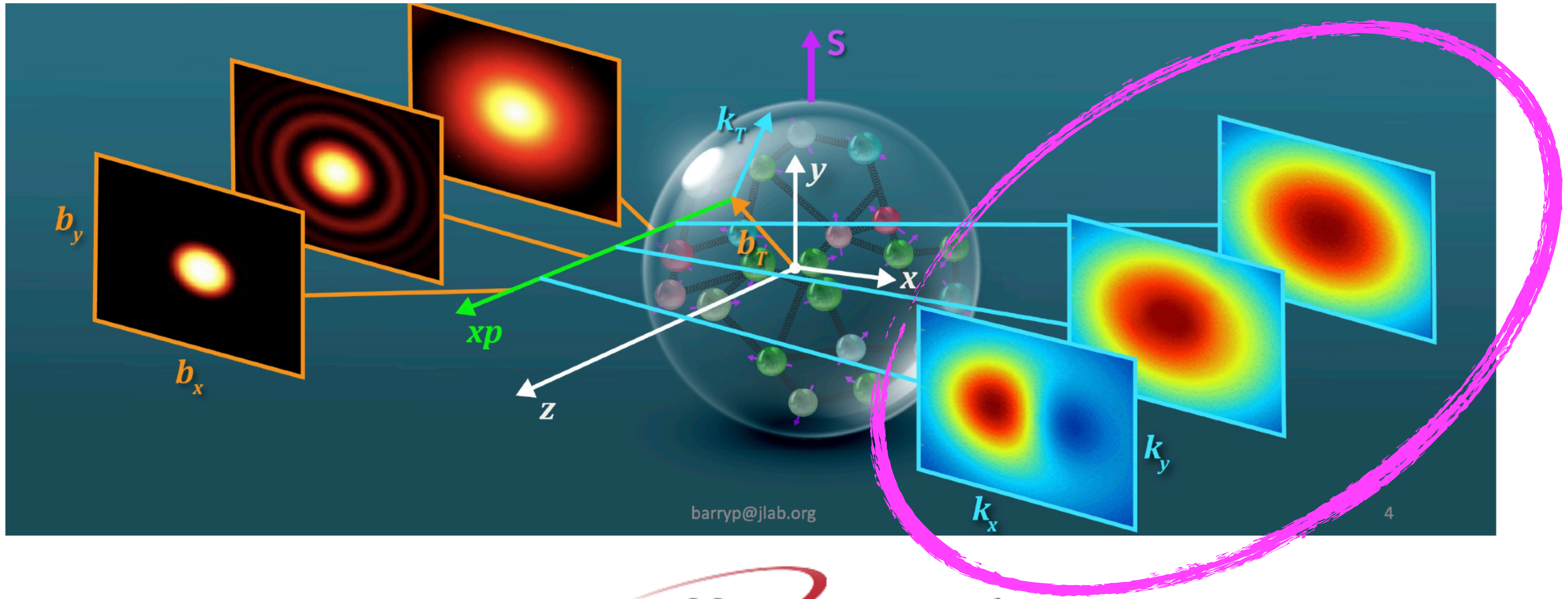
Partonic structure of hadrons from low to high $q_T \sim P_T/z$ in SIDIS

Towards Improved Hadron Femtography with Hard Exclusive Reactions 2025



Jefferson Lab

Leonard Gamberg July 30, 2025

U.S. DEPARTMENT OF ENERGY | Office of Science

- Paul Anderson, Nobuo Sato, Wally Melnitchouk, Richard Whitehill, Xiaochao Zheng (JAM Collaboration) & w/ Matteo Cerutti & Alessandro Bacchetta
- Zhongbo Kang, Owen Page (SULI), Ding-Yu Shao, John Terry, Fany Zhao

PennState Berks

Preamble

- Discuss $P_T\,(\text{or}\, q_T)$ dependence unpolarized observables in SIDIS

- SFs & observables:
  - $F_{UU,L}$ and R_{SIDIS} : of interest to EIC (& JLab TMD program 11 GeV, 22 GeV ...)
  - $F_{UU}^{\cos\phi_h}$ "Cahn Effect"

- Entails discussion factorization TMDs & collinear factorization
  - Collinear $P_T\,(\text{or}\, q_T) \sim Q$
  - TMD $q_T \ll Q$

1. Challange of Factorization at LP & NLP in the hard scale Q
2. Discussion: power counting $\&\ \exists$ a resummation formalism generaized CSS?
3. $\Longrightarrow$ Matching "low" (TMD) to "high" (collinear) $P_T\,(\text{or}\, q_T)$ spectrum

Preamble ...

- NLP "TMD-like" observables while suppressed by $(M/Q)^n$ wrt LP observables
  As sizable as LP in situations where Q not that large...e.g. kinematics of fixed-target experiments

- Understanding required for a complete description of "benchmark TMD processes" SIDIS, DY & e^+e^-...

- Of interest to probe physics of quark-gluon-quark correlations

- Experimental info SIDIS @ subleading TMDs available
  DESY/Zeus, Fermi-LAB, HERMES, COMPASS, JLab
- Opportunity for JLab TMD program EIC with its large kinematical coverage
  & for further theory progress in this area

◉ NB: Iff factorization can be established beyond "tree level" @ next to leading order
  -Global analysis in terms of NLP TMDs

Challenges of *NLP* TMDs

NLP TMD observables challenging in comparison to state-of-the-art of LP observables
Treatments in the literature mostly limited to tree-level until recently

*First SIDIS studies beyond tree level:
"Matches & Mis-matches" Bacchetta et al. JHEP 2008, Chen & Ma PLB 2017

<u>More recently</u>
A.P. Chen, J.P. Ma, PLB (2017)
Bacchetta et al. PLB 2019
MIT group, Gao, Ebert, Stewart JHEP 2022
Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209
Vladimirov, Rodini, Scimemi, et al JHEP 2021, 2022, JHEP 2023, PRD 2024, JHEP 2025
Balitsky 2023 rapidity TMD evolution

- In arXiv: e-Print:221.13209 present a systematic procedure for stress testing TMD factorization for DY & SIDIS at NLP using CSS formalism which addresses disagreements in the literature

TMD & Collinear factorization @ NLP w-w/o polarization (vast& incomplete …Literature)

F. Rivindal PLB 1972
Georgi Politzer PRL 1978
Cahn PLB 1978 (reply Georgi Politzer PRL 1978)

J.W. Qiu & G. Sterman NPB (1991)
A.Kotzinian NPB (1994
J. Levelt, P.Mulders Phys. Rev. D(1994)
R. Tangerman, P. Mulders hep-ph/9408305 [hep-ph] (1994)
P.Mulders, R. Tangerman, NPB 461(1996)
D. Boer, P. Mulders, Phys.Rev.D 57 (1998)
L. Gamberg, D. Hwang, A Metz, M. Schlegel, PLB (2006), uncanceled rapidity div. @tw3-factorization
Boer Vogelsang DY PRD 2006
Koike Nagashima Vogelsang SIDIS NPB 2006 Large P_T
A.Bacchetta, D. Boer, M. Diehl, P. Mulders JHEP (2008) factorization at NLP consistency checks on matching
A.P. Chen, J.P. Ma, Phys. Lett. B 768 (2017)
I. Feige, D.W. Kolodrubetz, I. Moult, I.W. Stewart, J. High Energy Phys. 11 (2017)
I. Balitsky, A. Tarasov, J. High Energy Phys. 07 (2017)
I. Balitsky, A. Tarasov, J. High Energy Phys. 05 (2018)
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, JHEP 12 (2018)
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, JHEP 04 (2019)
Moult, I.W. Stewart, G. Vita, arXiv:1905 .07411, 201
A. Bacchetta, G.Bozzi, Echevarria, Pisano, Prokudin, Radici, Physics Letters B 797 (2019)
A.Vladimirov Moos, Scimemi, & S.Rodini JHEP 2022
M. Ebert A. Gao I. Stewart JHEP 06 (2022)
S. Rodini, A. Vladimirov JHEP 08 (2022)
L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao arXiv: e-Print:221.13209 (2022)… in prep 2025
I.Balitsky, JHEP 03 (2023) and 2024
HSO Aslan,Boglione, Gonzalez-Hernandez, Rainaldi, Rogers, Simonelli PRD 2024
Transverse spin-dependence Qui Sterman collinear higher twist
X. Ji, J.W. Qiu, W. Vogelsang, and F. Yuan, PRL(2006), PLB 638 (2006),PRD (2006)

Advertisement

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

L. Gamberg, A. Metz, I. Stewart

w/ Tom Mehen

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e-Print:2304.03302 [hep-ph]

Focus on two NLP unpol. observables

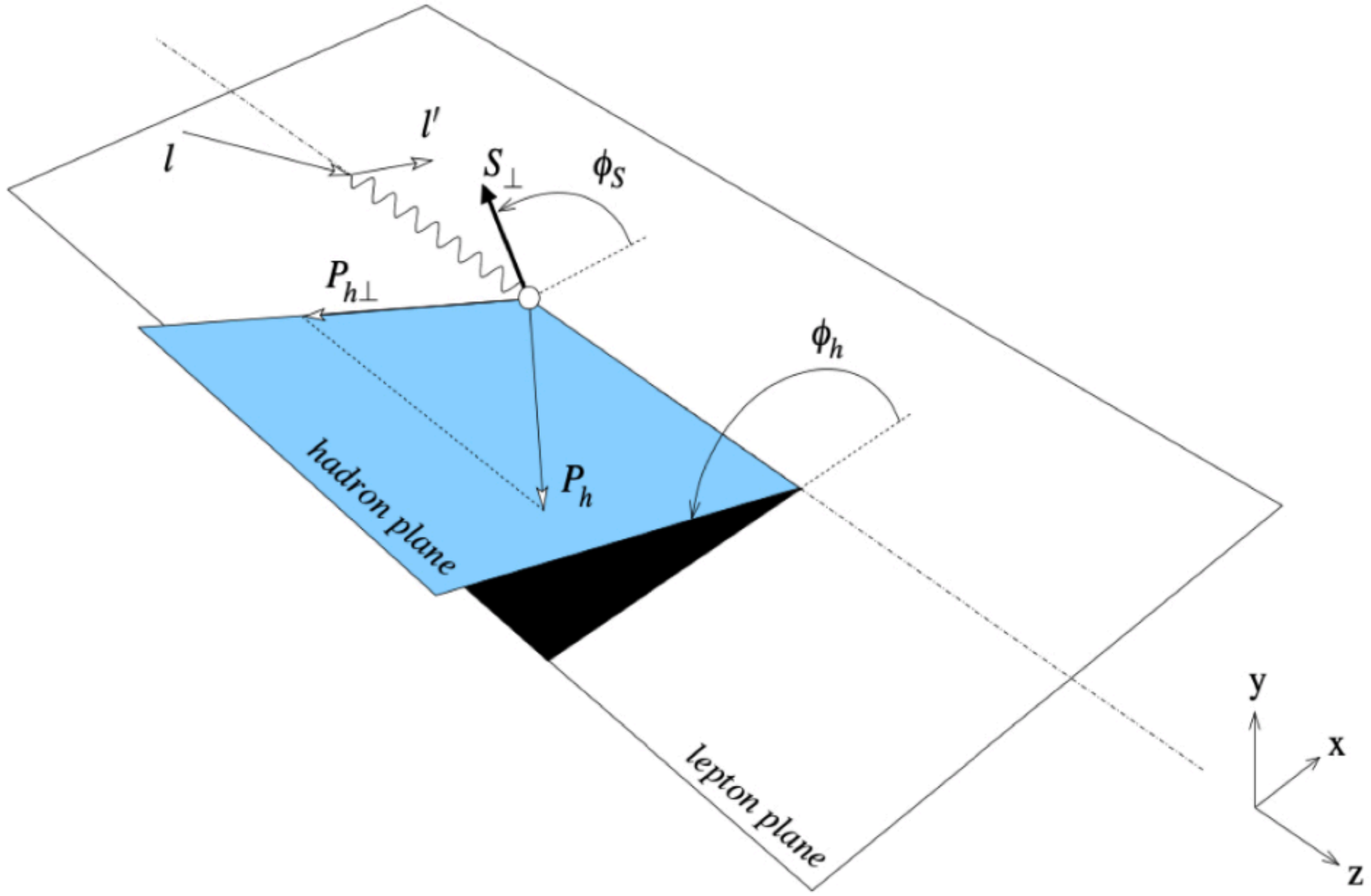
- $F_{UU,L}$ & $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$

Feynman "Photon-Hadron Phys." 1972, Ravndal, PLB 1973

- $F_{UU}^{\cos\phi_h} \rightarrow \langle \cos\phi_h \rangle$

Georgi & Cahn, PRL 1978, PLB 1978

  - Critique of perturbative QCD calculation of azimuthal dependence in SIDIS
  - Emphasize importance intrinsic k_T the early days/birth of "TMD physics"



R_{SIDIS} : $\exists$ TMD $P_T \sim p_\perp$ & or large $P_T \sim Q$ (or $q_T \approx P_T/z$)

- Interesting opportunity to study hadron structure, age old topic:
  Ratio of longitudinal and transverse $\gamma^*_{T/L}$ cross section (SIDIS)
  classic power supressed $(M/Q)^2$ in DIS ...

- suggested extension beyond DIS: Feynman 1972 "Photon Hadron Interactions"
  & Ravndal PLB 1973, Cahn 1989 PRD 1989:

$$R = \frac{\sigma_L}{\sigma_T} = \frac{4\left(m^2 + \langle p_\perp^2 \rangle\right)}{Q^2} \overset{?}{\Longrightarrow} \frac{F_L}{F_T} \overset{?}{\sim} \frac{F_{UU,L}}{F_{UU,T}}$$

$\langle p_\perp^2 \rangle$ intrinsic parton transverse momentum

R_{SIDIS} : ∃ TMD $P_T \sim p_\perp$ & or large $P_T \sim Q$ (or $q_T \approx P_T/z$)

- Often assumed $R_{DIS} \approx R_{SIDIS}$ however is independent of z, P_T and φ
- Hardly any measurements of $F_{UU,L}$
- In TMD pheno @ low transverse momentum often assumed negligible

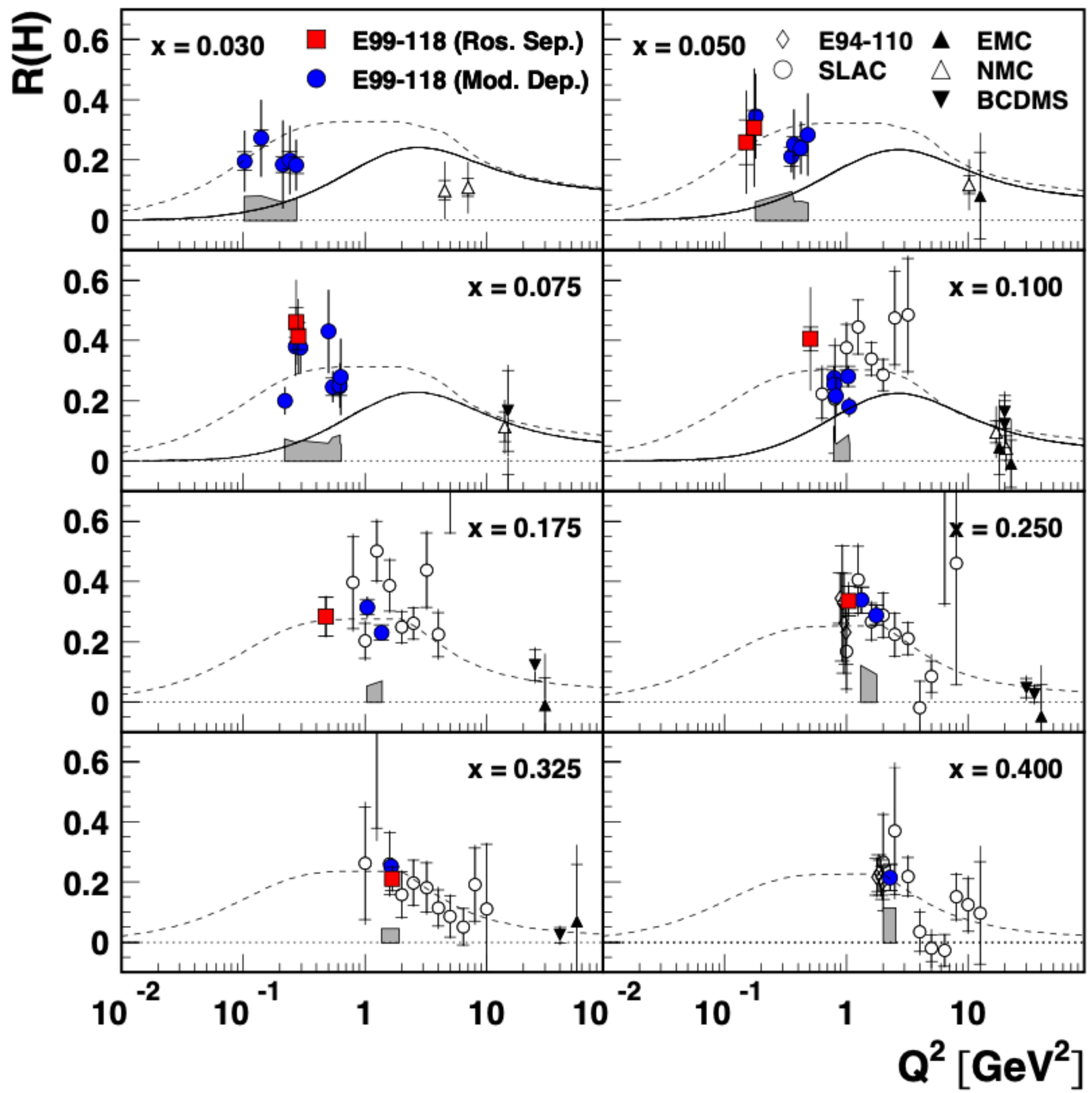
20^{th} century interpretation collinear DIS physics

Recall inclusive cross section in terms of σ_T and σ_L
or structure functions $F_L(F_2\,\&\,F_1)\;\&\;F_T(F_1)$,
via absorption of longitudinal vs. transverse photons

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{F_T} = \frac{1}{2xF_1}\left\{ F_2\left(1+\frac{4x^2M^2}{Q^2}\right) - 2xF_1 \right\}$$

- zero in the scaling limit $\lim_{Q^2\to\infty} R \approx \frac{4x^2M^2}{Q^2} \to 0\,,$
- non-trivial pQCD process $\{F_2 - 2xF_1\} \sim \frac{\alpha_s(Q)}{2\pi} C_2(F)\, x$

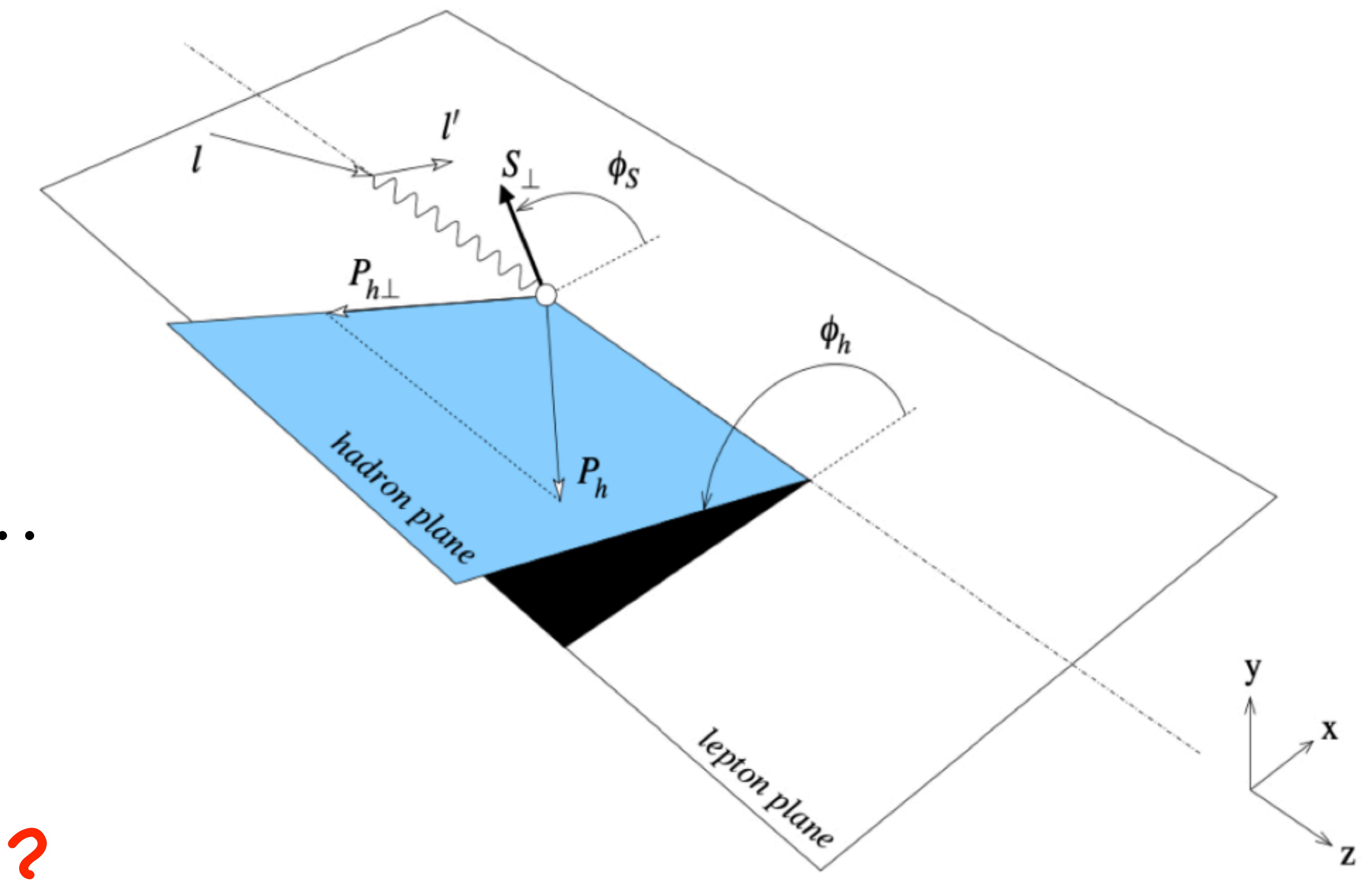
JLab exp. (E99-118) Prl 2007



Comparison of values of $R(x, Q^2)$ for hydrogen from JLab exp. (E99-118) to results of other exps.

21^{st} century interpretation: from DIS to SIDIS

$$R = \frac{\sigma_L}{\sigma_T} \longrightarrow \frac{F_{UU,L}}{F_{UU,T}}$$



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}F_{UU}^{\cos\phi} \ldots\right.$$

∃ TMD observable for $F_{UU,L}$?

$$F_{UU,T} = C[f_1 D_1]\,, \qquad F_{UU,L} = C[\,.\,?\,.\,]$$

$$\mathcal{C}\big[wfD\big] = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{q}_T\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

Tree level TMD factorization Mulders NPB 1996 Bacchetta et al. JHEP 2008

Review SIDIS cross section

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right. + \dots$$

Tree level TMD factorization Mulders NPB 1996 Bacchetta et al. JHEP 2008

$$F_{UU,T} = C[f_1 D_1]\ ,$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\,e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{q}_T\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

SIDIS cross section

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\,\frac{y}{2\,(1-\varepsilon)}\,\Big\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

Leading Quark TMDPDFs

Nucleon Spin | Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^\perp,\ g^\perp$	$e\ ,\ h$
	L	$f_L^\perp\ ,\ g_L^\perp$	$e_L\ ,\ h_L$
	T	$f_T,\ f_T^\perp,\ g_T,\ g_T^\perp$	$e_T,\ e_T^\perp,\ h_T,\ h_T^\perp$

Reminder: recall Quark correlator LP, NLP, NNLP

$$\Phi(x,p_T) = \frac{1}{2}\left\{ f_1 \not{n} + f_{1T}^{\perp} \frac{S_{T i}\epsilon^{ij} p_{T j}}{M} \not{n} + \ldots \right\}$$

$$+ \frac{M}{2P^+}\left\{ e + \ldots \qquad + f^{\perp} \frac{\not{p}_T}{M} \ldots \right\}$$

$$+ \frac{M^2}{(2P^+)^2}\left\{ \ldots \qquad ?? \qquad \ldots \right\}$$

n.b. "tree level" Mulders Tangerman 1996 Bacchetta 2007

Context TMD Correlator at tree level NNLP "twist 4"

Goeke, Metz, Schlegel PLB 2005

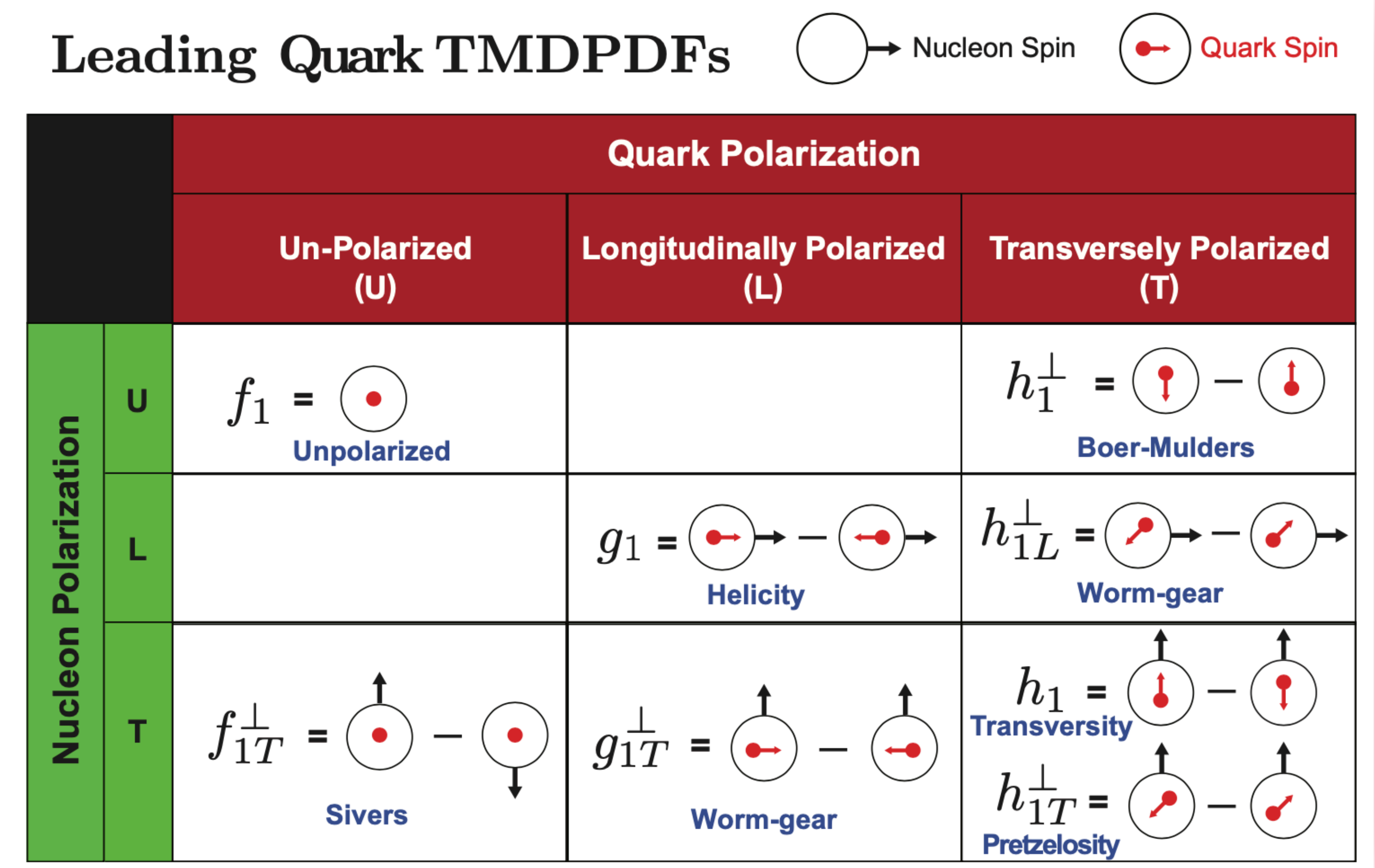
NNLP:

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[f_3(x,\vec{k}_T^2) - \frac{\varepsilon_T^{ij}k_{Ti}S_{Tj}}{M} f_{3T}^{\perp}(x,\vec{k}_T^2)\right]$$

Correlator at tree level @ "twist" 4

previously of academic interest

factorization "at best" unexplored



Subleading Quark TMDPDFs

Nucleon Polarization	Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
U	$f^\perp, g^\perp$	$e\,,\ h$
L	$f_L^\perp, g_L^\perp$	e_L, h_L
T	$f_T, f_T^\perp, g_T, g_T^\perp$	$e_T, e_T^\perp, h_T, h_T^\perp$

Recall *LP* & *NLP* :

$$\Phi^{[\gamma^+]} = f_1(x,\vec{k}_T^2) - \frac{\varepsilon_T^{ij}k_{Ti}S_{Tj}}{M} f_{1T}^{\perp}(x,\vec{k}_T^2),$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+}\left[\frac{k_T^i}{M}\left(f^\perp(x,\vec{k}_T^2) - \frac{\varepsilon_T^{jk}k_{Tj}S_{Tk}}{M} f_T^{\perp\prime}(x,\vec{k}_T^2)\right) + \ \ldots\right.$$

Context TMD Correlator at tree level "twist 4"

NNLP:

NNLP: some discussion in Bacchetta et al.
Matches and mismatches JHEP 2008
& recent discussion w/ M. Cerruti

$$F_{UU,L} = \frac{4M^2}{Q^2}\,\mathcal{C}\left[\frac{\boldsymbol{p}_T^2}{M^2}\,f_1 D_1\right]$$

Correlator at tree level @ "twist" 4

previously of academic interest

factorization is at best unexplored

Leading Quark TMDPDFs — Nucleon Spin, Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
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Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^\perp,\ g^\perp$	$e\ ,\ h$
	L	$f_L^\perp,\ g_L^\perp$	$e_L,\ h_L$
	T	$f_T,\ f_T^\perp,\ g_T,\ g_T^\perp$	$e_T,\ e_T^\perp,\ h_T,\ h_T^\perp$

R_{SIDIS} NLP TMDs ?

Bacchetta & Cerruti MAP
Eur. Phys. J. A (2024) 60:173

R_{sidis} esitimate MAP collaboration
sizable contribution up to 20%

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} \dots\right.$$

- ε ratio of longitudinal and transverse photon flux

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2 y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2 y^2}, \qquad \gamma^2 \equiv \frac{4M^2x^2}{Q^2}$$

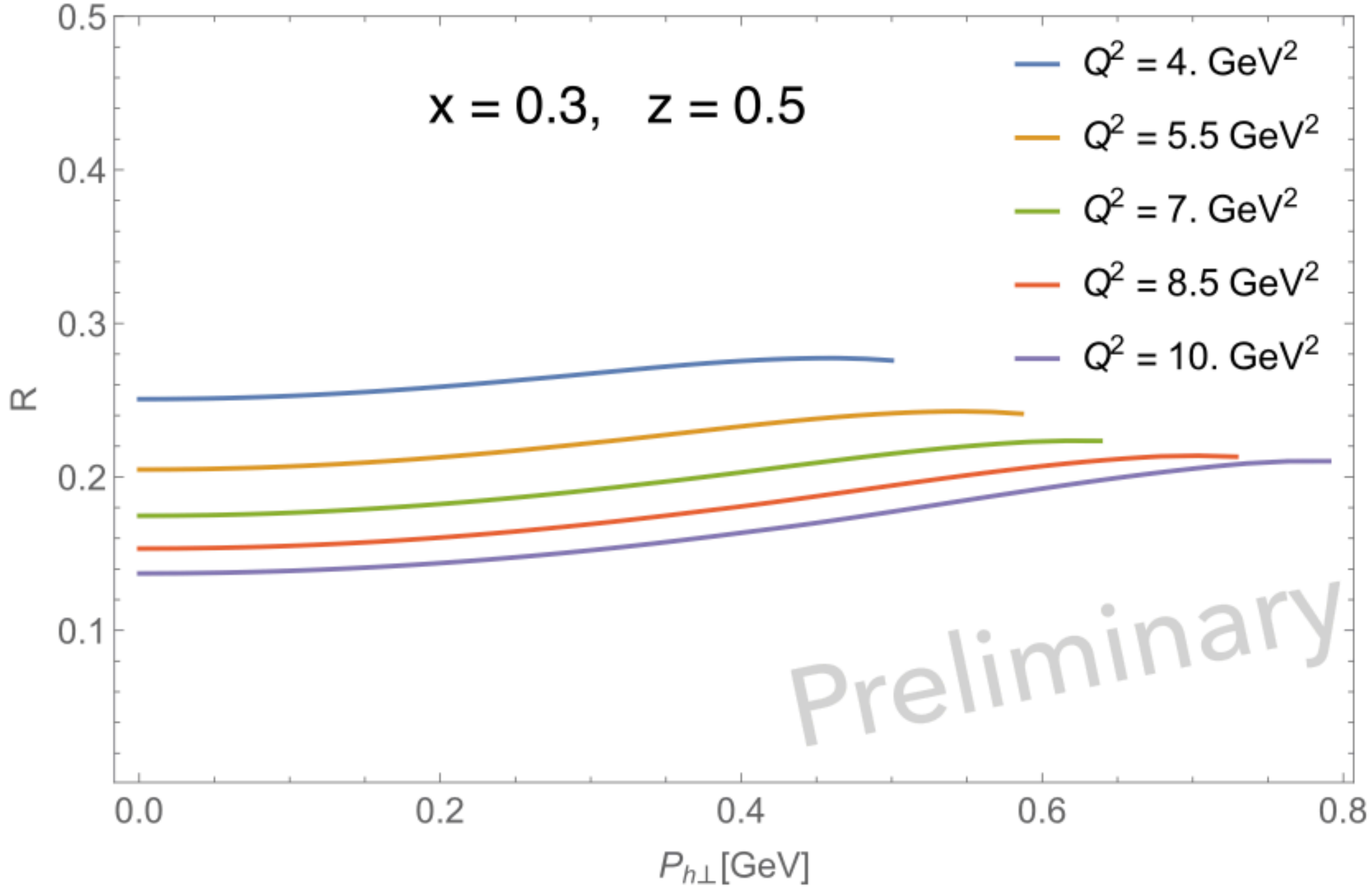


Fig. 18 Estimate of $R_{\text{SIDIS}} = F_{UU,L}/F_{UU,T}$ versus the hadron transverse momentum $P_T\,(P_{hT})$ at fixed values of x and z and for different values of Q^2, compatible with JLab22 kinematics, using MAP22 TMD analysis [134]

- Findings demonstrate $F_{UU,L}$ can't be ignored
- substantial & essential for an accurate interpretation of $F_{UU,T}$
- which is associated with LP TMDs
- SIDIS normalization in TMD factorization ?

What is the physics here ?

Take a Granular: look consider both small & large $q_T \approx \frac{P_{hT}}{z}$ structure functions

- $F_{UU,T}(x, z, q_T, Q)$ &
- $F_{UU,L}(x, z, q_T, Q)$

Key to understanding predictive power in SIDIS & global analysis
is whether the low & high q_T physics due to common mechanism ?

- Explore thru factorization low & high q_T
  If so, could have a match

i) consider power counting: <u>necessary conditions for match</u>
<u>ii</u>) direct calculation: <u>sufficient conditions for match</u>

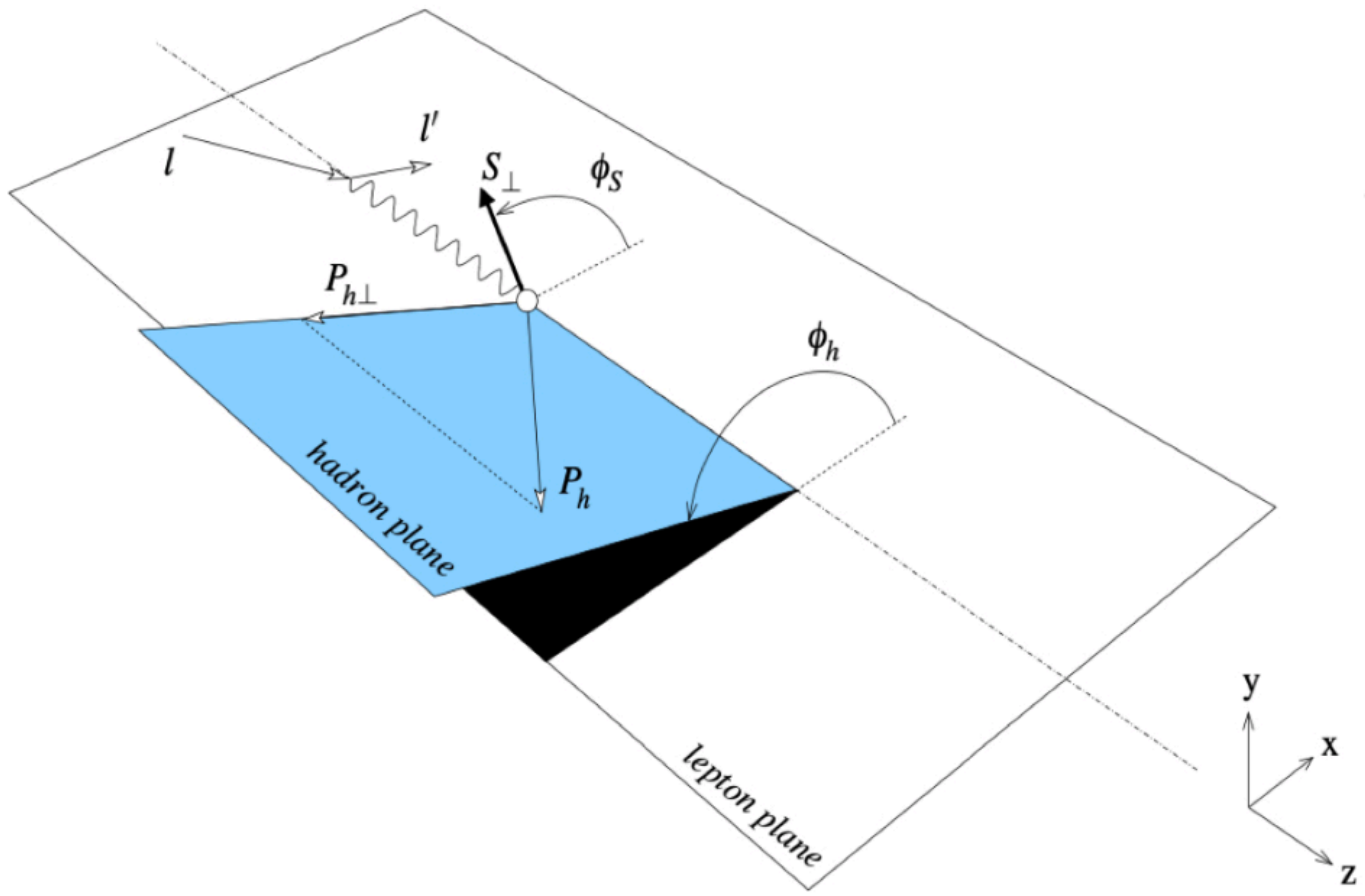
e.g. Allows simulataneous fits of colliner pdfs & TMDs

$$\tilde{f}(x, b_T; \mu, \zeta) = [C \otimes f](x, b_T; \mu_0, \zeta_0) \times e^{S_{\text{evo}}(b_T; \mu, \mu_0, \zeta, \zeta_0)} f_{\text{NP}}(x, b_T)$$

SIDIS: 2 theorems: TMD Factorization & Collinear $P_\perp$ Factorization

$nb\,,q_T = P_{h\perp}/z \equiv P_T/z$

- TMD: applicable $\Lambda_{QCD} \sim P_{h\perp} \ll Q$ Collinear: applicable $P_{h\perp} \sim Q \gg \Lambda_{QCD}$
- $P_{h\perp} \sim k_T$ or p_T intrinsic parton transverse momentum CS described via TMDs
- $P_{h\perp} \gg k_T$ or p_T generate transverse momentum in the final state as perturbative radiation & non-perturbative structure given via collinear pdfs & FFs



$$E'E_h \frac{d\sigma_{ep\to e'hX}}{d^3l'd^3P_h} \approx \hat{\sigma}_{eq\to e'q'} \otimes f_1 \widetilde{\otimes} D_{h/q'}.$$

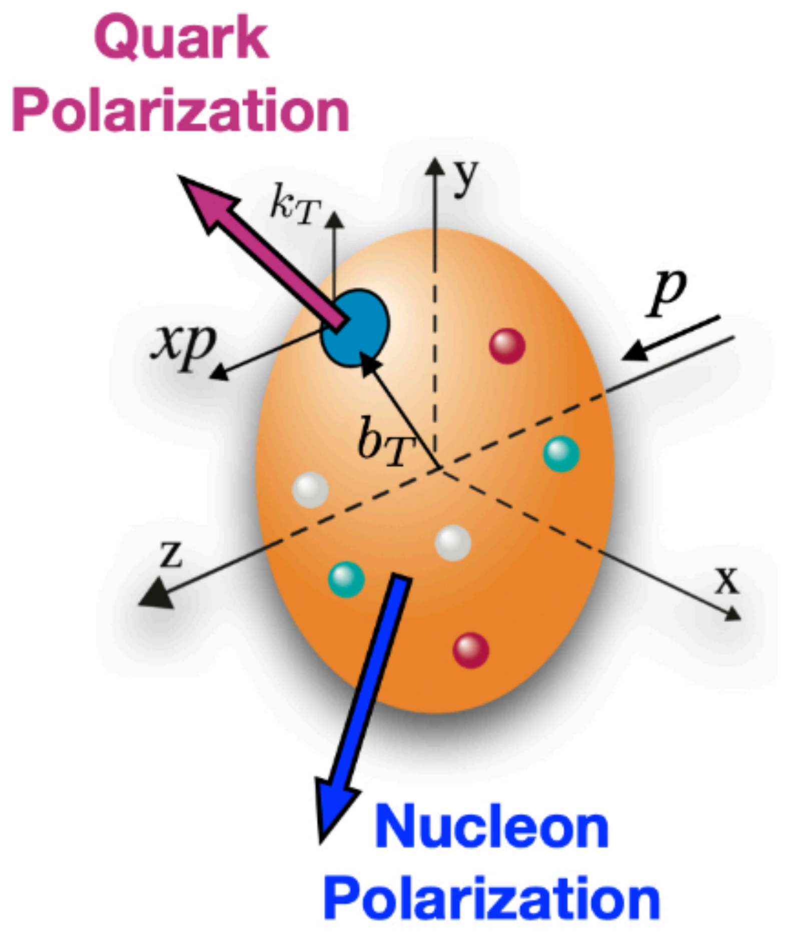
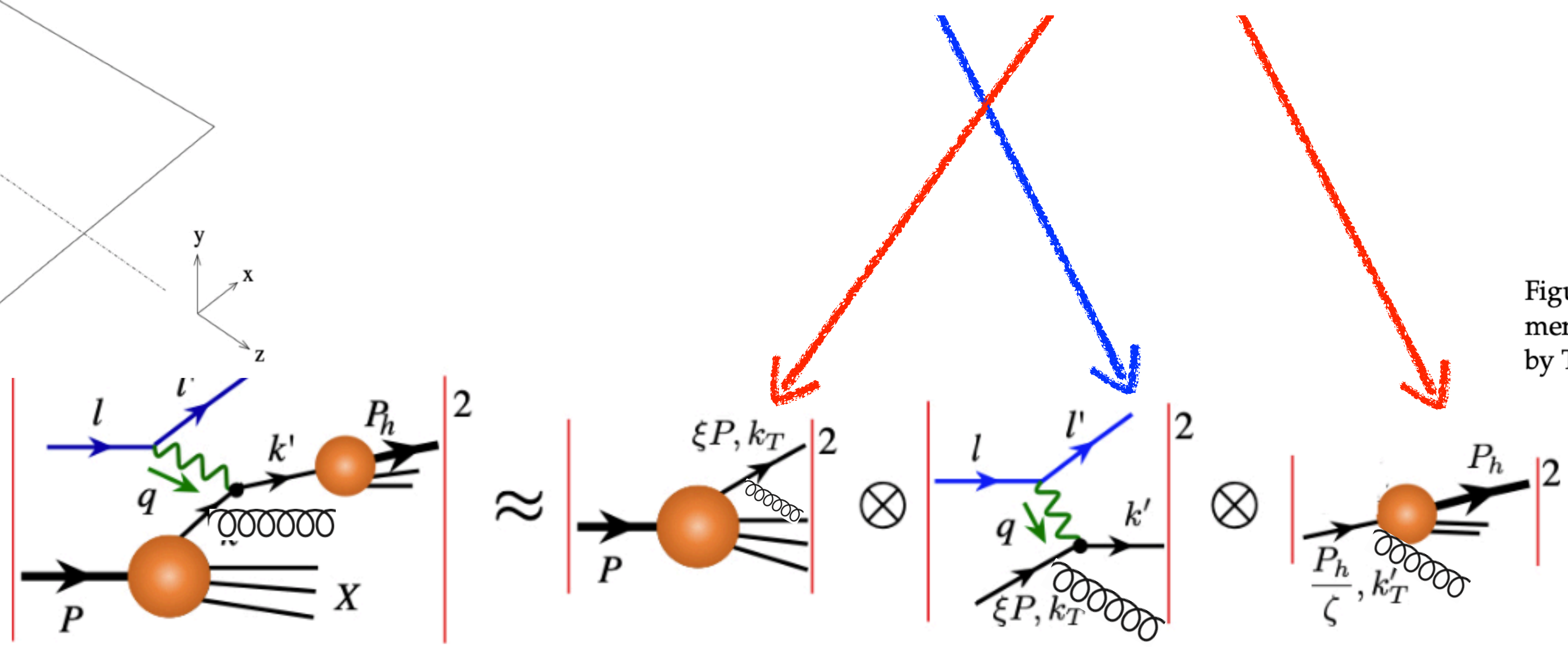


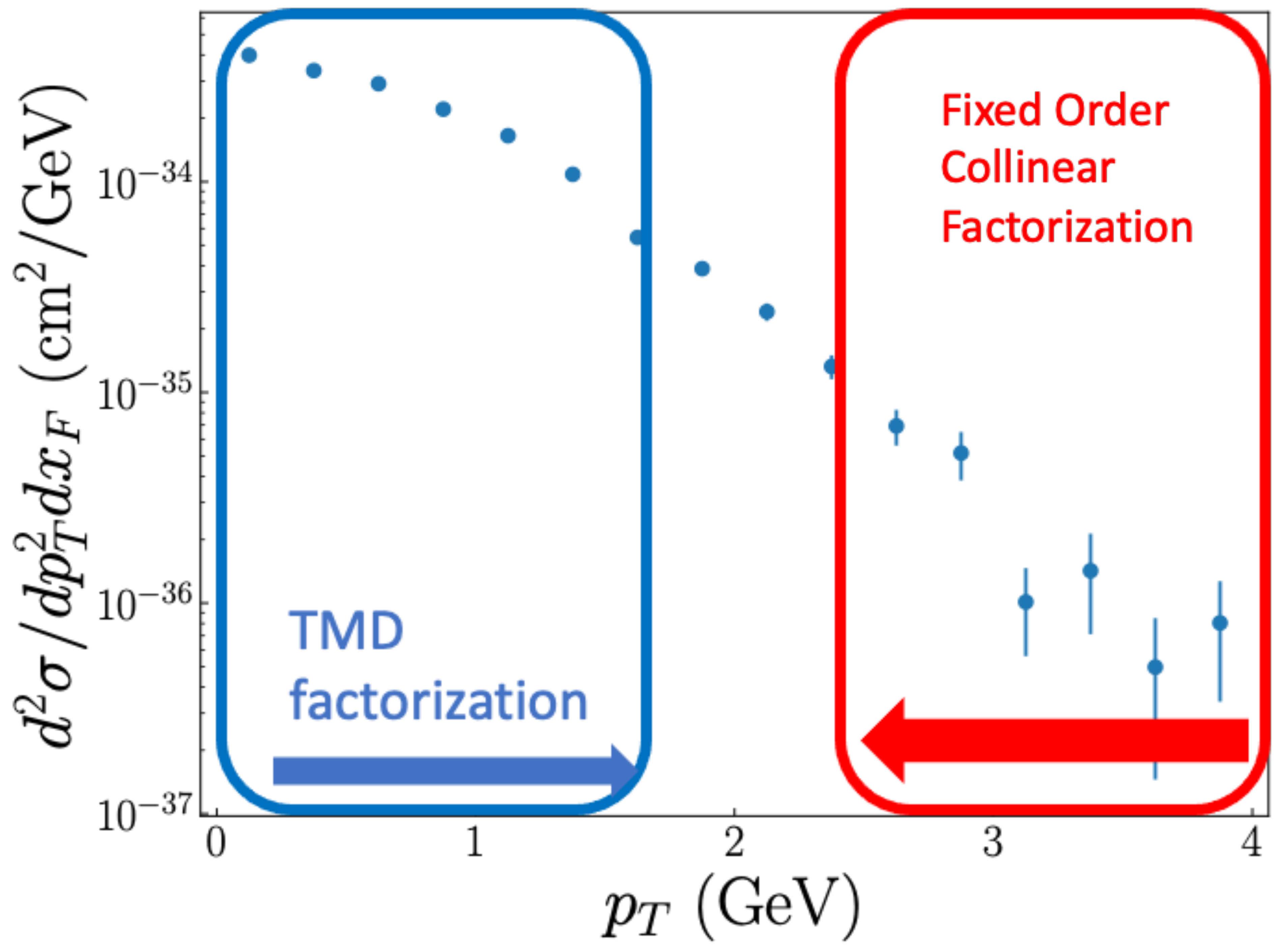
Figure 1.1: Illustration of the momentum and spin variables probed by TMD parton distributions.

$\sigma_{\rm SIDIS} \propto$



Implies matching of TMD & large $q_T \approx P_{hT}/z$ Rigorous proof @ Leading power

- **Factorization & Matching unpolarized** Collins Soper Sterman NPB (1985), Bozzi Catani Floraian Grazzini NPB (2006), Bacchetta, Boer, Diehl, Mulders JHEP (2008) Collins, Gamberg, Prokudin, Rogers, Sato, Wang PRD (2016) ....
- N.B. Transverse polarization Ji, Qiu, Vogelsang Yuan PRL (2006); PRD (2006)



- **<u>Cross section in terms of different "regions"</u>**
- W valid for $q_T \sim k_T \ll Q$ TMD factorization
- FO valid for $k_T \ll p_T \sim Q$ Collinear factorization
- AY subtracts d.c. & in principle,

  $AY \to W,\ p_T \to \infty$ and $AY \to FO, p_T \to 0$
- $Y \equiv \to FO - AY$

$$\frac{d\sigma(m \lesssim p_T \lesssim Q, Q)}{dy\, dq^2 d^2 p_T} = W(p_T, Q) - AY(p_T, Q) + FO(p_T, Q) + \mathcal{O}\left(\frac{M}{Q}\right)^c$$

i) consider power counting: necessary conditions for match

Power counting in regions

Bacchetta et al. JHEP 2008

Power counting $F_{XY,Z}$ → $F_{UU,L}$ & $F_{UU,T}$ ...

- Low TMD factorization

$M \sim q_T \ll Q$ $\quad F_{UU,T} \sim 1/M^2$ **Leading power**

$F_{UU,L} \sim 1/Q^2$ **Sub sub leading**

- High collinear factorization

$M \ll q_T \sim Q$ $\quad F_{UU,T} = \frac{1}{Q^2} \alpha_s \mathscr{F}\left[f_1 D_1\right]$ **Leading power**

$F_{UU,L} = \frac{1}{Q^2} \alpha_s \mathscr{F}\left[f_1 D_1\right]$ **Leading power**

Power counting in "intermediate region

Power counting $F_{UU,L}$

- High collinear factorization $\longrightarrow M \ll q_T \ll Q \longleftarrow$ - Low TMD factorization

$$M^2 F_{UU,L}^{M \ll q_T \ll Q} \sim \alpha_s \frac{q_T^2}{Q^2} \frac{M^2}{q_T^2} \mathcal{F}[f_1 D_1]$$

$$M^2 F_{UU,L}^{M \ll q_T \ll Q} \sim \alpha_s \frac{M^2}{q_T^2} \frac{q_T^2}{Q^2} \mathcal{F}[f_1 D_1]$$

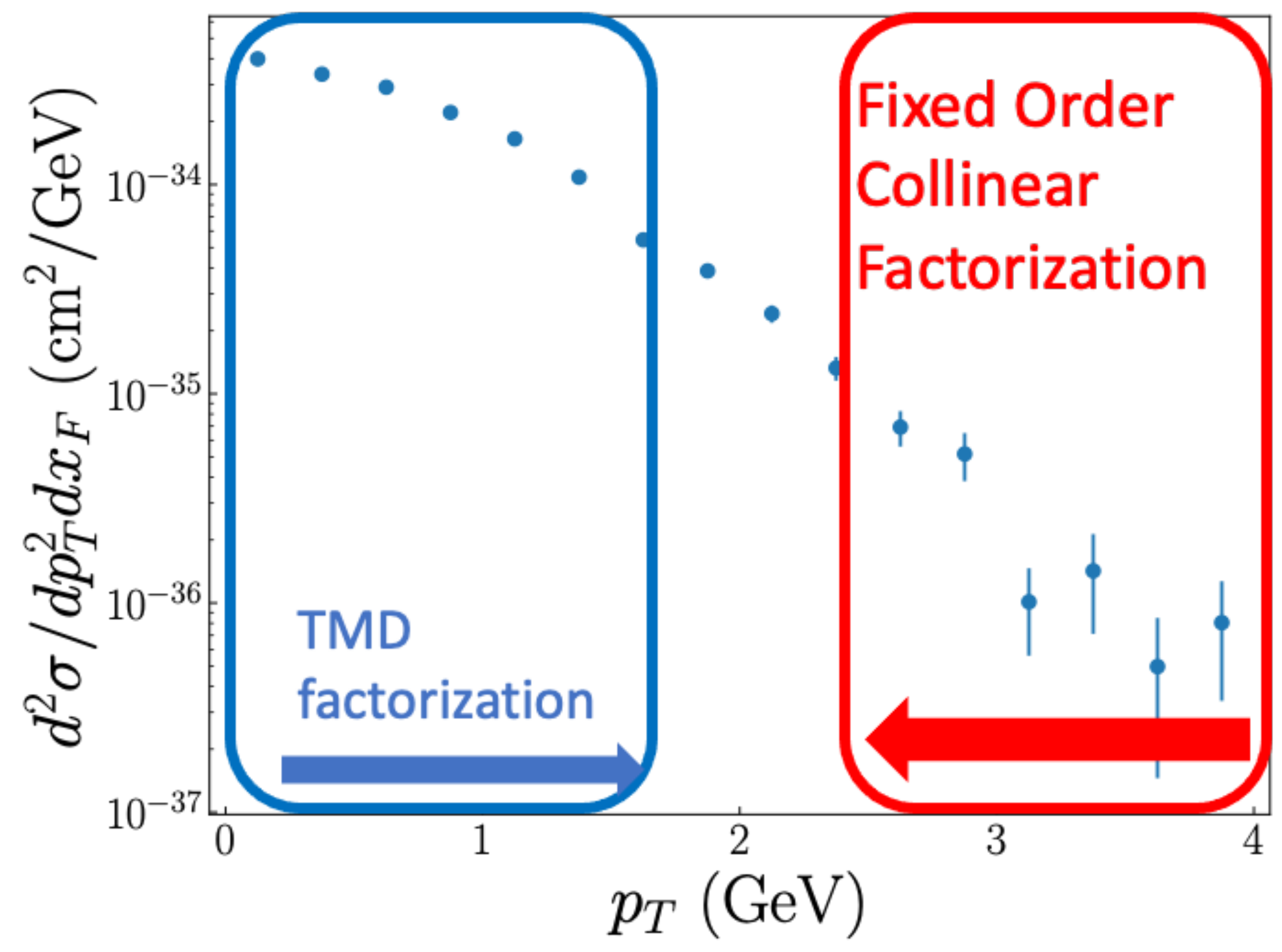
$\exists$ Match

$$F_{UU,L} \sim \frac{1}{Q^2} \alpha_s \mathcal{F}[f_1 D_1]$$

ii) direct calculation: sufficient conditions for match

"Mis"-Matches Factorization @ sub-leading power

- **Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Sterman NPB 1985**



$$\sigma_{UU,L}^{\cos\phi} \stackrel{?}{\to} W(p_T, Q) + Y(p_T, Q) + \mathcal{O}\left(\frac{M}{Q}\right)^c$$

- **Bacchetta, Boer, Diehl, Mulders JHEP (2008) Cahn Effect:**
  Mis-match/inconsistency breakdown of TMD factorization at NLP?

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)} \cos\phi + \int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

Attempt to match the high-q_T result for $F_{UU}^{\cos\phi_h}$ to low-q_T result at intermediate q_T consistency check on factorization framework that extends CSS to NLP

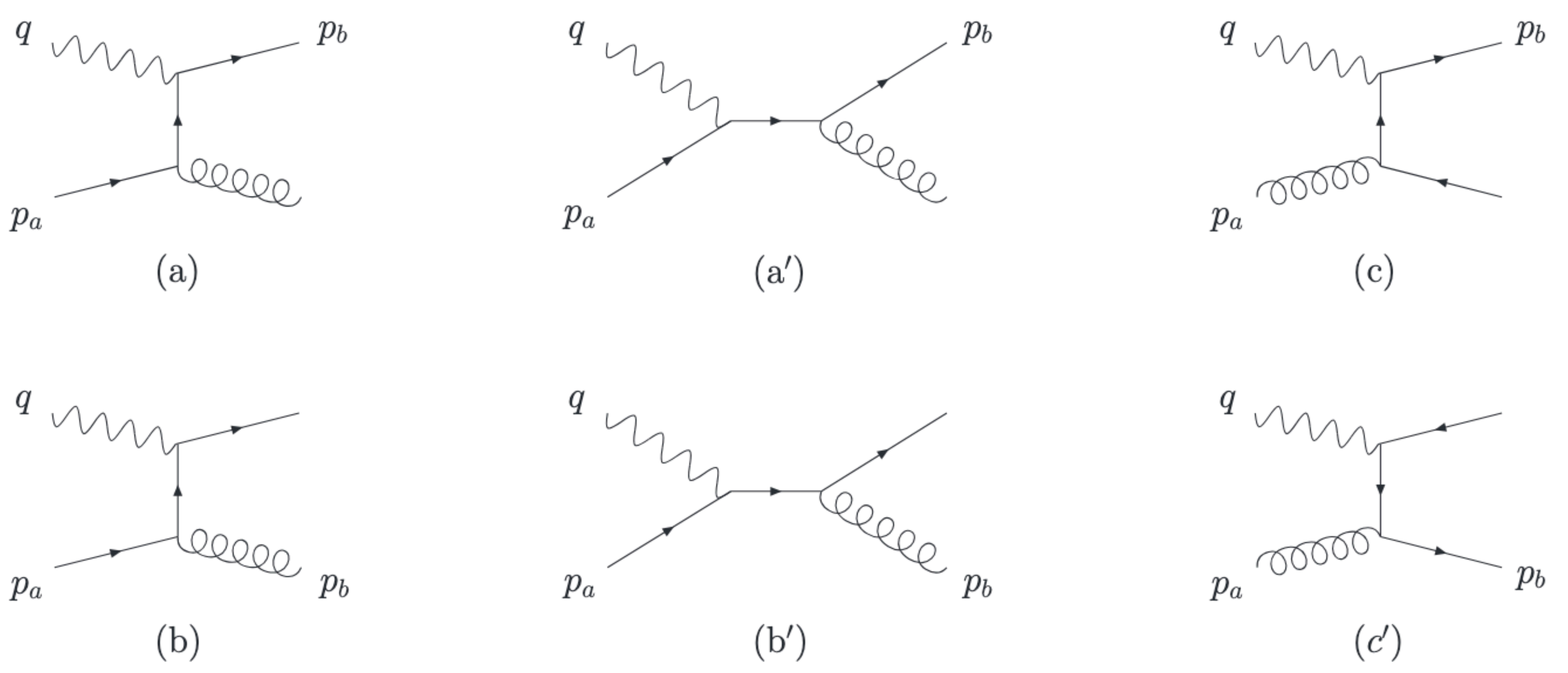
Consider $F_{UU,L}$ & R_{SIDIS} @ large P_T

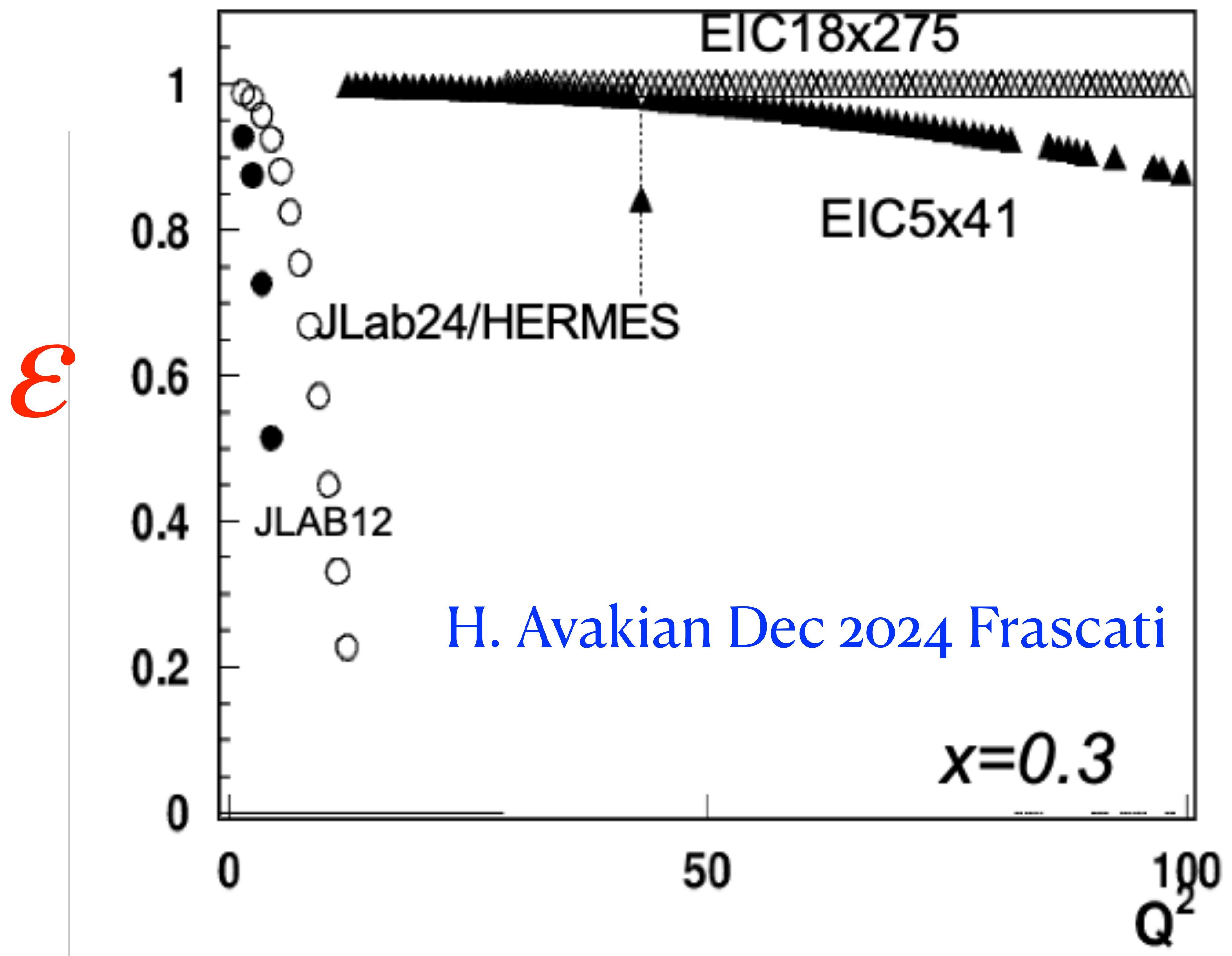
- @ large P_T , $F_{UU,L} \sim F_{UU}^{\cos 2\phi_h}$ — see Bacchetta et al. JHEP 2008 "Matches & Mis-matches":

  in principle hard gluon raditation — "collinear P_T factorization applies CSS 1985

  Catani et al. 1997-2015, Nadolsky, Vogelsang Koike NPB 2005 … many others

$$\frac{d\sigma}{dxdydzd\phi_h dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\, F_{UU,L} + \ldots\right.$$





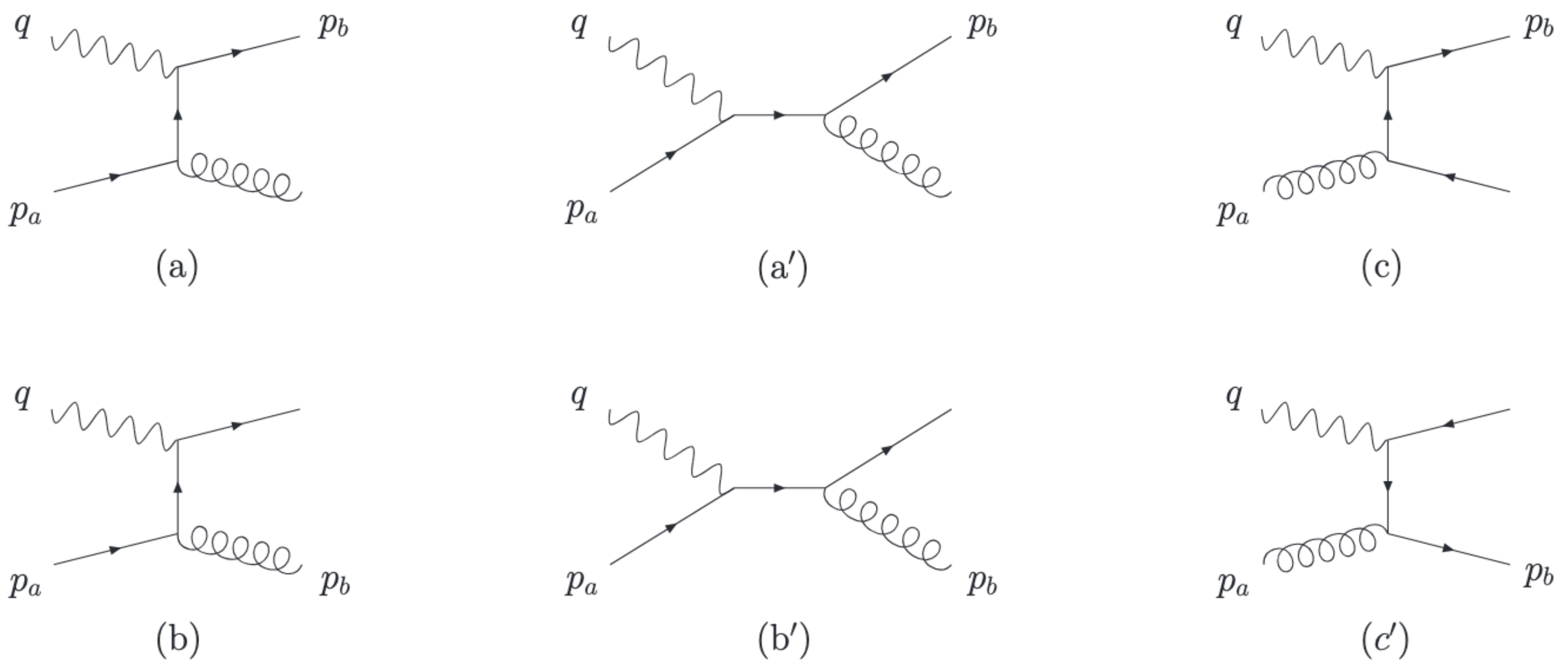
Large $q_T \sim P_{hT}/z$ well established factorization Leading POWER

$M \ll q_T \sim Q$

e.g.

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\, e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$

$$\times\left[f_1^a\!\left(\frac{x}{\hat{x}}\right)D_1^a\!\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right)D_1^g\!\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right)D_1^a\!\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$



- $\gamma^* q \to qg$ $\quad C_{UU}^{\cos 2\phi_h} = 4C_F\,\hat{x}\hat{z},$
- $\gamma^* q \to gq$ $\quad C_{UU}^{\cos 2\phi_h} = 4C_F\,\hat{x}\,(1-\hat{z}),$
- $\gamma^* g \to q\bar{q}$ $\quad C_{UU}^{\cos 2\phi_h} = 8T_R\,\hat{x}\,(1-\hat{x}),$

Large → intermediate → low q_T regions

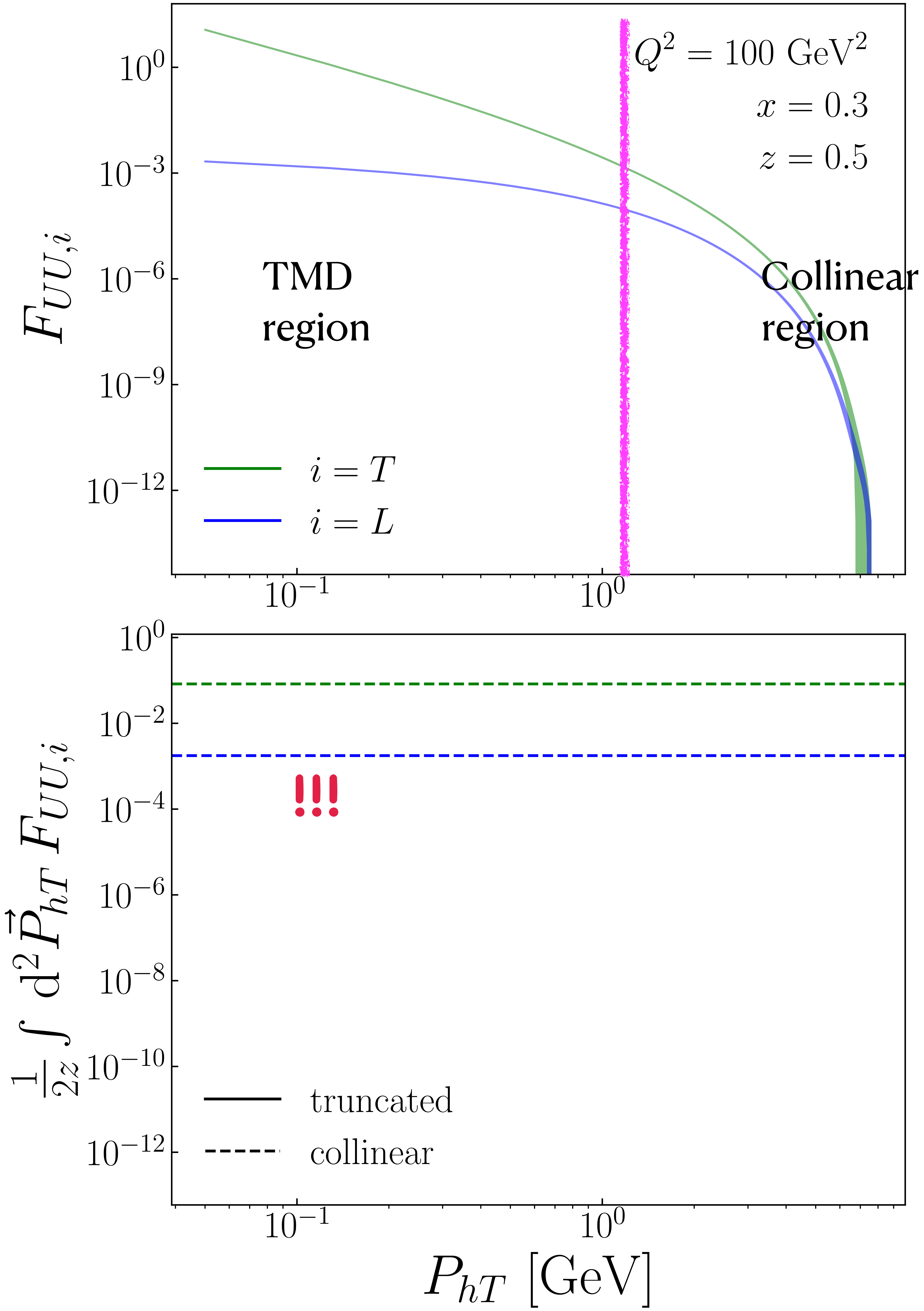
- $F_{UU,T}(x, z, q_T, Q)$ & $F_{UU,L}(x, z, q_T, Q)$

- **Collinear SIDIS vs. truncated moments**

$$\int_{P_{hT\,\min}^2/z^2}^{q_{T\,\max}^2} F(x, z, Q^2, P_{hT})$$

- $F_{UU,L}$ truncated moment converges "P_T **integrable**"
- $F_{UU,T}$ truncated moment as expected **diverges**

- Small TMD contribution ? Small power corrections ?



Reminder Power behavior $F_{UU,L}$ & $F_{UU,T}$ in AY region, $M \ll q_T \ll Q$

$M \ll q_T \sim Q$

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\, e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$

$$\times\left[f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^g\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$

Asymptotic region

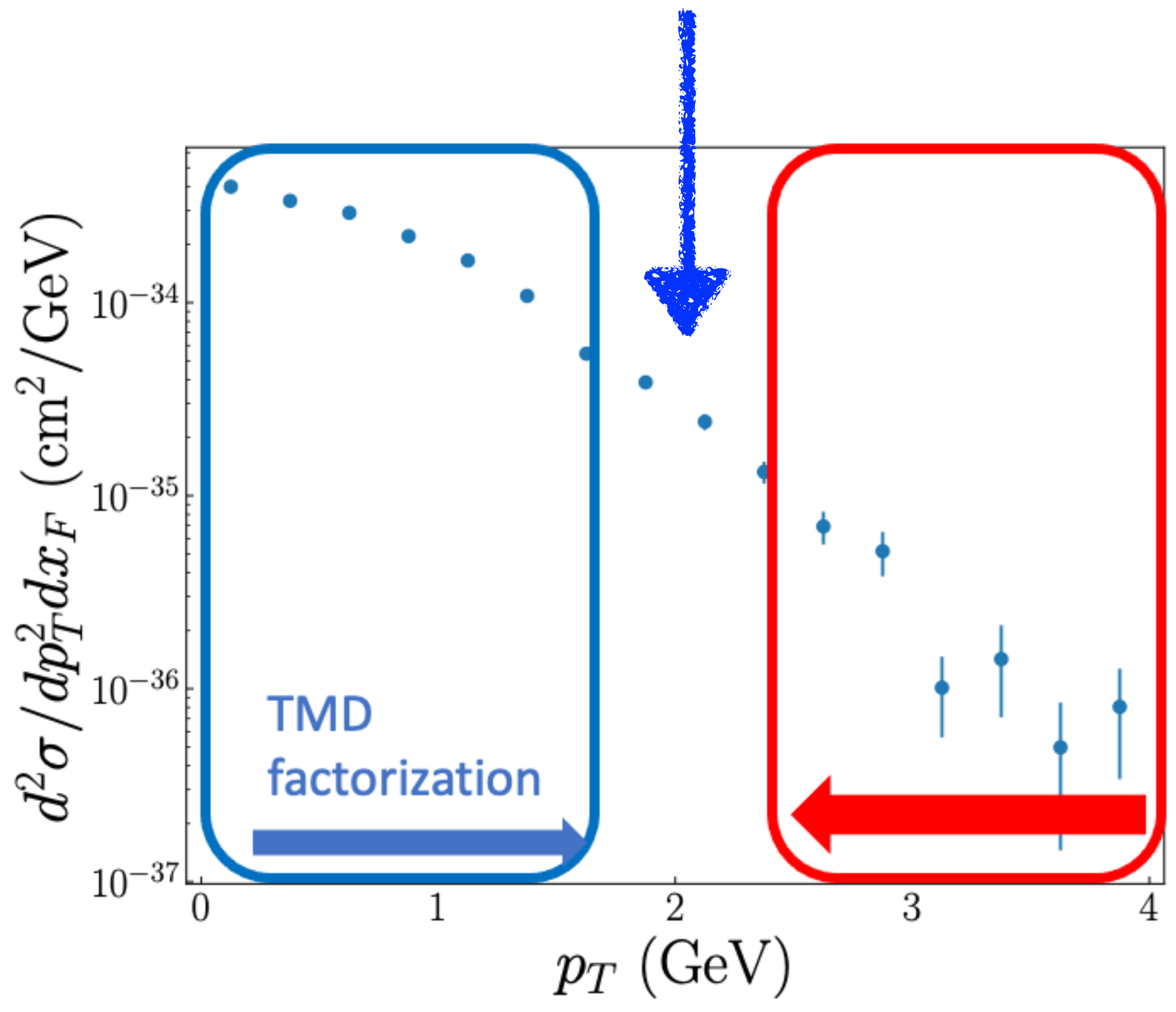
$M \ll q_T \ll Q$

$$F_{UU,T} = \frac{1}{q_T^2}\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\, e_a^2 \Big[f_1^a(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a\otimes P_{qq} + D_1^g \otimes P_{gq}\big)(z)$$

$$+ \big(P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g\big)(x)\, D_1^a(z)\Big]$$

$$F_{UU,L} = \frac{1}{Q^2}\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\, e_a^2 \Big[f_1^a(x)\, D_1^a(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,\big(D_1^a\otimes P''_{qq} + D_1^g \otimes P''_{gq}\big)(z)$$

$$+ \big(P''_{qq}\otimes f_1^a + P''_{qg}\otimes f_1^g\big)(x)\, D_1^a(z)\Big]$$



$$L\!\left(\frac{Q^2}{q_T^2}\right) \equiv C_F\left[2\ln\!\left(\frac{Q^2}{q_T^2}\right) - 3\right]$$

Can $F_{UU,L}$ be resummed ?

See e.g. Berger Qiu Zhang, PRD D65 (2002)

Truncated moments of R_{SIDIS} from large p_T

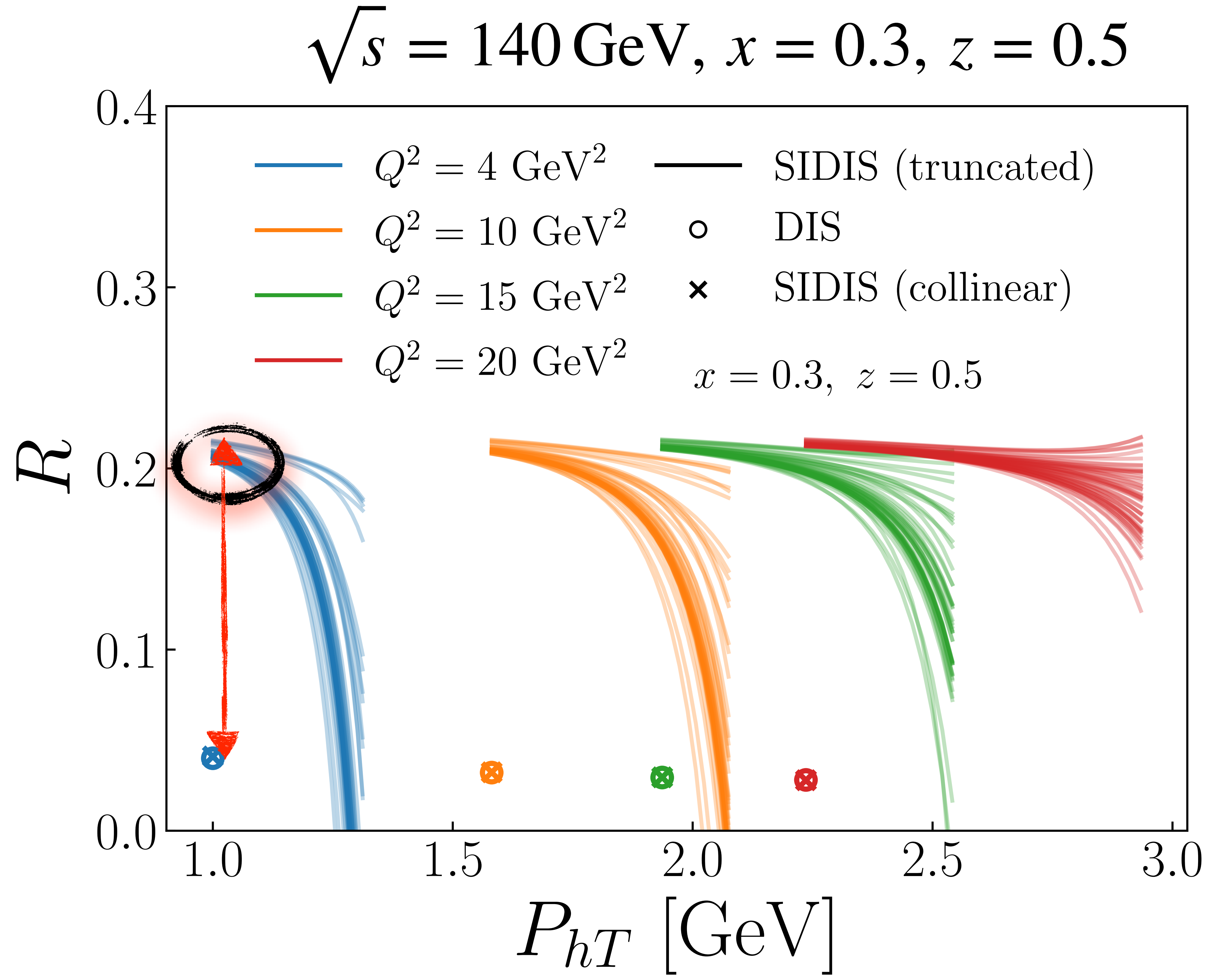
e.g.

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\, e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right) \times \left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})} \right]$$

$$R_{SIDIS} = \frac{F_{UU,L}}{F_{UU,T}}$$

SIDIS truncated moments

$$\int_{P_{hT\,\min}^2/z^2}^{q_{T\,\max}^2} F(x, z, Q^2, P_{hT})$$



Nb: Bands are generated by computing the observable on subset of JAM replicas (from recent W+charm analysis) & taking the mean ± standard deviation

Truncated moments of R_{SIDIS} from large p_T

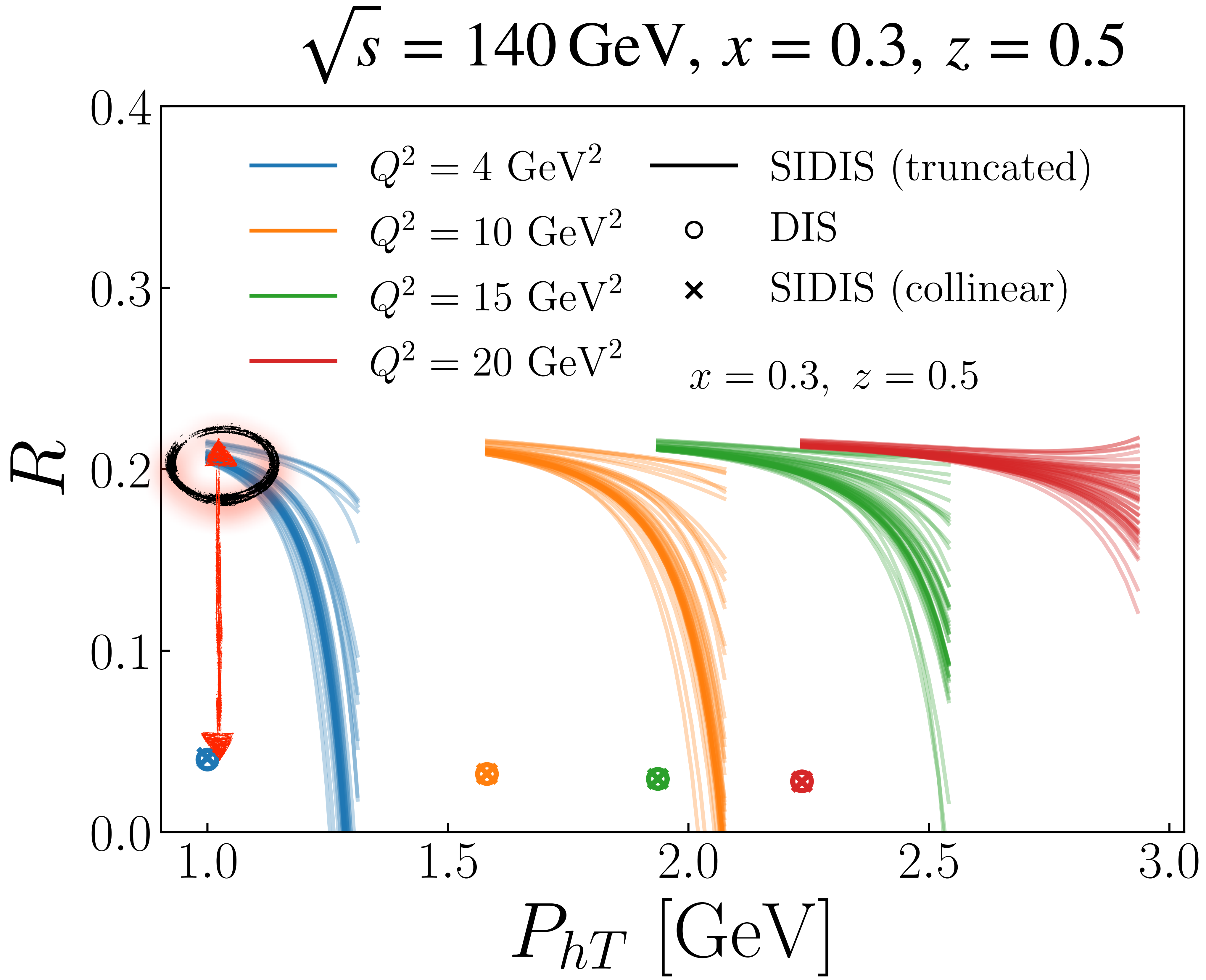
$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\,e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$
$$\times\left[f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^g\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$

$$R_{SIDIS} = \frac{F_{UU,L}}{F_{UU,T}}$$

Comments:

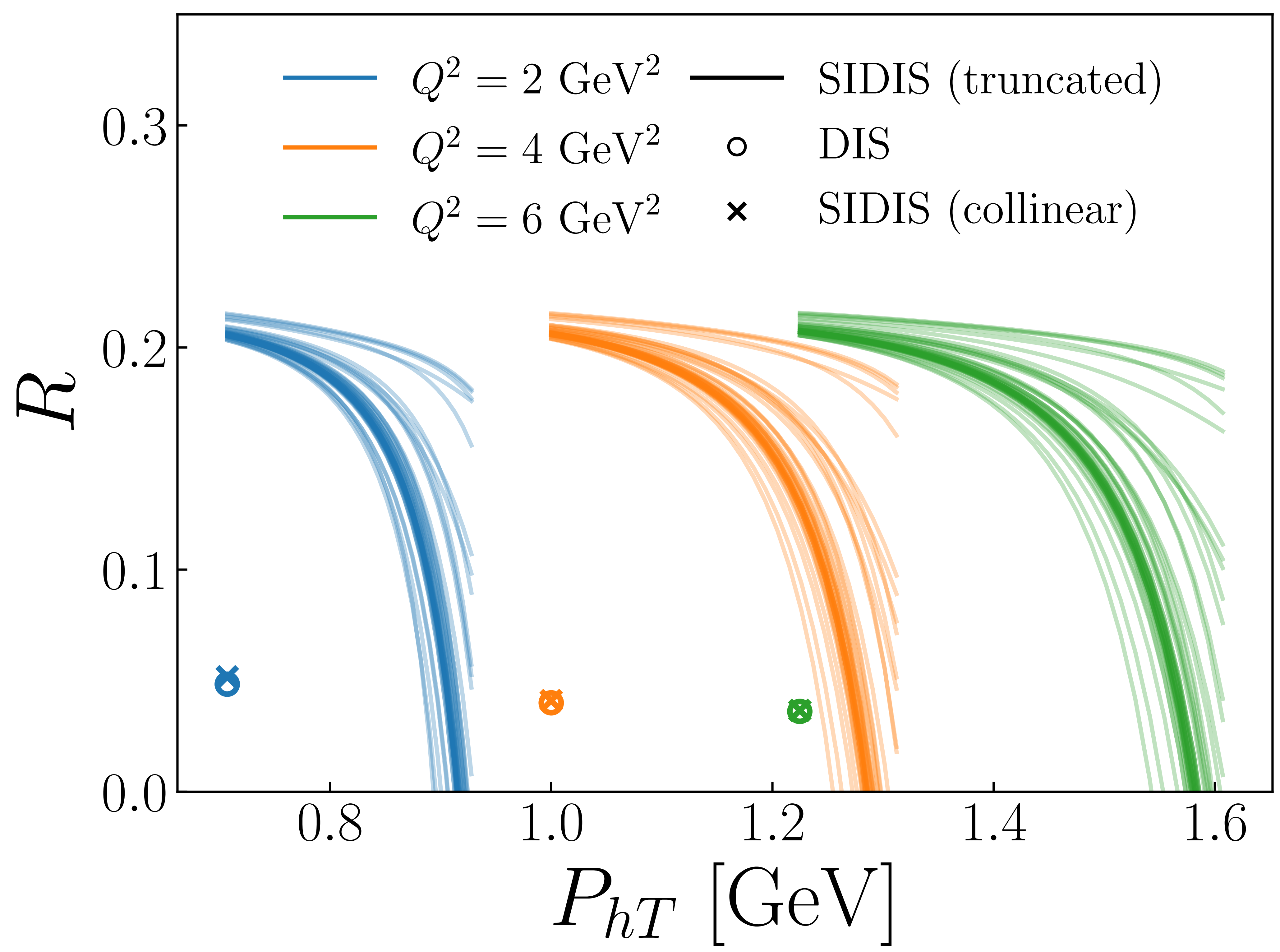
- Truncated moment is sig. larger than P_T integrated SIDIS—indication of
- power corrections ?
- TMD contribution ?

$$\int d^2\boldsymbol{q}_T \frac{d\sigma}{d^2\boldsymbol{q}_T \ldots} = \int d^2\boldsymbol{q}_T\, W + \int d^2\boldsymbol{q}_T\, Y.$$

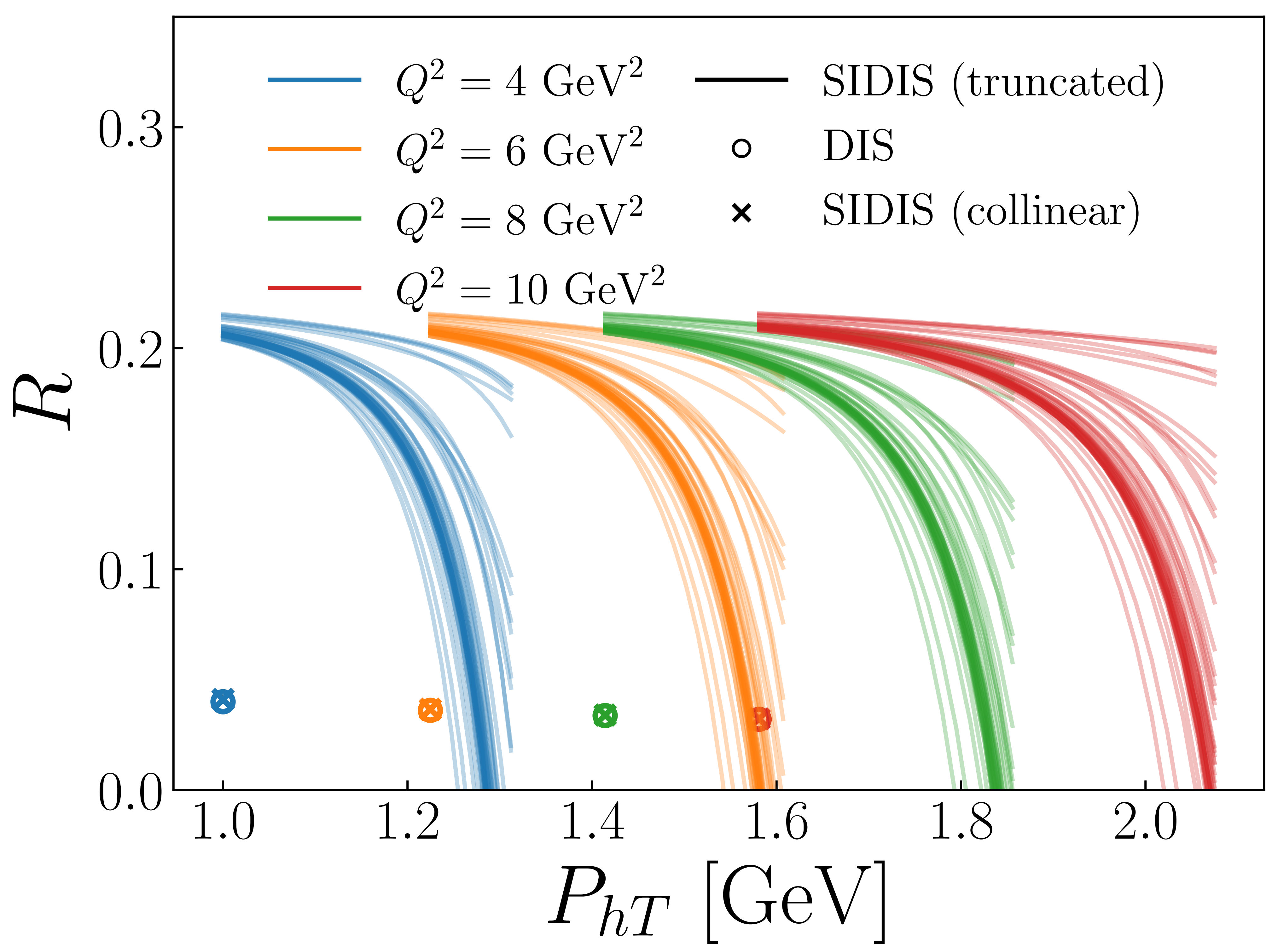


R_{SIDIS} & $\sigma_L \sim F_{UU,L}$ at large p_T

Jlab 11 GeV $x = 0.3$ & $z = 0.5$



Jlab 22 GeV $x = 0.3$ & $z = 0.5$

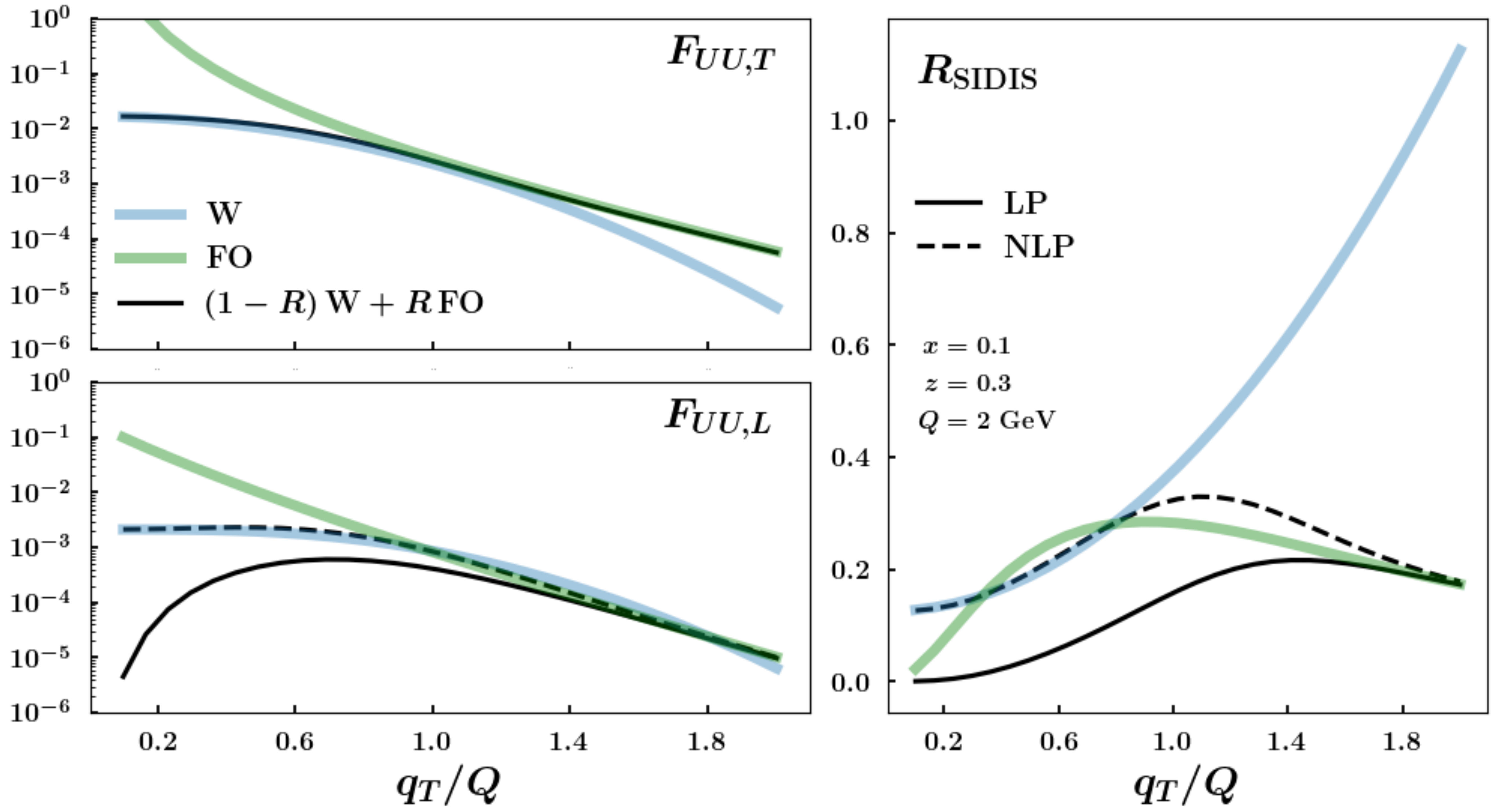


Stitching together TMD + FO with goal of estimating subleading power "resummed W" term

$$F_{UU,L} = (1-R) F_{UU,L}^{[M\sim q_T \ll Q]} + R\, F_{UU,L}^{[M \ll q_T \sim Q]} \overset{?}{\Longrightarrow} W + Y + \mathcal{O}\left(\frac{M}{Q}\right)^c$$

Where R is sigmoid function-transition function &

$$F_{UU,L}^{[M\sim q_T \ll Q]} = \frac{4M^2}{Q^2}\mathcal{C}\left[f_1 D_1\right] \qquad F_{UU,L}^{[M \ll q_T \sim Q]} = Fixed\,order$$

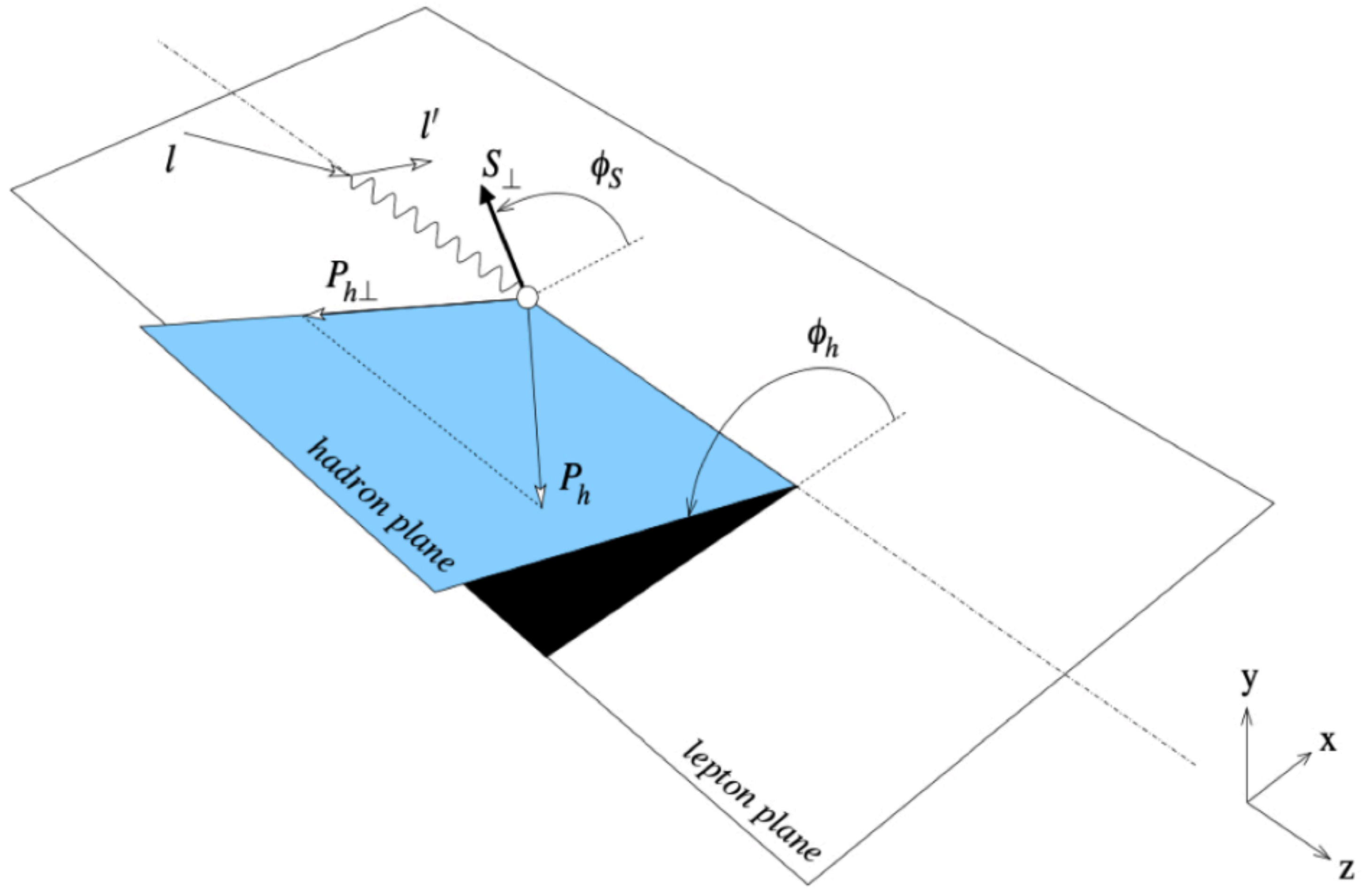


The observable $\langle \cos\phi \rangle$

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right.$$

Arise from interference of $L\,\&\,T$ photons
significant @ COMPASS HERMES

$$\int d\sigma^{(1)}\,\cos\phi = \int d^2P_T\,\cos\phi\,\frac{d\sigma}{dx_H\,dy\,dz_H\,d^2P_T}$$



Reminder

SIDIS Kinematics dictionary

$$Q^2 = -q^2,\quad \mathbf{P}_T = \mathbf{P}_{2T},\quad \phi,$$

$$x_H = \frac{Q^2}{2P_1\cdot q},\quad y = \frac{P_1\cdot q}{P_1\cdot k_1},\quad z_H = \frac{P_1\cdot P_2}{P_1\cdot q},$$

and the parton variables

$$x = \frac{x_H}{\xi} = \frac{Q^2}{2p_1\cdot q},\quad z = \frac{z_H}{\xi'} = \frac{p_1\cdot p_2}{p_1\cdot q}.$$

Cahn intrinsic k_T

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right.$$

TMD Collaboration

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

TMD Handbook 308

10 - Subleading TMDs

L. Gamberg, A. Metz, I. Stewart

L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao
arXiv: e-Print:221.13209
new work w/ O. Page (SULI), Z. Kang ...

Subleading Quark TMDPDFs

Nucleon Polarization	Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
U	$f^{\perp}, g^{\perp}$	$e\,,\ h$
L	$f_L^{\perp}, g_L^{\perp}$	e_L, h_L
T	$f_T, f_T^{\perp}, g_T, g_T^{\perp}$	$e_T, e_T^{\perp}, h_T, h_T^{\perp}$

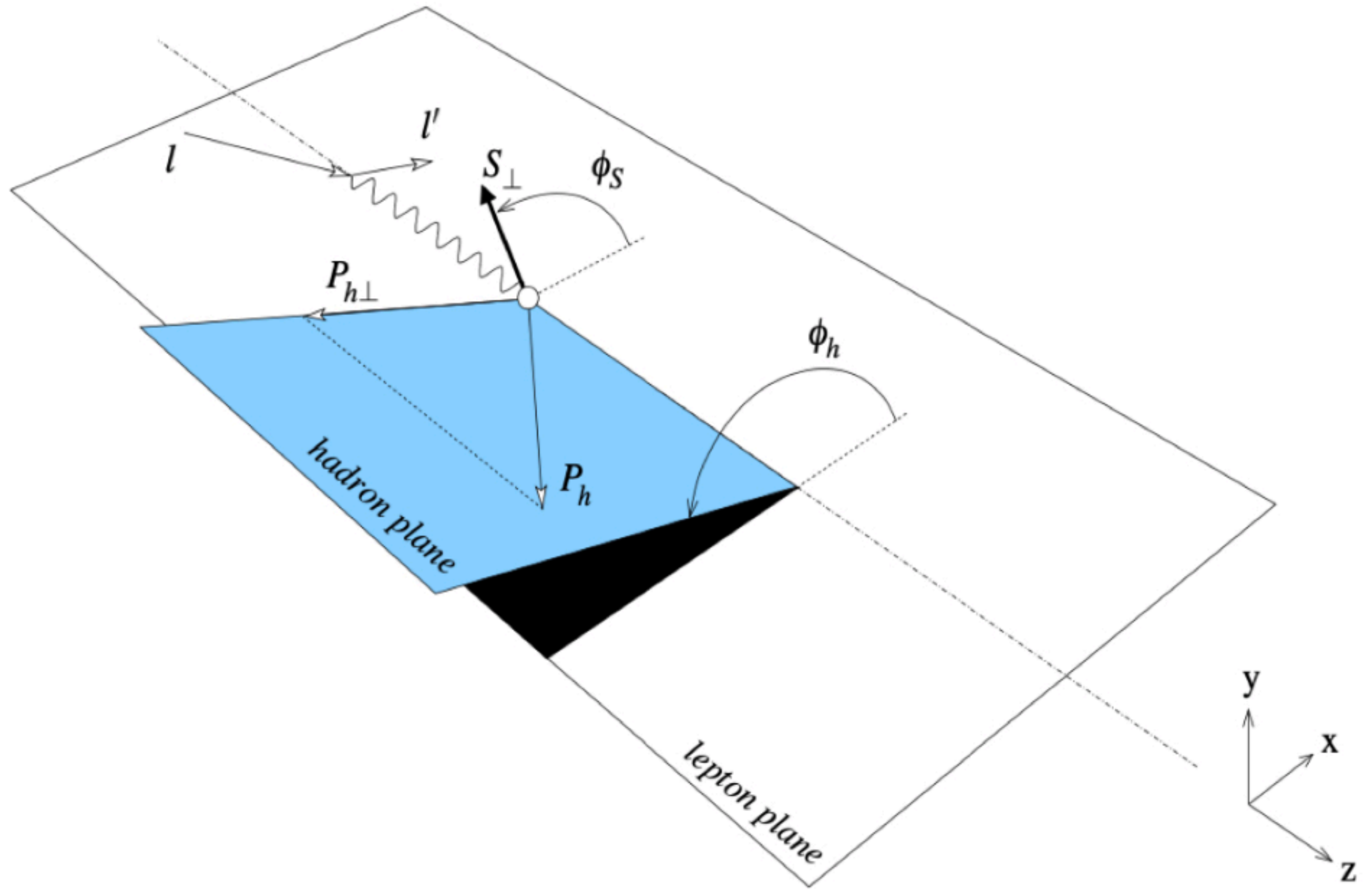
TMDs @ "twist-3 " NLP

The beginning of TMD physics? $\langle \cos \phi \rangle$

- **<u>Georgi Politzer, PRL 1978</u>** "Measurement $\langle \cos \phi \rangle$ provides clean test of predictions of PQCD

~12-15% …clean test of QCD "…since such effects would not arise as a result of limited transverse momentum associated with confined quarks…"

- **<u>Cahn, PLB 1978, (& earlier paper by Ravndal, PLB 1972)</u>**
Critique QCD calculation of azimuthal dependence
emphasize importance intrinsic $\boldsymbol{k}_T$ …

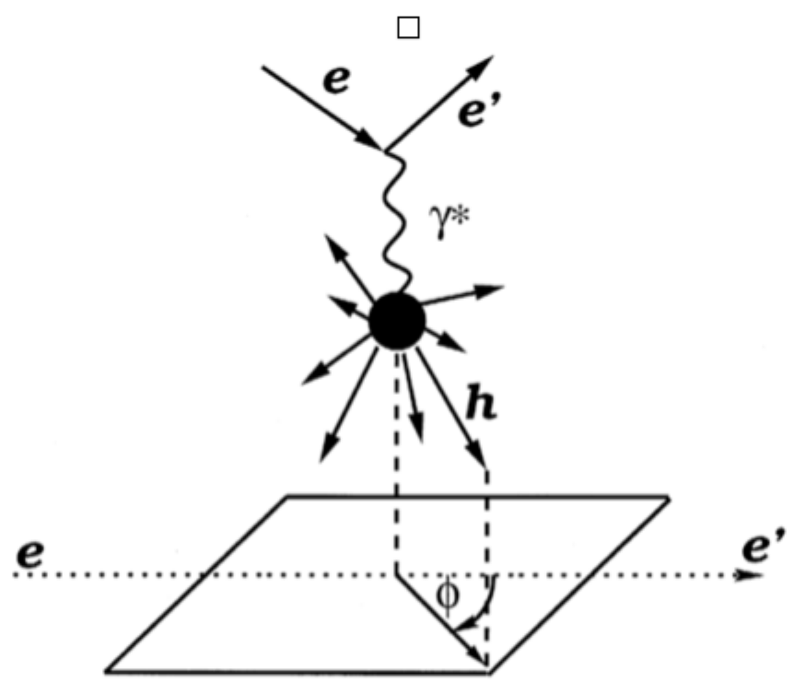


"…Results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics " (i.e. of G&P 78)

Clean tests of QCD ?

PHYSICAL REVIEW LETTERS

VOLUME 40 — 2 JANUARY 1978 — NUMBER 1



Clean Tests of Quantum Chromodynamics in μp Scattering

Howard Georgi
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

H. David Politzer
California Institute of Technology, Pasadena, California 91125

(Received 25 October 1977)

Hard gluon bremsstrahlung in μp scattering produces final-state hadrons with a large component of momentum transverse to the virtual-photon direction. Quantum chromodynamics can be used to predict not only the absolute value of the transverse momentum, but also its angular distribution relative to the muon scattering plane. The angular correlations should be insensitive to nonperturbative effects.

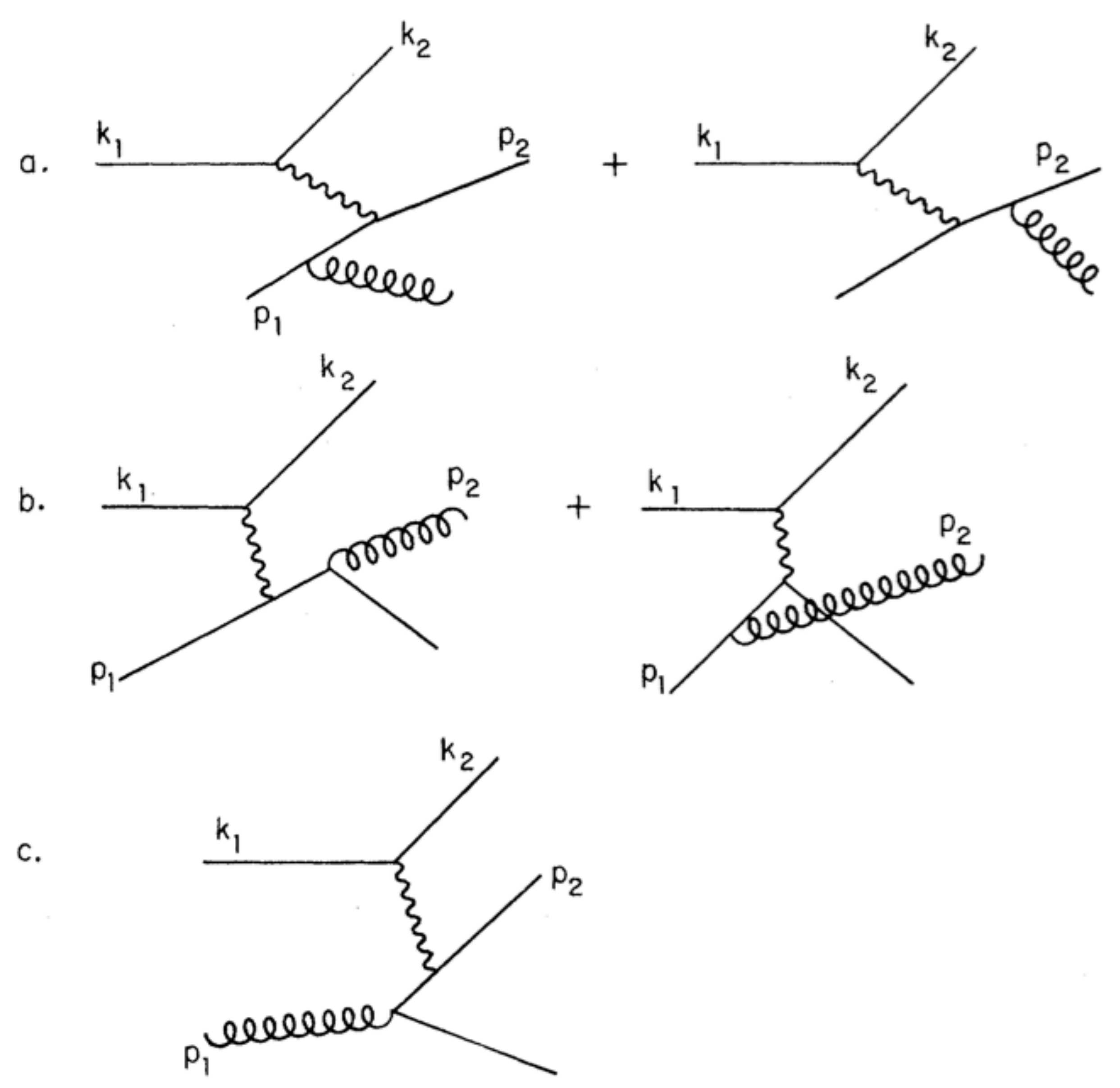


FIG. 1. Diagrams contributing to semi-inclusive μ-parton scattering to first order in α_s. k (p) denotes muon (parton) momentum. The wavy line is a virtual photon. The curly line is a gluon.

Pert. QCD $\alpha_s = g^2/4\pi$

$$\langle \cos\varphi \rangle_{ep} = -\frac{\alpha_s}{2}\kappa\sqrt{1-z}\,\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Cahn intrinsic k_T

Volume 78B, number 2,3 PHYSICS LETTERS 25 September 1978

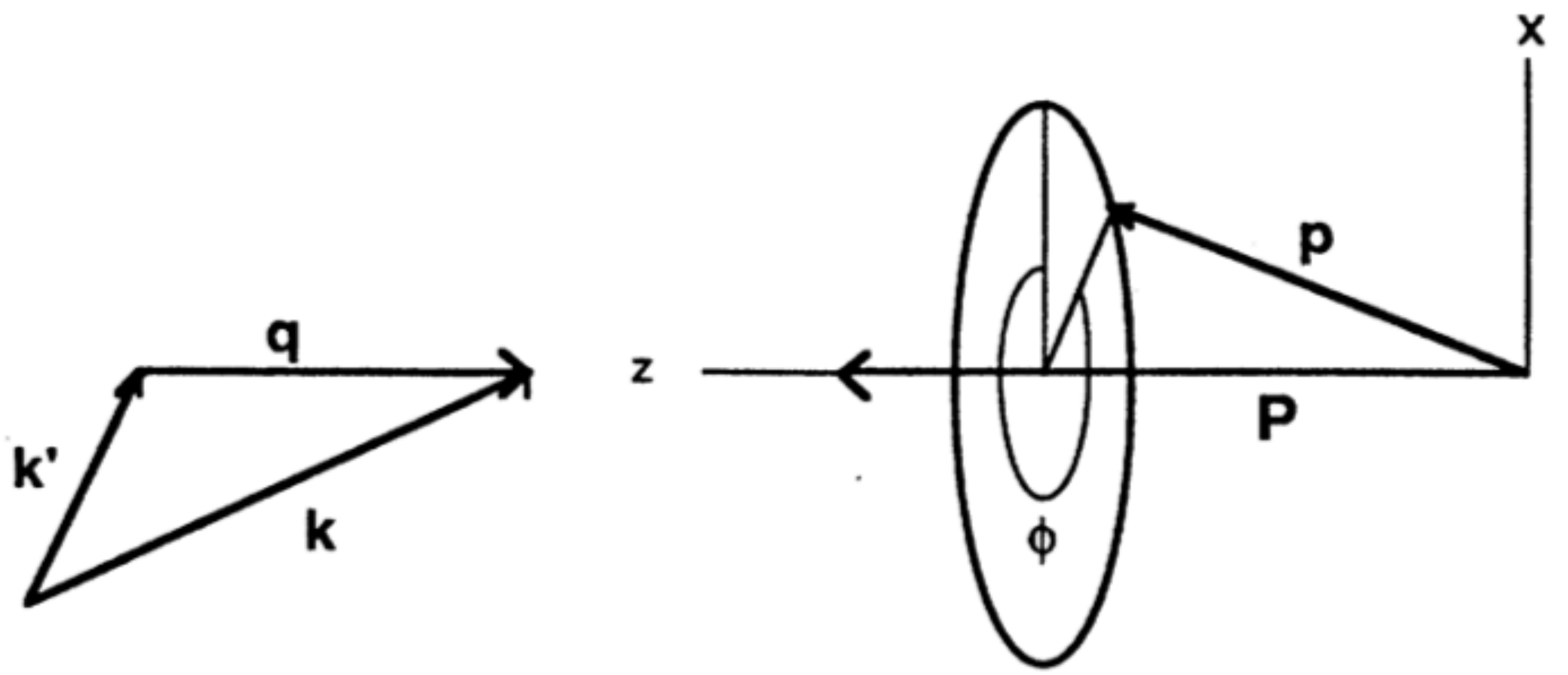
AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION☆

Robert N. CAHN
Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Received 5 June 1978

Semi-inclusive leptoproduction, $\ell + p \to \ell' + h + X$, is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in ep, νp and $\bar{\nu}$p scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.

Simple parton model argument allowing for transverse momentum in Mandelstam variables...



$$\sigma_{ep} \propto \hat{s}^2 + \hat{u}^2 \propto \left[1 - \frac{2p_\perp}{Q}\sqrt{1-y}\,\cos\phi\right]^2 + (1-y)^2\left[1 - \frac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right]^2$$

NLP ! $\frac{p_\perp}{Q}$

$$\langle \cos\phi \rangle_{ep} = -\left[\frac{2p_\perp}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Power counting in "intermediate region

Power counting $F_{UU}^{\cos\phi}$

- High collinear factorization $\longrightarrow M \ll q_T \ll Q \longleftarrow$ - Low TMD factorization

$$M^2 F_{UU}^{\cos\varphi}\Big|^{M\ll q_T\ll Q} \sim \alpha_s \frac{q_T}{Q}\frac{M^2}{q_T^2}\mathcal{F}[f_1 D_1]$$

$$M^2 F_{UU}^{\cos\varphi}\Big|^{M\ll q_T\ll Q} \sim \alpha_s \frac{M^2}{q_T^2}\frac{q_T}{Q}\mathcal{F}[f_1 D_1]$$

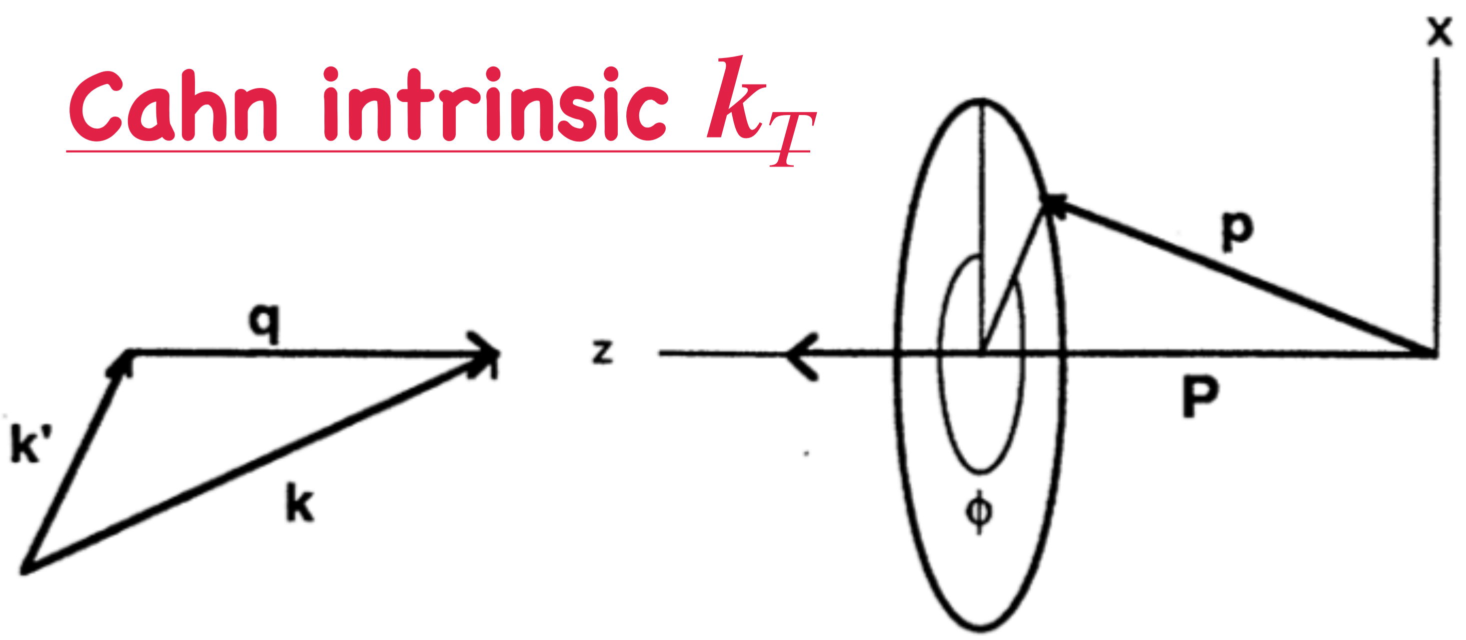
$\exists$ Match

$$F_{UU}^{\cos\phi_h} \sim \frac{1}{Q\,q_T}\alpha_s\,\mathcal{F}[f_1 D_1]$$

Two mechanisms ? *Matching* ...

Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Sterman NPB 1985

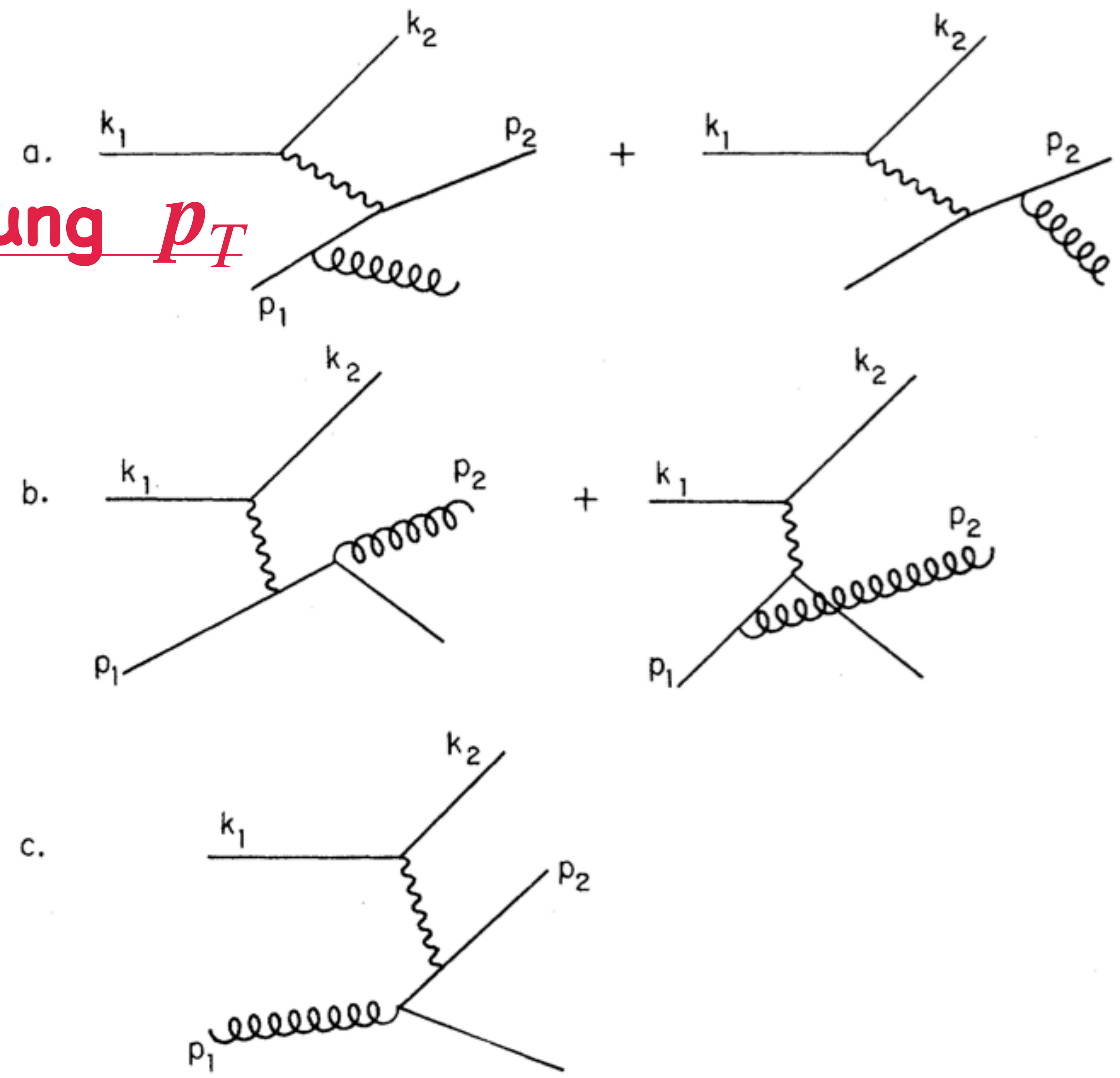
Cahn intrinsic k_T

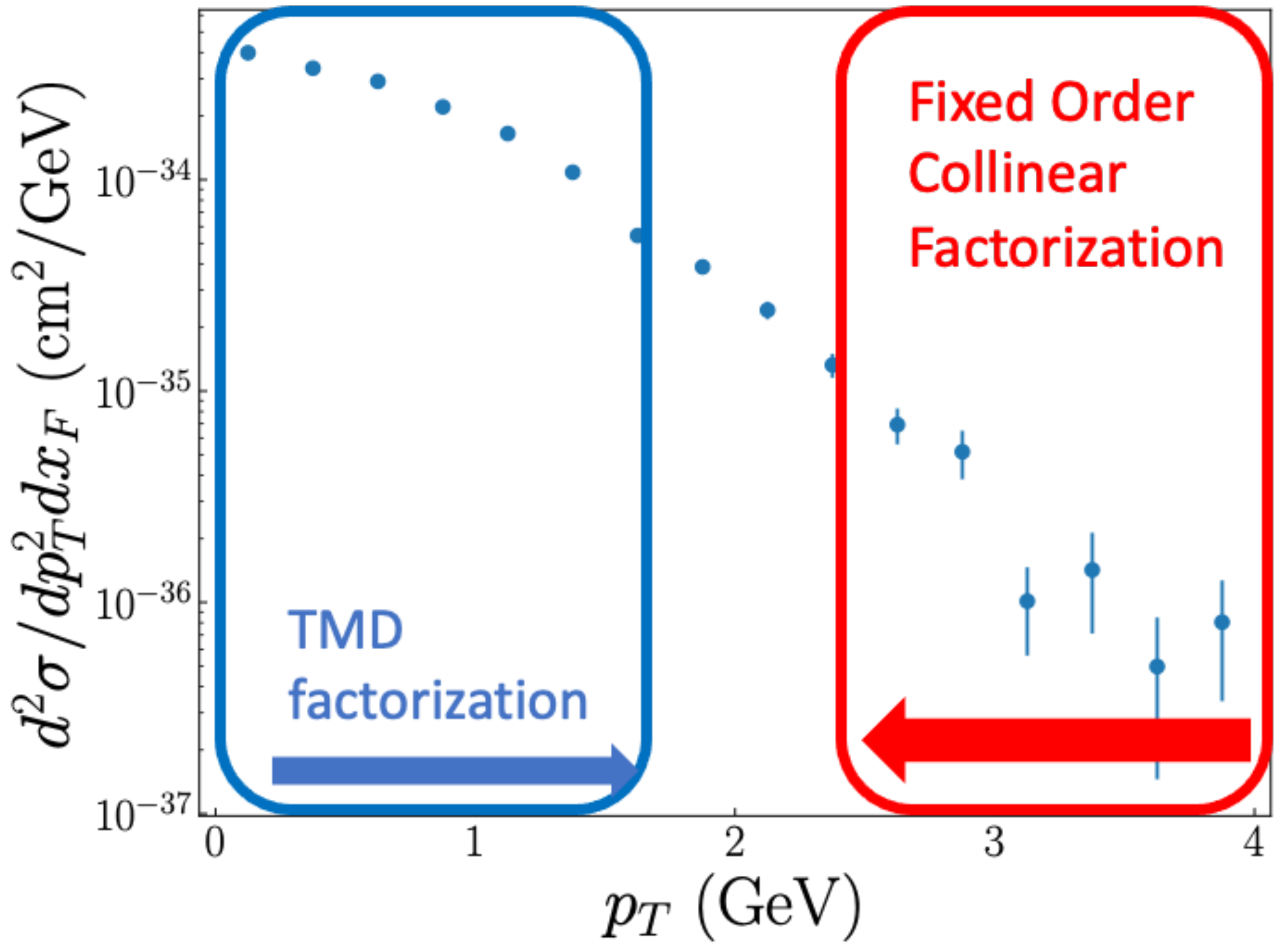


- **"TMD" region**

$(p_T \sim k_T) \sim q_T \ll Q$

Georgi & Politzer
hard gluon bremsstrahlung p_T





- **"Collinear " region**

$\Lambda_{qcd} \ll q_T \sim Q$

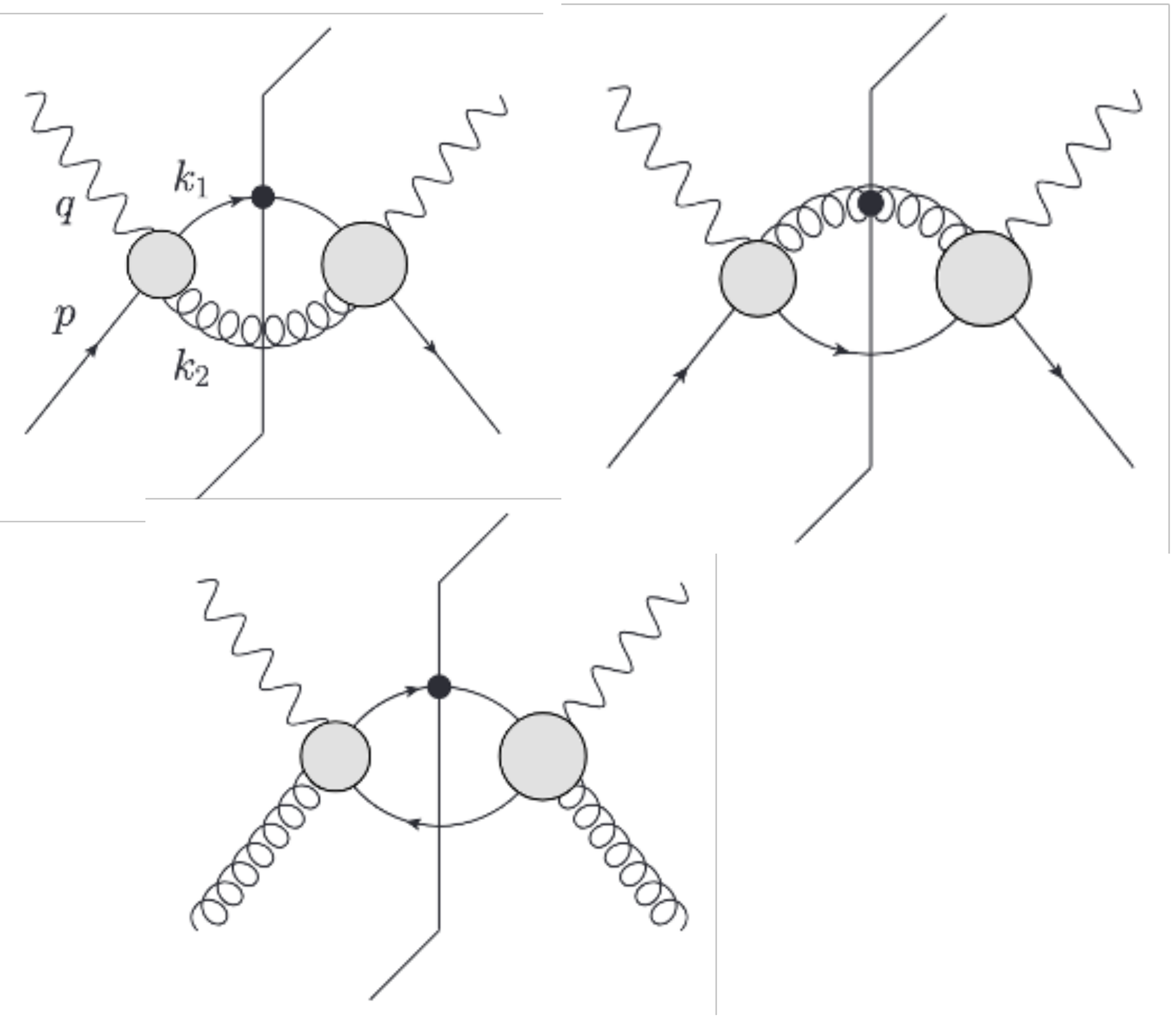
Comprehensive study of matching the hi & low Q_T in the AY (overlap) region in SIDIS was carried out by Bacchetta, Boer, Diehl, Mulders JHEP (2008) attention was given to azimuthal and polarization dependence

We have explored lg q_T angular modulations Cahn effect & $\cos\varphi$ & $\cos 2\varphi_h$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}q_T^2\,\mathrm{d}\varphi} = \frac{\pi\alpha^2 yz}{4Q^2}\Big[\underbrace{\sinh^2\vartheta F_{UU,L} - \frac{1}{2}(2+\sinh^2\vartheta)F_{UU,T}}_{\sigma_0} - \underbrace{\sinh 2\vartheta F_{UU}^{\cos\varphi}}_{\sigma_1}\cos\varphi + \underbrace{\frac{1}{2}\sinh^2\vartheta F_{UU}^{\cos 2\varphi}}_{\sigma_2}\cos 2\varphi\Big]$$

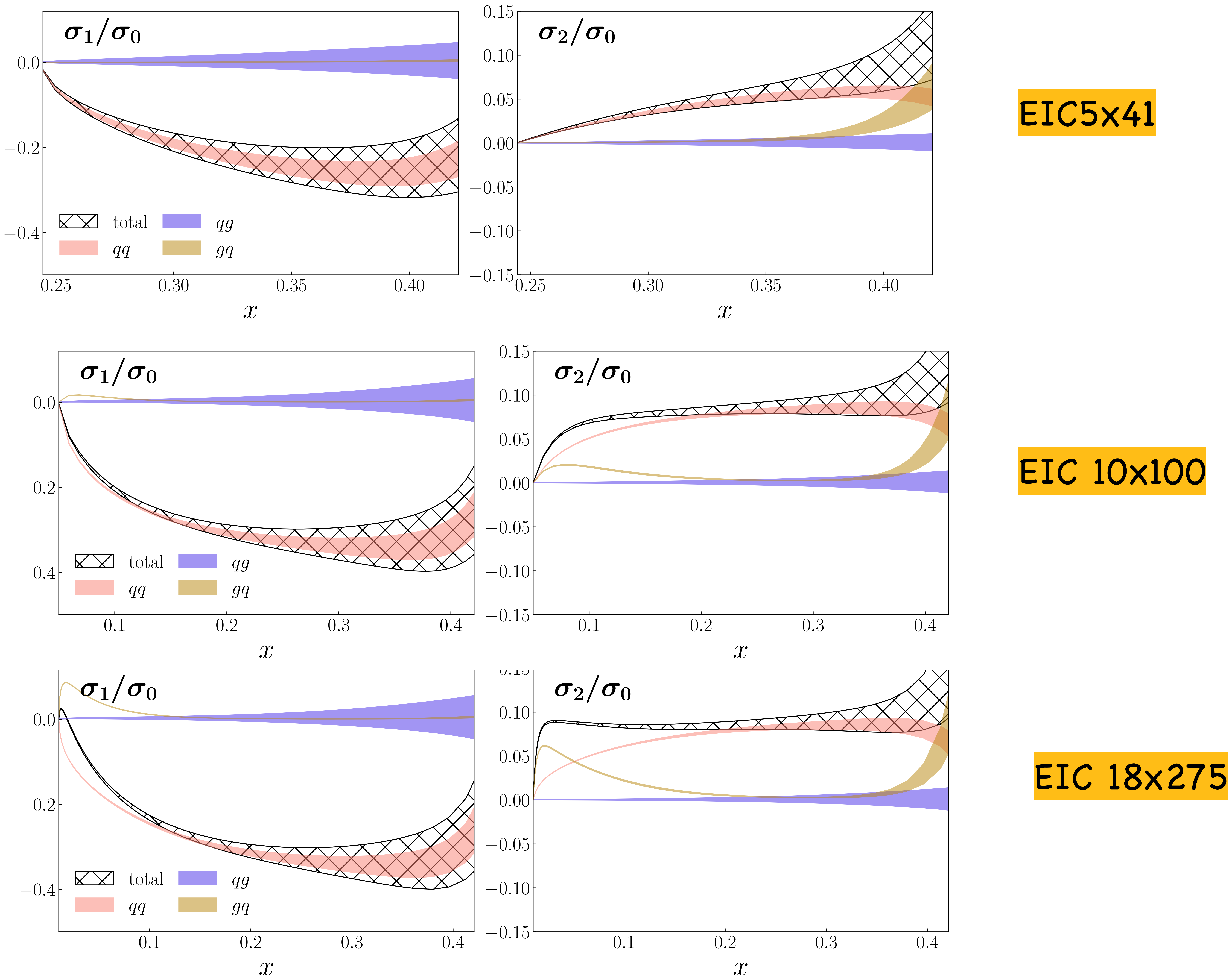
e.g.

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \times \left[f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^g\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$



- $\gamma^* q \to qg$ $\quad C_{UU}^{\cos\phi_h} = -4C_F\left[\hat{x}\hat{z} + (1-\hat{x})(1-\hat{z})\right]\frac{Q}{q_T},$ $\quad C_{UU}^{\cos 2\phi_h} = 4C_F\,\hat{x}\hat{z},$
- $\gamma^* q \to gq$ $\quad C_{UU}^{\cos\phi_h} = 4C_F\left[\hat{x}(1-\hat{z}) + (1-\hat{x})\,\hat{z}\right]\frac{1-\hat{z}}{\hat{z}}\frac{Q}{q_T},$ $\quad C_{UU}^{\cos 2\phi_h} = 4C_F\,\hat{x}\,(1-\hat{z}),$
- $\gamma^* g \to q\bar{q}$ $\quad C_{UU}^{\cos\phi_h} = -4T_R\,(2\hat{x}-1)\,(2\hat{z}-1)\,\frac{1-\hat{x}}{\hat{z}}\frac{Q}{q_T}$ $\quad C_{UU}^{\cos 2\phi_h} = 8T_R\,\hat{x}\,(1-\hat{x}),$

w/ P. Anderson, W. Melntitchok, N. Sato, R. Whitehill (JAM)


σ_1/σ_0
σ_2/σ_0
total
qg
qq
gq
x
EIC5x41
σ_1/σ_0
σ_2/σ_0
total
qg
qq
gq
x
EIC 10x100
σ_1/σ_0
σ_2/σ_0
total
qg
qq
gq
x
EIC 18x275

Summary

We explore NLP $(M/Q)^n$ contributions in large q_T and TMD regions via power counting and factorization theorems

- NLP factorization based on "*TMD formalism*"
- Considier R_{SIDIS} & revisit "Cahn effect" & matching related to early importance intrinsic k_T
  - *"Intrinsic"NLP TMDs related thru EOM in terms "kinematic" & "dynamical"*
    —extend the tree level Amsterdam formalism and beyond leading order
    CSS, Ji Ma Yuan, Abyat Rogers, framework vs. SCET and Background Field Methods
- Consider RG consistency of matching to collinear factorization & issues of resummation
  - Bacchetta, Boer, Diehl, Mulders JHEP 2008, Bacchetta et al. PLB 2019
- Report progress in this *necessary condition* NLP factorization (not yet sufficient)

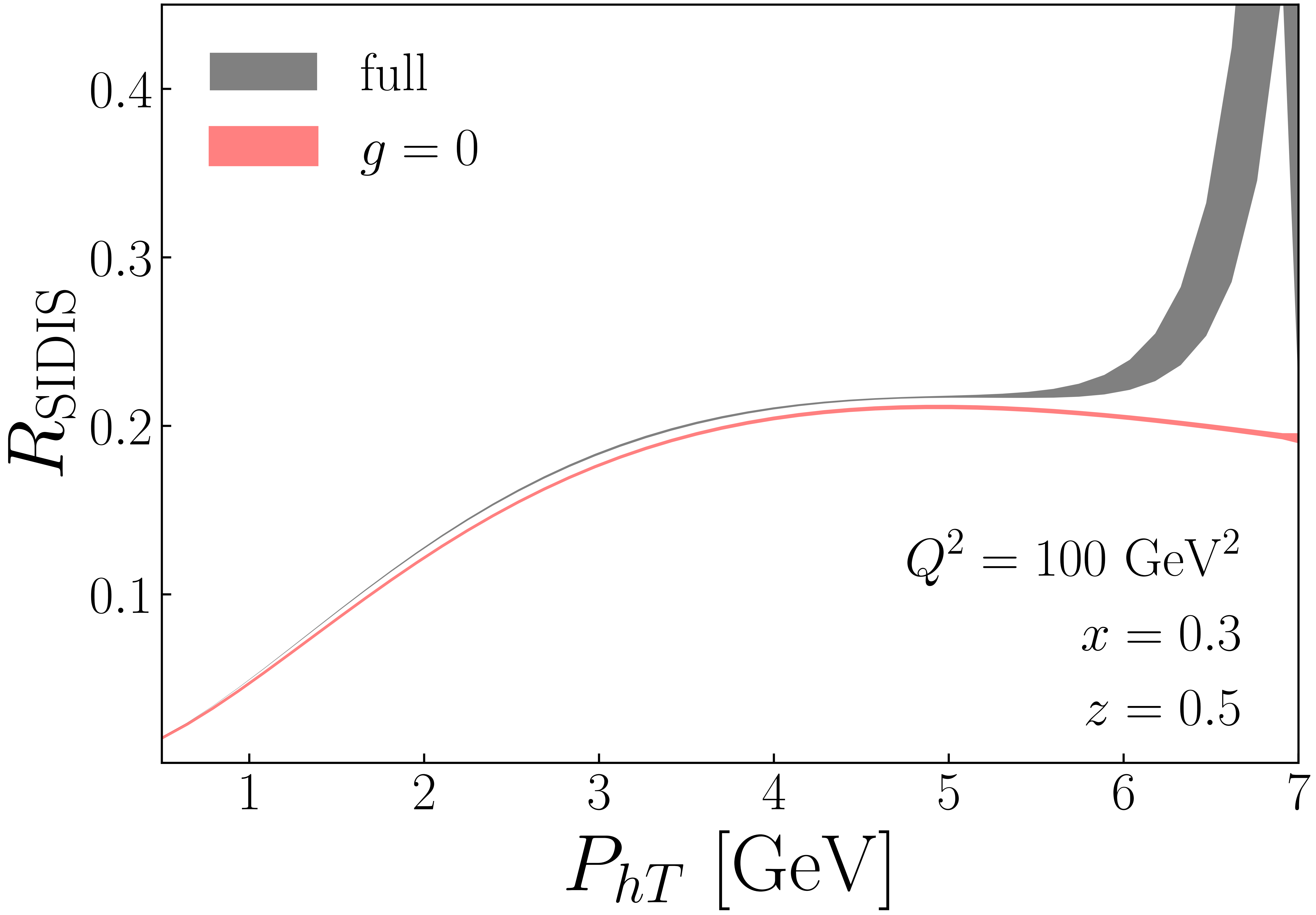
- In doing so, we provide basis for performing global analysis & phenomenology of one the earliest observables used to study intrinsic 3-D momentum structure of the nucleon
  —For Jlab, EIC, COMPASS, studies of momentum imaging of transverse nucleon structure

Thank You

R_{SIDIS} & $\sigma_L \sim F_{UU,L}$ at large p_T

- gluon contribute large uncertainty @ hi-x (see delta function)
- $g \to 0$ gluon PDF set to zero
- R_{SIDIS} could be useful to pin down the g @ $lg\ x$

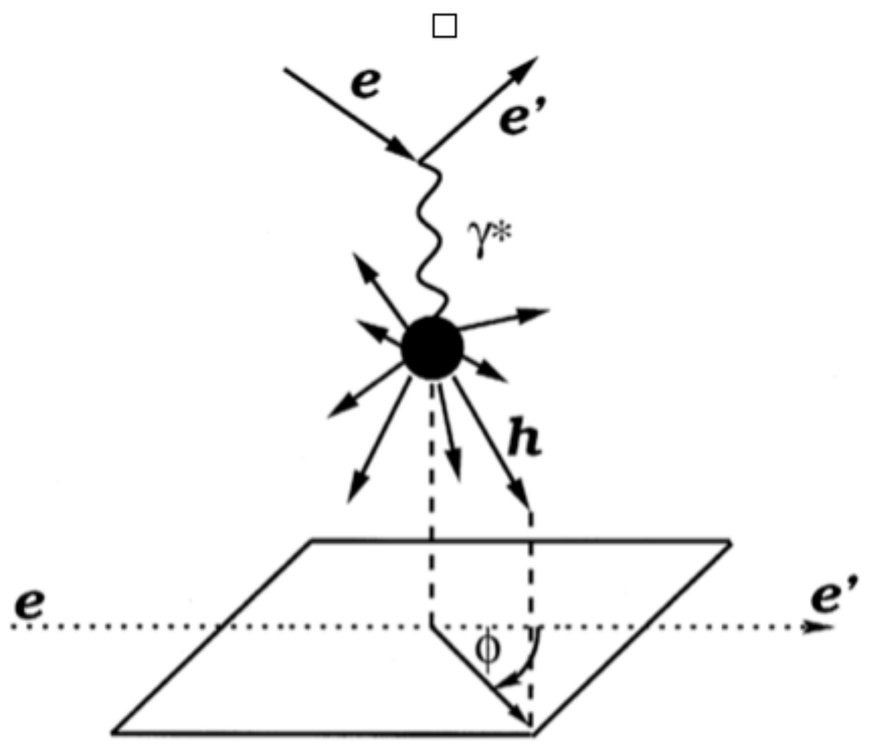
$$\text{FO} = \sum_q e_q^2 \int_{\frac{q_T^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x} H(\xi)\ \boldsymbol{f}_q(\xi,\mu)\ \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$



+ **Attention**: $\left(\frac{q_T^2}{Q^2}\frac{xz}{1-z} + x\right) < \xi < 1$
+ **large q_T probes large ξ in PDFs**
+ Can be useful in collinear global fits

Recent DATA

$$\frac{d\sigma^{ep\to ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

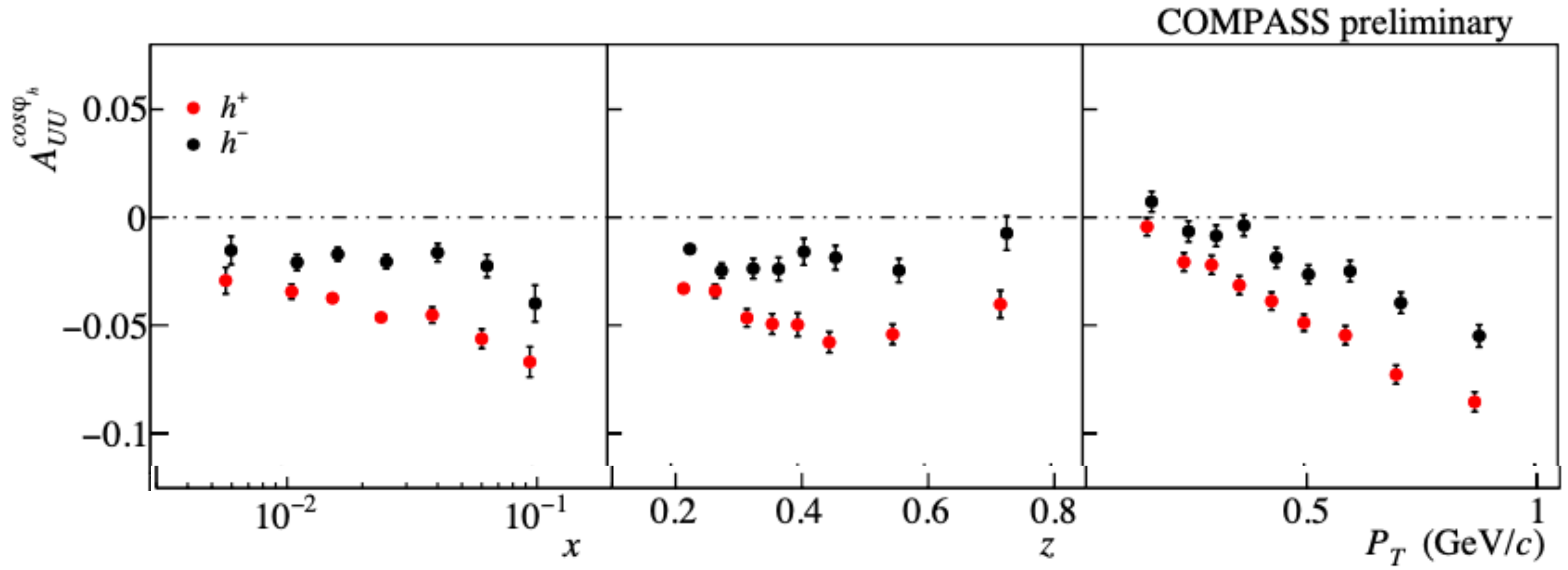


$(p_T \sim k_T) \sim q_T \ll Q$

"TMD region"

COMPASS, Nucl. Phys. B 886 (2014) 1046

& 2016 2017 data



HERMES, Phys. Rev. D 87 (2013) 012010