GPDs through
Universal
Moment
Parametrization

The GUMP FRAMEWORK

A moment based approach to GPD Pheno

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GUMP Collaboration

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CENTER for NUCLEAR FEMTOGRAPHY

Jefferson Lab

Outline

- *GPDs and Challenges of the GPD Pheno*
  - *Parametrization*
  - *Inverse problem*

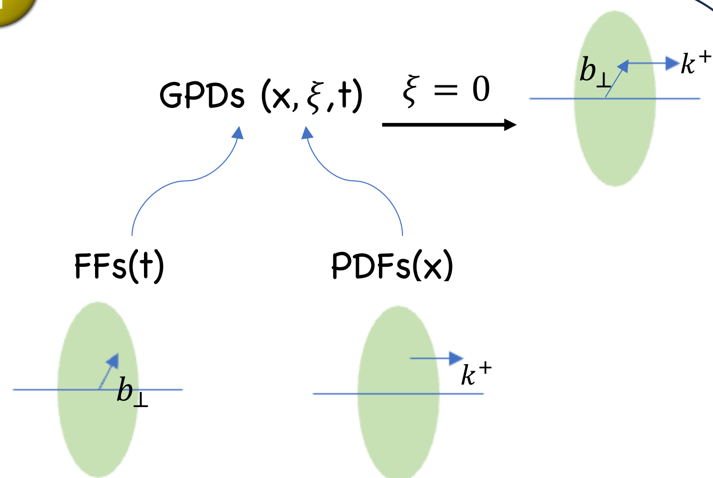
- *The GUMP Project*
  - *Conformal wave expansion of GPDs*

- *DVMP and DVCS*

- *TFFs from ρ meson production*

GPDs…

1



- Impact parameter distributions

  $q(x, b_\perp) = FT(-\Delta_T^2)\ GPD(x, \xi = 0, -\Delta_T^2)$

- Charge and magnetization distributions:

  $\int dx\ GPD(x, \xi, t) \rightarrow em\ FFs(t)$

- Longitudinal momentum distribution:

  $GPD(x, \xi = 0, t = 0) = PDF(x)$

2

GPDs (x, ξ, t) → Gravitational form factors

$$\int dx\ x\ GPDs(x, \xi, t) \rightarrow gr\ FFs(t)$$

- Mass and energy distributions
- Angular momentum distribution
- Pressure distribution
- Shear force distribution

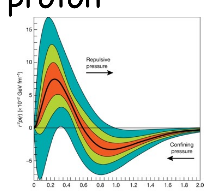
From GPDs we learned:

- Mass radius of the proton: $R_m = 0.55\ fm$

  Kharzeev (*PhysRevD*. 104.0540151)
  Polyakov, Schweitzer (2018) 1805.06596

- Pressure distribution of the proton

  V. D. Burkert, L. Elouadrhiri & F. X. Girod 2018
  https://doi.org/10.1038/s41586-018-0060-z



GPDs…

1

GPDs (x, ξ, t) $\xi = 0$ → $b_\perp$, k^+

FFs(t) → $b_\perp$

PDFs(x) → k^+

- Impact parameter distributions

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  $\int dx\ GPD(x, \xi, t) \to em\ FFs(t)$

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That much information comes at a cost!

2

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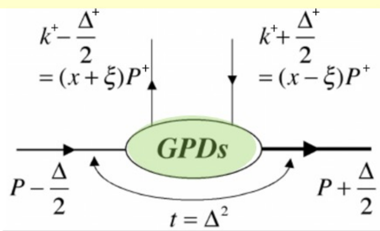
The Challenges of GPD pheno

Parametrizing the GPDs

3 variables at fixed scale: x, ξ, t

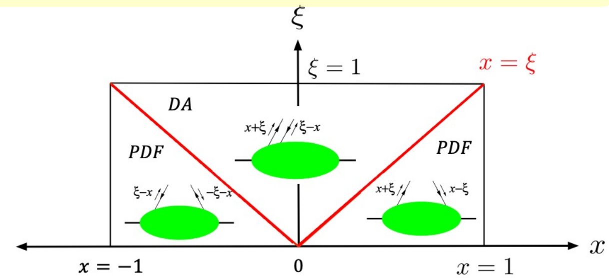


x:Average longitudinal momentum fraction

ξ:Longitudinal momentum transfer fraction

t: Momentum transfer squared

Two kinematic regions: *DGLAP and ERBL (or PDF or DA)*



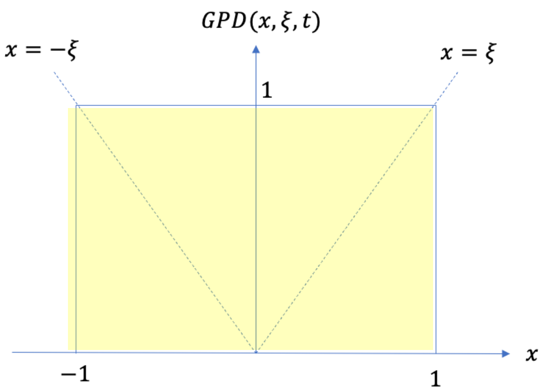
Constraints on GPDs(x, ξ, t)

- **Kinematical limits on the variables** : $t \in \left[-\infty, \frac{4\xi^2 M^2}{1-\xi^2}\right], \quad x \in [-1,1], \quad \xi \in [-1,1]$
- **Continuity at $x = \xi$:** Continuity between the two kinematic regions
- **Polynomiality** $\mathcal{F}_n(\xi,t) \equiv \int_{-1}^{1} dx\, x^{n-1} GPD(x,\xi,t) = \sum_{k=0,even}^{n} \xi^k F_{n,k}(t)$
- **The forward limit:** $\lim_{\xi,t\to 0} GPD(x,\xi,t) = PDF(x)$

We need to parametrize a multi dimensional function of three variables, (x, ξ, t), that is defined in two kinematical regions and must obey certain theoretical constraints

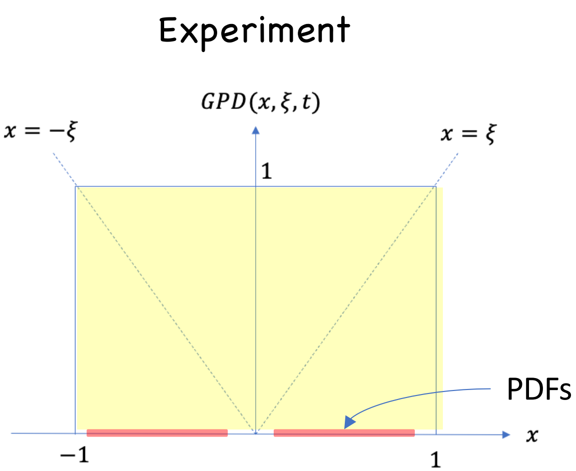
The Challenges of GPD pheno

The Inverse Problem



The Challenges of GPD pheno

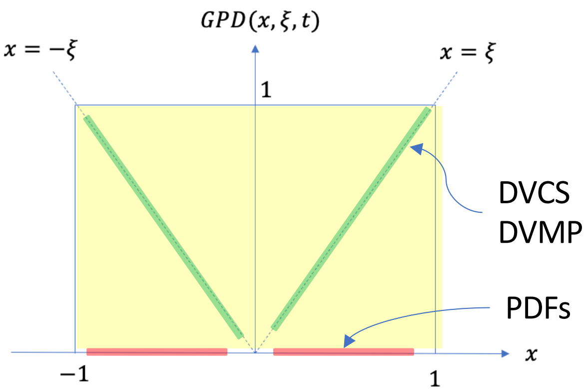
The Inverse Problem



The Challenges of GPD pheno

The Inverse Problem

Experiment



Experimental data give information only on $x = \pm\xi$

$*$ *Neglecting the real parts of the CFFs and TFFs*

The Challenges of GPD pheno

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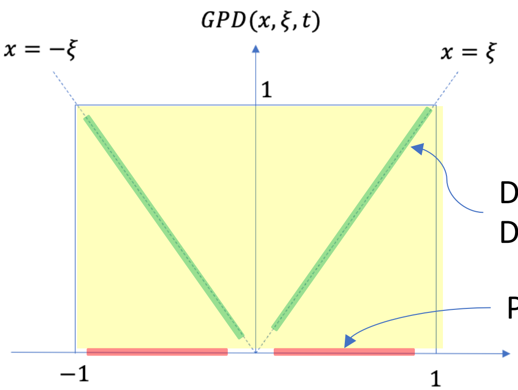

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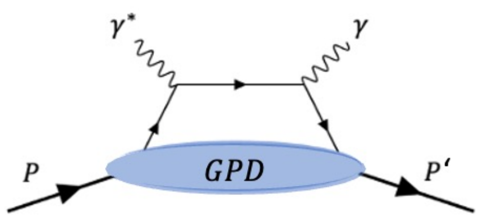
Why only $x = \pm\xi$?



$$d\sigma_{\rm DVCS} \propto |\mathcal{H}(\xi,t)|^2$$

$$\mathcal{H}(\xi,t) = \int_{-1}^{1} H(x,\xi,t)\left(\frac{1}{\xi - x - i\epsilon}\right) dx$$

$$= \underbrace{\mathrm{PV}\int_{-1}^{1} dx\,\frac{H(x,\xi,t)}{\xi - x}}\, + i\pi \underbrace{\int_{-1}^{1} dx\, H(x,\xi,t)\,\delta(\xi - x)}_{GPD\ at\ x=\xi} .$$

The Challenges of GPD pheno

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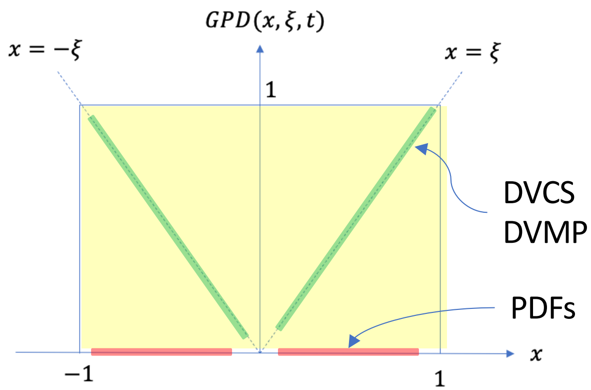

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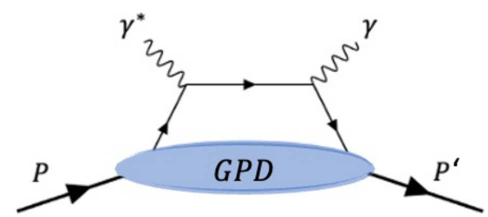

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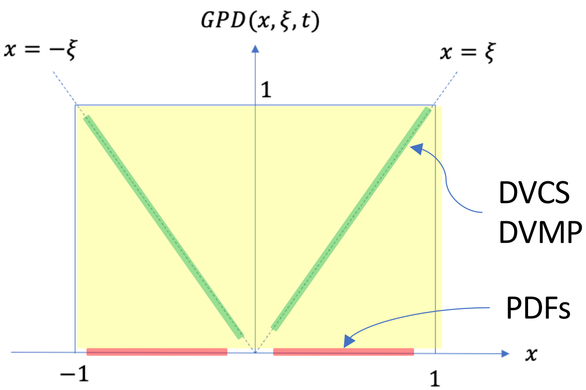
How to take the GPDs off the $x = \pm\xi$ line: Single diffractive hard exclusive processes. Qiu & Yu 2407.11304

The Challenges of GPD pheno

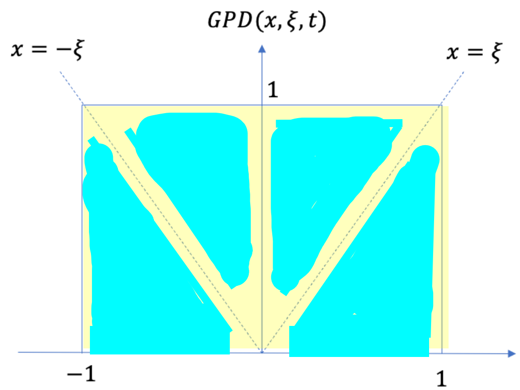
The Inverse Problem

Experiment

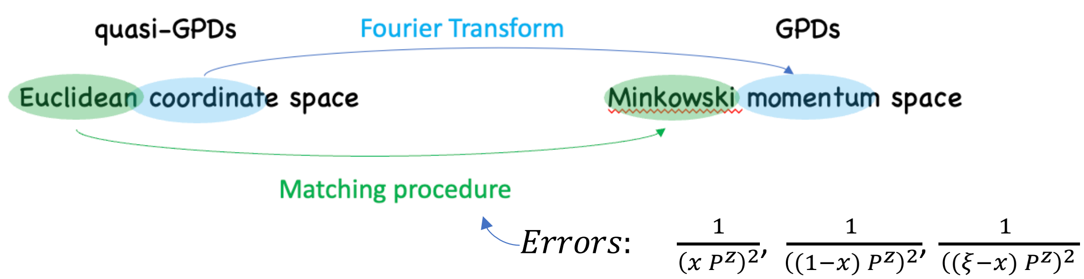


Experimental data give information only on $x = \pm\xi$

Lattice



Lattice has limitations on $x = \pm\xi$

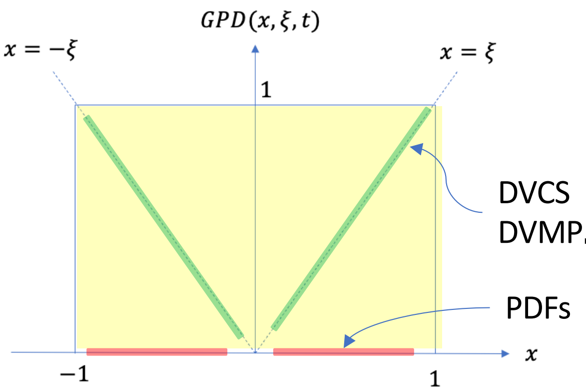


The Challenges of GPD pheno

The Inverse Problem

THE GUMP PROJECT

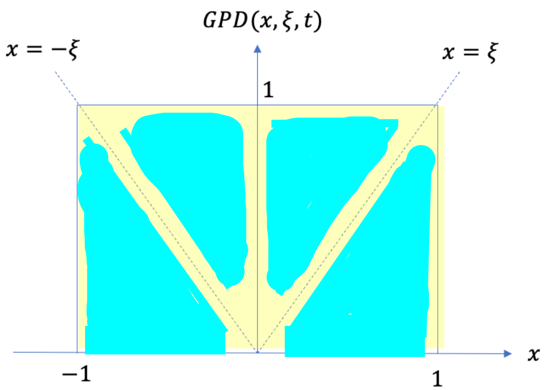
Experiment



Experimental data give information only on $x = \pm\xi$

+

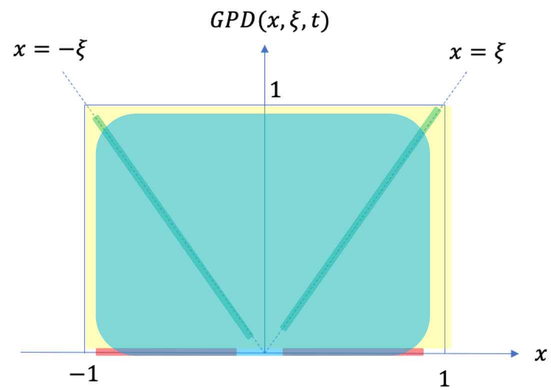
Lattice



Lattice has limitations on $x = \pm\xi$

=

Experiment+Lattice



Experimental data and lattice calculations are complementary!

Combining all the possible constraints on GPDs, including both experimental and lattice calculations and parametrizing the t dependence of the conformal moments of valence and sea distributions

Conformal wave expansion of GPDs

- *Consider we expand the GPD in terms of a complete set of polynomials*: $F(x,\xi,t) = \sum \rho_C(x) C_i(x) F_i^C(\xi,t)$,

- *Then the expansion coeffcients*, $F_i^C(\xi,t)$ *are simply the moments of* $F(x,\xi,t)$: $F_i^C(\xi,t) = \int_{-1}^{1} \mathrm{d}x C_i(x) F(x,\xi,t)$.

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Evolution in x-space	Evolution in Conformal space
Integro-differential (difficult) $\frac{d\, F(x,\xi,t,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{-1}^{1} \frac{dx'}{\lvert\xi\rvert} V^{(0)}(\frac{x}{\xi},\frac{x'}{\xi})\, F(x',\xi,t,Q^2) + \mathcal{O}(\alpha(Q^2)^2) + \cdots$	Matrix multiplicative (easier) $\frac{d\, F_n(\xi,t,Q^2)}{d\ln(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} \gamma_n^{(0)} F_n(\xi,t,Q^2) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 \sum_m \gamma_{nm}^{(1)}\, F_m(\xi,t,Q^2) + \cdots$

LO Evolution:
The evolution is ***diagonal*** *in conformal space,*
and there is ***no mixing*** *of conformal moments.*

NLO and Beyond:
The evolution is ***non − diagonal*** *in conformal space,*
there is ***mixing*** *of conformal moments*
but it is still easier than the coordinate space evolution
because it is matrix multiplicative

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\mathcal{F}_1\\ \mathcal{F}_2\\ \vdots\\ \mathcal{F}_n\end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\begin{pmatrix}\gamma_1^{(0)} & 0 & 0 & \dots & 0\\ 0 & \gamma_2^{(0)} & 0 & \dots & 0\\ 0 & 0 & \gamma_3^{(0)} & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \dots & \gamma_n^{(0)}\end{pmatrix}\begin{pmatrix}\mathcal{F}_1\\ \mathcal{F}_2\\ \vdots\\ \mathcal{F}_n\end{pmatrix} + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2\begin{pmatrix}\gamma_{11}^{(1)} & \gamma_{12}^{(1)} & \dots & \gamma_{1n}^{(1)}\\ 0 & \gamma_{22}^{(1)} & \dots & \gamma_{2n}^{(1)}\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & \gamma_{nn}^{(1)}\end{pmatrix}\begin{pmatrix}\mathcal{F}_1\\ \mathcal{F}_2\\ \vdots\\ \mathcal{F}_n\end{pmatrix} + \cdots$$

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$\rho_j(x,\xi)$: Partial wave function

Conformal moment

Normalization constant to match the Mellin moment in the forward limit

Weight function of the Gegenbauer polynomials

Gegenbauer polynomials

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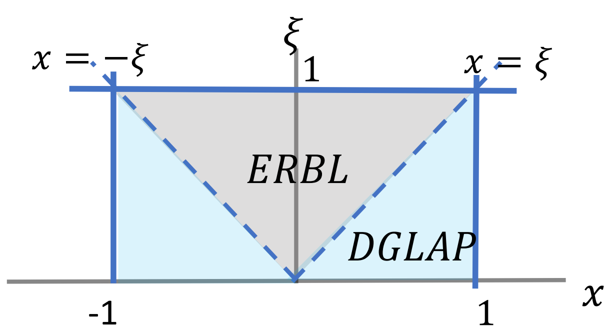
This series diverges!
Cannot represent GPDs



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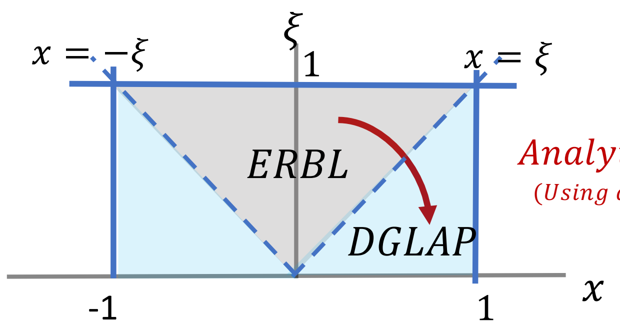

Analytic continuation
(Using asymptotic behaviour)

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Analytic continuation
(Using asymptotic behaviour)

Mellin Barnes Integral: $F(x,\xi,t) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \frac{p_j(x,\xi)}{\sin(\pi[j+1])} \mathcal{F}_j(\xi,t)$

Muller and Schafer hep-ph/0509204 (2005)

Parametrization of GPDs

Inverse conformal transform

$$F(x,\xi,t) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)$$

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↓ *polynomiality*

$$\mathcal{F}_j(\xi,t) = \sum_{k=0}^{j+1}\xi^k \mathcal{F}_{j,k}(t) = \mathcal{F}_{j,0}(t) + \xi^2 \underbrace{\mathcal{F}_{j,2}(t)}_{R_2\mathcal{F}_{j,0}} + \xi^4 \underbrace{\mathcal{F}_{j,4}(t)}_{R_4\mathcal{F}_{j,0}} + \cdots$$

Parametrization of GPDs

Inverse conformal transform

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$$\mathcal{F}_{j,0}(t) = \underbrace{N_0\, B(j+1-\alpha_0, 1+\beta_0)}_{\text{Common ansatz for PDFs}} \underbrace{\frac{j+1-\alpha_0}{j+1-\alpha_0-\alpha'_0 t}}_{\text{Regge Trajectory}} \overbrace{e^{-b|t|}}^{\text{Observed exp. fall off}}$$

Observed exp. fall off

Common ansatz for PDFs

$$f(x) = N_0\; x^{-\alpha_0}\; (1-x)^{\beta_0}$$

Regge Trajectory

$$x^{-\alpha(t)}$$

$$\alpha(t) = \alpha_0 + \alpha'_0 t$$

5 parameters in the semi – forward limit

GUMP PAPERS

Generalized parton distributions through universal moment parameterization: zero skewness case

Yuxun Guo (Maryland U.), Xiangdong Ji (Maryland U. and Unlisted, US), Kyle Shiells (Unlisted, US) (Jul 12, 2022)

Published in: *JHEP* 09 (2022) 215 • e-Print: 2207.05768 [hep-ph]

Analysis of t-dependent PDFs which correspond to GPDs in the $\xi \to 0$ limit.

Generalized parton distributions through universal moment parameterization: non-zero skewness case

Yuxun Guo (Maryland U. and LBNL, NSD), Xiangdong Ji (Maryland U.), M. Gabriel Santiago (Ctr. Nucl. Femtography, Washington, DC), Kyle Shiells (Manitoba U.), Jinghong Yang (Maryland U.) (Feb 14, 2023)

Published in: *JHEP* 05 (2023) 150 • e-Print: 2302.07279 [hep-ph]

Extending the framework to allow for the global analysis at non-zero skewness.

On convergence properties of GPD expansion through Mellin/conformal moments and orthogonal polynomials

Hao-Cheng Zhang (Shandong U.), Xiangdong Ji (Maryland U.) (Aug 7, 2024)

Published in: *Nucl.Phys.B* 1010 (2025) 116762 • e-Print: 2408.04133 [hep-ph]

Deriving an asymptotic condition on the conformal moments of GPDs to satisfy the boundary condition at x=1

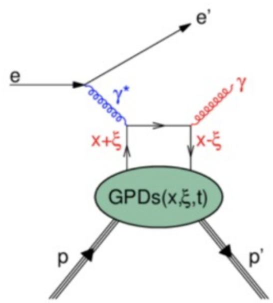
Small-x gluon GPD constrained from deeply virtual J/ψ production and gluon PDF through universal-moment parameterization

Yuxun Guo (LBNL, NSD), Xiangdong Ji (Maryland U.), M. Gabriel Santiago (Unlisted, US and Maryland U.), Jinghong Yang (Maryland U.), Hao-Cheng Zhang (Shandong U.) (Sep 25, 2024)

Gluon GPDs from $DVJ/\psi P$ using HERA data at NLO

Current project: Global analysis combining DVCS and DVMP + lattice data at NLO
Fatma Aslan, Yuxun Guo, Xiangdong Ji, M. Gabriel Santiago

DVCS and DVMP; complementary rather than redundant

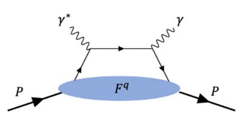
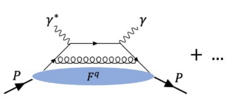
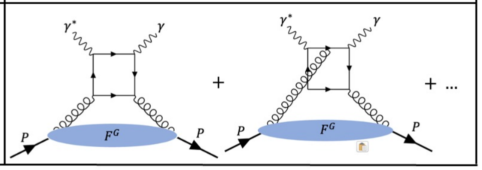


DVCS

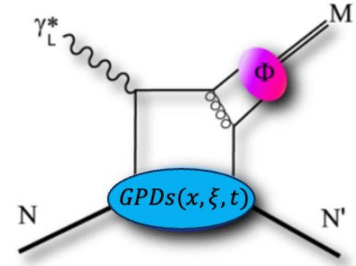
$$e\,p \rightarrow e'\,p'\,\gamma$$

A lepton scatters off the proton by exchanging a virtual photon, and in the final state, a real photon is emitted

- ❑ *The gluons start contributing the hard part at NLO. Their impact at LO arises only through evolution.*

	Quark GPD	Gluon GPD
LO		*There is no gluon contribution at LO!*
NLO		

- ❑ *Contributions from vector GPDs H & E and axial vector GPDs $\tilde{H}$ & $\tilde{E}$*

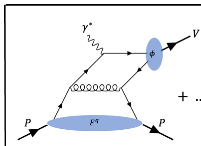
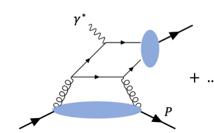
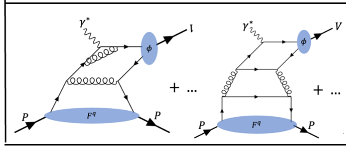
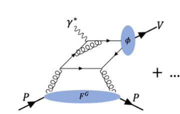


DVV_LP

$$e\,p \rightarrow e'p'M$$

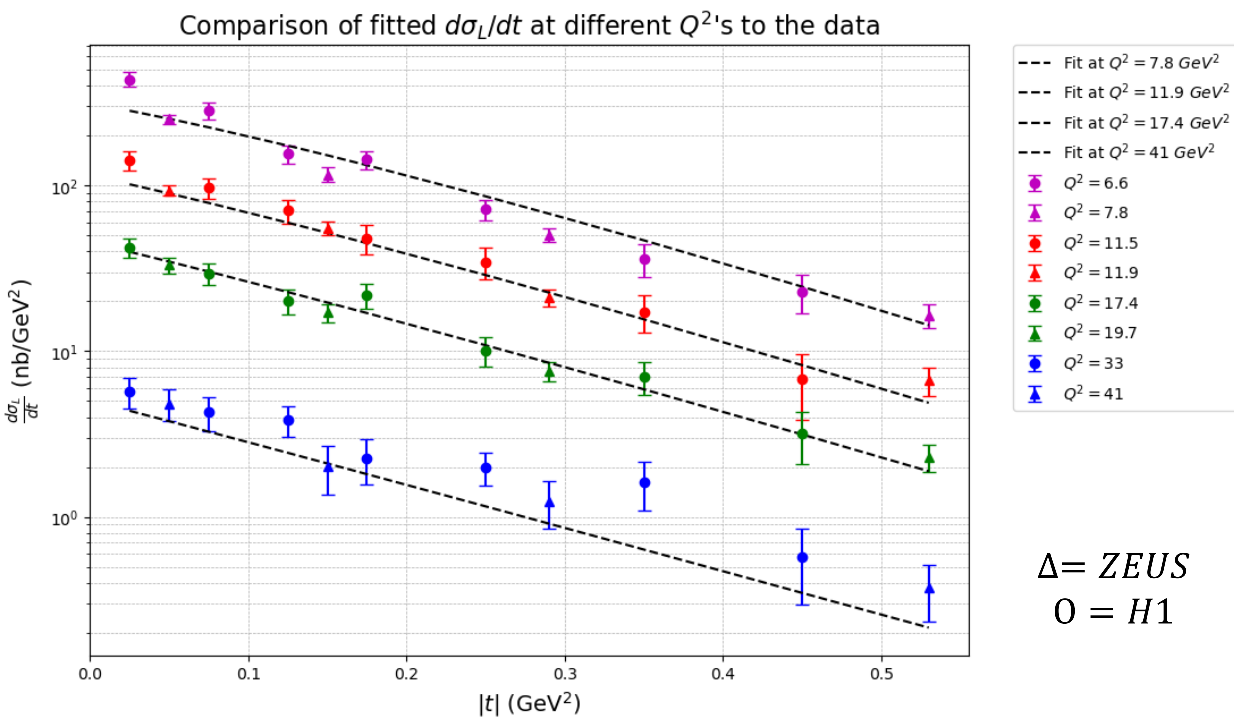
A lepton scatters off the proton by exchanging a virtual photon, and in the final state, a meson is produced

- ❑ *The gluons start contributing to the hard part at LO.*

	Quark GPD	Gluon GPD
LO		
NLO		

- ❑ *Vector GPDs H and E contribute to vector meson production.*

Fitting the DVρ_LP cross section using HERA data



$\Delta = ZEUS$
$0 = H1$

- We are making an L\T separation of the differential cross section

$$\sigma_{\text{tot}} = \sigma_T + \varepsilon \sigma_L$$

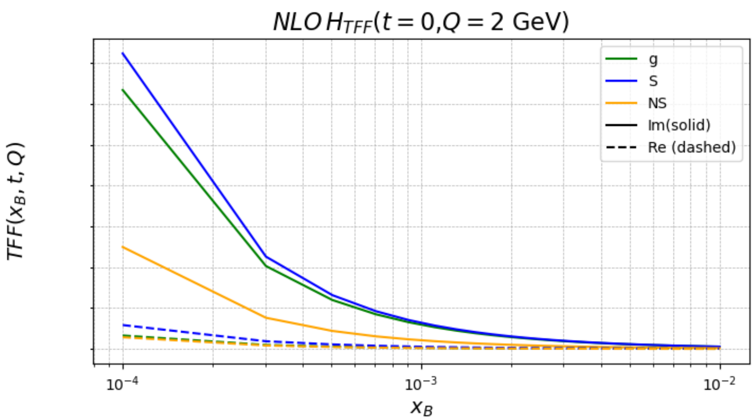
$$R \equiv \frac{\sigma_{\text{L}}^{\rho^0}}{\sigma_{\text{T}}^{\rho^0}}$$

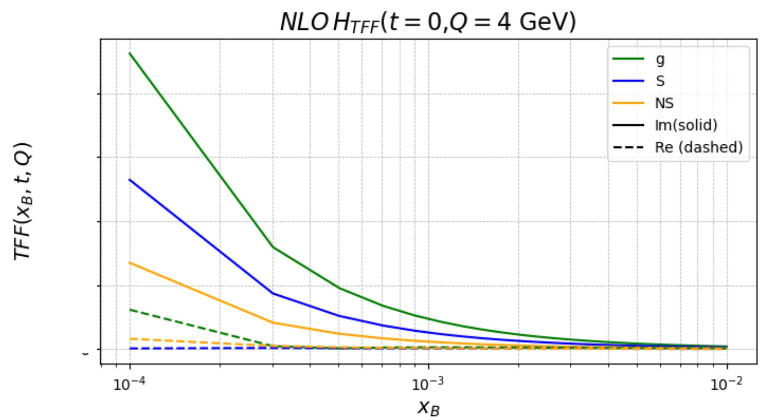
- We are using only HERA data for DVMP

The Real and Imaginary parts of the transition form factor at NLO

Singlet=$(u+\bar{u})+(d+\bar{d})$

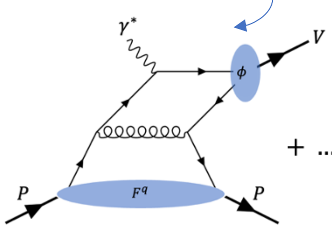
Non-singlet= -$(u+\bar{u})+(d+\bar{d})$



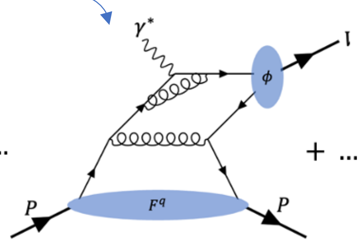


TFF = Hard part ⊗ GPD (x,ξ)
perturbative
(LO+NLO…)

TFF = Hard part ⊗ Evolution ⊗ GPD (x,ξ)
perturbative perturbative
(LO+NLO+…) (LO+NLO+…)



+ …



+ …

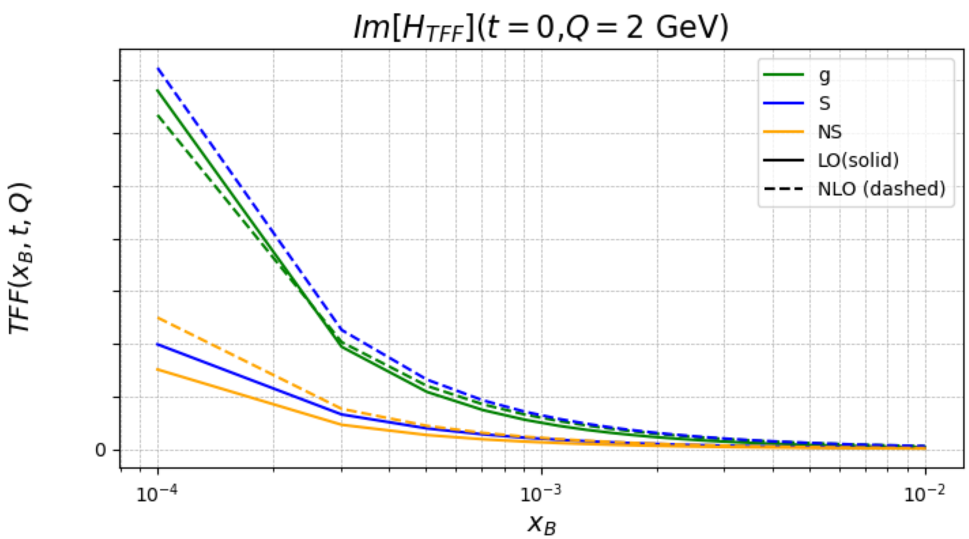
- At LO, real part is negligibly small:

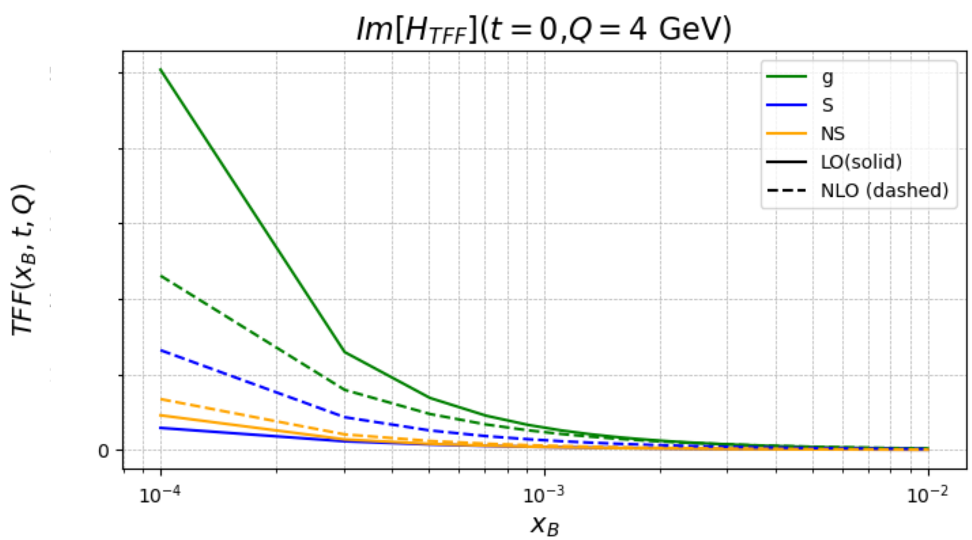
$$\mathcal{H}(\xi,t)=\int_{-1}^{1}H(x,\xi,t)\left(\frac{1}{\xi-x-i\epsilon}\right)dx = \underbrace{\mathrm{PV}\int_{-1}^{1}dx\,\frac{H(x,\xi,t)}{\xi-x}}+i\pi\underbrace{\int_{-1}^{1}dx\,H(x,\xi,t)\,\delta(\xi-x)}_{H(x,x,t)}\,.$$

- This is also the case at NLO

Imaginary part of the transition form factor at LO and NLO

Singlet=$(u+\bar{u})+(d+\bar{d})$
Non-singlet= $-(u+\bar{u})+(d+\bar{d})$





Significant contribution from NLO!

THE GUMP PROJECT

Challenges of GPD Pheno	*The GUMP project*
Parametrization	*Working with conformal moments of GPDs (easy evolution, polynomiality, ...)*
Inverse problem	*Combining the experimental and lattice data*

DVρP at NLO

- The real part of the TFFs are negligible also at NLO
- Significant contributions from NLO to the imaginary part of the TFFs
- Global analysis combining DVCS and DVMP + lattice data

Outlook

- Implementation of Next-to-leading order NRQCD factorization of Jpsi production and photo-productions
- Bayesian and ML analysis for GUMP/GPD
- String-based parameterization and phenomenological studies
- phi-production and strange quark distributions
- Implement large-xi and threshold production into GPD analysis
- Next-to-leading order refactorization of Deeply virtual J/psi Production
- EIC simulations with GUMP
- Kinematic higher twist effects in DVCS and DVMP
- …
- …

THANK YOU!

BACK UP...

The GPDs that contribute to DVCS and DVMP

DVCS: $H, E, \widetilde{H}, \tilde{E}$ (Jlab+HERA data)
DVMP: Depends on the final state meson (HERA data)

HERA data: $\phi, J/\psi, \rho$

$DVMP$

$DV\ P\ P$

No HERA data

$Mesons: \pi, K, \eta, \eta'$

Leading twist GPDs : $\widetilde{H}, \tilde{E}$

$DV\ V\ P$

No HERA data for ω

$Mesons: \rho, \omega, \phi, J/\psi$

Good for gluon GPDs only

$J/\psi \rightarrow c\bar{c}$ $\phi \rightarrow s\bar{s}$ $\rho \rightarrow u\bar{u} - \mathrm{d}\bar{d}$ ✔

We did not include the s flavor in the code yet

The GPDs that contribute to DVCS and DVMP

DVCS: $H, E, \widetilde{H}, \tilde{E}$ (Jlab+HERA data)

DVMP: Depends on the final state meson (HERA data)

HERA data: $\phi, J/\psi, \rho$

$DVMP$

$DV\ P\ P$

No HERA data

~~$Mesons$: π, K, η, η'~~

~~Leading twist GPDs : $\widetilde{H}, \tilde{E}$~~

$DV\rho\ P$

$DV\rho_L\ P$

$$\sigma_L = \sigma(\gamma_L^* P \to \rho_L^0 P)$$

Leading twist GPDs : H, E

$DV\ \rho_T\ P$

$$\sigma_T = \sigma(\gamma_T^* P \to \rho_T^0 P)$$

No factorization (higher twist)

$$\frac{d\sigma_{tot}}{dt} = \epsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt}$$

$$R \equiv \frac{\sigma_L}{\sigma_T}$$

$$\frac{d\sigma_\mathrm{L}}{dt} = \frac{\frac{d\sigma_\mathrm{tot}}{dt}}{\left(\epsilon + \frac{1}{R}\right)}$$

The SCHC assumption: polarization of the meson follows that of the photon

$$\gamma_\mathrm{L}^* \to \mathrm{V_L}, \quad \gamma_\mathrm{T}^* \to \mathrm{V_T}$$

EVOLUTION

The operator E_{jl} governing the perturbative evolution of the singlet vector GPDs - Representative picture

Leading order structure

$$\begin{pmatrix} \boxed{\begin{pmatrix} \gamma_{11}^{\Sigma\Sigma(0)} & \gamma_{11}^{\Sigma G(0)} \\ \gamma_{11}^{G\Sigma(0)} & \gamma_{11}^{GG(0)} \end{pmatrix}} & 0 & 0 & \cdots \\ 0 & \boxed{\begin{pmatrix} \gamma_{22}^{\Sigma\Sigma(0)} & \gamma_{22}^{\Sigma G(0)} \\ \gamma_{22}^{G\Sigma(0)} & \gamma_{22}^{GG(0)} \end{pmatrix}} & 0 & \cdots \\ 0 & 0 & \boxed{\begin{pmatrix} \gamma_{33}^{\Sigma\Sigma(0)} & \gamma_{33}^{\Sigma G(0)} \\ \gamma_{33}^{G\Sigma(0)} & \gamma_{33}^{GG(0)} \end{pmatrix}} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Next to leading order structure

$$\begin{pmatrix} \boxed{\begin{pmatrix} \gamma_{11}^{\Sigma\Sigma(1)} & \gamma_{11}^{\Sigma G(1)} \\ \gamma_{11}^{G\Sigma(1)} & \gamma_{11}^{GG(1)} \end{pmatrix}} & \boxed{\begin{pmatrix} \gamma_{12}^{\Sigma\Sigma(1)} & \gamma_{12}^{\Sigma G(1)} \\ \gamma_{12}^{G\Sigma(1)} & \gamma_{12}^{GG(1)} \end{pmatrix}} & \boxed{\cdots} & \cdots \\ 0 & \boxed{\begin{pmatrix} \gamma_{22}^{\Sigma\Sigma(1)} & \gamma_{22}^{\Sigma G(1)} \\ \gamma_{22}^{G\Sigma(1)} & \gamma_{22}^{GG(1)} \end{pmatrix}} & \boxed{\cdots} & \cdots \\ 0 & 0 & \boxed{\begin{pmatrix} \gamma_{33}^{\Sigma\Sigma(1)} & \gamma_{33}^{\Sigma G(1)} \\ \gamma_{33}^{G\Sigma(1)} & \gamma_{33}^{GG(1)} \end{pmatrix}} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Singlet evolution

$$\begin{pmatrix} H_j^{\Sigma}(\xi,t,\mu^2) \\ H_j^{G}(\xi,t,\mu^2) \end{pmatrix} = \mathbb{E}_{jl}(\mu,\mu_0;\xi) \begin{pmatrix} H_l^{\Sigma}(\xi,t,\mu_0^2) \\ H_l^{G}(\xi,t,\mu_0^2) \end{pmatrix}$$

$$\mathbb{E}_{jl}(\mu,\mu_0;\xi) = \sum_{a,b=\pm} \left[\delta_{ab} \boldsymbol{P}_j^a \delta_{jl} + \frac{\alpha_s(\mu)}{2\pi} \left(\boldsymbol{\mathcal{A}}_j^{(1)ab}(\mu,\mu_0)\delta_{jl} + \boldsymbol{\mathcal{B}}_{jl}^{(1)ab}(\mu,\mu_0)\,\xi^{j-l} \right) + O(\alpha_s^2) \right] \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{-\frac{\lambda_l^b}{\beta_0}}$$

Non-Singlet evolution

Reduce the matrix-valued quantities in the singlet evolution to scalar values associated with quark contributions

$$\boxed{{}^{\mathrm{NS}}\gamma_j^{(0)} = {}^{\Sigma\Sigma}\gamma_j^{(0)}\,, \qquad {}^{\mathrm{NS}}\gamma_j^{(1)} = {}^{\Sigma\Sigma}\gamma_j^{(1)}}$$