Exclusive and Semi-Inclusive Pions: QED Effects

Workshop
TOWARDS IMPROVED HADRON TOMOGRAPHY WITH HARD EXCLUSIVE REACTIONS
Jefferson Lab
Newport News, USA

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Coordination with Harut Avakian (JLab) is acknowledged

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- Motivation & Introduction
- Background
- Assumptions & Calculations
- Results
- Conclusions

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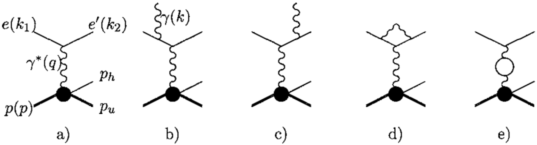
- Photon emission is a part of any electron scattering process: accelerated charges radiate
- Typical magnitudes for QED corrections for exclusive processed are around -20-30% due to large log enhancement factor $\log \frac{Q^2}{m_e^2}$ and kinematic cuts
- Two-photon exchange corrections - a part of QED corrections unaccounted for - are estimated at 1-4% at different angles - can be directly measured
- Exclusive electron scattering processes such as p(e,e'h_1)h_2 are actually inclusive p(e,e'h_1)h_2 nγ, where an infinite number of low-energy photons can be generated
- Low-energy photons do not affect polarization observables, thanks to Low theorem

Afanasev, Akushevich, Burkert, Joo, Phys.Rev.D**66**, 074004 (2002)

- Conventional RC, precise treatment of phase space, no peaking approximation, no dependence on hard/soft photon separation; extension to DVMP is straightforward;
- Can be used for any exclusive electroproduction of 2 hadrons, e.g., d(e,e'p)n (EXCLURAD code)



- Fortran code EXCLURAD is available at www.jlab.org/RC
- Used for data analysis at JLab, COMPASS, MAMI,...

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QED Corrections for (Exclusive) Electroproduction of Pions

Sample results from EXCLURAD

- QED corrections to unpolarized cross sections reach tens of per cent
- Corrections are dependent on both polar and azimuthal angles of outgoing hadron (pion), which affects extraction of resonance parameters in the resonance region and GPDs in the deep-virtual region

AFANASEV *et al.* PHYSICAL REVIEW D **66**, 074004 (2002)

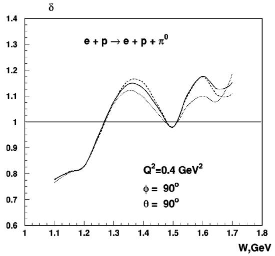

FIG. 3. *W* dependence of RC to the cross section of neutral pion

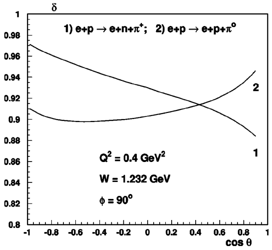

FIG. 5. RC to the cross section as a function of cos θ.

- QED corrections due to real-photon emission are smaller for polarization asymmetries

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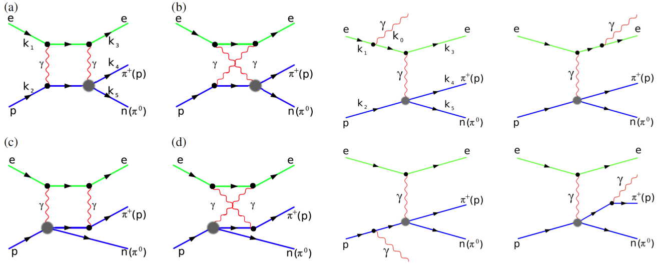
- Ge/Gm polarization vs Rosenbluth discrepancy is agreed to be partly due to two-photon exchange (resulting from about 5 per cent missing systematic correction at high momentum transfers (see for review A Afanasev, PG Blunden, D Hasell, BA Raue, Prog. Part. Nucl. Phys., 2017
- JLab experiment Katich et al., Phys.Rev.Lett. 113 (2014)022502 reveals about 5 per cent polarization asymmetries in DIS on 3He that are zero in one-photon exchange approximation
- Proposed positron beamline at JLab will provide a direct probe for two-photon effects via measurements of electron-positron asymmetries

Two-Photon Exchange Corrections for Electroproduction of Pions

Afanasev, Aleksejevs, Barkanova, Phys.Rev. D88: 053008, 2013

- Calculated previously neglected QED corrections from two-photon exchange
- Used a soft-photon approximation, results expressed in terms of Passarino-Veltman integrals



- Computed corrections result in about 5 per cent variation of cross section from backward to forward scattering angles
- Conclusion: Important for the analysis of angular dependences, $\cos(\phi)$ moments in particular

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Two-Photon Exchange Corrections for Electroproduction of Pions

Afanasev, Aleksejevs, Barkanova, Phys.Rev. D88: 053008, 2013

- Angular dependencies of two-photon corrections affect σ_L/σ_T extraction

AFANASEV, ALEKSEJEVS, AND BARKANOVA PHYSICAL REVIEW D **88,** 053008 (2013)

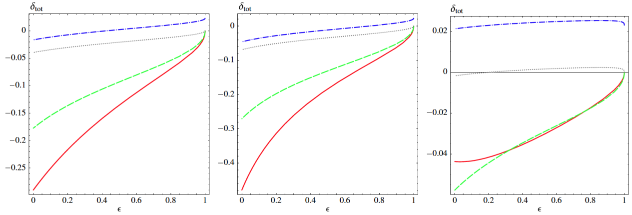

FIG. 5 (color online). π^0 electroproduction two-photon box correction (for detected proton) dependencies on virtual photon degree of polarization parameter ϵ for momentum transfers $Q^2 = 3.0\ \mathrm{GeV}^2$ (left plot), $Q^2 = 7.0\ \mathrm{GeV}^2$ (middle plot), and $Q^2 = 0.4\ \mathrm{GeV}^2$ (right plot). All plots are given for $\phi_4 = 90°$ and $\theta_4 = 90°$ and $W = 1.232$ GeV. Dot-dashed curve, SPT; dotted curve, SPT with $\alpha\pi$ subtracted; dashed curve, SPMT; solid curve, FM approach.

- These effects can be directly measured with proposed positron beamline at JLab
- Two-photon correction times two = electron-positron scattering asymmetry

Theory Challenges

- Both soft and hard photons in the loop integral are present
- Soft photons do not resolve the quark/parton structure
- Soft/hard scale separation is necessary
- We used Grammer-Yennie procedure for soft/hard separation - as in AA, Brodsky, Carlson, Chen, Vanderhaeghen, PRD72, 013008 (2005)
- The results become dependent on soft-hard separation scheme, QED and QCD have to be consistently combined
- Not all of the contributions are factorizable in terms of GPDs

Semi-Inclusive electroproduction and TMD studies

x-section for eN→e'hX assuming one-photon exchange

from Bacchetta et al, 1703.10157

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$
$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$
$$+ S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$
$$+ S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$
$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}$$

SIDIS phenomenology based on several assumptions[1], including:

- One-photon exchange dominates;
- Transverse photon cross section dominates, and F_{UU}^L can be ignored.

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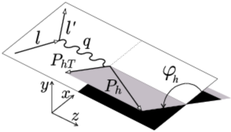
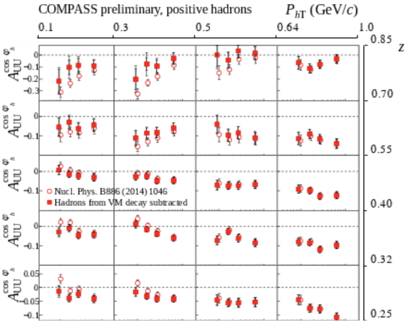
[1] Bacchetta et al. JHEP06(2017)081

TMDs in SIDIS, Azimuthal modulation





COMPASS data

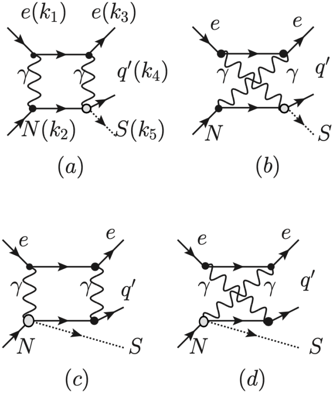
Considering the correction δ^{TPE},

$$\frac{d\sigma_{tot}}{dxdzdQ^2d^2P_T} \equiv d\sigma_{tot} = d\sigma_{exp}/(1+\delta^{TPE})$$
$$\sim (1-\delta^{TPE})\{K(y)[(1+\epsilon\frac{F_{UU,L}}{F_{UU,T}}) + \sqrt{2\epsilon(1+\epsilon)}\cos 2\phi\frac{F_{UU}^{\cos(2\phi)}}{F_{UU,T}}$$
$$+\epsilon\cos\phi\frac{F_{UU}^{\cos\phi}}{F_{UU,T}}]\} \tag{1}$$

with x is Bjorken-x, transverse momentum of the detected meson P_T, Q^2 relates to the momentum transfer of the virtual photon.

Assumptions & Calculations

$$e(k_1) + N(k_2) \to e(k_3) + q'(k_4) + S(k_5),$$



For quark-diquark model, q' represents quark and S represents diquark.

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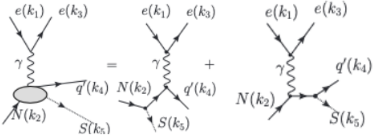
Assumptions & Calculations



Born-level one photon models, which equals to the sum of the "quark graph" and the "proton pole graph". q' and S stand for quark and diquark.[2]

[2] Afanasev and Carlson, Phys. Rev. D 74.114027 (2004)

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Assumptions & Calculations

Using soft-photon approximation (SPT [3]) by neglecting the momentum for one of the photon while calculating the amplitude, such that

$$M^{2\gamma} = M^{1\gamma} \cdot \sum_{l} [\frac{-e^2}{2\pi} \cdot \sum_{i,j} (2k_i \cdot k_j) \cdot C_0(\{k_i, m_i\}, \{\mp k_j, m_j\}) \tag{2}$$

$$= \sum_{l=N,q',s} \sum_{i=a,b,c} M^{1\gamma} M_{l,i,box}, \tag{3}$$

where the Passarino-Veltman three-point scalar integral

$$C_0(\{k_i, m_i\}, \{k_j, m_j\}) = \frac{1}{i\pi^2} \int d^4q \frac{1}{q^4} \cdot \frac{1}{(k_i - q)^2 - m_i^2} \cdot \frac{1}{(k_j - q)^2 - m_j^2}. \tag{4}$$

The correction

$$\delta_{box} = \frac{2Re[M^{2\gamma} M^{1\gamma\dagger}]}{|M^{1\gamma}|^2} = 2Re[\sum_{l;i} M_{l,i,box}]. \tag{5}$$

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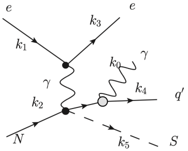
Assumptions & Calculations

Infrared divergence of two-photon exchange is canceled by interference between emission from electron lines and hadron lines (hadron Bremsstrahlung)



One of the possibilities for the hadron Bremsstrahlung process [4].

[4] Afanasev et al. Phys. Rev. D 88, 053008 (2013)

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Results

- $E_{lab} = 10.6$ GeV;
- $Q^2 \approx 2.5$ GeV^2;
- $y < 0.75$ to avoid the region most susceptible to radiative effects and lepton-pair symmetric background;
- $x = 0.31$ (the invariant mass $W \approx 2.7$ GeV);
- $z = 0.5$;
- The polar angle of the detected meson is $\cos\theta = 0.8$ ($P_T \approx 0.35$) for P_T independent figures;
- The azimuthal angle of the detected meson is defined as $\phi = \pi/6$ for the figures that are ϕ independent; SF from Lund model
- $F_{UU,L}/F_{UU,T} \approx 0.2$;
- $F_{UU}^{\cos\phi}/F_{UU,T} \approx -0.05$;
- $F_{UU}^{\cos(2\phi)}/F_{UU,T} \approx 0.1$.

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Results (see arXiv:2504.17123 and PRD **111**, 113008 (2025))

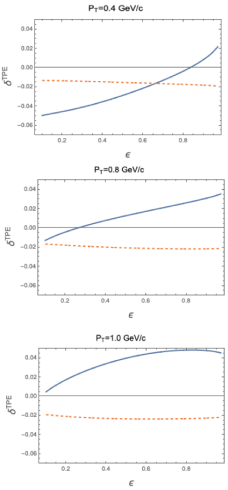

FIG. 5. TPE correction δ^{TPE} as a function of ϵ for fixed values of transverse momentum $P_T = 0.4, 0.8, 1.0$ (GeV/c) at $z = 0.7$, $Q^2 = 2.5$ GeV^2, $x = 0.31$, and $\phi = \pi/6$. The blue solid line and the orange dashed line indicate the detected meson is π^+ and ρ^+, respectively.

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Results (see arXiv:2504.17123 and PRD **111**, 113008 (2025)))

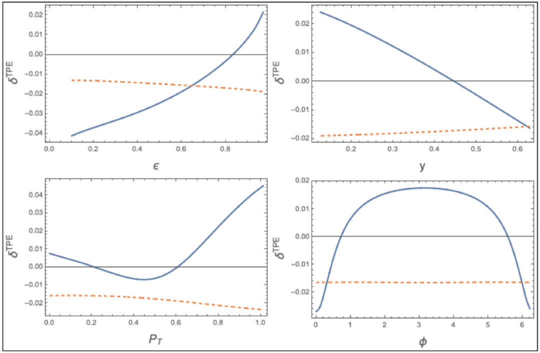

FIG. 6. Dependence of TPE correction δ^{TPE} on the virtual photon ϵ, electron's relative energy loss y, transverse momentum P_T, and azimuthal angle ϕ with $E_{lab} = 10.6$ GeV, $Q^2 = 2.5$ GeV^2, the mean value $\langle x_{BJ} \rangle = 0.31$, $z = 0.7$, using kinematics for projected experiments [29, 30]. The blue solid line and the orange dashed line represent the detected mesons are π^+ and ρ^+ meson, respectively.

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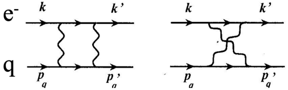
Next Step: Hard Two-Photon Exchange

- Hard TPE on a parton
- Remove Soft-Photon Exchange at parton level and include it at hadronic level following AA, Brodsky, Carlson, Chen, Vanderhaeghen, PRL 93:122301,2004; PRD 92:013008,2005
- Two-photon amplitude for a (massless) quark



$$A^{2\gamma}_{eq\to eq} = \frac{e_q^2}{t}\frac{\alpha_{em}}{2\pi}(V_\mu^e \otimes V_\mu^q \times f_V + A_\mu^e \otimes A_\mu^q \times f_A),$$

$$V_\mu^{e,q} = \bar{u}_{e,q}\gamma_\mu u_{e,q},\; A_\mu^{e,q} = \bar{u}_{e,q}\gamma_\mu\gamma_5 u_{e,q}$$

$$f_V = -2[\log(-\frac{u}{s}) + i\pi]\log(-\frac{t}{\lambda^2}) - \frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) - \frac{1}{u}\log(-\frac{s}{t})] +$$

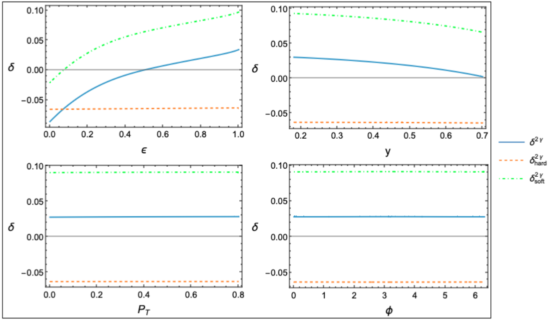
$$+\frac{(u^2 - s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) + \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{u^2 - s^2}{2su}$$

$$f_A = -\frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) + \frac{1}{u}\log(-\frac{s}{t})] +$$

$$+\frac{(u^2 - s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) - \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{t^2}{2su}$$

Soft+Hard Two-Photon Exchange: Results

- Addition of "hard" TPE appears to *reduce* the total effect; ($Q^2 \approx 3.7$ GeV^2; $x_{BJ} = 0.31$; $z = 0.7$; $P_T \approx 0.3$ for P_T independent figures; $\phi = \pi/6$ for ϕ independent figures. Notice stronger dependence on electron's variables ϵ, y)



The orange dashed curves represent calculations based on the hard-photon exchange contribution, $\delta^{2\gamma}_{hard}$. The dot-dashed green curves indicate the soft-photon contribution, $\delta^{2\gamma}_{soft}$. The blue solid curves show the full TPE corrections, $\delta^{2\gamma}$.

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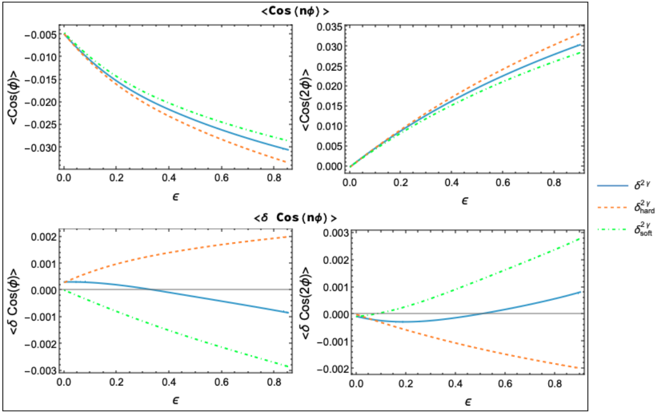
- Addition of "hard" TPE *reduces* the total effect for the azimuthal asymmetries, as well



Cosine moments as a function of polarization factor ϵ, computed using $m_{q'} = 0$ and $m_s = m_p$ and kinematics of the future experiment in JLab E12-06-010C.

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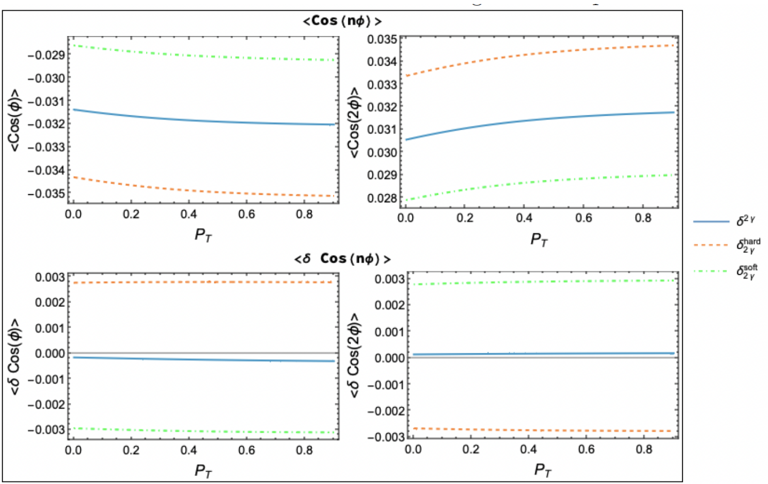
Soft+Hard Two-Photon Exchange: Results

- Negligible variation with P_T



- Two-photon exchange (TPE) effects alter angular dependence of cross sections in DVMP and SIDIS
- Their measurement is necessary for extracting GPDs, TMDPDFs and FFs, can be done with *positron beams* at CEBAF
- TPE corrections computed for projected JLab measurements of L/T separation and azimuthal asymmetries, TPE corrections of the two-photon exchange are in the range of $\sim \pm 5\%$ for y, ϵ & P_T dependence; $\cos(\phi), \cos(2\phi)$ are affected by $\leq$0.5%
  - Addition of hard two-photon exchange at a parton level *reduces* the total correction
  - Next steps: Spin asymmetries in SIDIS; "hard" TPE for (exclusive) DVMP

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