Hard exclusive π^0 muoproduction at COMPASS

Towards Improved Hadron Tomography with Hard Exclusive Reactions, 28-31 July 2025

Karolína Lavičková* on behalf of the COMPASS collaboration

*FNSPE CTU in Prague

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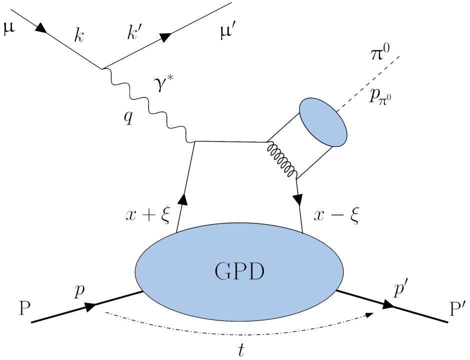
Hard exclusive π^0 muoproduction



Sensitive to the GPDs

- $\tilde{H}(x, \xi, t)$ and $\tilde{E}(x, \xi, t) \rightarrow$ chiral-even (conserving the parton helicity)
- $H_{\mathrm{T}}(x, \xi, t)$ and $\bar{E}_{\mathrm{T}}(x, \xi, t) = 2\tilde{H} + E_{\mathrm{T}} \rightarrow$ chiral-odd (parton helicity flip)
  - H_{T} related to tranversity and $\bar{E}_{\mathrm{T}}$ related to Boer-Mulders PDFs

Cross section for hard exclusive π^0 production

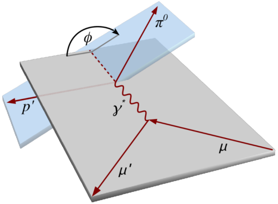
$$\frac{d^2\sigma^{\gamma^* p}}{dt d\phi} = \frac{1}{2\pi}\left[\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \epsilon\cos(2\phi)\frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi)\frac{d\sigma_{LT}}{dt} \mp |P| \sqrt{2\epsilon(1-\epsilon)}\sin(\phi)\frac{d\sigma'_{LT}}{dt}\right]$$

$$\frac{d\sigma_T}{dt} \propto \left[(1-\xi^2)|\langle H_T\rangle|^2 - \frac{t'}{8m_p^2}|\langle \bar{E}_T\rangle|^2\right]$$

$$\frac{d\sigma_L}{dt} \propto \left[(1-\xi^2)|\langle \tilde{H}\rangle|^2 - 2\xi^2 \mathrm{Re}\left[\langle \tilde{H}\rangle * \langle \tilde{E}\rangle\right] - \frac{t}{4m_p^2}\xi^2|\langle \tilde{E}\rangle|^2\right]$$

$$\frac{d\sigma_{TT}}{dt} \propto \frac{t'}{16m_p^2}|\langle \bar{E}_T\rangle|^2$$

$$\frac{d\sigma_{LT}}{dt} \propto \xi\sqrt{1-\xi^2}\sqrt{-t'}\ \mathrm{Re}\left[\langle H_T\rangle * \langle \tilde{E}\rangle\right]$$



Cross section for hard exclusive π^0 production

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$$\frac{d\sigma_T}{dt} \propto \left[(1-\xi^2)|\langle H_T\rangle|^2 - \frac{t'}{8m_p^2}|\langle \bar{E}_T\rangle|^2\right]$$

$$\frac{d\sigma_L}{dt} \propto \left[(1-\xi^2)|\langle \tilde{H}\rangle|^2 - 2\xi^2 \mathrm{Re}\left[\langle \tilde{H}\rangle * \langle \tilde{E}\rangle\right] - \frac{t}{4m_p^2}\xi^2|\langle \tilde{E}\rangle|^2\right]$$

$$\frac{d\sigma_{TT}}{dt} \propto \frac{t'}{16m_p^2}|\langle \bar{E}_T\rangle|^2$$

$$\frac{d\sigma_{LT}}{dt} \propto \xi\sqrt{1-\xi^2}\sqrt{-t'}\ \mathrm{Re}\left[\langle H_T\rangle * \langle \tilde{E}\rangle\right]$$

The last term cancels out upon averaging when using μ^+ and μ^- beams of exactly opposite polarization.

Cross section for hard exclusive π^0 production

$$\frac{d^2\sigma^{\gamma^* p}}{dt d\phi} = \frac{1}{2\pi}\left[\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \epsilon\cos(2\phi)\frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi)\frac{d\sigma_{LT}}{dt}\right]$$

$$\frac{d\sigma_T}{dt} \propto \left[(1-\xi^2)|\langle H_T\rangle|^2 - \frac{t'}{8m_p^2}|\langle \bar{E}_T\rangle|^2\right]$$

$$\frac{d\sigma_{TT}}{dt} \propto \frac{t'}{16m_p^2}|\langle \bar{E}_T\rangle|^2$$

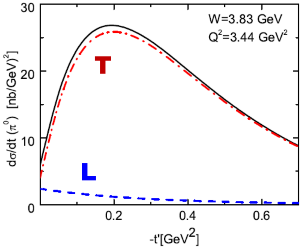
- In exclusive π^0 production, the GPDs enter the amplitude in the flavor combination $F = \frac{1}{\sqrt{2}}(\frac{2}{3}F^u + \frac{1}{3}F^d)$
- $\tilde{E}^u$ and $\tilde{E}^d$, $\tilde{H}^u$ and $\tilde{H}^d$ or H_T^u and H_T^d have opposite signs
- $\bar{E}_T^u$ and $\bar{E}_T^d$ are evaluated with the same sign
  $\Rightarrow$ **exclusive π^0 production is dominated by $\bar{E}_T$**

Cross section for hard exclusive π^0 production

$$\frac{\mathrm{d}^2\sigma^{\gamma^* p}}{\mathrm{d}t\mathrm{d}\phi} = \frac{1}{2\pi}\left[\frac{\mathrm{d}\sigma_\mathrm{T}}{\mathrm{d}t} + \epsilon\frac{\mathrm{d}\sigma_\mathrm{L}}{\mathrm{d}t} + \epsilon\cos(2\phi)\frac{\mathrm{d}\sigma_\mathrm{TT}}{\mathrm{d}t} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi)\frac{\mathrm{d}\sigma_\mathrm{LT}}{\mathrm{d}t}\right]$$

$$\frac{\mathrm{d}\sigma_\mathrm{T}}{\mathrm{d}t} \propto \left[(1-\xi^2)|\langle H_\mathrm{T}\rangle|^2 - \frac{t'}{8m_\mathrm{p}^2}|\langle \bar{E}_\mathrm{T}\rangle|^2\right]$$

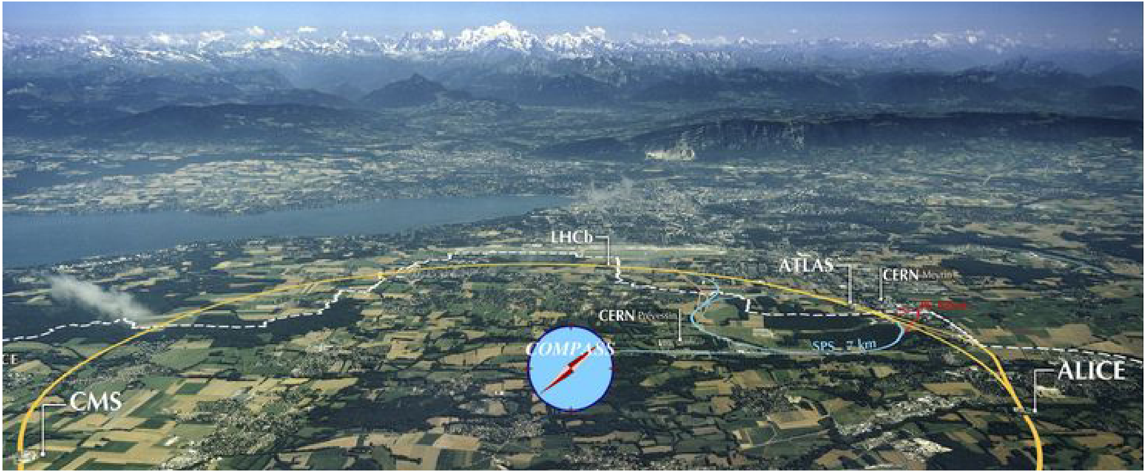
$$\frac{\mathrm{d}\sigma_\mathrm{L}}{\mathrm{d}t} \propto \left[(1-\xi^2)|\langle \tilde{H}\rangle|^2 - 2\xi^2\mathrm{Re}\left[\langle \tilde{H}\rangle * \langle \tilde{E}\rangle\right] - \frac{t}{4m_\mathrm{p}^2}\xi^2|\langle \tilde{E}\rangle|^2\right]$$



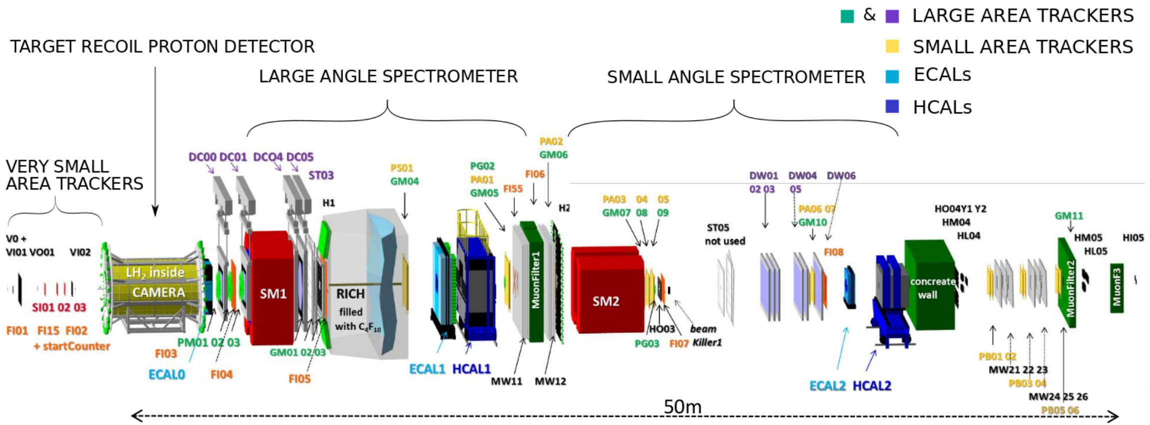
S. Goloskokov, P. Kroll EPJC47 (2011)

- GPDs decrease with $-t' = -(t - t_0)$, $t_0 \approx 10^{-2} \Rightarrow -t' \approx |t|$
- $\frac{\mathrm{d}\sigma_\mathrm{L}}{\mathrm{d}t}$ decrease with $-t'$ (dominated by $\tilde{H}$)
- $\frac{\mathrm{d}\sigma_\mathrm{T}}{\mathrm{d}t}$ increase at small $-t'$, then decrease (maximum due to $t'|\langle \bar{E}_\mathrm{T}\rangle|^2$)

The COMPASS fixed target experiment at CERN (2002 - 2022)



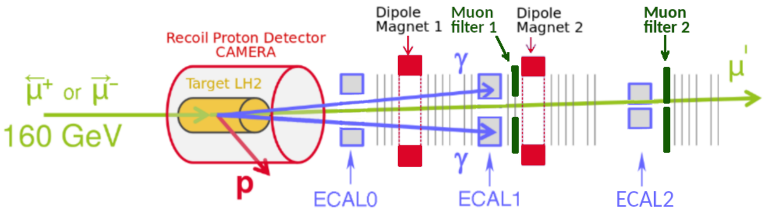
Experimental setup for GPD run



Measurement of the hard exclusive π^0 muoproduction at COMPASS

Beam polarisation
$\sim \pm 80$ %

Unpolarized target



- Collected data for HEMP and DVCS
  - 2012 - pilot run (results published in PLB 805 135454)
  - **2016 - this work** (same beam intesity but 2.3 times larger statistics than 2012)
  - 2017 - analysis started recently (2016+2017 $\approx$ 9 times larger statistics than 2012)

Cross section determination

$$\left\langle \frac{\mathrm{d}^2\sigma^{\gamma^* p}}{\mathrm{d}t\mathrm{d}\phi} \right\rangle^{\pm}_{\Delta\Omega} = \left(\sum_{i=1}^{N^{\Delta\Omega}_{data^\pm}} \frac{1}{\Gamma(\Omega_i)a(\Omega_i)\mathcal{L}^{\pm}\Delta\Omega} - f^{\pm} \sum_{i=1}^{N^{\Delta\Omega}_{LEPTO^\pm}} \frac{1}{\Gamma(\Omega_i)a(\Omega_i)\mathcal{L}^{\pm}\Delta\Omega} \right)$$

- phase space element $\Delta\Omega = \Delta|t|\Delta\Phi\Delta\nu\Delta Q^2$
  - $0.08 < |t| < 0.64$ (GeV/c)^2
  - $-\pi < \phi < \pi$ rad
  - $6.4 < \nu < 40$ GeV (virtual photon energy)
  - $1 < \mathrm{Q}^2 < 8$ (GeV/c)^2
- virtual photon flux Γ, acceptance a, luminosity $\mathcal{L}$
- background normalization

$$f^{\pm} = r^{\pm}_{LEPTO} \cdot \frac{N_{data^\pm}}{N_{LEPTO^\pm}}$$

r_{LEPTO} is a fraction of SIDIS background evaluated using LEPTO MC and exclusive HEPGEN++ MC

Cross section determination

$$\left\langle \frac{\mathrm{d}^2\sigma^{\gamma^* p}}{\mathrm{d}t\mathrm{d}\phi} \right\rangle^{\pm}_{\Delta\Omega} = \left(\sum_{i=1}^{N^{\Delta\Omega}_{data\pm}} \frac{1}{\Gamma(\Omega_i)a(\Omega_i)\mathcal{L}^{\pm}\Delta\Omega} - f^{\pm} \sum_{i=1}^{N^{\Delta\Omega}_{LEPTO\pm}} \frac{1}{\Gamma(\Omega_i)a(\Omega_i)\mathcal{L}^{\pm}\Delta\Omega} \right)$$

- phase space element $\Delta\Omega = \Delta|t|\Delta\Phi\Delta\nu\Delta Q^2$
  - $0.08 < |t| < 0.64$ (GeV/c)^2
  - $-\pi < \phi < \pi$ rad
  - $6.4 < \nu < 40$ GeV (virtual photon energy)
  - $1 < \mathrm{Q}^2 < 8$ (GeV/c)^2
- virtual photon flux Γ, acceptance a, luminosity $\mathcal{L}$
- spin-independent cross section

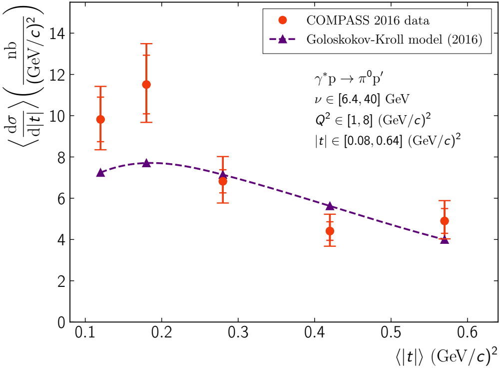
$$\left\langle \frac{\mathrm{d}^2\sigma^{\gamma^* p}}{\mathrm{d}t\mathrm{d}\phi} \right\rangle_{\Delta\Omega} = \frac{1}{2} \left(\left\langle \frac{\mathrm{d}^2\sigma^{\gamma^* p}}{\mathrm{d}t\mathrm{d}\phi} \right\rangle^{+}_{\Delta\Omega} + \left\langle \frac{\mathrm{d}^2\sigma^{\gamma^* p}}{\mathrm{d}t\mathrm{d}\phi} \right\rangle^{-}_{\Delta\Omega} \right)$$

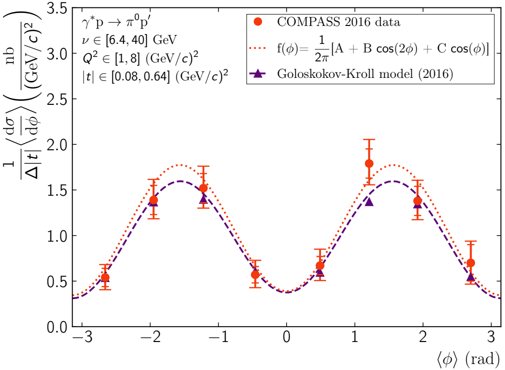
Results

Overview: results for the spin-independent cross section from 2016 data

- Full kinematic domain: $\nu \in (6.4, 40)$ GeV and $Q^2 \in (1, 8)$ $(\mathrm{GeV}/c)^2$
- Comparison to previously published results in reduced kinematic domain: $\nu \in (8.5, 28)$ GeV and $Q^2 \in (1, 5)$ $(\mathrm{GeV}/c)^2$
- Study of the ϕ-dependent cross section in different $|t|$-ranges
- Dependence of the cross section on virtual photon energy ν
- Dependence of the cross section on photon virtuality Q^2

Cross section for $\nu \in (6.4,40)$ GeV and $Q^2 \in (1,8)$ $(\mathrm{GeV}/c)^2$, $\langle x_{Bj} \rangle = 0.134$





S. Goloskokov, P. Kroll EPJC47 (2011) + private communication

$$\left\langle \frac{d\sigma_T}{d|t|} + \epsilon \frac{d\sigma_L}{d|t|} \right\rangle = \left(6.7 \pm 0.3_{\mathrm{stat}} {}^{+\,0.9}_{-\,0.8}\Big|_{\mathrm{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

$$\left\langle \frac{d\sigma_{TT}}{d|t|} \right\rangle = \left(-4.4 \pm 0.5_{\mathrm{stat}} {}^{+\,0.3}_{-\,0.3}\Big|_{\mathrm{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

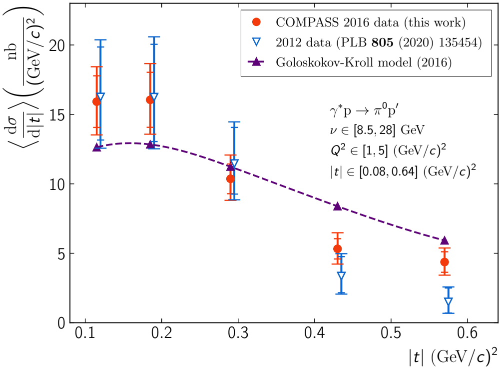
$$\left\langle \frac{d\sigma_{LT}}{d|t|} \right\rangle = \left(0.1 \pm 0.2_{\mathrm{stat}} {}^{+\,0.1}_{-\,0.1}\Big|_{\mathrm{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

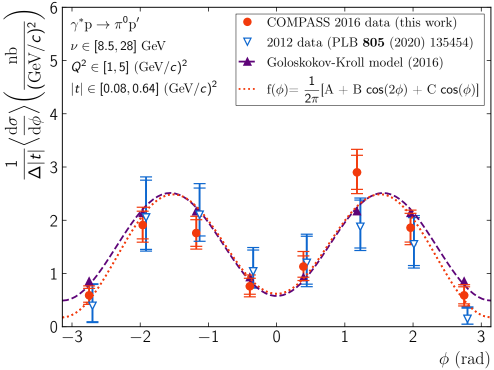
$\langle \epsilon \rangle = 0.997$

$$\frac{1}{2\pi}\left[\frac{d\sigma_T}{d|t|} + \epsilon \frac{d\sigma_L}{d|t|} + \epsilon\cos(2\phi)\frac{d\sigma_{TT}}{d|t|} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi)\frac{d\sigma_{LT}}{d|t|}\right]$$

- $\frac{d\sigma_{TT}}{d|t|}$ as large as $\frac{d\sigma_T}{d|t|} + \epsilon \frac{d\sigma_L}{d|t|} \Rightarrow$ Importance of $\bar{E}_T$

Cross section for $\nu \in (8.5, 28)$ GeV and $Q^2 \in (1, 5)$ $(\mathrm{GeV}/c)^2$, $\langle x_{Bj} \rangle = 0.1$





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2012 data:

$$\left\langle \frac{d\sigma_T}{d|t|} + \epsilon \frac{d\sigma_L}{d|t|} \right\rangle = \left(8.1 \pm 0.9_{\text{stat}} {}^{+\,1.1}_{-\,1.0}\Big|_{\text{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

$$\left\langle \frac{d\sigma_{TT}}{d|t|} \right\rangle = \left(-6.0 \pm 1.3_{\text{stat}} {}^{+\,0.7}_{-\,0.7}\Big|_{\text{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

$$\left\langle \frac{d\sigma_{LT}}{d|t|} \right\rangle = \left(1.4 \pm 0.5_{\text{stat}} {}^{+\,0.3}_{-\,0.2}\Big|_{\text{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

$\langle \epsilon \rangle = 0.996$

2016 data:

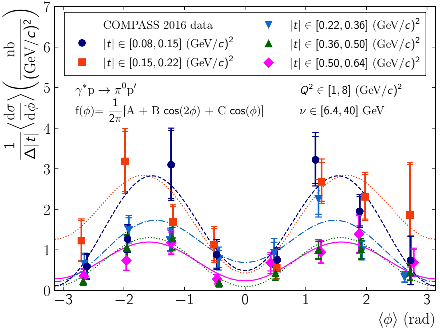
$$\left\langle \frac{d\sigma_T}{d|t|} + \epsilon \frac{d\sigma_L}{d|t|} \right\rangle = \left(9.0 \pm 0.5_{\text{stat}} {}^{+\,1.1}_{-\,1.0}\Big|_{\text{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

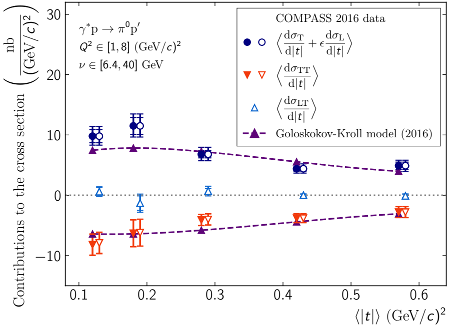
$$\left\langle \frac{d\sigma_{TT}}{d|t|} \right\rangle = \left(-6.6 \pm 0.8_{\text{stat}} {}^{+\,0.5}_{-\,0.5}\Big|_{\text{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

$$\left\langle \frac{d\sigma_{LT}}{d|t|} \right\rangle = \left(0.7 \pm 0.3_{\text{stat}} {}^{+\,0.4}_{-\,0.4}\Big|_{\text{sys}}\right) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^2}$$

$\langle \epsilon \rangle = 0.996$

Study of the ϕ-dependent cross section in different $|t|$-ranges





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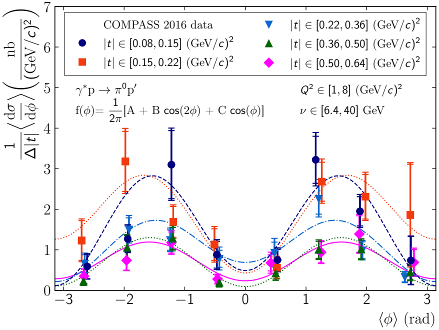
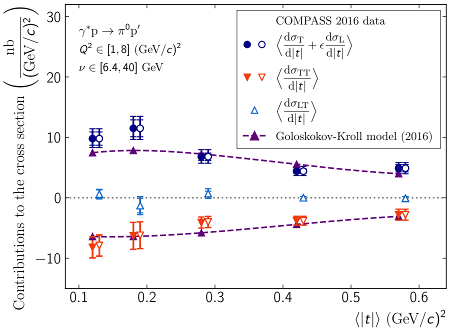
	$\langle Q^2\rangle$ [(GeV/c)^2]	$\langle\nu\rangle$ [GeV]	$\langle\lvert t\rvert\rangle$ [(GeV/c)^2]	$\langle x_{Bj}\rangle$	$\langle\epsilon\rangle$
$\lvert t\rvert \in [0.08, 0.15]$	1.93	11.76	0.12	0.104	0.996
$\lvert t\rvert \in [0.15, 0.22]$	2.11	10.32	0.18	0.123	0.997
$\lvert t\rvert \in [0.22, 0.36]$	2.33	9.86	0.28	0.140	0.997
$\lvert t\rvert \in [0.36, 0.50]$	2.41	9.29	0.42	0.150	0.998
$\lvert t\rvert \in [0.50, 0.64]$	2.65	9.35	0.57	0.165	0.998

Study of the ϕ-dependent cross section in different $|t|$-ranges

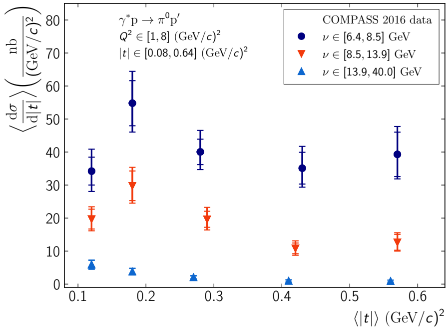


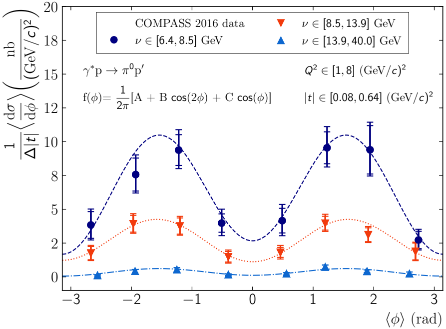


S. Goloskokov, P. Kroll EPJC47 (2011) + private communication

- $\frac{d\sigma_{LT}}{d|t|}$ compatible with zero within the stat. uncertainties
  $\Rightarrow$ the solid points correspond to the fit of only the 2 first contributions
- Large and opposite contributions of $\frac{d\sigma_T}{d|t|} + \epsilon\frac{d\sigma_L}{d|t|}$ and $\frac{d\sigma_{TT}}{d|t|}$
- Results could indicate the dominance of $\bar{E}_T$ compared to the other involved GPDs

Dependence of the cross section on virtual photon energy ν

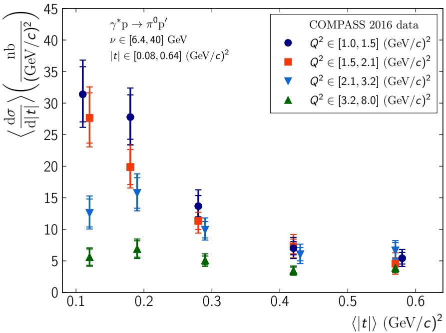


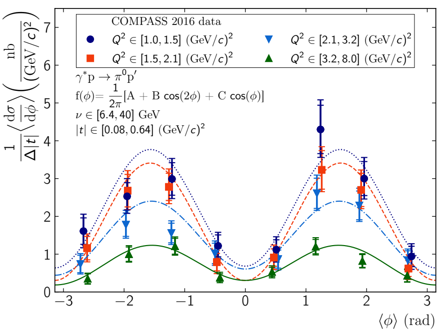


	$\langle \nu \rangle$ [GeV]	$\langle Q^2 \rangle$ [(GeV/c)^2]	$\langle x_{Bj} \rangle$	$\langle \epsilon \rangle$
$\nu \in [6.4, 8.5]$	7.35	2.15	0.156	0.999
$\nu \in [8.5, 13.9]$	10.32	2.50	0.131	0.998
$\nu \in [13.9, 40.0]$	21.08	2.09	0.057	0.989

- At small x_{Bj} (large ν), the maximum for the evolution in $|t|$ is no more visible
  $\Rightarrow$ significant longitudinal contribution $\frac{d\sigma_L}{d|t|}$

Dependence of the cross section on photon virtuality Q^2

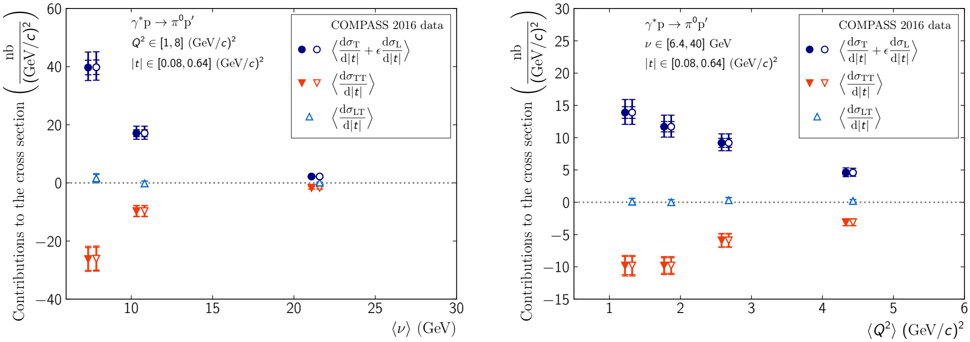




	$\langle Q^2 \rangle$ [(GeV/c)^2]	$\langle \nu \rangle$ [GeV]	$\langle x_{Bj} \rangle$	$\langle \epsilon \rangle$
$Q^2 \in [1.0, 1.5]$	1.22	10.54	0.072	0.997
$Q^2 \in [1.5, 2.1]$	1.77	9.81	0.109	0.997
$Q^2 \in [2.1, 3.2]$	2.58	9.82	0.157	0.997
$Q^2 \in [3.2, 8.0]$	4.33	10.39	0.247	0.997

- At small x_{Bj} (small Q^2), the maximum for the evolution in $|t|$ is no more visible $\Rightarrow$ significant longitudinal contribution $\frac{d\sigma_L}{d|t|}$

Contributions to the cross section as a function of ν and Q^2



- Strong decrease of the abs. values of $\frac{d\sigma_T}{d|t|} + \epsilon \frac{d\sigma_L}{d|t|}$ and $\frac{d\sigma_{TT}}{d|t|}$ with increasing ν, weaker decrease with increasing Q^2
- Valuable inputs for modelling GPDs, especially description of the major $\bar{E}_T$ contribution
- Should facilitate the study of NLO corrections and higher twist contributions

Summary

- Significant transverse–transverse interference contribution could indicate clear experimental evidence for the chiral-odd GPD $\bar{E}_{\mathrm{T}}$
- Non-negligible contribution of longitudinally polarised photons is suggested at $x_{Bj} < 0.1$
- Results provide valuable input for future modelling of GPDs

- Paper with results from 2016 data submitted to PLB; preprint arXiv:2412.19923 (2024)

- Prospects: analysis of 2017 data started recently
  - 2017 data $\approx$ 3 times larger statistics than 2016
    $\Rightarrow$ 2016+2017 enable separate study from μ^+ and μ^- beams $\Rightarrow$ determination of $\frac{\mathrm{d}\sigma'_{\mathrm{LT}}}{\mathrm{d}t}$

BACKUP

Event selection for $\mu p \to \mu' \pi^0 p'$

- Vertex candidate
  - μ and μ' associated to the vertex
  - requirements on μ and μ'

- π^0 candidate from 2 photons
  - pair of neutral clusters in ECALs
  - energy thresholds
  - timing requirement

- Proton candidate
  - CAMERA
  - exclusivity conditions

- $|\Delta\varphi| < 0.4$ rad
- $|\Delta p_T| < 0.3$ GeV/c
- $|M_x^2| < 0.3\ (\mathrm{GeV}/c^2)^2$
- $|\Delta z_A| < 16$ cm

- Kinematics
  - Kinematic fit
  - $6.4 < \nu < 40$ GeV
  - $1 < Q^2 < 8\ (\mathrm{GeV}/c)^2$
  - $0.08 < |t| < 0.64\ (\mathrm{GeV}/c)^2$

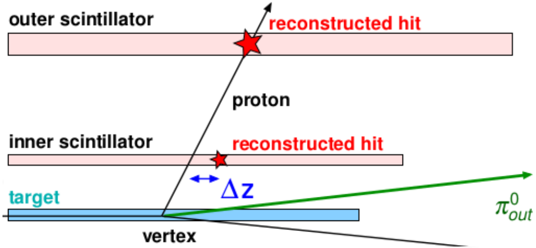
- Events with only one combination: *vertex candidate* + π^0 *candidate* + *proton candidate* pass the selection

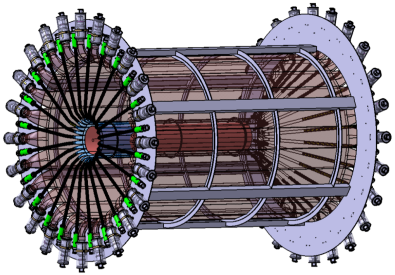
Exclusive variables measured either by the spectrometer or by CAMERA

- $P'_{spec} = P + k - k' - p_{\pi^0}$

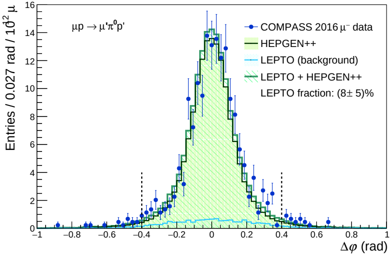
- $\Delta\varphi = \varphi_{CAM}$ - φ_{spec}
- $\Delta p_T = p_{T_{CAM}}$- $p_{T_{spec}}$
- $M_x^2 = (P + k - k' - p_{\pi^0} - P'_{CAM})^2$

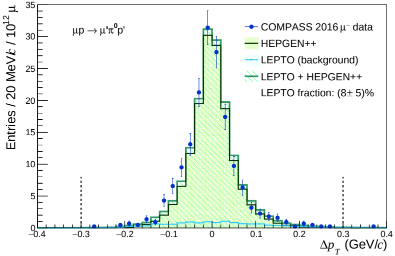
- $|\Delta\varphi| < 0.4$ rad
- $|\Delta p_T| < 0.3$ GeV/c
- $|M_x^2| < 0.3\ (\mathrm{GeV}/c^2)^2$
- $|\Delta z_A| < 16$ cm

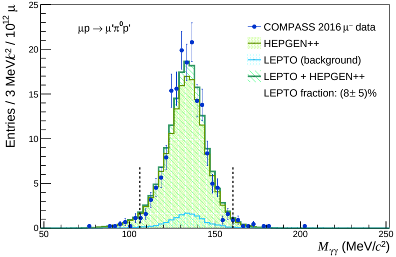




Example of exclusive variables and π^0 mass







virtual photon flux:

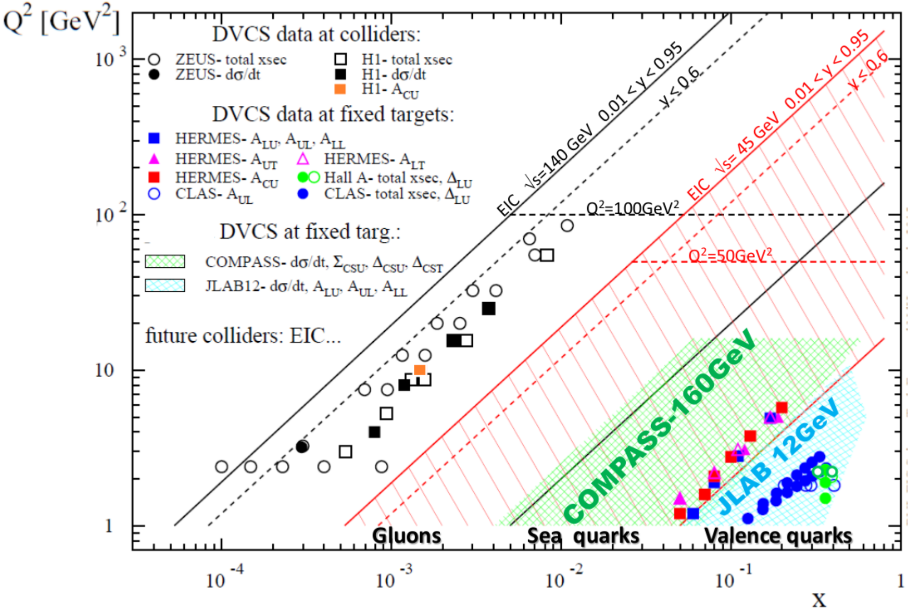
$$\Gamma(Q^2,\nu) = \frac{\alpha_{em}(1-x_{Bj})}{2\pi Q^2 y E_\mu}\left[y^2\left(1-\frac{2m_\mu^2}{Q^2}\right)+\frac{2}{1+Q^2/\nu^2}\left(1-y-\frac{Q^2}{4E_\mu^2}\right)\right]$$

virtual photon polarization parameter:

$$\epsilon = \frac{1-y-\frac{1}{4}y^2Q^2/\nu^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}y^2Q^2/\nu^2}$$

the longitudinal momentum fraction transfer ξ can be approximated in the COMPASS kinematic regime at small x_{Bj} by

$$\xi \approx \frac{x_{Bj}}{2-x_{Bj}}.$$



Q^2 [GeV^2]
DVCS data at colliders:
ZEUS- total xsec
ZEUS- dσ/dt
H1- total xsec
H1- dσ/dt
H1- A_CU
DVCS data at fixed targets:
HERMES- A_LU, A_UL, A_LL
HERMES- A_UT
HERMES- A_LT
HERMES- A_CU
Hall A- total xsec, Δ_LU
CLAS- A_UL
CLAS- total xsec, Δ_LU
DVCS at fixed targ.:
COMPASS- dσ/dt, Σ_CSU, Δ_CSU, Δ_CST
JLAB12- dσ/dt, A_LU, A_UL, A_LL
future colliders: EIC...
EIC √s=140 GeV 0.01 < y < 0.95
y < 0.6
Q^2=100GeV^2
EIC √s= 45 GeV 0.01 < y < 0.95
y < 0.6
Q^2=50GeV^2
COMPASS-160GeV
JLAB 12GeV
Gluons
Sea quarks
Valence quarks
10^3
10^2
10
1
10^-4
10^-3
10^-2
10^-1
1
x

Kinematic fit

- Measurements of exclusive processes at COMPASS are over-constrained
- This can be used to improve the resolution on the measured kinematic quantities by the usage of a kinematically constrained fit
- In case of exclusive π^0 analysis, the fit can be used to improve signal to background ratio

- The goal is to minimize the least square function $\chi^2(\vec{k}) := (\vec{k}_{fit} - \vec{k})^T \widehat{C}^{-1}(\vec{k}_{fit} - \vec{k})$
- $\vec{k}$ is the vector of measured quantities (e.g. variables such as momentum and transverse position corresponding to the incoming/scattered muon, initial/final proton and lower/higher energetic photon)
- $\widehat{C}$ is the covariance matrix corresponding to the measured quantities
- The Lagrange multiplier method with constraints g_i is used for the minimization:

$$L(\vec{k}, \vec{\alpha}) = \chi^2(\vec{k}) + 2\sum_{i=1}^{N} \alpha_i g_i. \qquad (1)$$

Kinematic fit - introduction - kinematic constraints

- **Momentum and energy conservation**

$$g_i = p^{fit}_{\mu,i} - p^{fit}_{\mu\prime,i} - p^{fit}_{p\prime,i} - p^{fit}_{\gamma_h,i} - p^{fit}_{\gamma_l,i} = 0, \;\; i = 1,2,3$$

$$g_4 = E^{fit}_{\mu} + m_p c^2 - E^{fit}_{\mu\prime} - E^{fit}_{p\prime} - E^{fit}_{\gamma_h} - E^{fit}_{\gamma_l} = 0$$

- **Common vertex for all tracks (except initial and final state proton)**

$$g_{5+i} = p^{fit}_{j,3}\left(x_v - x^{fit}_j\right) - p^{fit}_{j,1}\left(z_v - z^{fit}_j\right) = 0 \;\; (i,j) \in \{(0,\mu),(2,\mu\prime),(4,\gamma_h),(6,\gamma_l)\}$$

$$g_{6+i} = p^{fit}_{j,3}\left(y_v - y^{fit}_j\right) - p^{fit}_{j,2}\left(z_v - z^{fit}_j\right) = 0, \;\; (i,j) \in \{(0,\mu),(2,\mu\prime),(4,\gamma_h),(6,\gamma_l)\}$$

- **Constraints for final state proton**

$$g_{13+i} = p^{fit}_{p\prime,3}\left(x^{fit}_j - x_v\right) - p_{p\prime,1}\left(z^{fit}_j - z_v\right) = 0, \;\; (i,j) \in \{(0,A),(2,B)\}$$

$$g_{14+i} = p^{fit}_{p\prime,3}\left(y^{fit}_j - y_v\right) - p_{p\prime,2}\left(z^{fit}_j - z_v\right) = 0, \;\; (i,j) \in \{(0,A),(2,B)\}$$

- **Mass constraint** $g_{17} = (E^{fit}_{\gamma_h} + E^{fit}_{\gamma_l})^2 - (p^{fit}_{\gamma_h} + p^{fit}_{\gamma_l})^2 - m^2_{\pi^0} = 0$