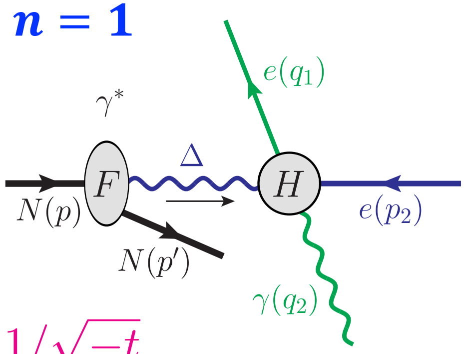


$\Delta = p_1 = p - p'$

$=$

Leading power (LP)

$n = 1$

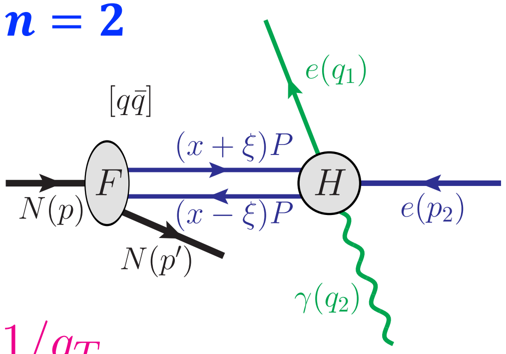


$1/\sqrt{-t}$

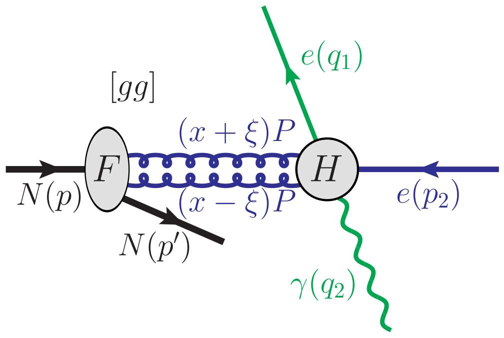
$+$

Next-to-leading power (NLP)

$n = 2$



$+$

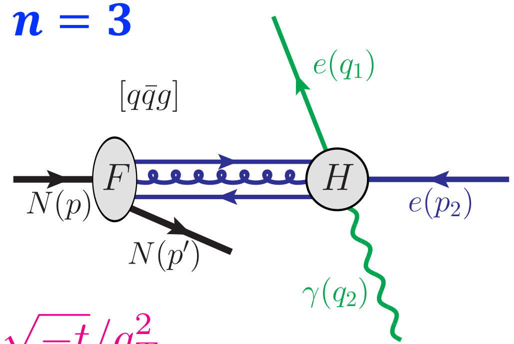


$1/q_T$

NNLP

$+$

$n = 3$



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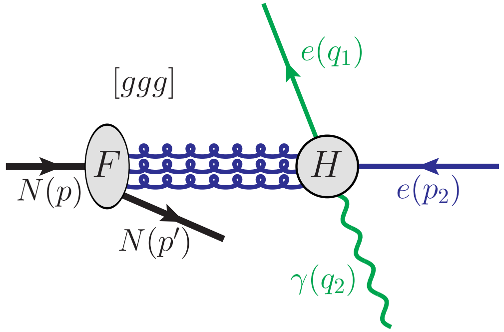
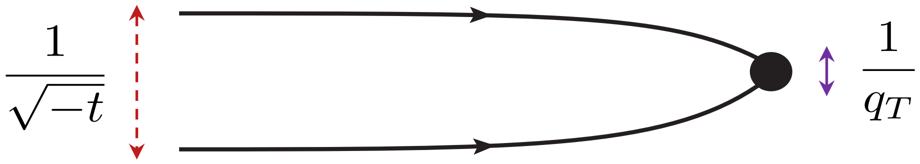


$\sqrt{-t}/q_T^2$

One more physically polarized parton in A^*

one more suppression of $\sqrt{-t}/q_T$



- **Consistent power counting**
- **Channel expansion = power expansion**

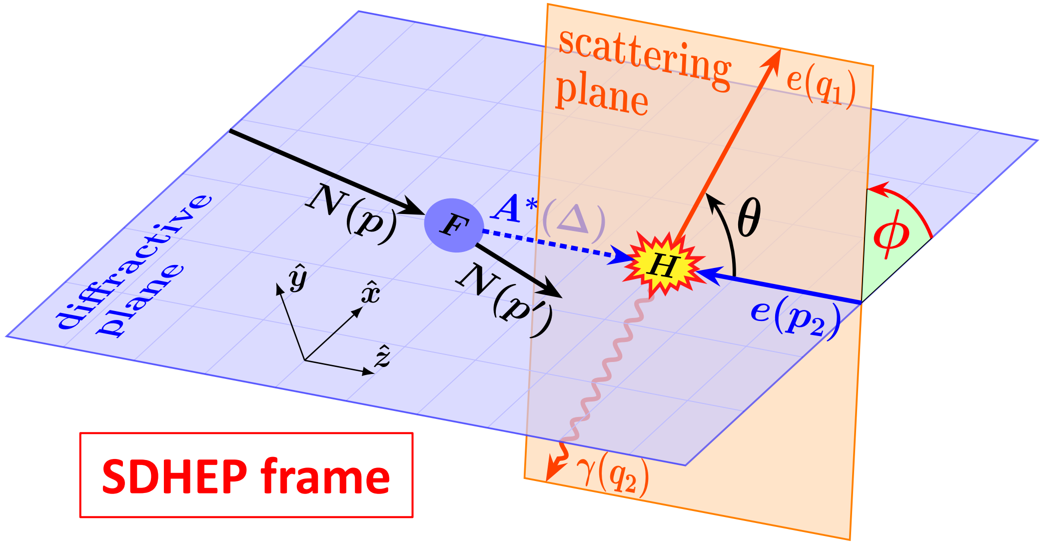
NNNLP…

$+\cdots \; (n > 3) \quad \mathcal{O}\left(-t/q_T^3\right)$

SDHEP frame and ϕ distribution

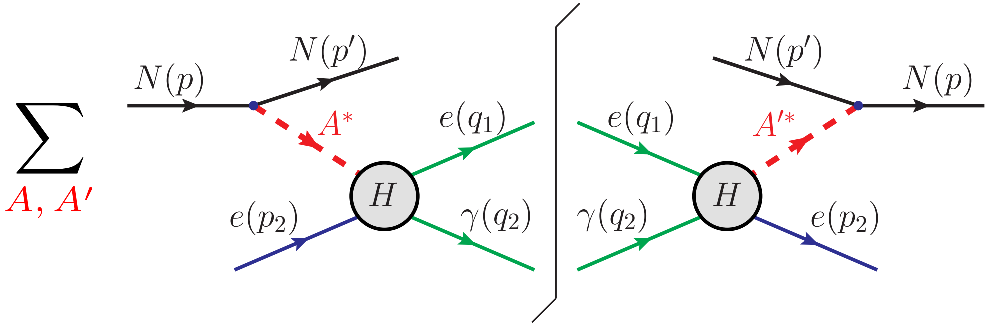
SDHEP frame observables: $(t,\ \xi,\ \theta,\ \phi)$

$$\mathcal{M}(t,\xi,\phi_S,\theta,\phi) = \sum_{A^*} e^{i(\lambda_A-\lambda_e)\phi} F_{N\to NA^*} \otimes G_{A^*e\to e\gamma}$$



SDHEP frame

$|\mathcal{M}|^2$



Interference of $(\lambda_A,\ \lambda_A')$ channels

$$\Delta\lambda_A = \lambda_A - \lambda_A'$$

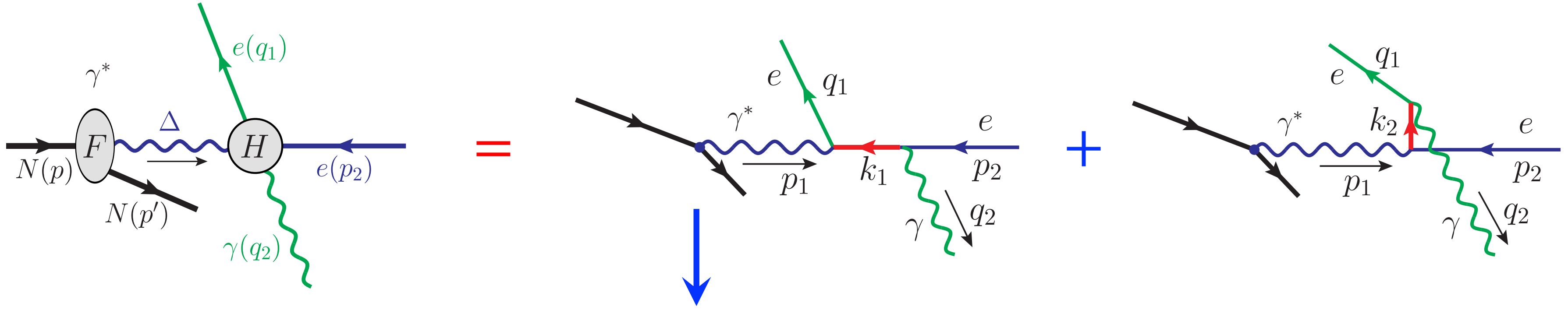
$\cos[(\Delta\lambda_A)\phi]$

$\sin[(\Delta\lambda_A)\phi]$

- **ϕ distribution is determined by A^* spin states!**
- **Both A^* and e are along z axis, so there is no ϕ in denominators.**

$\boldsymbol{n=1}$: $\boldsymbol{\gamma^*}$ channel --- BH subprocess

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \dots$**



$$F_N^\mu(p,p') = \langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p',s')\left[F_1(t)\,\gamma^\mu - F_2(t)\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\right]u(p,s)$$

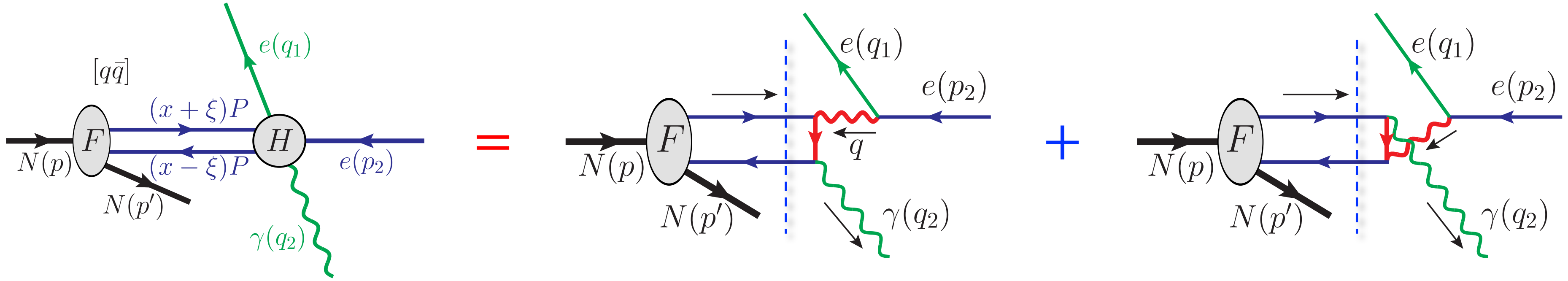
$$\mathcal{M}^{[1]} = \frac{-e}{t}\,F_N^\mu(p,p')\,G_\mu^\gamma(\Delta,p_2,q_1,q_2) = \frac{e}{t}\left[\sum_{\lambda=\pm1}(F_N\cdot\epsilon_\lambda^*)(\epsilon_\lambda\cdot G^\gamma) - 2(F_N\cdot n)(\bar{n}\cdot G^\gamma)\right]$$

$\lambda_A^\gamma = \pm 1$ $\qquad\qquad$ $\lambda_A^\gamma = 0$

- **Only the transverse polarization γ_T^* is at LP** $\quad \mathcal{O}(1/\sqrt{-t})$
- **The longitudinal polarization γ_L^* is at NLP** $\quad \mathcal{O}(1/q_T)$ $\longleftrightarrow$ **Combine with $n=2$ (DVCS)**

$n = 2$: $[q\bar{q}]$ channel --- DVCS (twist-2)

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



$$\mathcal{M}^{[2]} \simeq \sum_q \int_{-1}^{1} dx \left[F^q(x,\xi,t)\, G^q(x,\xi;\hat{s},\theta,\phi) + \widetilde{F}^q(x,\xi,t)\, \widetilde{G}^q(x,\xi;\hat{s},\theta,\phi) \right] + \mathcal{O}\left(\sqrt{-t}/q_T^2\right)$$

GPDs (H, E): defined with γ^+   **$(\widetilde{H}, \widetilde{E})$: defined with $\gamma^+\gamma_5$**

$$\lambda_A^{q\bar{q}} = 0$$

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_{\mathrm{I}}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\mathrm{II}}: \quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0); \quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0)$ **+ $[gg]$ (high order)**

NNLP: …

❑ Cross section level

$$|\mathcal{M}_{\mathrm{I}} + \mathcal{M}_{\mathrm{II}} + \cdots|^2 = \underbrace{|\mathcal{M}_{\mathrm{I}}|^2}_{\mathrm{LP}} + \underbrace{2\,\mathrm{Re}\,(\mathcal{M}_{\mathrm{I}}\mathcal{M}_{\mathrm{II}}^*)}_{\mathrm{NLP}} + \cdots$$

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_{\rm I}$: $A^* = \gamma_T^*$ $(\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}$: (1) $A^* = \gamma_L^*$ $(\lambda_A^\gamma = 0)$; (2) $A^* = [q\bar{q}]$ $(\lambda_A^q = 0)$ $+ [gg]$ **(high order)**

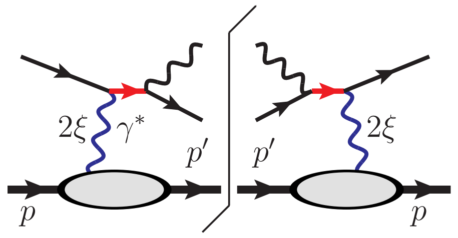
NNLP: …

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

$$\boxed{\begin{array}{c}\cos[(\Delta\lambda_A)\phi] \\ \sin[(\Delta\lambda_A)\phi]\end{array}} \qquad \Delta\lambda_A = \lambda_A - \lambda_A'$$

LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**



"twist-2"

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\quad \mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

NLP $\quad \mathcal{M}_{\rm II}: \quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0); \quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0)$ $\quad + [gg]$ **(high order)**

NNLP: …

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

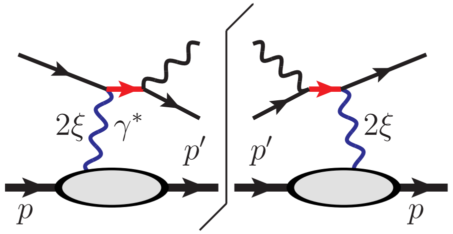
$$\boxed{\begin{array}{c}\cos[(\Delta\lambda_A)\phi] \\ \sin[(\Delta\lambda_A)\phi]\end{array}} \qquad \Delta\lambda_A = \lambda_A - \lambda_A'$$

LP $\quad |\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ $\quad$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**

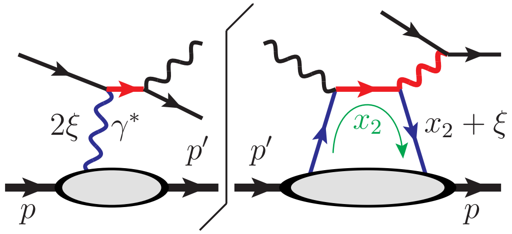
NLP $\quad |\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ $\quad \Rightarrow$ **$\cos\phi$ or $\sin\phi$ modulation.**

Interference of different numbers of particles.

Unique signal of GPDs.



"twist-2"



"twist-3"

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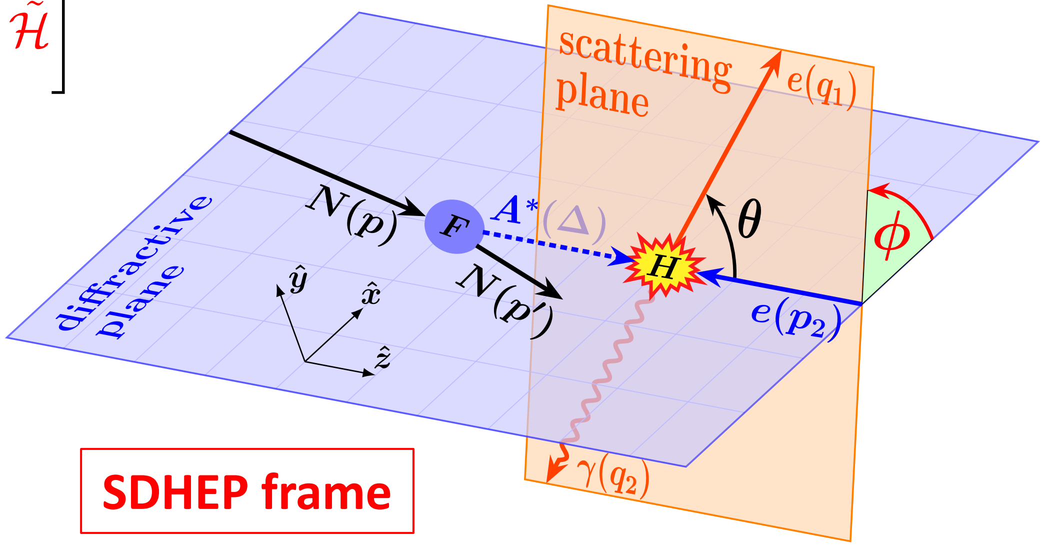
Cross section within NLP: unpolarized proton

$$\frac{d\sigma}{dt\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1 + \underbrace{A_{UU}^{\text{NLP}}(t,\xi,\cos\theta)\cos\phi + \lambda_e A_{UL}^{\text{NLP}}(t,\xi,\cos\theta)\sin\phi}_{\text{Interference of } \gamma_T^* \text{ and GPD moments}}\Big]$$

$$A_{UU}^{\text{NLP}} \propto \frac{2\sin\theta}{\xi}\left(F_1^2 - \frac{t}{4m^2}F_2^2\right) - \frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left[F_1\,\text{Re}\,\mathcal{H} - \frac{t}{4m^2}F_2\,\text{Re}\,\mathcal{E} + \xi\,(F_1+F_2)\,\text{Re}\,\tilde{\mathcal{H}}\right]$$

$$A_{UL}^{\text{NLP}} \propto \frac{3-\cos\theta}{\sin\theta}\left[F_1\,\text{Im}\,\mathcal{H} - \frac{t}{4m^2}F_2\,\text{Im}\,\mathcal{E} + \xi\,(F_1+F_2)\,\text{Im}\,\tilde{\mathcal{H}}\right]$$



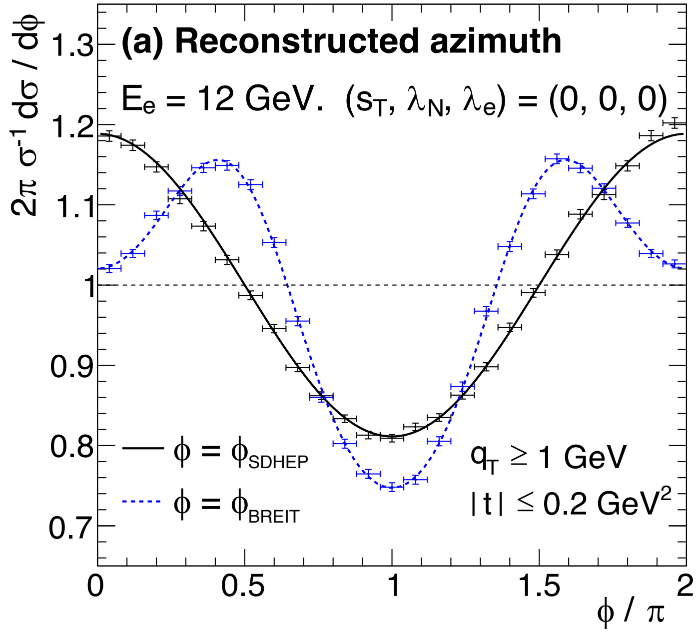
SDHEP frame

Cross section within NLP: unpolarized proton

$$\frac{d\sigma}{dt\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta}\cdot\left[1+\boxed{A_{UU}^{\text{NLP}}(t,\xi,\cos\theta)\cos\phi}+\lambda_e A_{UL}^{\text{NLP}}(t,\xi,\cos\theta)\sin\phi\right]$$

Generate 10^6 events and reconstruct

unpolarized



(normalized)

SDHEP fit to $1.00 + 0.190\cos\phi$ ⟹ $\langle A_{UU}^{\text{NLP}}\rangle = 0.190$

Breit fit to $1.00 + 0.15\cos\phi - 0.12\cos 2\phi - 0.01\cos 3\phi + 0.01\cos 4\phi$

- **No straightforward interpretation of the coefficients.**
- **Need to introduce more gears in GPD extraction.**

[A.V. Belitsky et al., 2002]
[B. Kriesten et al., 2020, 2022]
[Y. Guo et al., 2021, 2022]
...

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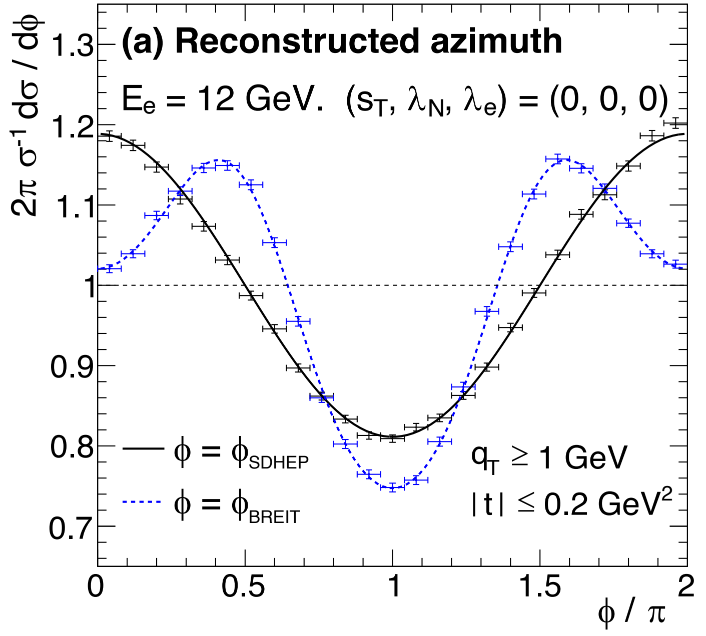
Cross section within NLP: unpolarized proton

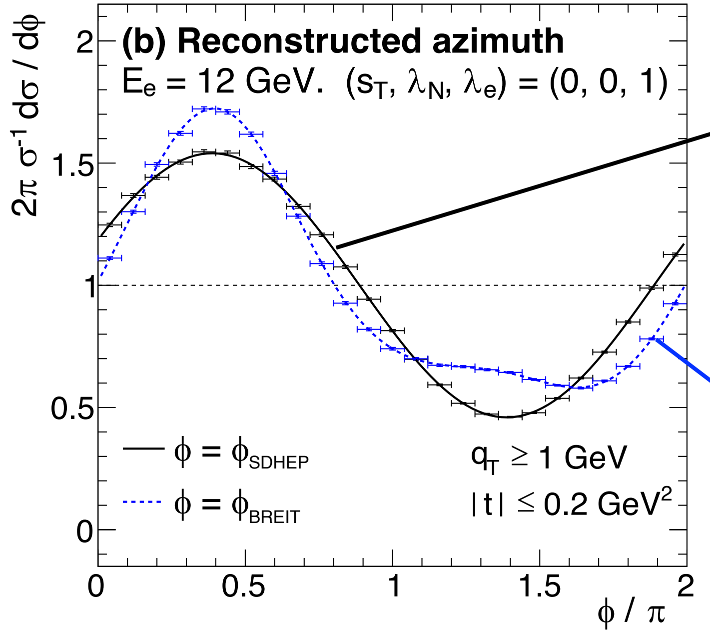
$$\frac{d\sigma}{dt\, d\xi\, d\cos\theta\, d\phi} = \frac{1}{2\pi}\frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta}\cdot\left[1+\boxed{A_{UU}^{\rm NLP}(t,\xi,\cos\theta)\cos\phi + \lambda_e A_{UL}^{\rm NLP}(t,\xi,\cos\theta)\sin\phi}\right]$$

Generate 10^6 events and reconstruct

single electron polarization



(normalized)



$$1.00 + 0.19\cos\phi + 0.51\sin\phi$$

$$\langle A_{UU}^{\rm NLP}\rangle = 0.19$$
$$\langle A_{UL}^{\rm NLP}\rangle = 0.51$$

Not clear how to fit...

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Cross section within NLP: introducing proton spin

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e\lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP}\cos\phi_S$$
$$+ \left(A_{UU}^{\rm NLP} + \lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$
$$+ s_T\left(A_{TU,1}^{\rm NLP}\cos\phi_S\sin\phi + A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right)$$
$$+ \lambda_e s_T\left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi + A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

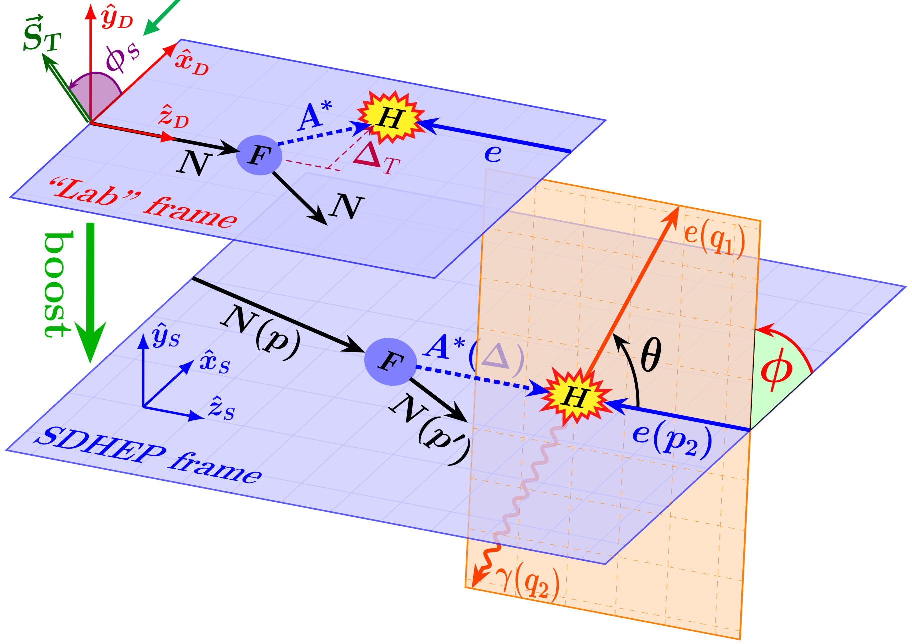
In the experimental setting (fixed lab frame),

- **Nucleon spin vector** $\vec{s}_N = (s_T, 0, \lambda_N)$
- **Electron spin vector** $\vec{s}_e = (0, 0, \lambda_e)$

Subscripts: (nucleon, electron)

U = Unpolarized
L = Longitudinally polarized
T = Transversely polarized

ϕ_S: of nucleon transverse spin in the "Lab" frame



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Cross section within NLP: full polarization

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \boxed{\frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}} \cos\phi_S} + \left(A_{UU}^{\text{NLP}} + \lambda_e \lambda_N A_{LL}^{\text{NLP}}\right) \cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right) \sin\phi$$
$$+ s_T \left(A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi\right)$$
$$+ \lambda_e s_T \left(A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi\right) \Big]$$

LP: from γ_T^* squared

$$\frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} = \frac{\alpha_e^3}{(1+\xi)^2} \frac{m^2}{s\, t^2} \Sigma_{UU}^{\text{LP}}$$

$$\Sigma_{UU}^{\text{LP}} = \left[\frac{1}{\sin^2(\theta/2)} + \sin^2(\theta/2)\right] \left[\left(\frac{1-\xi^2}{2\xi^2} \frac{-t}{m^2} - 2\right)\left(F_1^2 - \frac{t}{4m^2} F_2^2\right) - \frac{t}{m^2}(F_1 + F_2)^2\right]$$

$$A_{LL}^{\text{LP}} = \frac{1}{\Sigma_{UU}^{\text{LP}}} \cdot \left[\frac{1}{\sin^2(\theta/2)} - \sin^2(\theta/2)\right] (F_1 + F_2) \left[F_1 \left(\frac{-t}{\xi m^2} - \frac{4\xi}{1+\xi}\right) - \frac{t}{m^2} F_2\right]$$

$$A_{TL}^{\text{LP}} = \frac{1}{\Sigma_{UU}^{\text{LP}}} \cdot \frac{\Delta_T}{2m} \left[\frac{1}{\sin^2(\theta/2)} - \sin^2(\theta/2)\right] (F_1 + F_2) \left[-4F_1 + \frac{1+\xi}{\xi} \frac{-t}{m^2} F_2\right]$$

Quadratic in (F_1, F_2)

Control the rate (unpolarized cross section). No ϕ modulation.

Cross section within NLP: full polarization

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP} \cos\phi_S$$

$$+ \left(A_{UU}^{\rm NLP} + \lambda_e \lambda_N A_{LL}^{\rm NLP}\right) \cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right) \sin\phi$$

$$+ s_T \left(A_{TU,1}^{\rm NLP} \cos\phi_S \sin\phi + A_{TU,2}^{\rm NLP} \sin\phi_S \cos\phi\right)$$

$$+ \lambda_e s_T \left(A_{TL,1}^{\rm NLP} \cos\phi_S \cos\phi + A_{TL,2}^{\rm NLP} \sin\phi_S \sin\phi\right) \Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

No contribution to the rate,

$\Longrightarrow$ **only to azimuthal modulations** $(\cos\phi, \sin\phi)$

Cross section within NLP: full polarization

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}} \cos\phi_S$$

$$+ \left(A_{UU}^{\text{NLP}} + \lambda_e \lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi$$

$$+ s_T \left(A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi\right)$$

$$+ \lambda_e s_T \left(A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi\right) \Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

$$A_{XX}^{\text{NLP}} = \frac{1}{\Sigma_{UU}^{\text{LP}}} \cdot \left(\frac{-t}{m\sqrt{\hat{s}}}\right) \Sigma_{XX}^{\text{NLP}}$$

$$\Sigma_{UU}^{\text{NLP}} = \frac{\Delta_T}{2m}\frac{1+\xi}{\xi}\left[\frac{2\sin\theta}{\xi}\left(F_1^2 - \frac{t}{4m^2}F_2^2\right) - \frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_1 \cdot \operatorname{Re} V_{\mathcal{F}}\right)\right],$$

$$\Sigma_{LL}^{\text{NLP}} = -\frac{\Delta_T}{m}\left[\sin\theta(F_1+F_2)\left(\frac{1+\xi}{\xi}F_1 + F_2\right) + \frac{3-\cos\theta}{\sin\theta}\left(M_2 \cdot \operatorname{Re} V_{\mathcal{F}}\right)\right],$$

$$\Sigma_{TL,1}^{\text{NLP}} = 2\sin\theta\,(F_1+F_2)\left[F_1 + \left(\frac{\xi}{1+\xi} + \frac{t}{4\xi m^2}\right)F_2\right] + \frac{2(3-\cos\theta)}{\sin\theta}\left(M_3 \cdot \operatorname{Re} V_{\mathcal{F}}\right),$$

$$\Sigma_{TL,2}^{\text{NLP}} = 2\sin\theta\,(F_1+F_2)\left(F_1 + \frac{t}{4m^2}F_2\right) - \frac{2(3-\cos\theta)}{\sin\theta}\left(M_4 \cdot \operatorname{Re} V_{\mathcal{F}}\right),$$

$$\Sigma_{UL}^{\text{NLP}} = -\frac{\Delta_T}{m}\frac{1+\xi}{\xi}\frac{3-\cos\theta}{\sin\theta}\left(M_1 \cdot \operatorname{Im} V_{\mathcal{F}}\right),$$

$$\Sigma_{LU}^{\text{NLP}} = -\frac{\Delta_T}{2m}\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_2 \cdot \operatorname{Im} V_{\mathcal{F}}\right),$$

$$\Sigma_{TU,1}^{\text{NLP}} = \frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_3 \cdot \operatorname{Im} V_{\mathcal{F}}\right),$$

$$\Sigma_{TU,2}^{\text{NLP}} = \frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_4 \cdot \operatorname{Im} V_{\mathcal{F}}\right).$$

- **Linear in GPD moments** $V_{\mathcal{F}} = (\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}})^T$
- **Controlled by the real matrix M, same for real and imaginary parts of GPD moments**

  $M_i = (M_{i1}, M_{i2}, M_{i3}, M_{i4})$ **(see next slide)**
- **8 asymmetries $\Leftrightarrow$ 8 (real) GPD moments**

Cross section within NLP: full polarization

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP} \cos\phi_S$$

$$+ \left(A_{UU}^{\rm NLP} + \lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$

$$+ s_T \left(A_{TU,1}^{\rm NLP}\cos\phi_S \sin\phi + A_{TU,2}^{\rm NLP} \sin\phi_S \cos\phi\right)$$

$$+ \lambda_e s_T \left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi + A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right) \Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

$$M = \begin{array}{c} \begin{bmatrix} F_1 & -\dfrac{t}{4m^2}F_2 & \xi(F_1+F_2) & 0 \\ (1+\xi)(F_1+F_2) & \xi(F_1+F_2) & \dfrac{1+\xi}{\xi}F_1 & -\xi F_1 - (1+\xi)\dfrac{t}{4m^2}F_2 \\ \xi(F_1+F_2) & \left(\dfrac{\xi^2}{1+\xi}+\dfrac{t}{4m^2}\right)(F_1+F_2) & -\xi F_1 + \dfrac{t}{4m^2}\dfrac{1-\xi^2}{\xi}F_2 & -\left(\dfrac{\xi^2}{1+\xi}+\dfrac{t}{4m^2}\right)F_1 - \dfrac{\xi t}{4m^2}F_2 \\ \xi(F_1+F_2) & \dfrac{\xi t}{4m^2}(F_1+F_2) & -\xi F_1 + \dfrac{t}{4m^2}\dfrac{1-\xi^2}{\xi}F_2 & -\left(\xi+\dfrac{t}{4\xi m^2}\right)F_1 - \dfrac{\xi t}{4m^2}F_2 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow M_1 \\ \leftarrow M_2 \\ \leftarrow M_3 \\ \leftarrow M_4 \end{array}$$

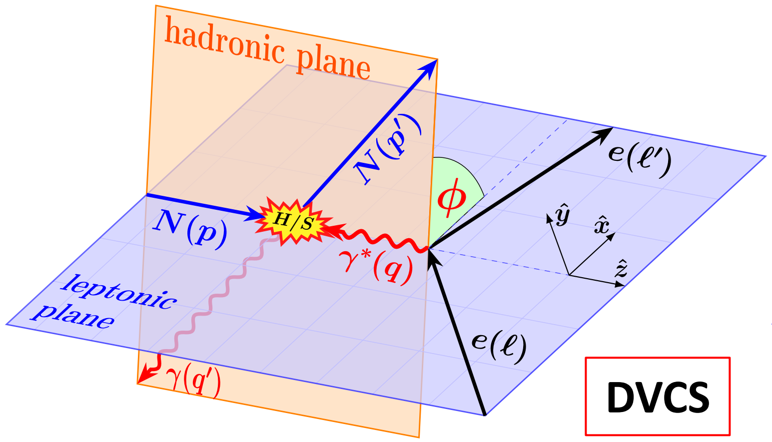
$$\Rightarrow M \cdot \begin{bmatrix} \mathcal{H} \\ \mathcal{E} \\ \widetilde{\mathcal{H}} \\ \widetilde{\mathcal{E}} \end{bmatrix} = \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \\ \hat{V}_4 \end{bmatrix} \leftarrow$$

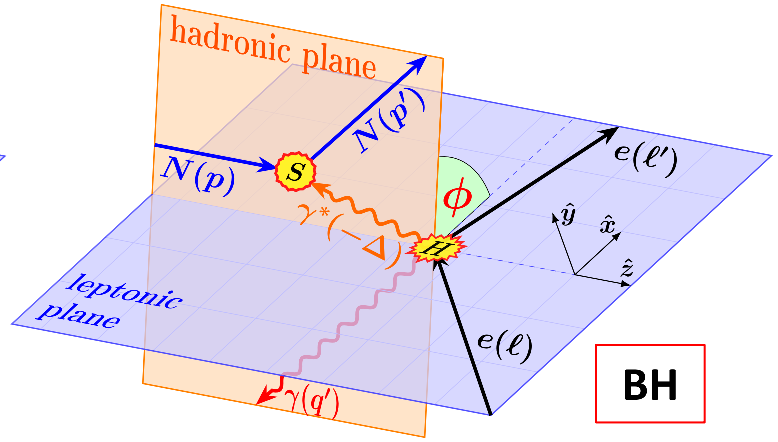
Reconstructed from experiments
(complex valued)

$\det M \neq 0$ $\Rightarrow$ **Unique solution for GPD moments!**

Summary --- SDHEP frame vs. Breit frame

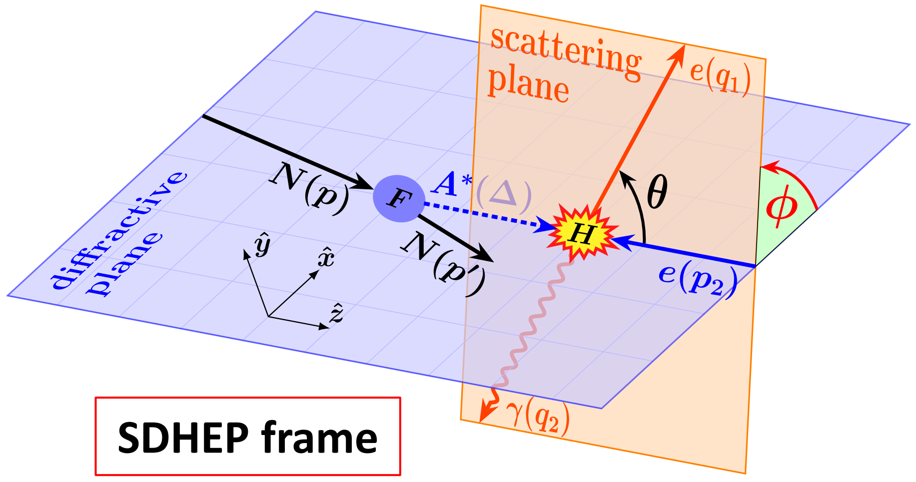
❑ Breit frame: centered around $\gamma^*(q)$





- Center around a high-virtuality state
- Mixes soft and hard scales
- Inconsistent for DVCS and BH
- Complicated ϕ modulations

❑ SDHEP frame: centered around $A^*(\Delta)$



- Center around a low-virtuality state
- Clear physical picture: scale separation
- Unifies γ^* and GPD channels coherently: $A^* = \gamma^*, [q\bar{q}], \ldots$
- Clear azimuthal distributions
- Unified frame for ALL SDHEPs

Thank you!