Recent developments of GPDs on the lattice

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Towards Improved Hadron Femtography with Hard Exclusive Reactions 2025

Jefferson Lab

July 28 - 31, 2025

OUTLINE

A. Methods to access GPDs from lattice QCD

B. Recent results for proton GPDs
- twist-2 transversity GPDs
- twist-3 GPDs
- additional physical information

C. Synergy with phenomenology

D. Concluding remarks

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A. Methods to access GPDs from lattice QCD

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

B. Recent results for proton GPDs

- twist-2 transversity GPDs
- twist-3 GPDs
- additional physical information

Twist-2 $(f_i^{(0)})$

Nucleon \ Quark	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

C. Synergy with phenomenology

(Selected) Twist-3 $(f_i^{(1)})$

Nucleon \ $\mathcal{O}$	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

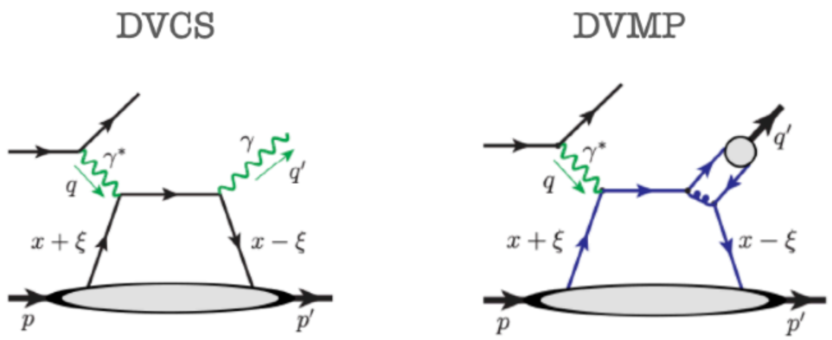
D. Concluding remarks

Generalized Parton Distributions

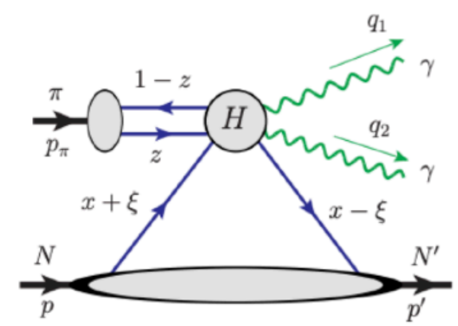
[M. Burkardt, PRD62 071503 (2000), hep-ph/0005108] [M. V. Polyakov, PLB555 (2003) 57, hep-ph/0210165]

★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)

★ exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$



[X.-D. Ji, PRD 55, 7114 (1997)]



[J. Qiu et al, arXiv:2205.07846]

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathcal{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

A number of talks in this Workshop

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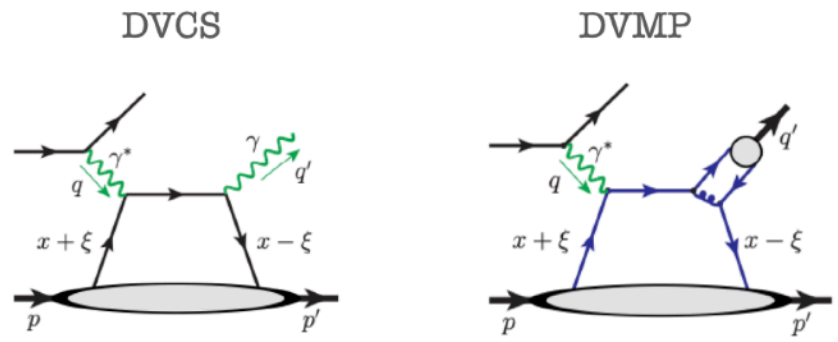

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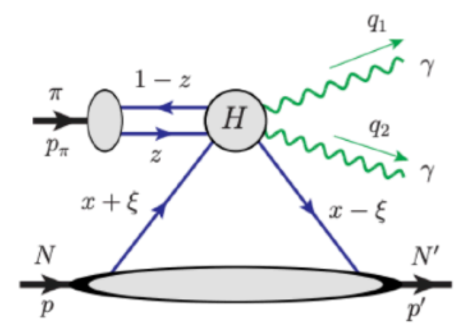

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★ Essential to complement the knowledge on GPD from lattice QCD

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

Accessing information on PDFs/GPDs

★ Parton model: physical picture valid for infinite momentum frame

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

★ PDFs via matrix elements of nonlocal light-cone operators ($-t^2 + \vec{r}^2 = 0$)

$$f(x) = \frac{1}{4\pi} \int dy^- e^{-ixP^+y^-} \langle P, S | \bar{\psi}_f \gamma^+ \mathcal{W} \psi_f | P, S \rangle$$

★ Light-cone correlations inaccessible from Euclidean lattices ($\tau^2 + \vec{r}^2 = 0$)

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A. Mellin moments (local OPE expansion)

local operators

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\, q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty} \frac{1}{n!} z_{\alpha_1} \ldots z_{\alpha_n} \left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1} \left\{ \gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} A_{n,i}(t) - i\frac{\Delta_\alpha \sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} B_{n,i}(t) \right\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\text{ even}} \Big) \Big\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\text{ even}} \Big] U(P)$$

See talks by Y. Zhao and D. Richard

Reconstruction of PDFs/GPDs very challenging

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B. Matrix elements of nonlocal operators (quasi-GPDs, pseudo-GPDs)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Nonlocal operator with Wilson line

This talk

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N} E(x,\xi,t)\right\} U(P) + \text{ht}$$

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C. Other methods

Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

$$\mathscr{M}(P_f, P_i, z) = \langle N(P_f) \,|\, \bar{\Psi}(z)\, \Gamma\, \mathscr{W}(z,0)\Psi(0) \,|\, N(P_i)\rangle_\mu$$

Calculation very taxing!

- length of the Wilson line (z) } PDFs, GPDs
- nucleon momentum boost (P_3) } PDFs, GPDs
- momentum transfer (t) } GPDs
- skewness (ξ) } GPDs

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[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]
[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

pseudo-ITD

[A. Radyushkin, PRD 96, 034025 (2017)]

$$\tilde{q}_\Gamma^{\rm GPD}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\,\mathscr{M}(P_f, P_i, z)$$

$$\mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathscr{M}(\nu, \xi, t; z_3^2)}{\mathscr{M}(0,0,0; z^2)} \qquad (\nu = z \cdot p)$$

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Matching in momentum space (Large Momentum Effective Theory)

pseudo-ITD [A. Radyushkin, PRD 96, 034025 (2017)]

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Matching in ν space

$$Q(\nu,\mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x,\mu^2)$$

Light-cone PDFs & GPDs

Calculation very taxing!

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quasi-PDFs

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Matching resembles factorization:

$$\sigma_{\text{DIS}}(x, Q^2) = \sum_i \left[H^i_{\text{DIS}} \otimes f_i\right](x, Q^2)$$

$$\tilde{q}^{\text{GPD}}_\Gamma(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\,\mathcal{M}(P_f, P_i, z)$$

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A new approach to GPDs from lattice QCD

(leading twist)

GPDs on the lattice: the unpolarized case

★ Off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \bar{\psi}(-\tfrac{z}{2})\,\gamma^+ \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})\,| p;\lambda\rangle \Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization in two leading twist GPDs

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

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★ Potential parametrization (γ^+ inspired)

$$F^{[\gamma^3]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^3 H_{\mathrm{Q}(0)}(x,\xi,t;P^3) + \frac{i\sigma^{3\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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→ finite mixing with scalar
[Constantinou & Panagopoulos (2017)]

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→ reduction of power corrections in fwd limit
[Radyushkin, PLB 767, 314, 2017]

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γ^0 ideal for PDFs

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$$F^{[\gamma^3]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\Big[$$  $$t;P^3) + \frac{i\sigma^{3\mu}\Delta_\mu}{2M}E_{Q(0)}(x,\xi,t;P^3)\Big]u(p,\lambda)$$

→ finite mixing with scalar
[Constantinou & Panagopoulos (2017)]

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→ reduction of power corrections in fwd limit
[Radyushkin, PLB 767, 314, 2017]

γ^0 ideal for PDFs

γ^0 parametrization is prohibitively expensive

GPDs on the lattice: the unpolarized case

★ Off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z}\langle p';\lambda'|\bar\psi(-\tfrac{z}{2})\gamma^+ \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization in two leading twist GPDs

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar u(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

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Let's rethink calculation of GPDs !

Definition of GPDs on Euclidean lattice

K. Cichy, 2024 Workshop

★ Parametrization of matrix elements in Lorentz invariant amplitudes

Vector [S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

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Advantages

- Applicable to any kinematic frame and have definite symmetries
- Lorentz invariant amplitudes A_i can be related to the standard GPDs
- Quasi GPDs may be redefined (Lorentz covariant) to eliminate $1/P_3$ contributions

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Light-cone GPDs using lattice correlators in non-symmetric frames

Proof of Concept Calculation

Twisted-mass fermions & clover

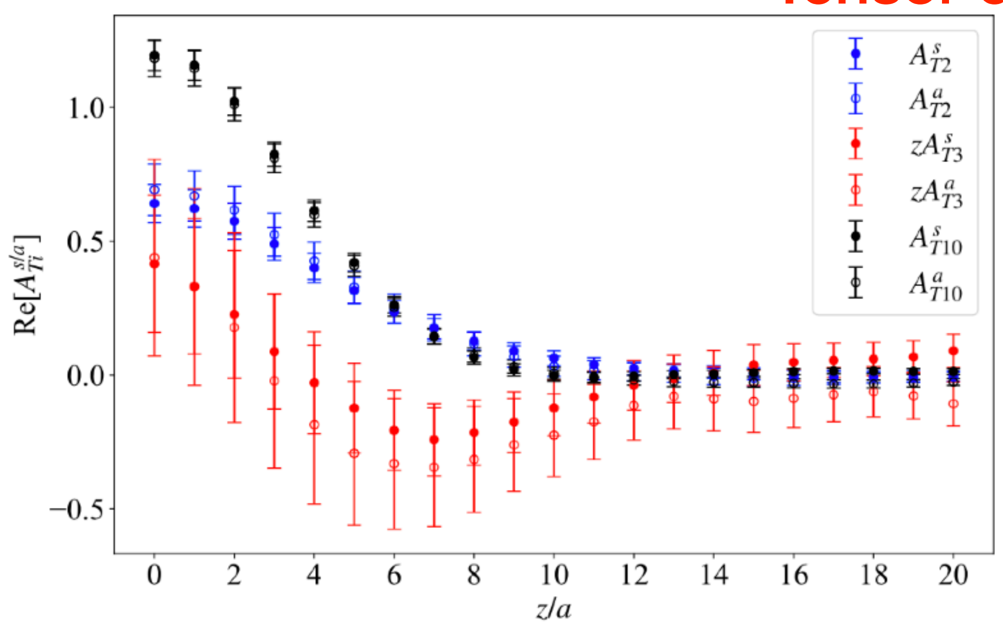
Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

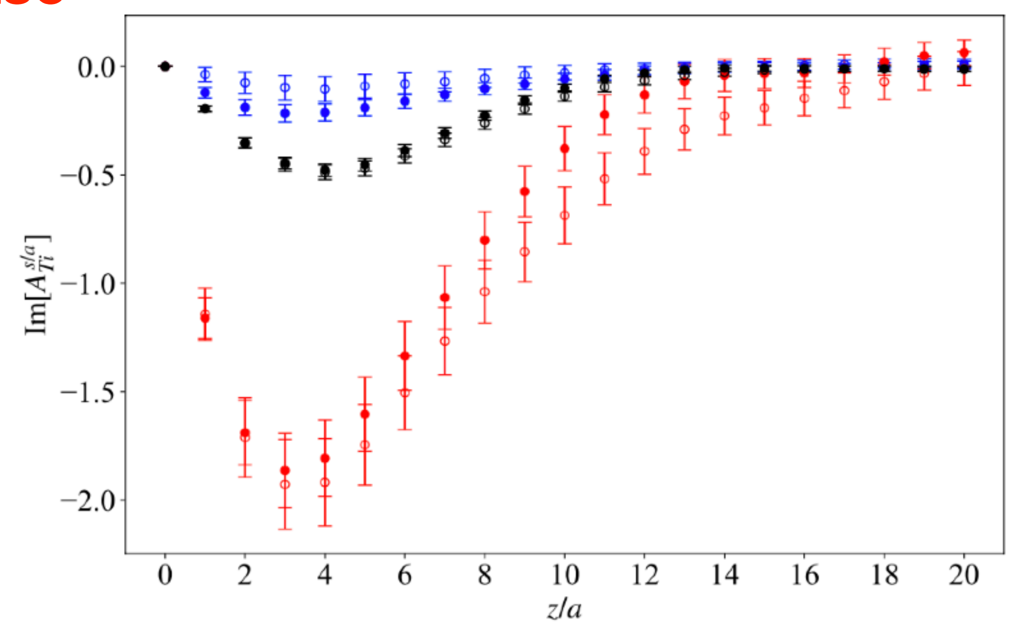
Test at zero skewness

- **symmetric frame:** $\vec{p}^{\,s}_f = \vec{P} + \vec{Q}/2\,, \quad \vec{p}^{\,s}_i = \vec{P} - \vec{Q}/2 \quad -t^s = \vec{Q}^2 = 0.69\, GeV^2$
- **asymmetric frame:** $\vec{p}^{\,a}_f = \vec{P}\,, \quad \vec{p}^{\,a}_i = \vec{P} - \vec{Q} \quad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\, GeV^2$

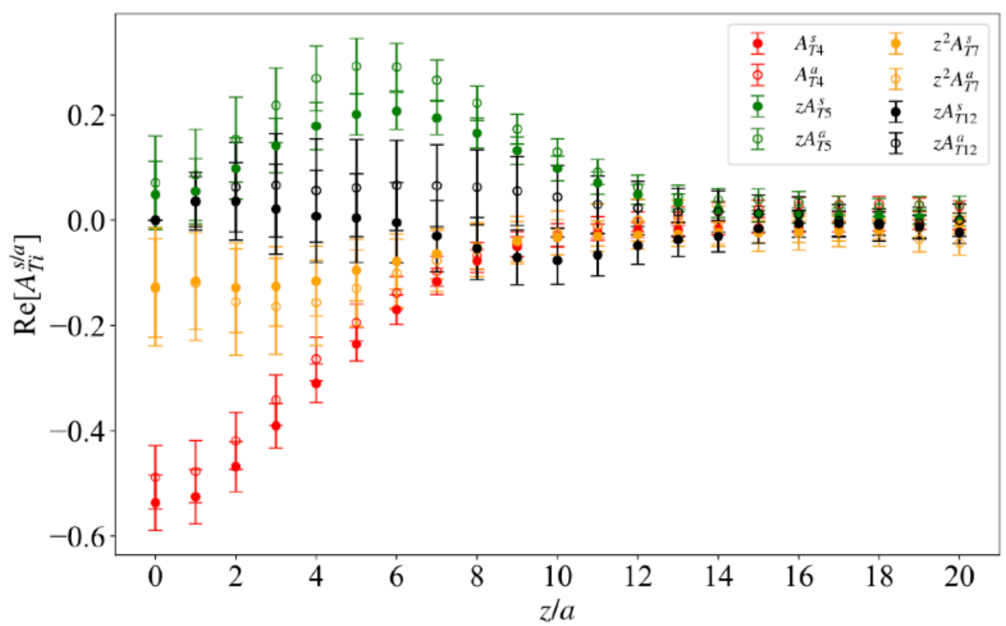
Tensor case

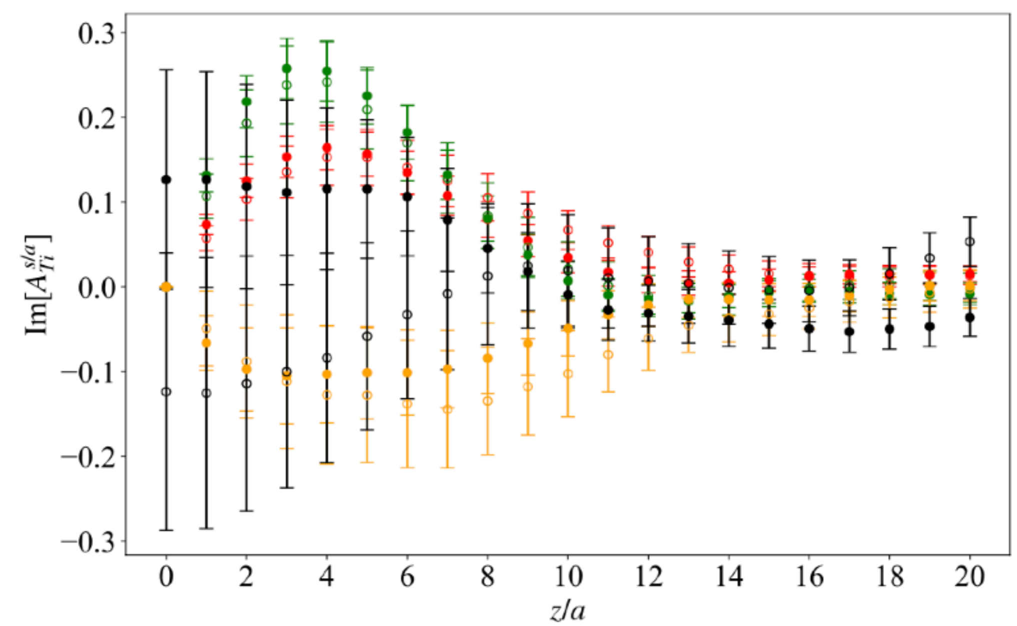
Dominant magnitude





Smaller magnitude





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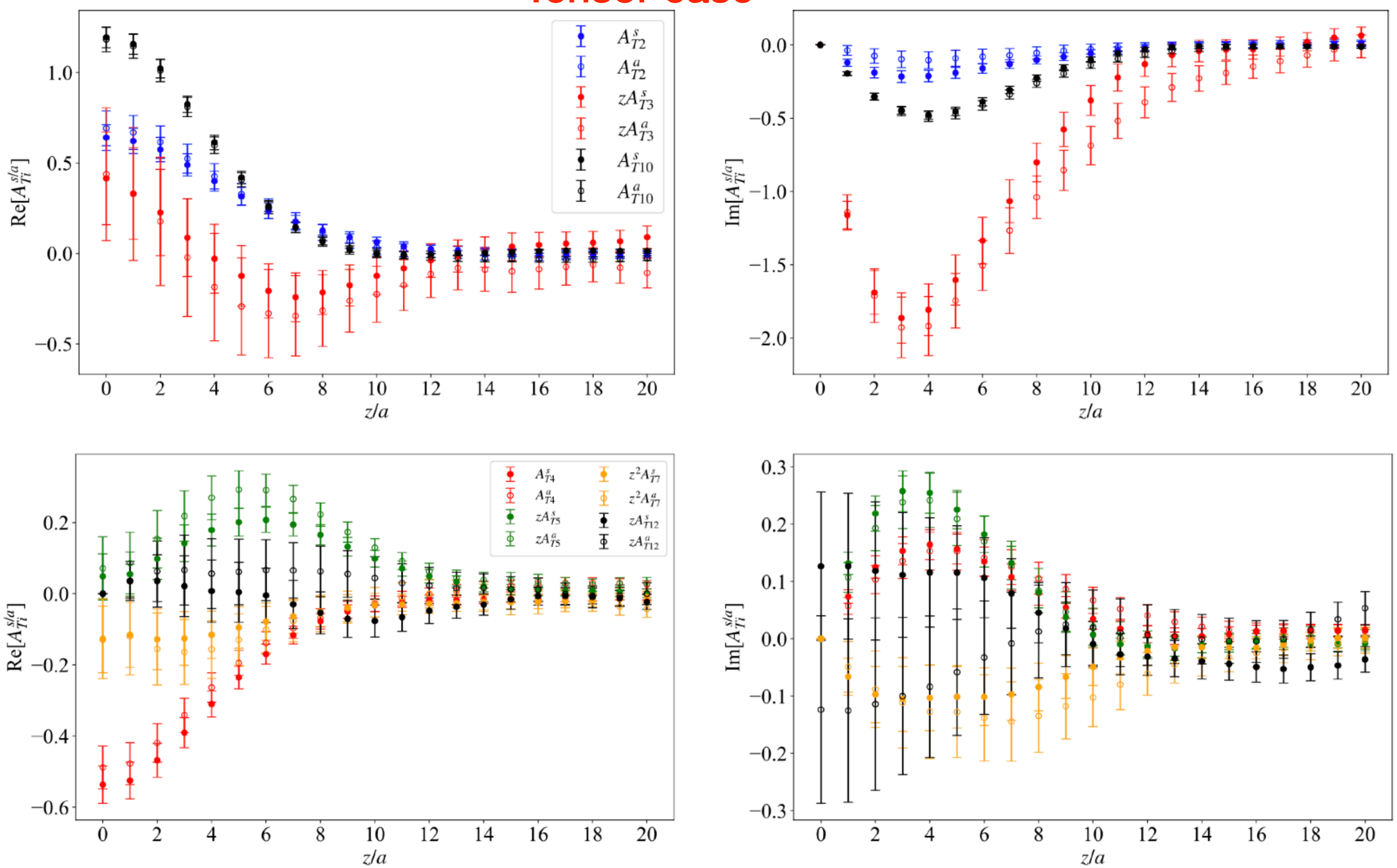

Indeed frame independence

Proof of Concept Calculation

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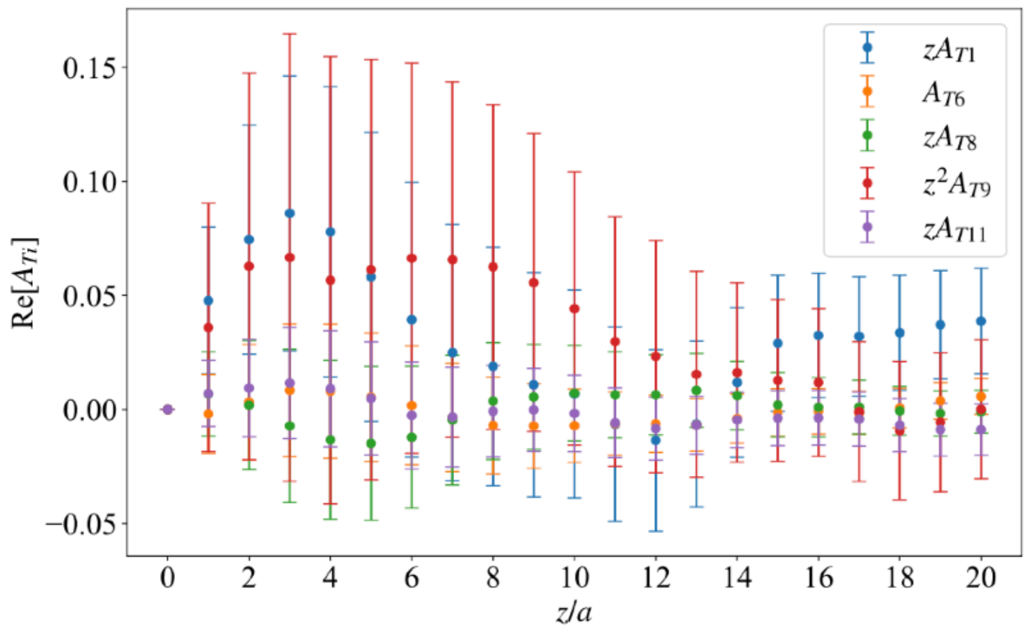
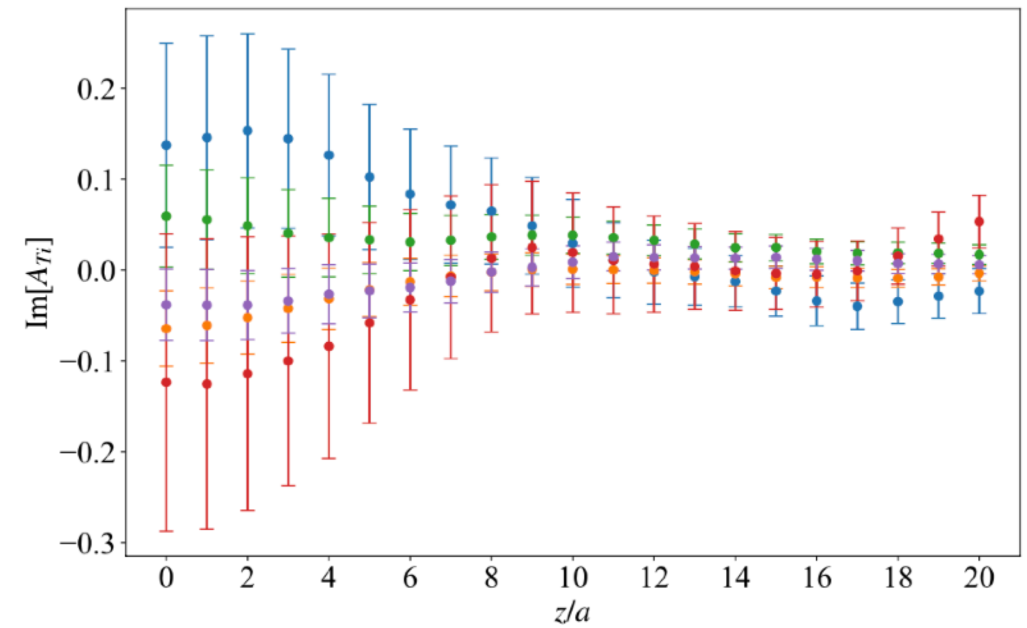
Tensor case

Theoretically zero at ξ=0





Indeed frame independence

Beyond Exploration

★ Symm. frame: separate calculation for each $\vec{Q}$

★ Asymm. frame: Two classes of $\vec{Q}$: $(Q_x,0,0), (Q_x, Q_y,0)$

frame	P_3 [GeV]	Δ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

★ Momentum transfer range is very optimistic
(some values have enhanced systematic uncertainties)

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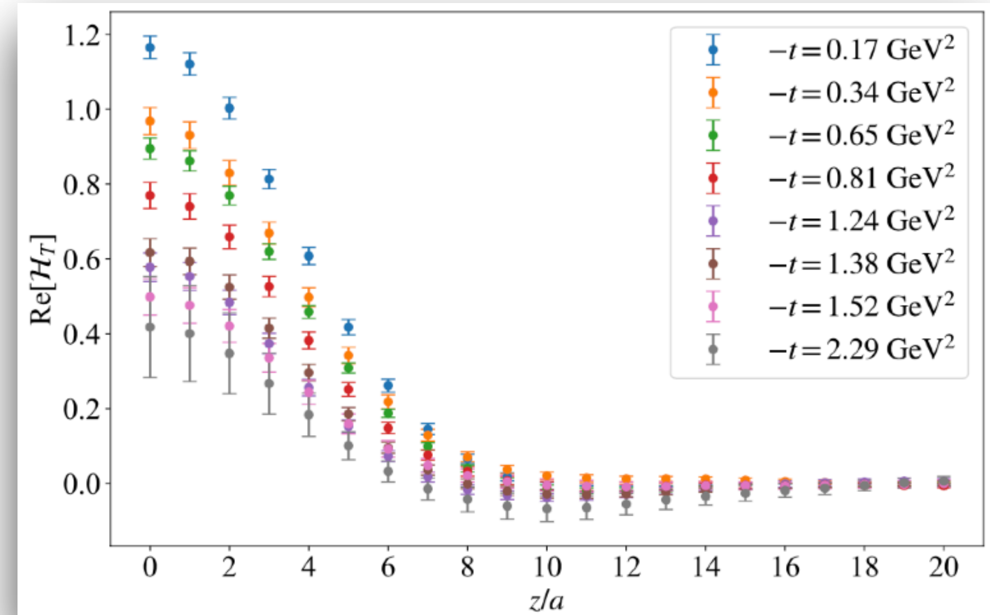
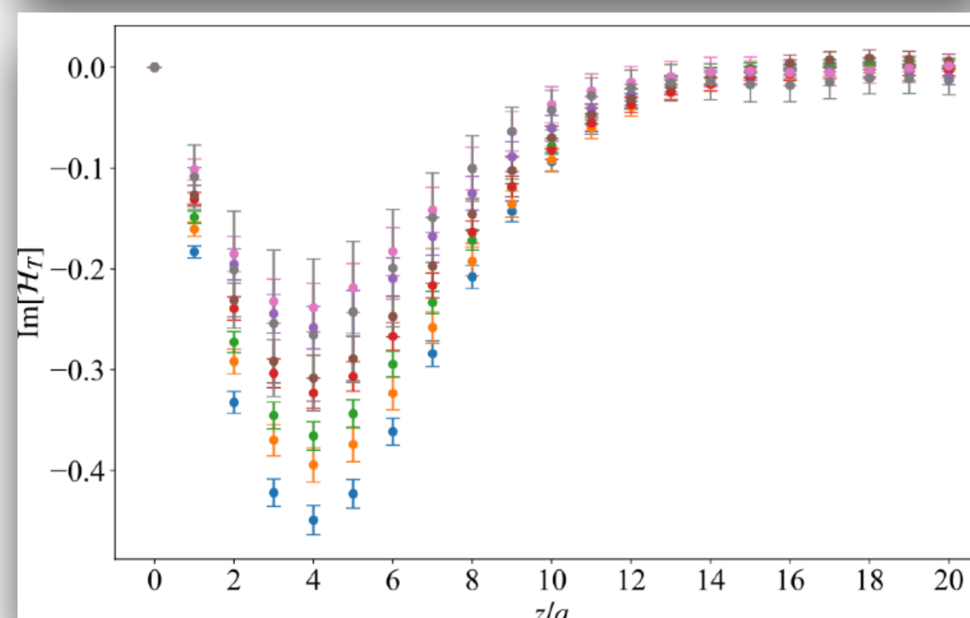
asymmetric frame





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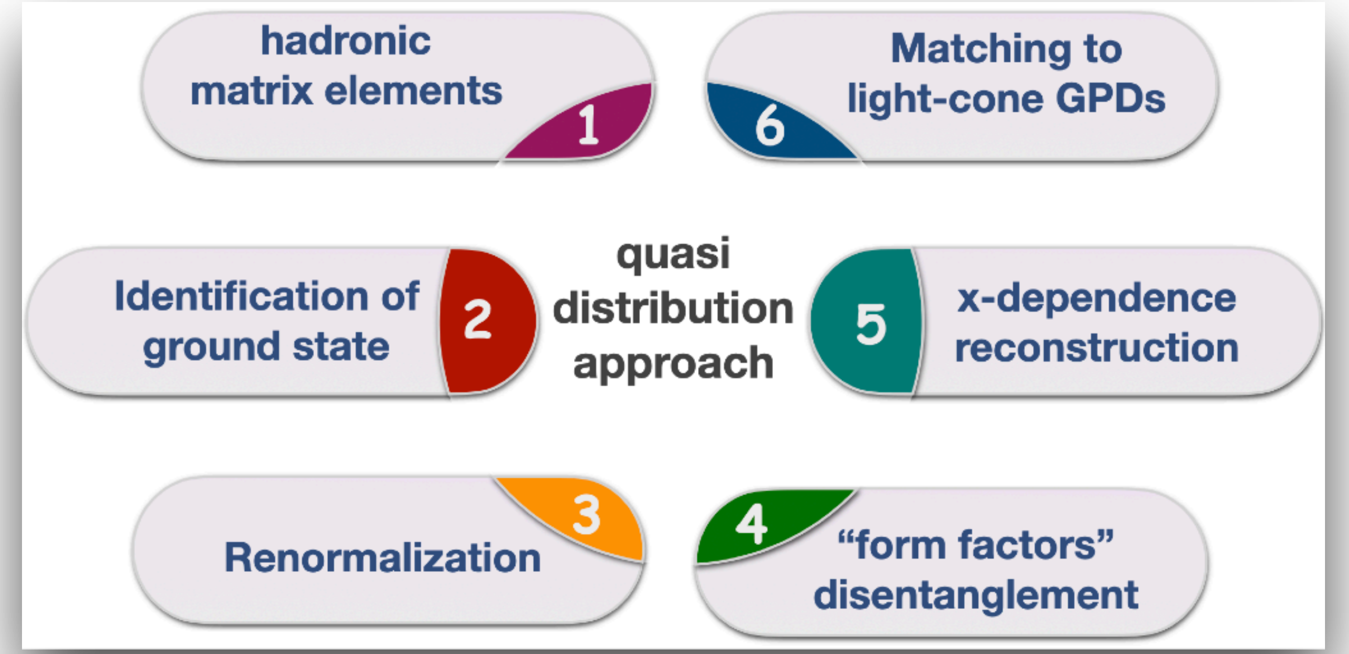
Beyond Exploration

Apply non-trivial steps



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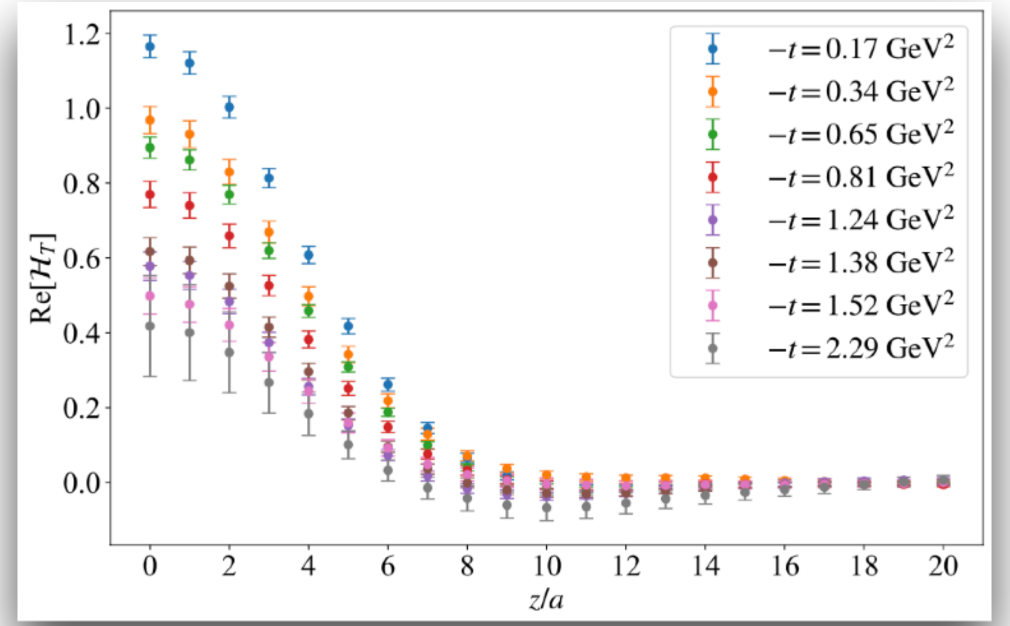

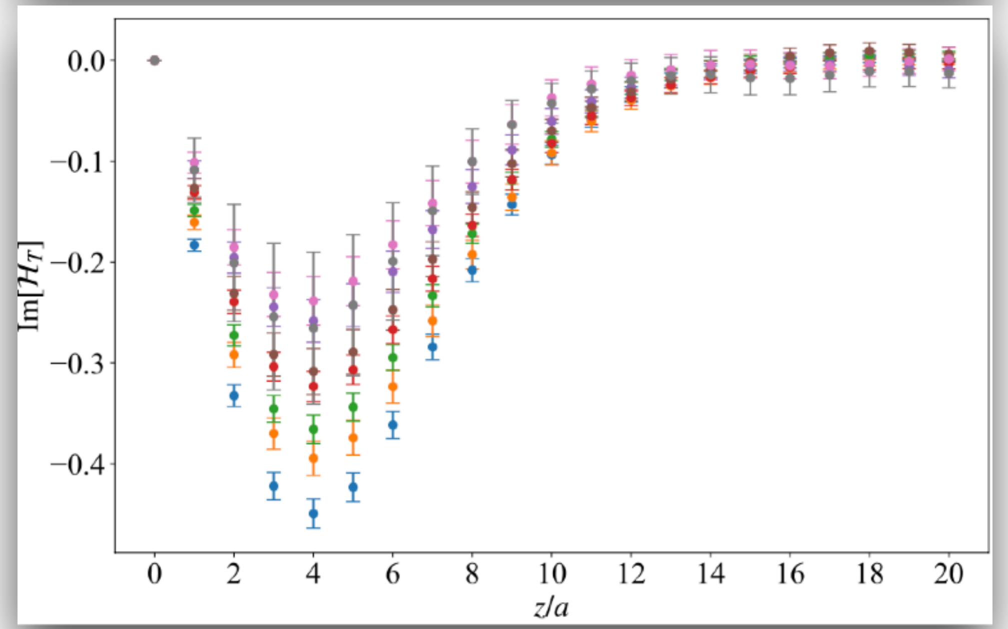

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Definition of GPDs

Lorentz invariant definition

$$\mathcal{H}_T = -2A_{T2}\left(1+\frac{P^2}{M^2}\right) + A_{T4} + A_{T10}$$

$$\mathcal{E}_T = 2A_{T2} - A_{T4}$$

$$\widetilde{\mathcal{H}}_T = -A_{T2}$$

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Standard definition (σ^{3j})

$$\mathcal{H}_T^s = -2A_{T2}\left(1+\frac{P^2}{M^2}\right) + A_{T4} - zA_{T8}\left(\frac{E_f^2 - E_i^2}{2P_3}\right) + A_{T10}$$

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- ★ Definitions for H_T, E_T identical between frames at $\xi = 0$
- ★ $\widetilde{E}_T(\xi = 0) = 0, \quad (A_{T6} = A_{T8} = 0 \text{ at } \xi = 0)$
- ★ Definitions for $\widetilde{H}_T$ have small numerical differences

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- ★ Definitions for $\widetilde{H}_T$ have small numerical differences

Definition of GPDs

Lorentz invariant definition

$$\mathcal{H}_T = -2A_{T2}\left(1+\frac{P^2}{M^2}\right) + A_{T4} + A_{T10}$$

$$\mathcal{E}_T = 2A_{T2} - A_{T4}$$

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Standard definition (σ^{3j})

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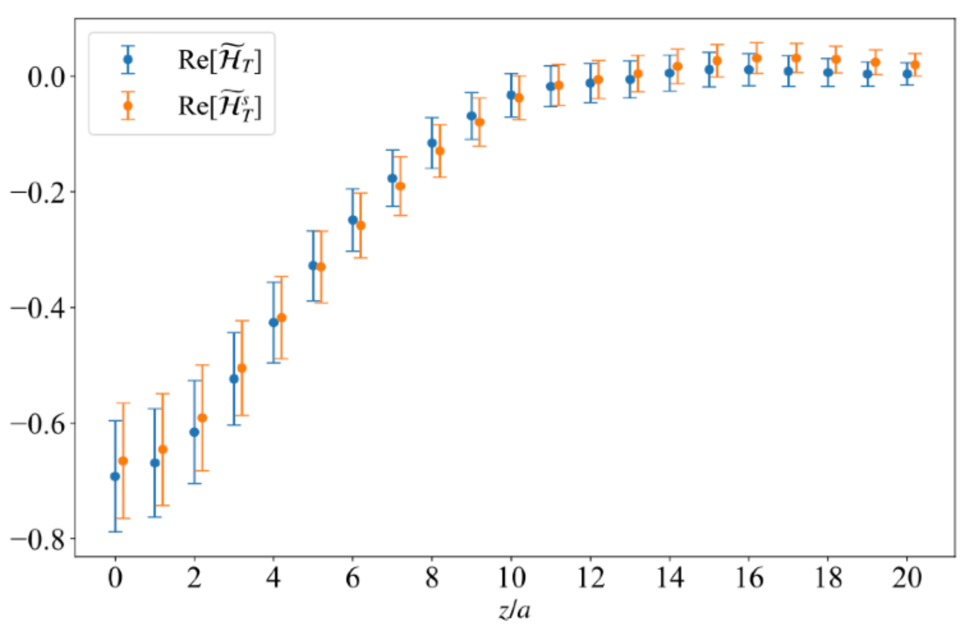
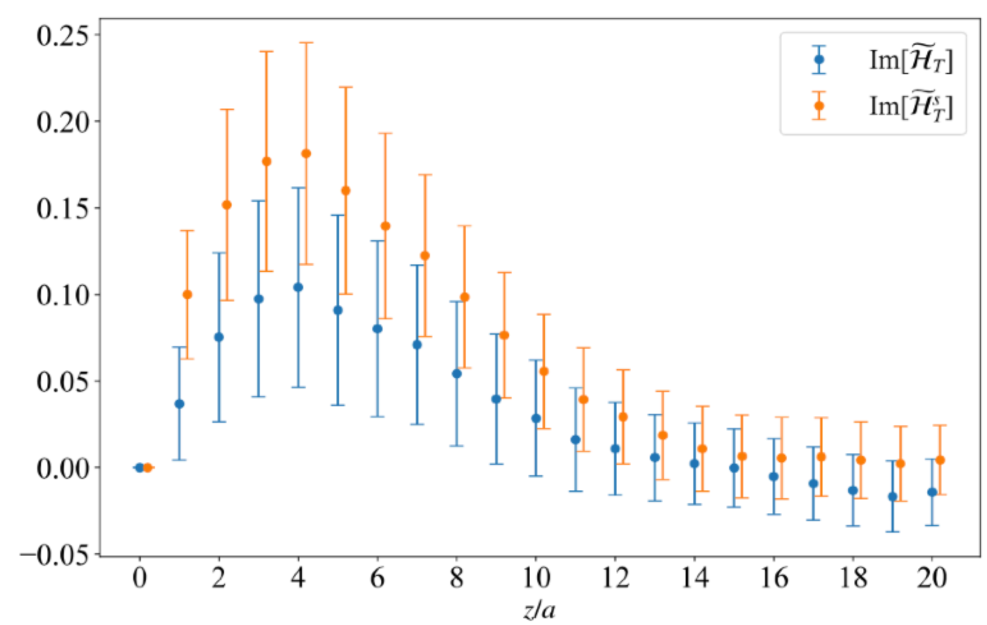
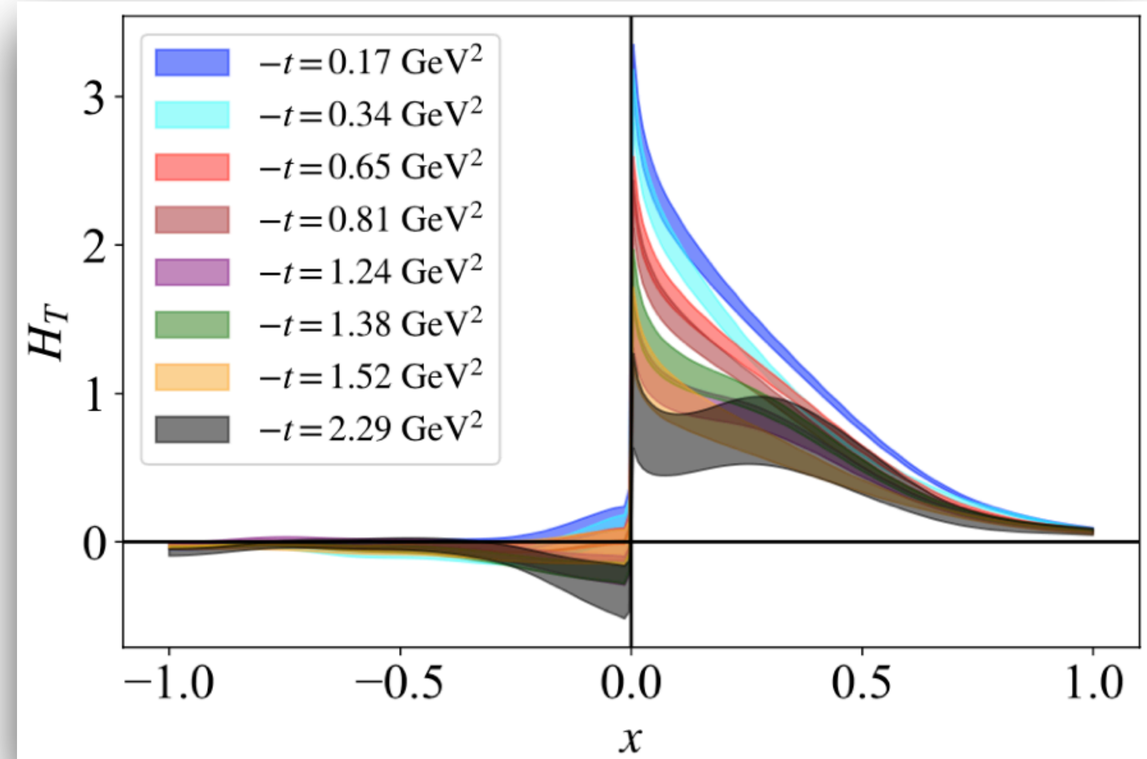
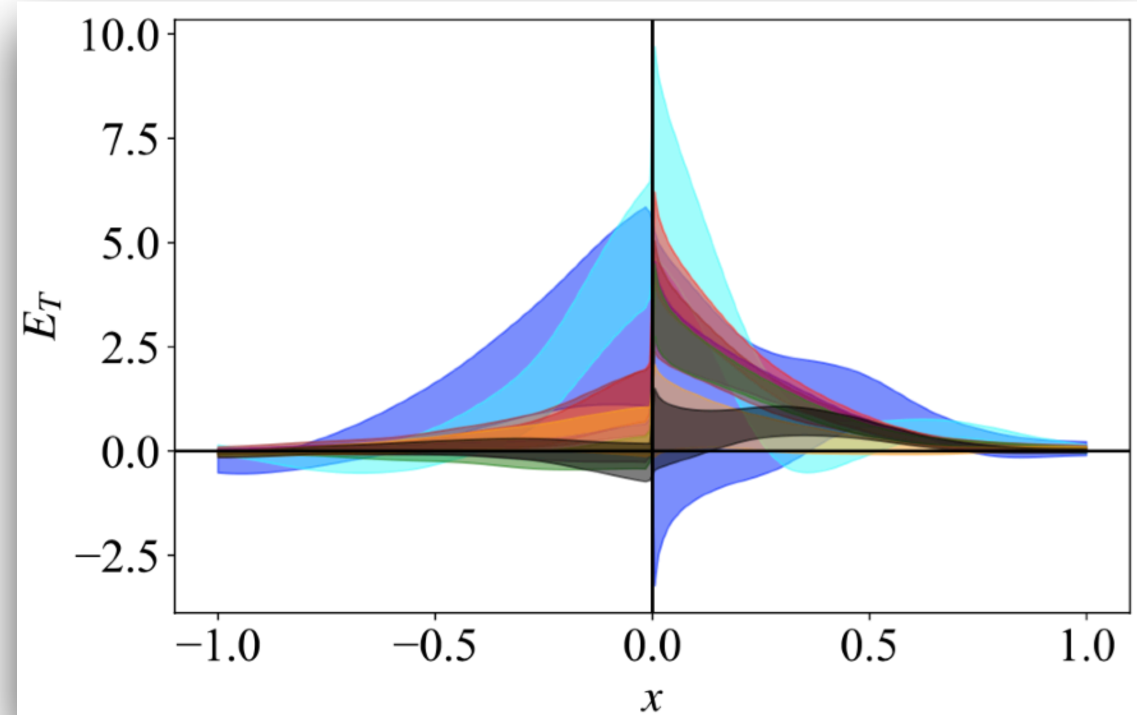


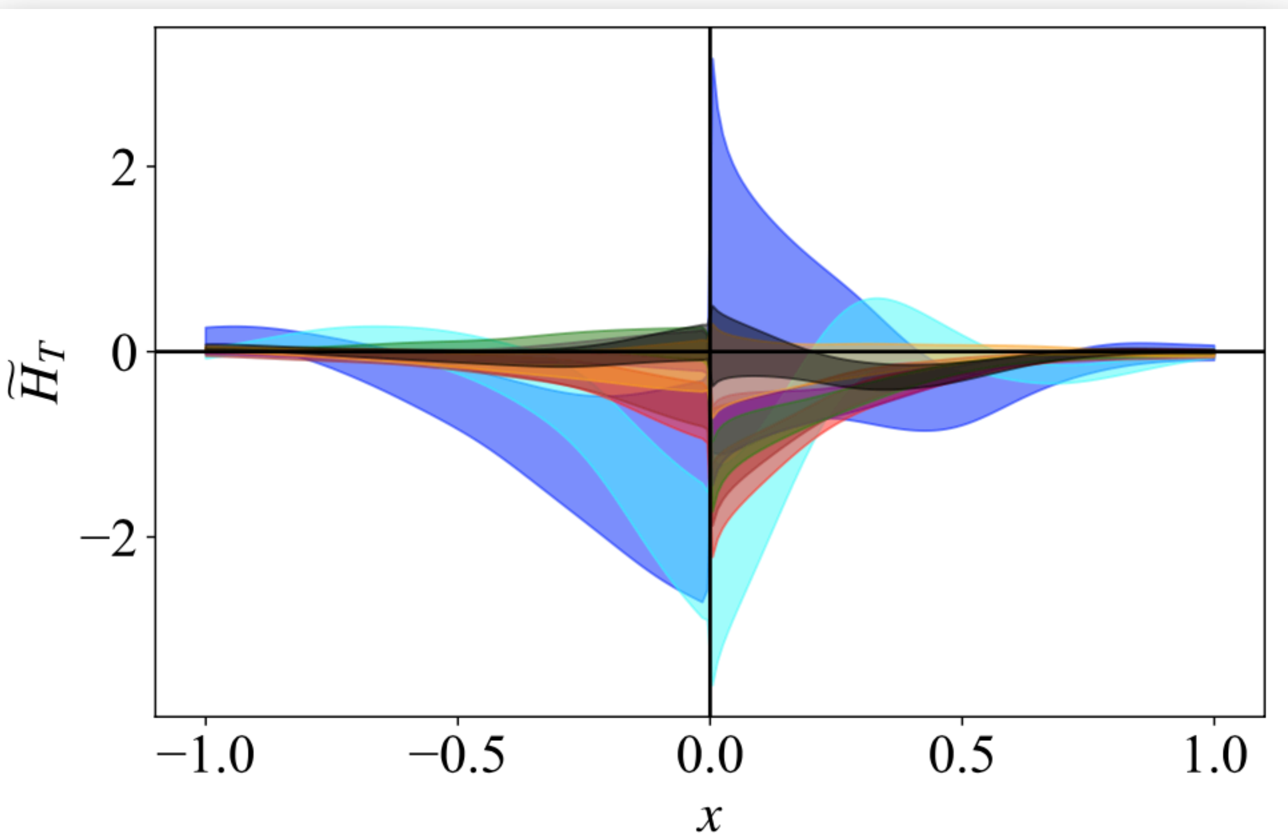


Representative example
($-t^a = 0.64$ GeV^2)

Final Results







- ★ $+x$ ($-x$) region: quarks (anti-quarks)
- ★ anti-quark region susceptible to more systematic uncertainties
- ★ small- and large-x region not reliably extracted
- ★ large -t values unreliable but free

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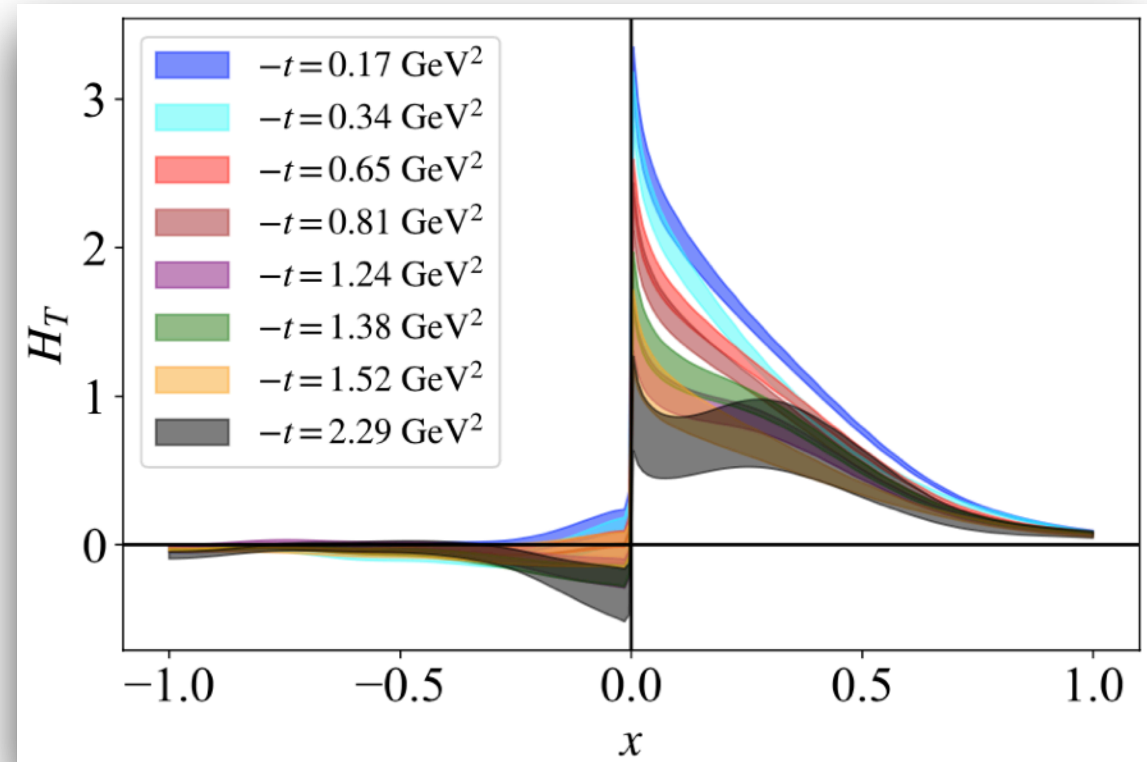

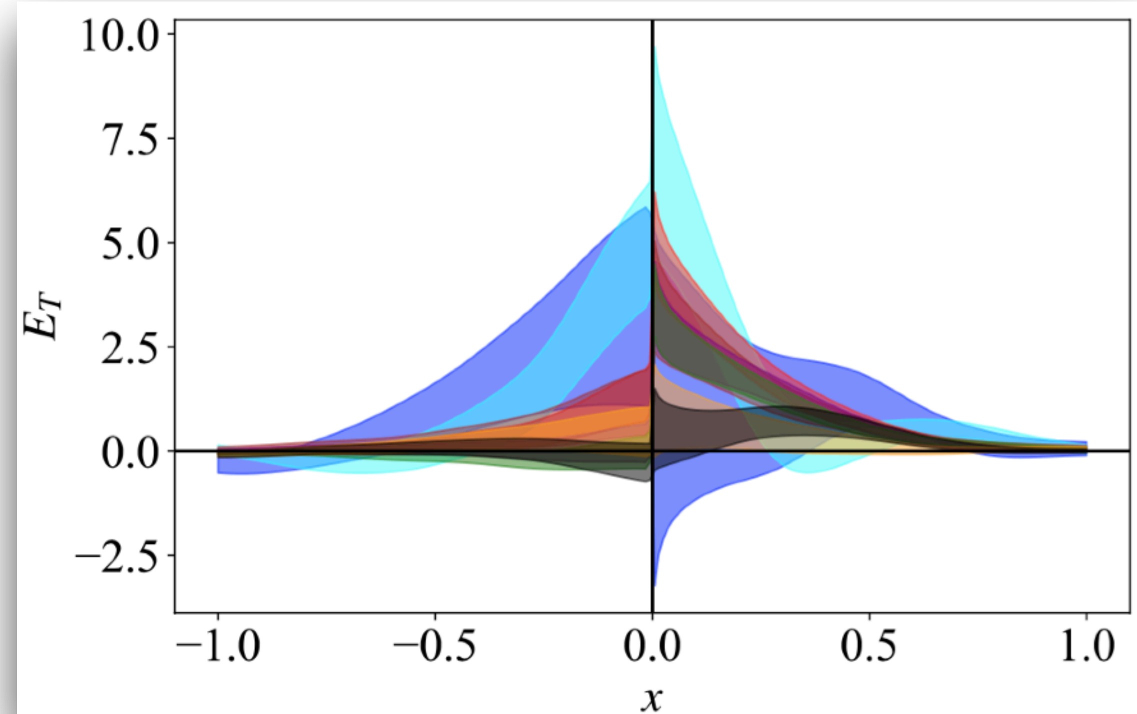

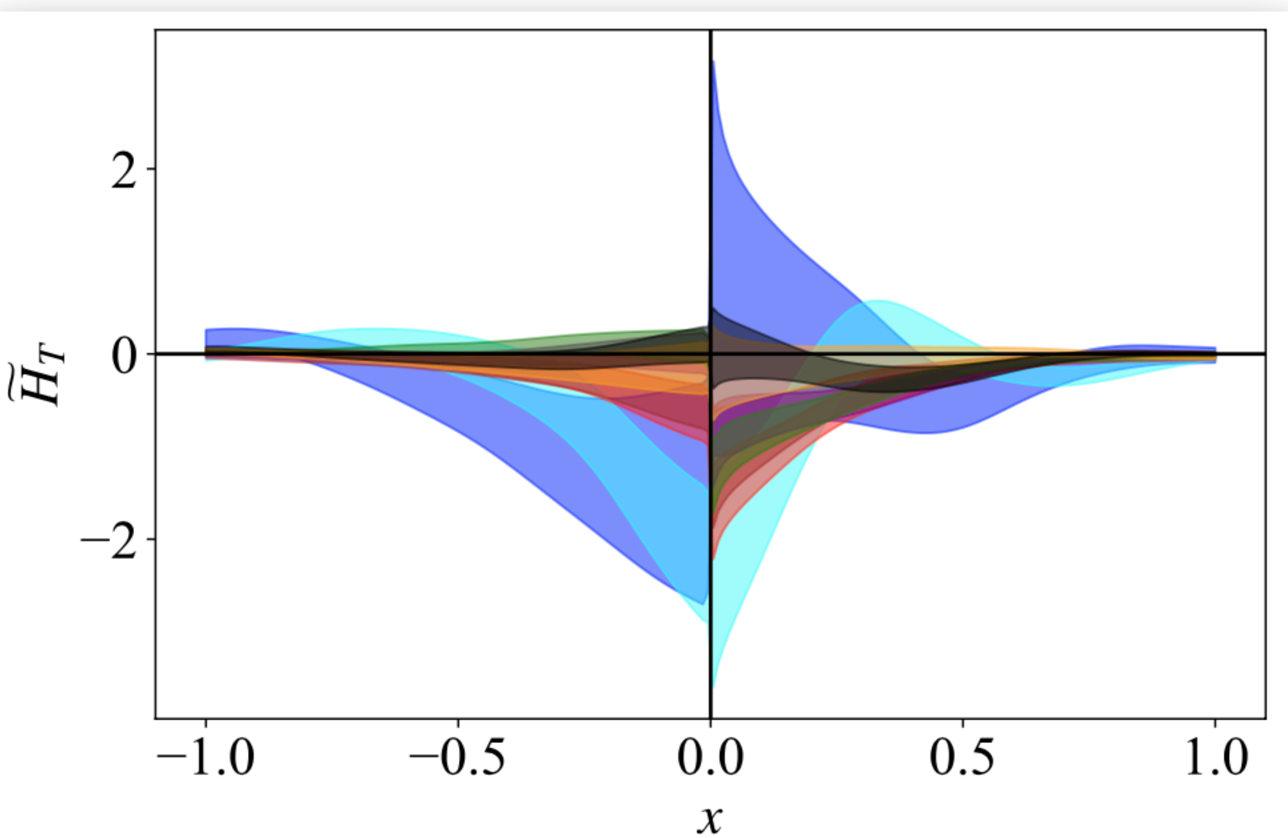

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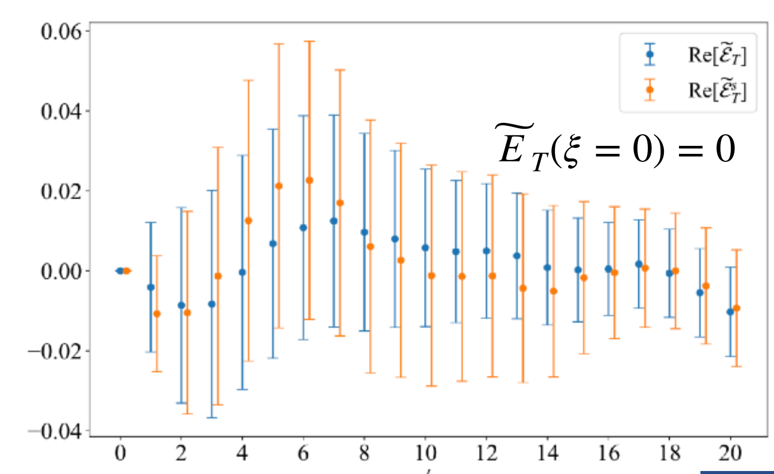
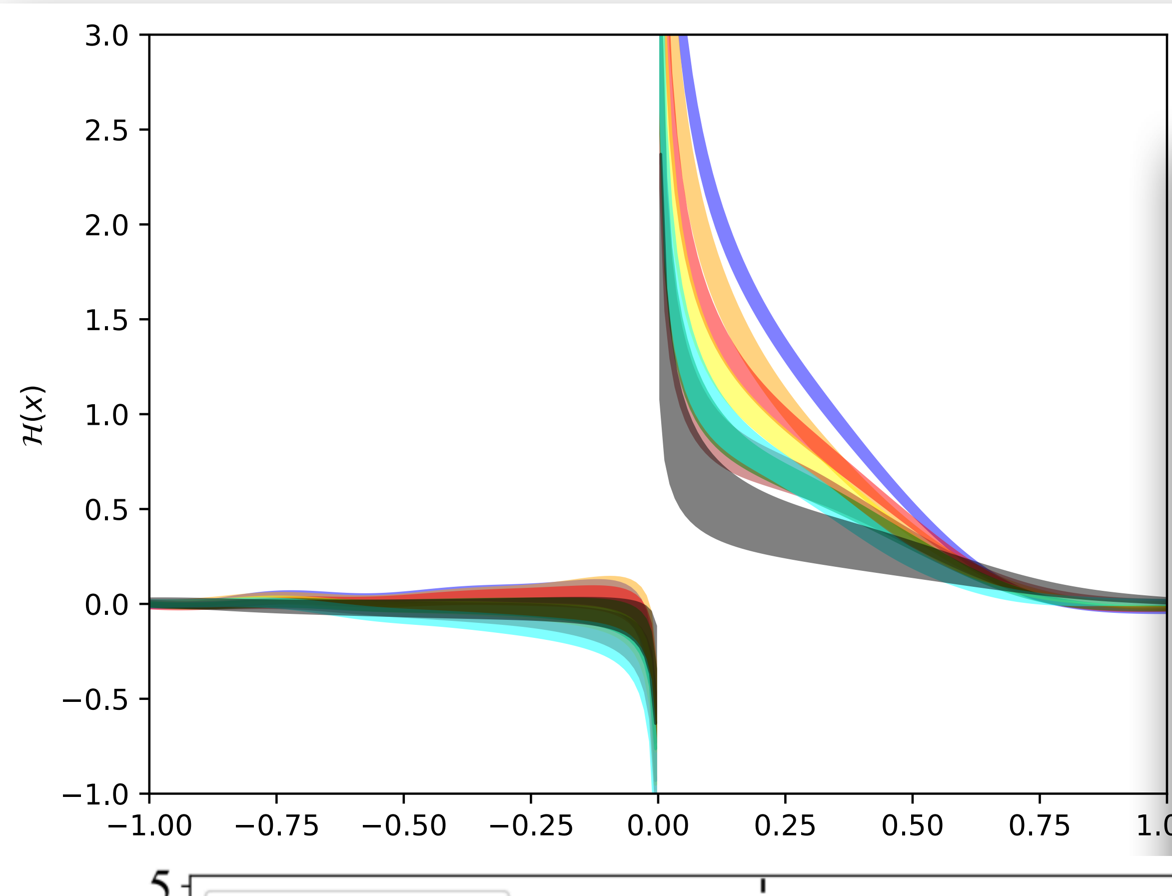
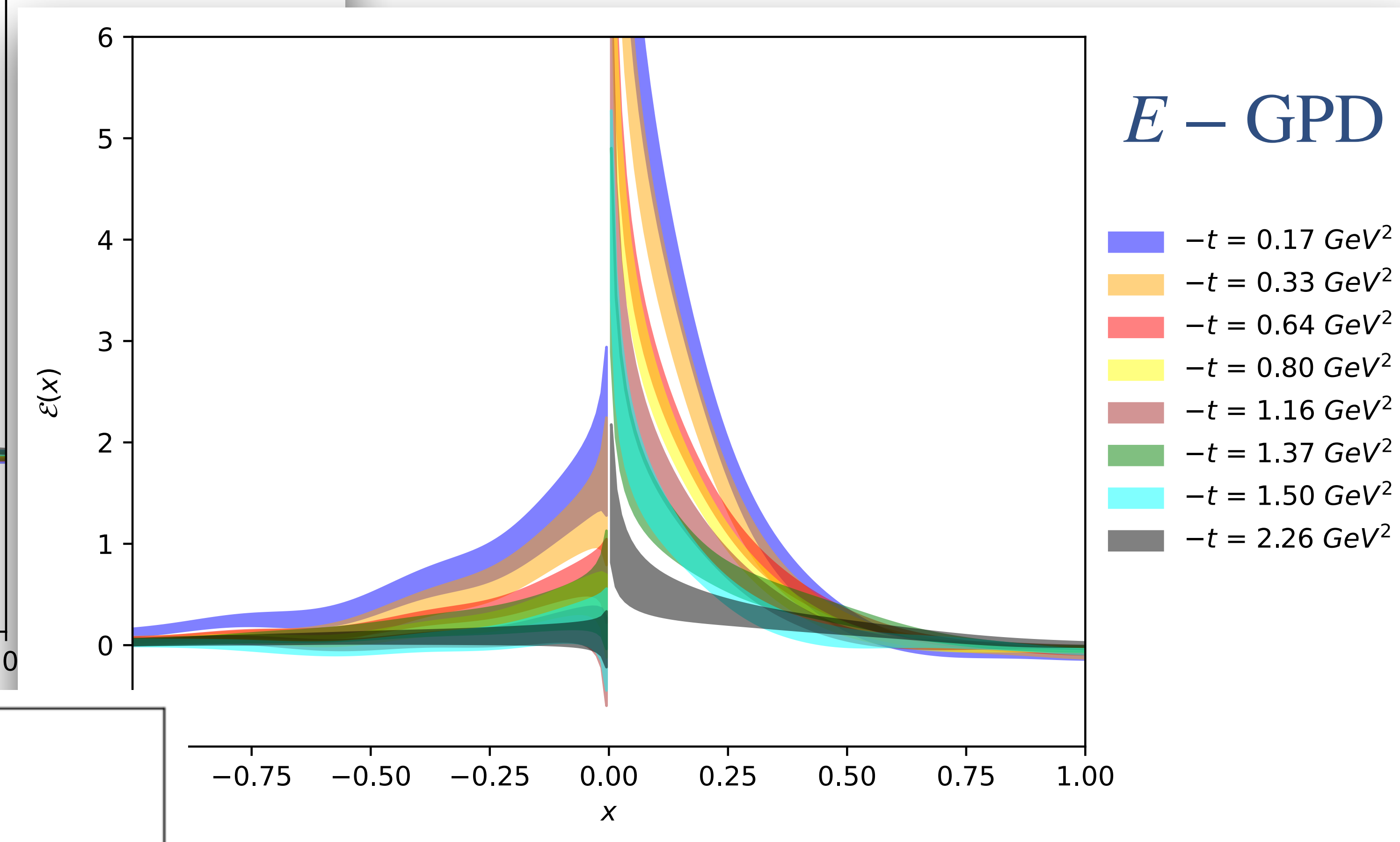
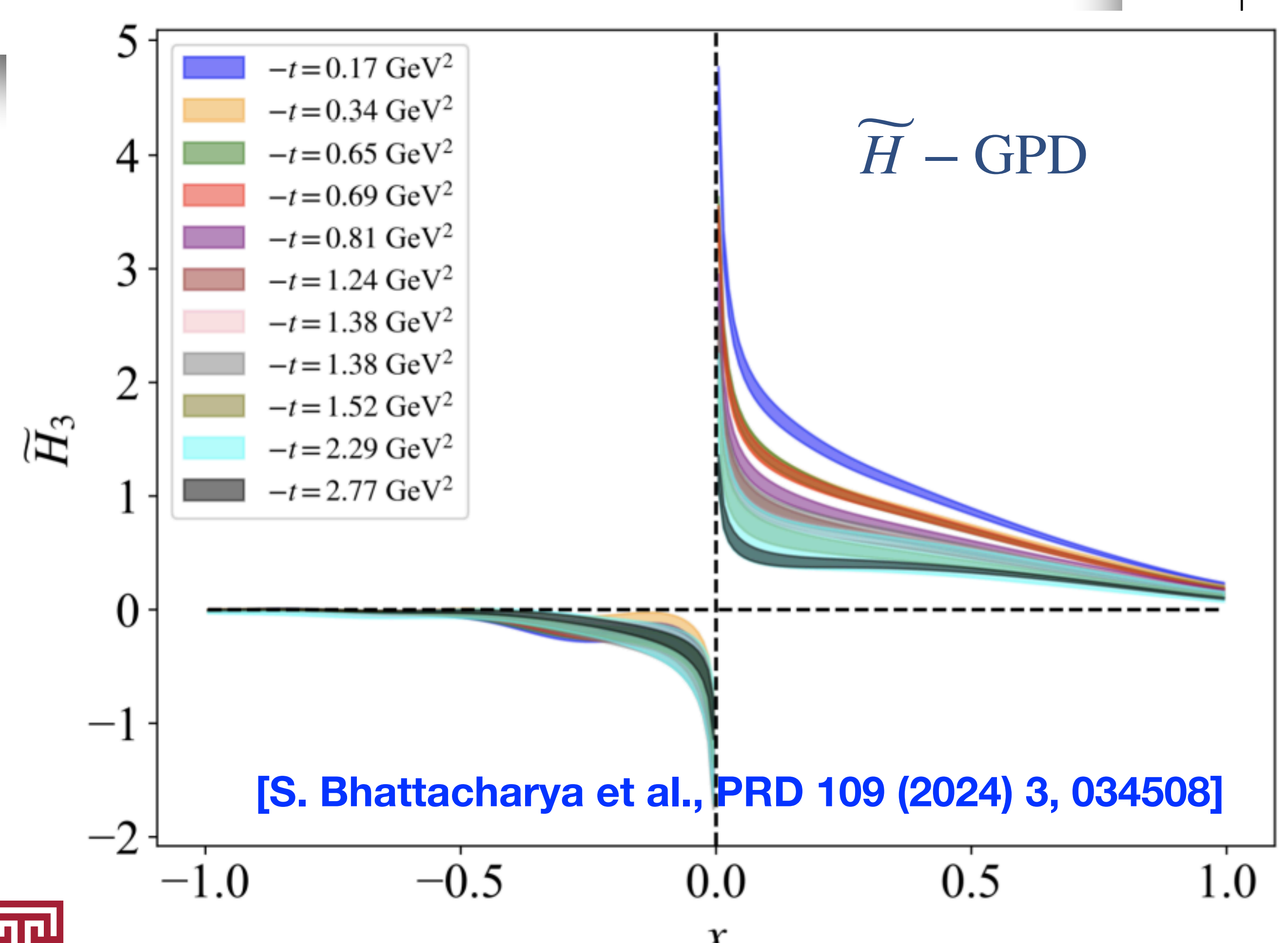


Reminder: Unpolarized & Helicity GPDs

[S. Bhattacharya et al., PRD 106 (2022) 11, 114512]







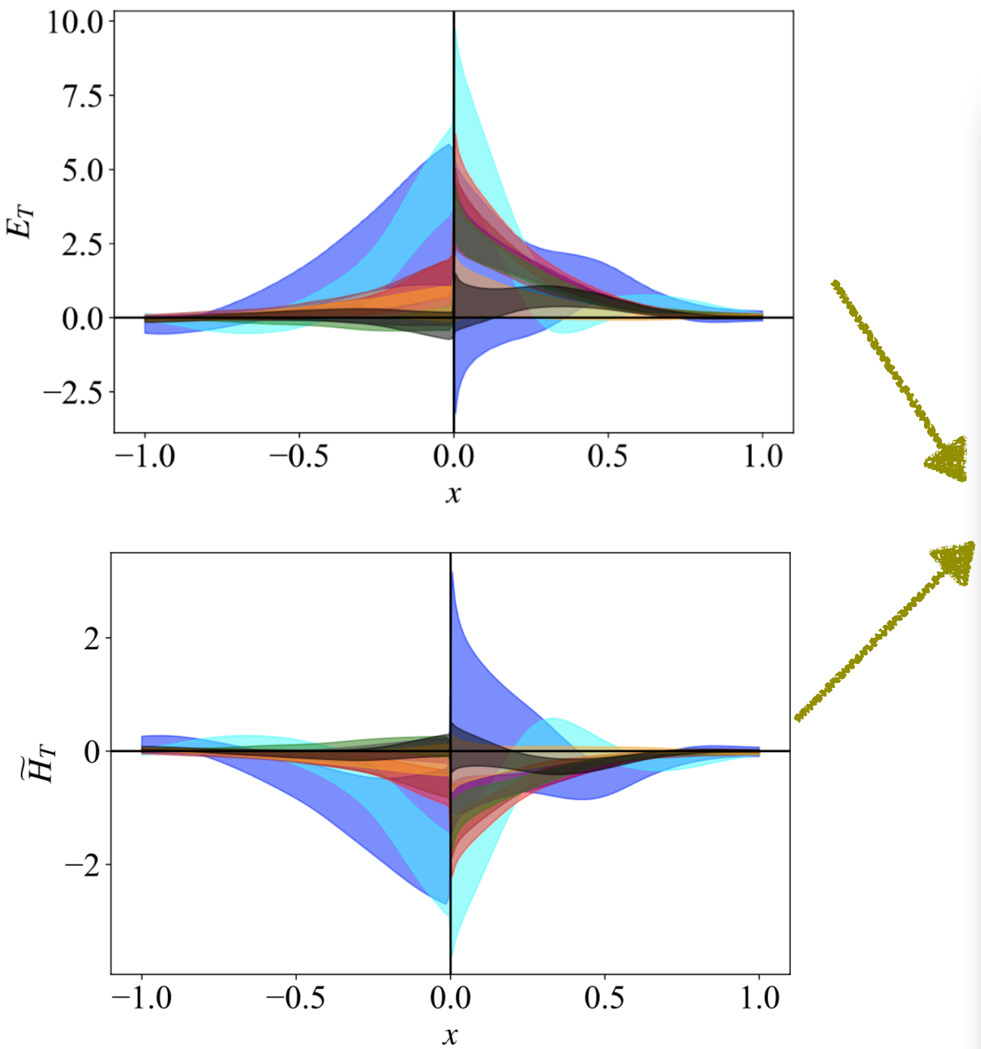
[S. Bhattacharya et al., PRD 109 (2024) 3, 034508]

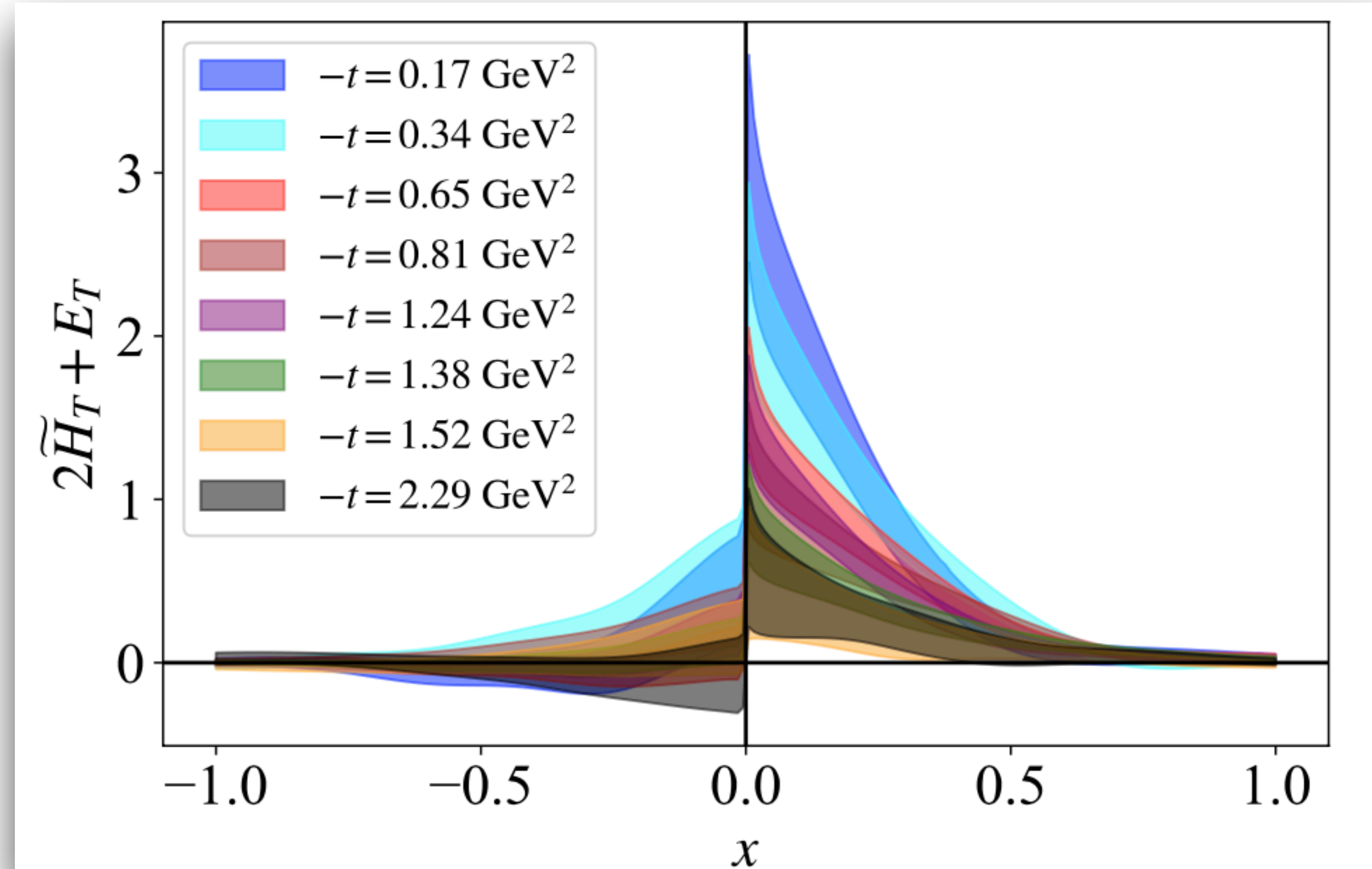
- ★ Signal for H_T comparable with H, $\widetilde{H}$
- ★ $\widetilde{E}(\xi = 0)$: inaccessible

Physical Interpretation

E_T : No physical interpretation as a density

$\widetilde{H}_T$: connection with quadrupole deformation of distribution of $\perp$pol. quarks in $\perp$ pol. proton





- ★ $E_T + 2\widetilde{H}_T$ related to transverse spin structure of the proton

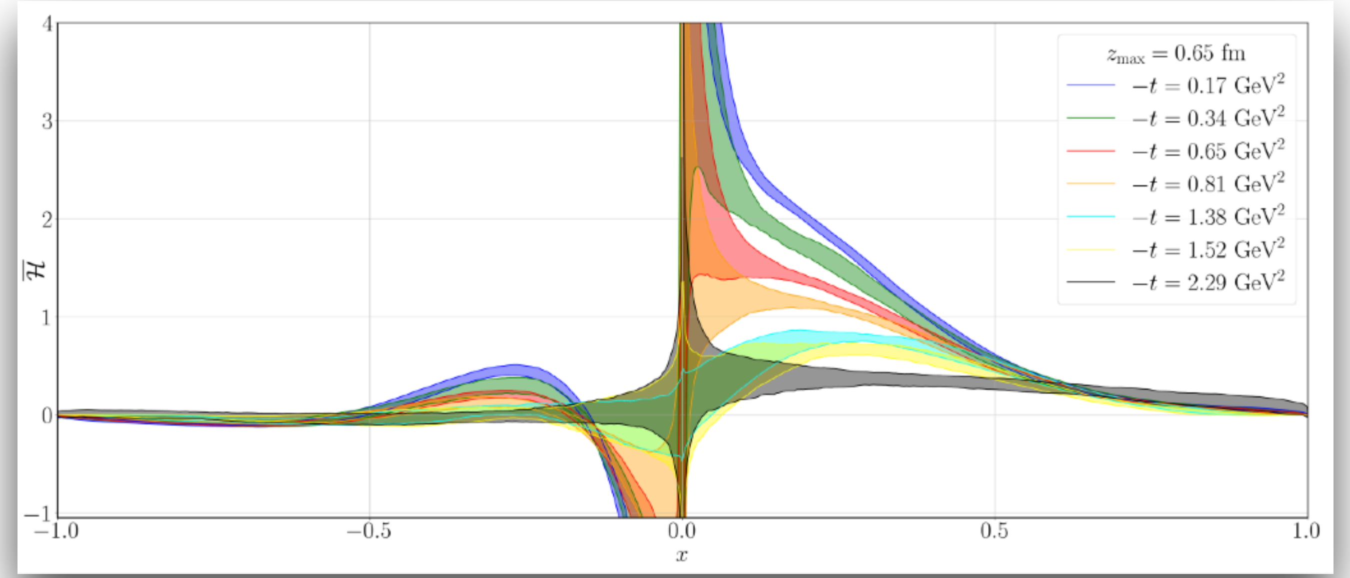
[M. Burkardt, PRD 72, 094020, 2005]

- ★ Impact parameter space: describes the deformation in the distribution of transversely polarized quarks within an unpolarized proton.
- ★ $k_T = \int dx \left(E_T(x,0,0) + 2\widetilde{H}_T(x,0,0) \right)$: size of dipole moment given by distribution
- ★ $E_T + 2\widetilde{H}_T = -A_4$ (good signal)

From Raw Data to Rich Insights

Alternative approach: pseudo-ITD

Example: unpolarized GPDs case



[Battacharya et al., PRD 110 (2024) 5, 054502]

Different steps between approaches:
- renormalization
- x-dependence reconstruction
- matching formalism

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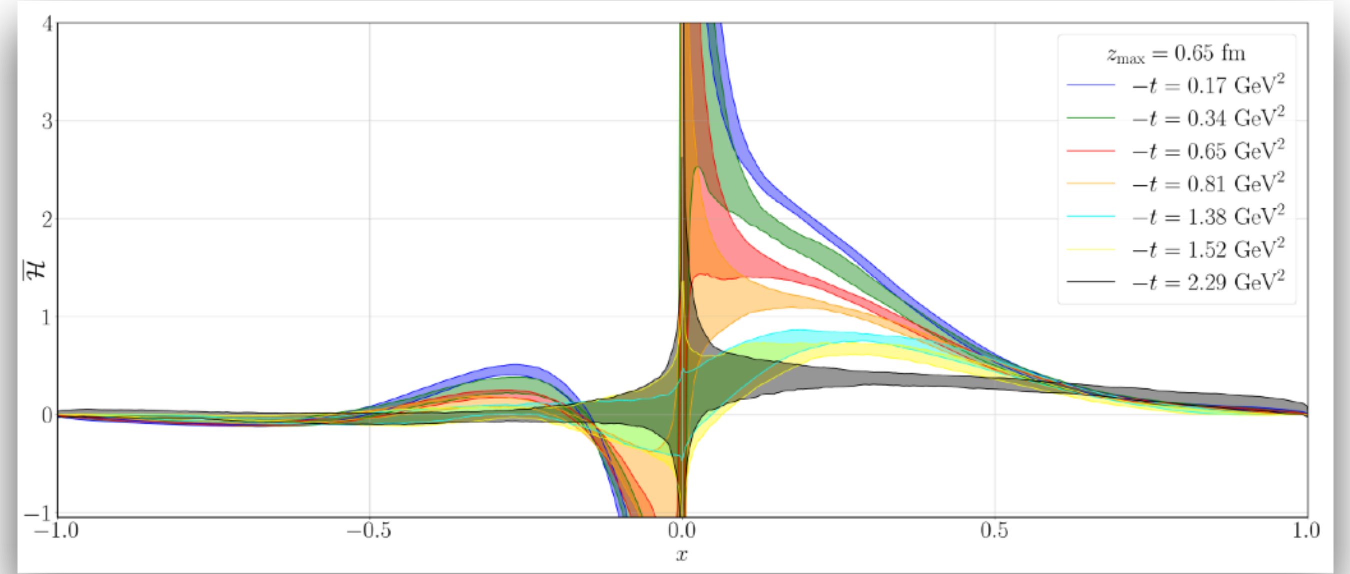
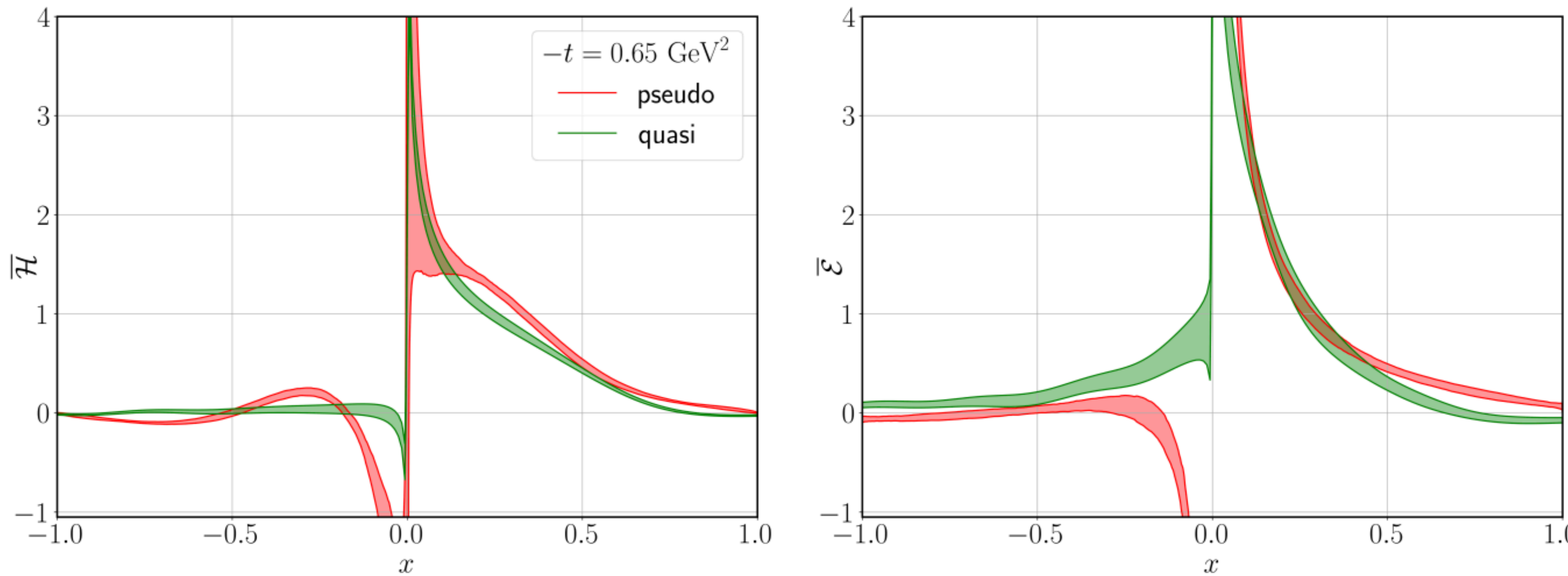


[Battacharya et al., PRD 110 (2024) 5, 054502]

Different steps between approaches:
- renormalization
- x-dependence reconstruction
- matching formalism

★ Comparison between methods helps assess systematic effects



★ x<0 and small-x regions susceptible to systematic effects

★ Comparison only includes statistical uncertainties

Mellin moments

See, X. Gao's talk in 2023 Workshop

Example: unpolarized GPDs case

★ Leading-twist factorization formula

$$\mathcal{M}(z,P,\Delta) \equiv \frac{\mathcal{F}(z,P,\Delta)}{\mathcal{F}(z,P=0,\Delta=0)} = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\overline{\mathrm{MS}}}(\mu^2 z^2)}{C_0^{\overline{\mathrm{MS}}}(\mu^2 z^2)} \langle x^n \rangle + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2 z^2)$$

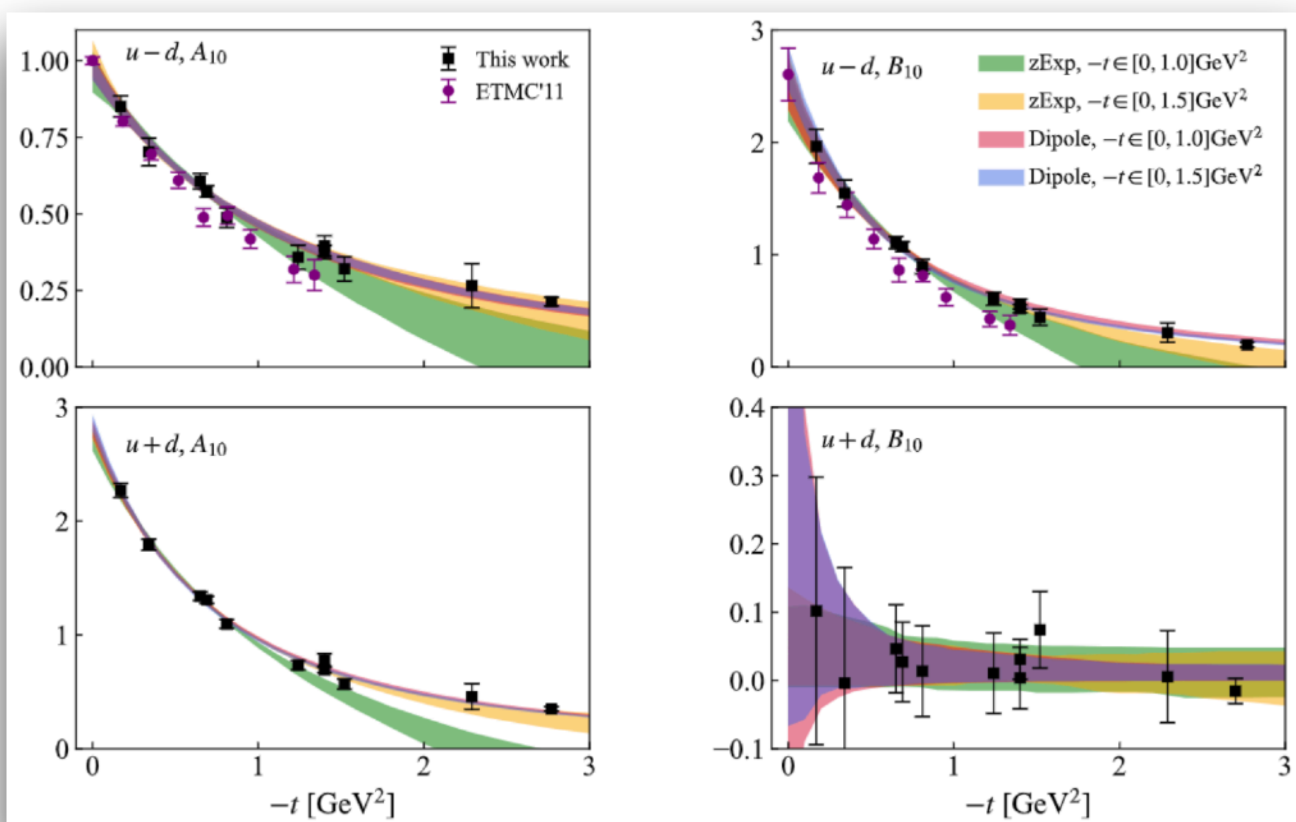
★ Avoid power-divergent mixing of multi-derivative operators

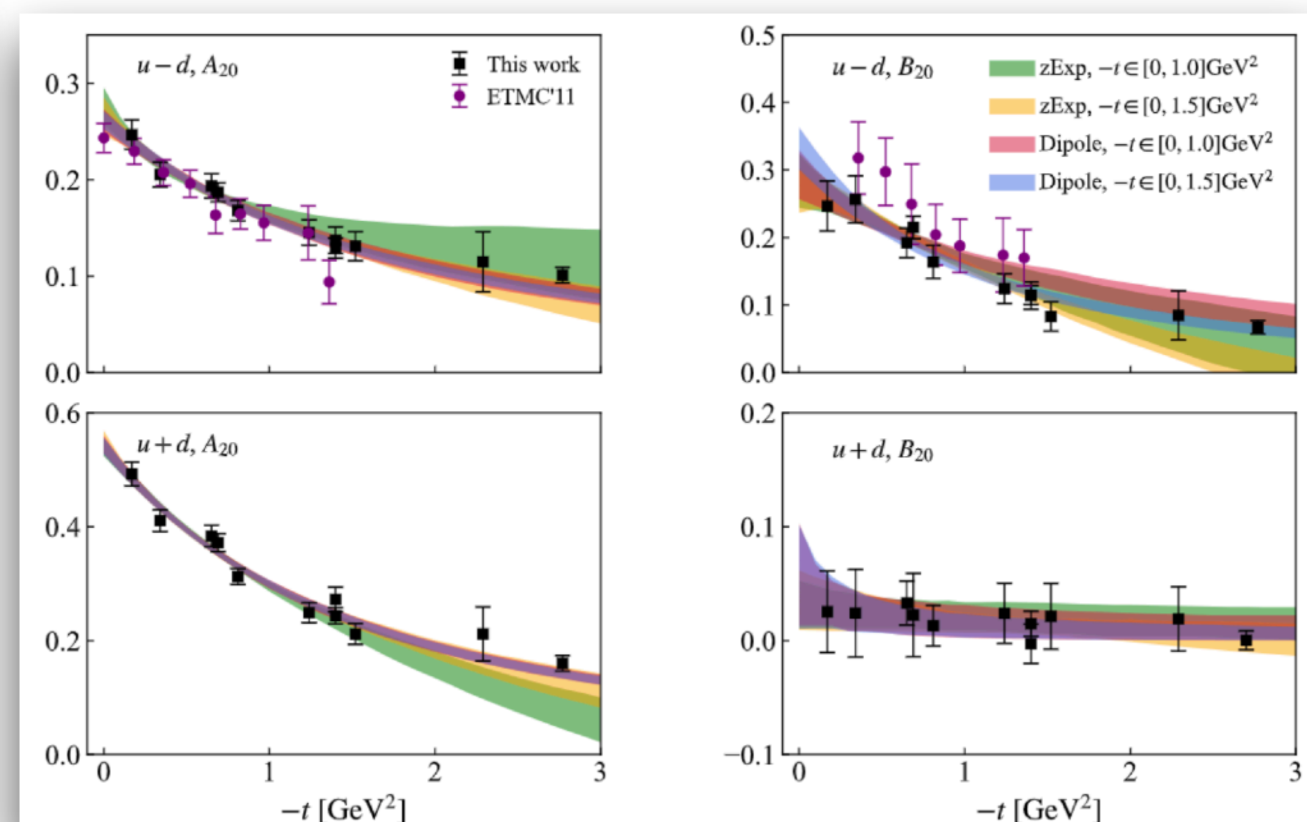
★ Wilson coefficients known to NLO (or NNLO)

★ Both isovector and isoscalar (ignores disconnected; found tiny)

[C. Alexandrou et al., PRD 104 (2021) 5, 054503]

[S. Bhattacharya et al., PRD 108 (2023) 1, 014507; arXiv:2410.03539]





Beyond leading twist

See, M. Constantinou's talk in 2024 Workshop

- ★ Lack density interpretation, but have physical interpretation
- ★ Contain information about quark-gluon correlations inside hadrons
- ★ Sensitive to soft dynamics
- ★ Appear in QCD factorization theorems for various observables
- ★ Challenging to probe experimentally and isolate from leading-twist
  [Defurne et al., PRL 117, 26 (2016); Defurne et al., Nature Commun. 8, 1 (2017)]
- ★ Can be as sizable as leading twist

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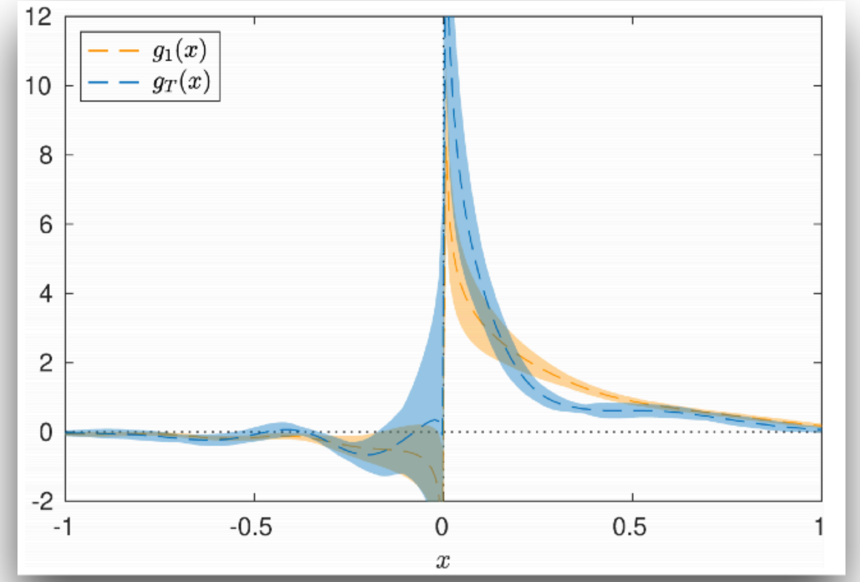

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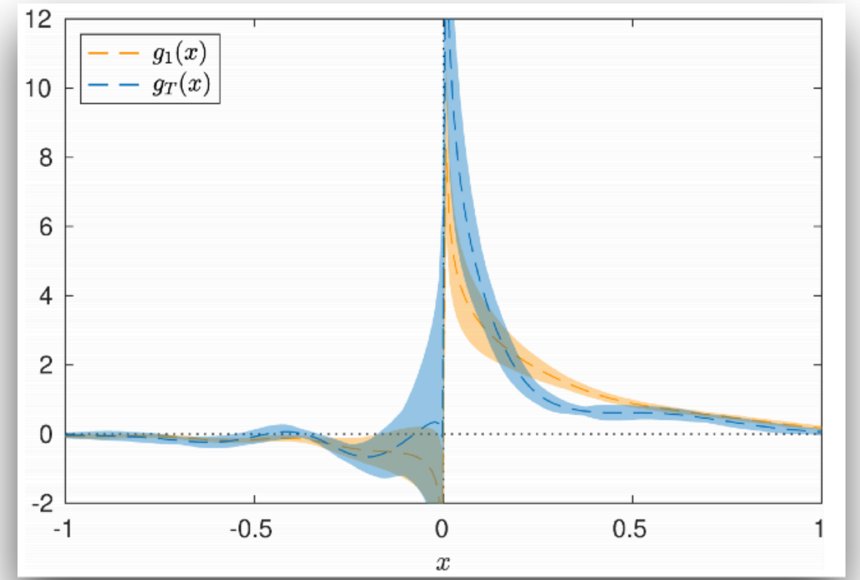


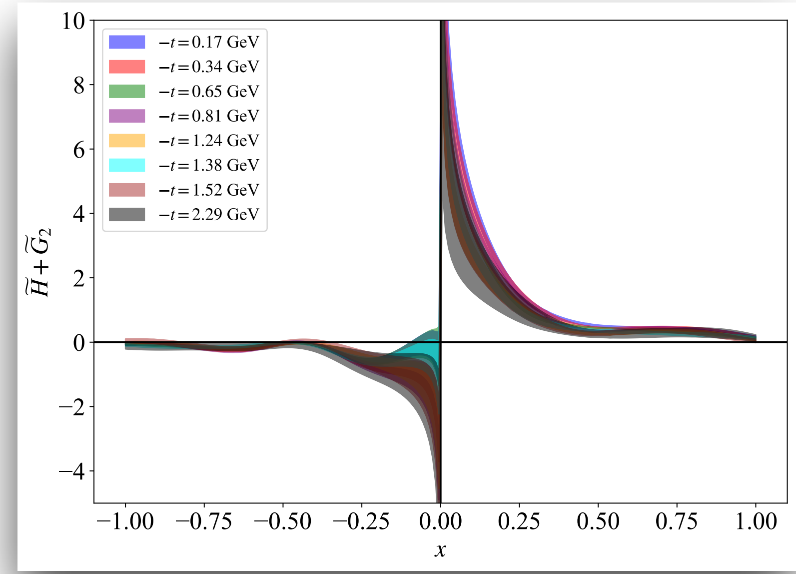
★ Extraction of twist-3 distributions from lattice QCD is very challenging

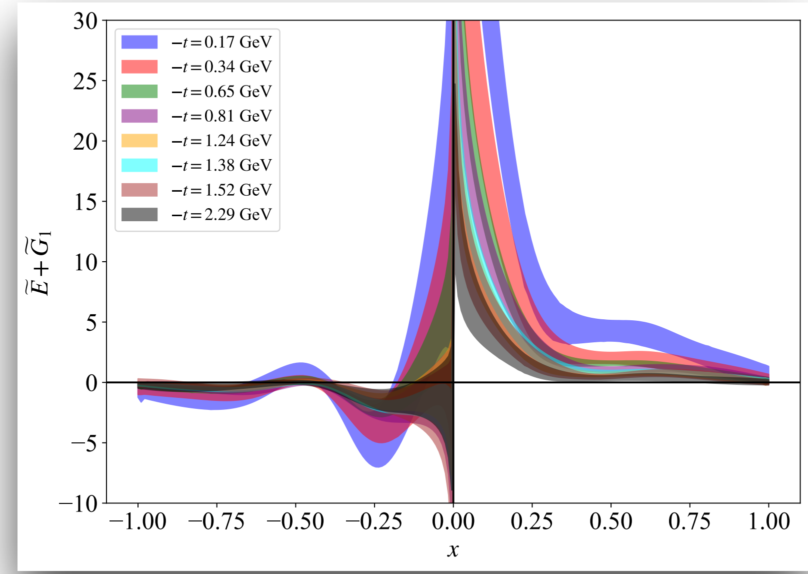
★ Mixing with q-g-q correlators; matching:
[V. Braun et al., JHEP 05 (2021) 086; JHEP 10 (2021) 087]

★ Kinematic twist-3 contributions to pseudo & quasi GPDs to restore translation invariance
[V. Braun et al., JHEP 10 (2023) 134]

Amplitude decomposition

Example: axial twist-3 GPDs





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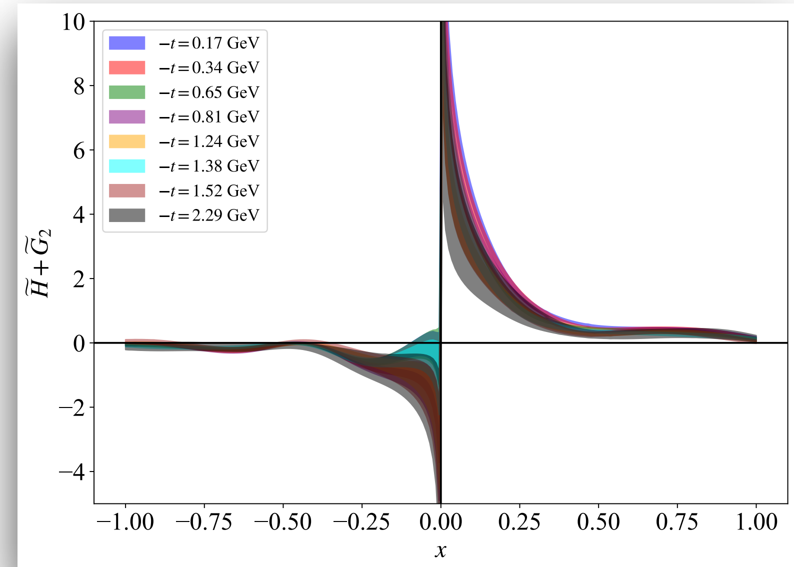
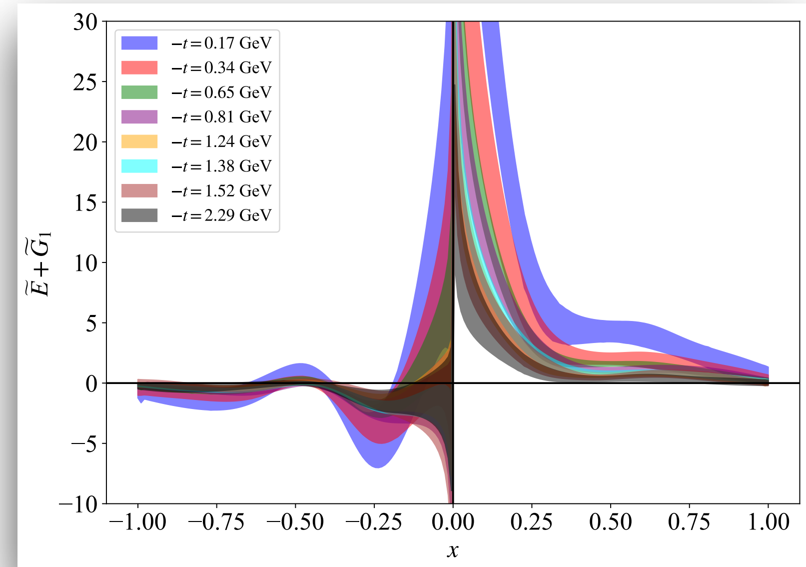
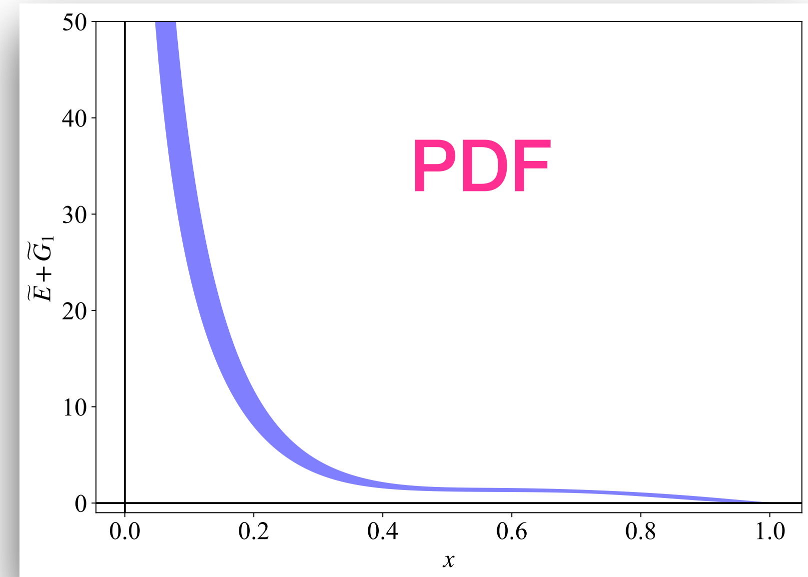




Parametrization, gives info, e.g., access to $\widetilde{E}$-GPD even at zero skewness



$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t) \qquad \int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,,$$

Amplitude decomposition

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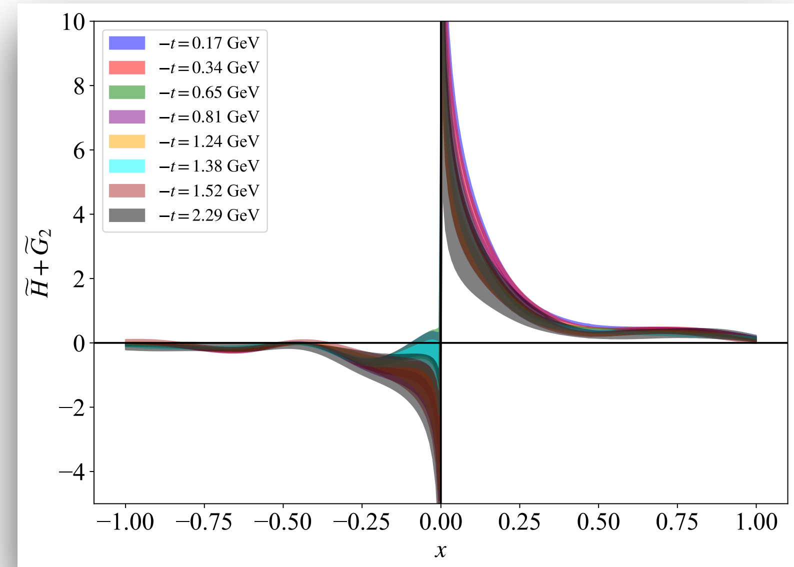

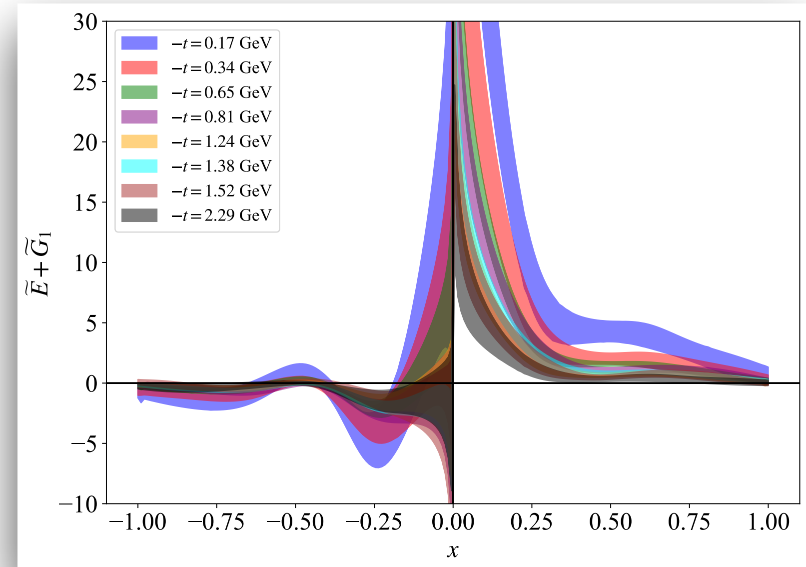

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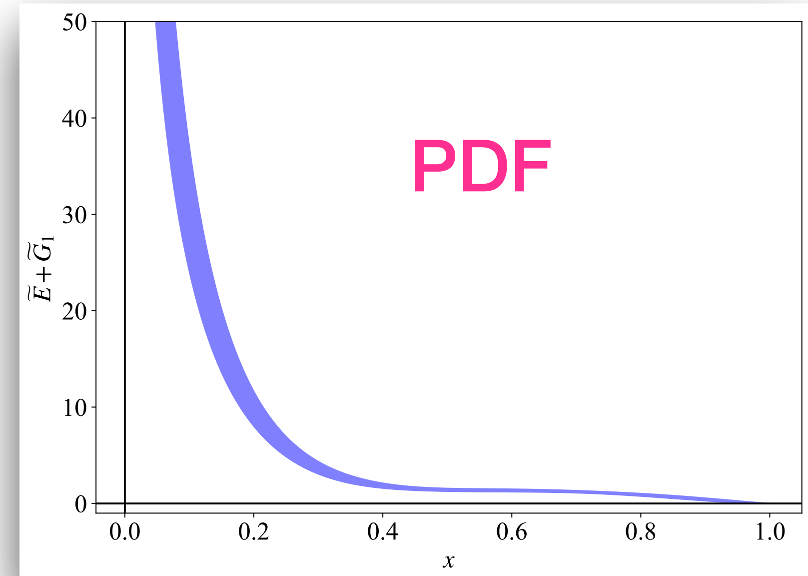

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Similar methodology for tensor twist-3 GPDs (chiral odd)

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p')\left(\gamma^+\gamma_5\,\widetilde{H}_2' + \frac{P^+\gamma_5}{M}\,\widetilde{E}_2'\right)u(p)$$

[Meissner et al., *JHEP* 08 (2009) 056]

★ Fwd limit (h_L) may be accessed via:

- double-polarized Drell-Yan process
[R. Jaffe, PRL 67 (1991) 552-555; Y. Koike et al., PLB 668 (2008) 286]

- di-hadron single spin asymmetries (CLAS)
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Amplitude decomposition

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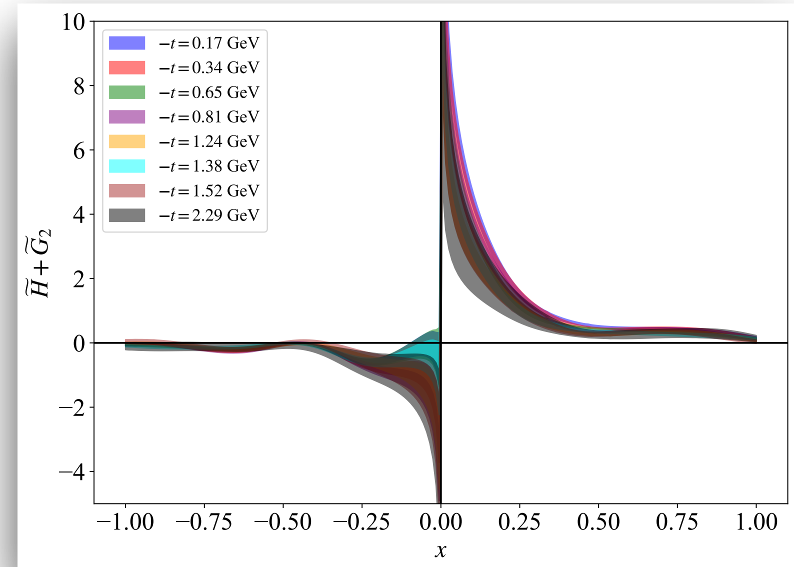

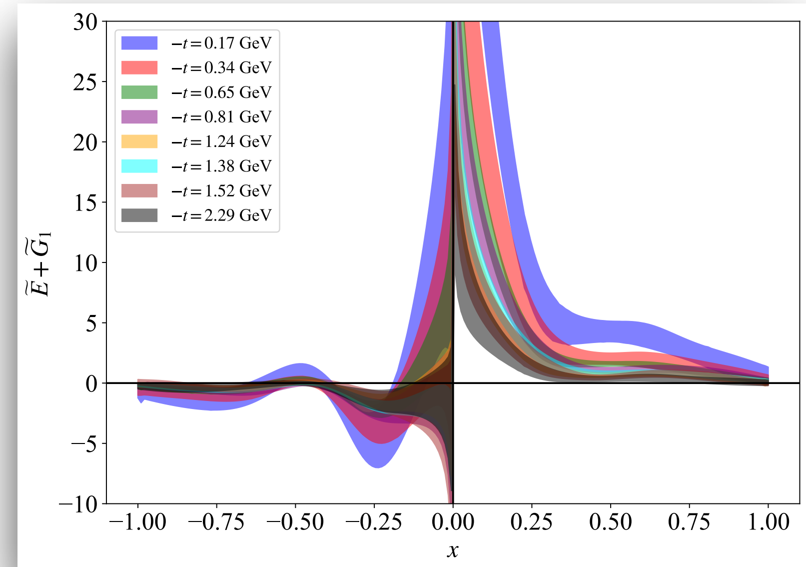

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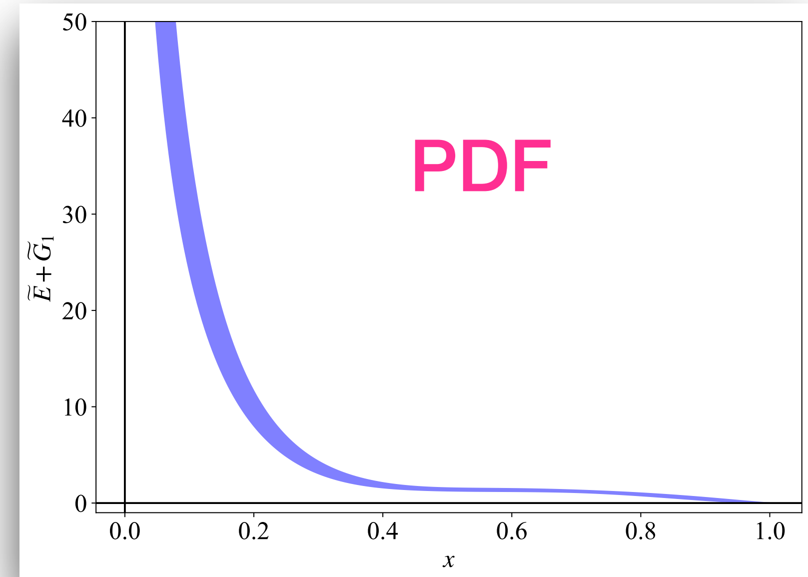

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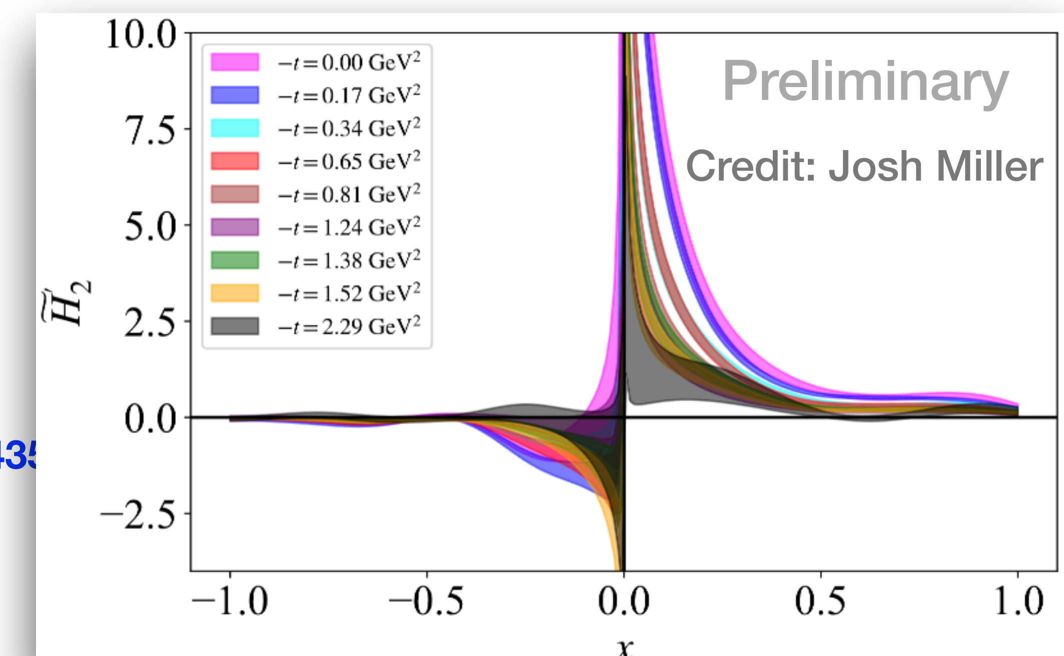

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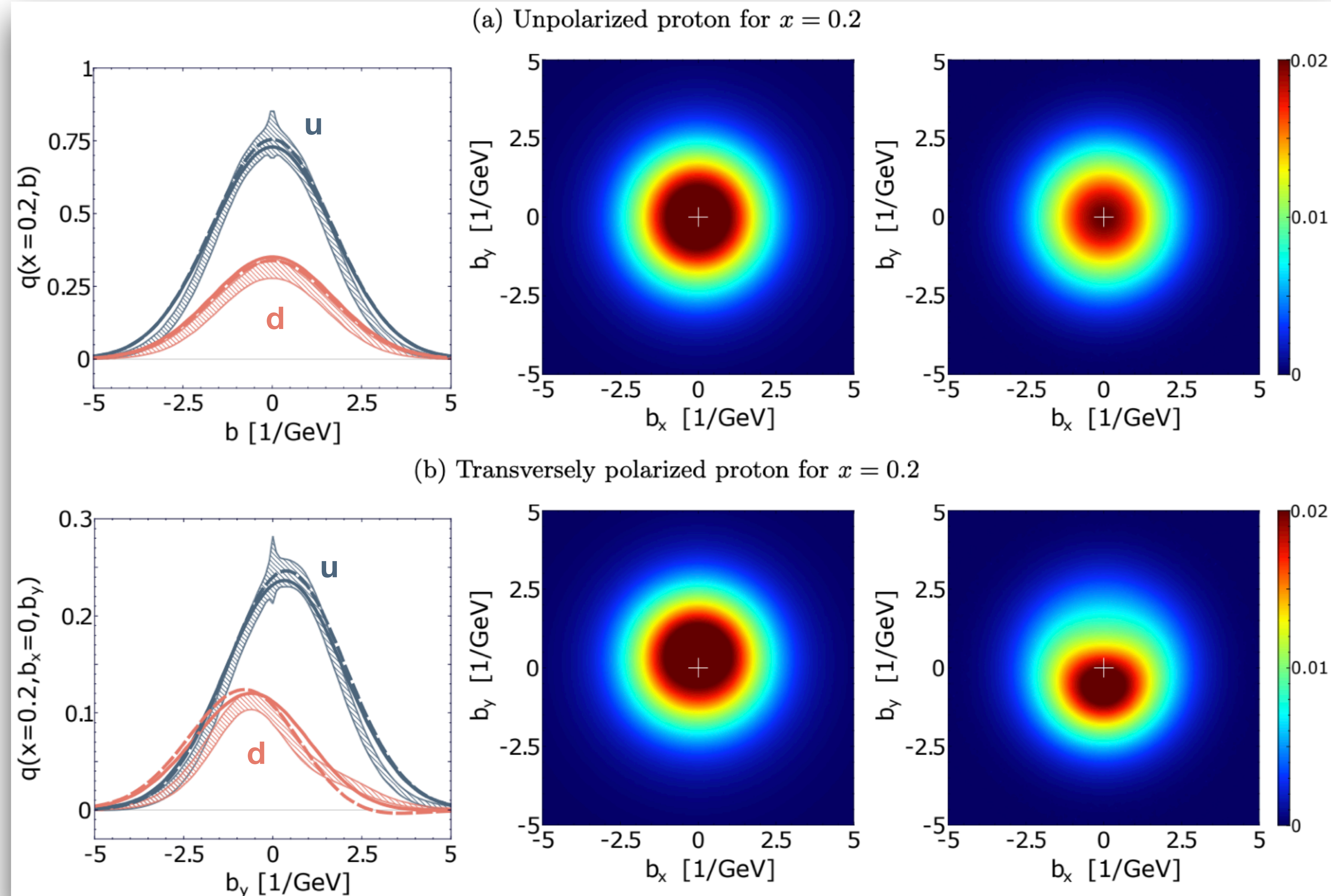
Synergy/Complementarity of lattice and phenomenology

Toward synergy for GPDs

[K. Cichy et al., PRD 110 (2024) 11, 114025]

Example: unpolarized GPDs



- GK (solid line),
- VGG (dashed line)

- Good agreement for up quark; reasonable agreement for down quark
- Further study needed on how to combine lattice results with data

How do lattice QCD data fit in the overall effort for hadron tomography?

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

QUARK-GLUON TOMOGRAPHY COLLABORATION

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Award Number:
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See related talks @ this Workshop

- From FFs to GPDs, Y. Zhao (Monday)
- String-based approach to GPDs, I. Zahed (Monday)
- Transition GPDS, Ch. Weiss (Tuesday)
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PHENOMENOLOGY
LATTICE QCD
THEORY

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"Open database for GPD analyses",
V.D. Burkert et al., accepted in Eur. Phys. J. C,
(manuscript led by M. Higuera-Angulo and P. Sznajder)

* Other GPD global analysis efforts:

- Gepard [https://gepard.phy.hr/]
- PARTONS [https://partons.cea.fr]
- EXCLAIM [https://exclaimcollab.github.io/web.github.io/#/]

Concluding Remarks

- ★ New developments in several promising directions
- ★ Extensive program in extracting GPDs from lattice QCD
- ★ New methods can optimize computational resources
- ★ Access beyond leading-twist GPDs feasible from lattice QCD (q-g-q challenging)
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Thank you

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DOE Early Career Award
Grant No. DE-SC0020405 &
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U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

NERSC

ASCR Leadership Computing Challenge ALCC

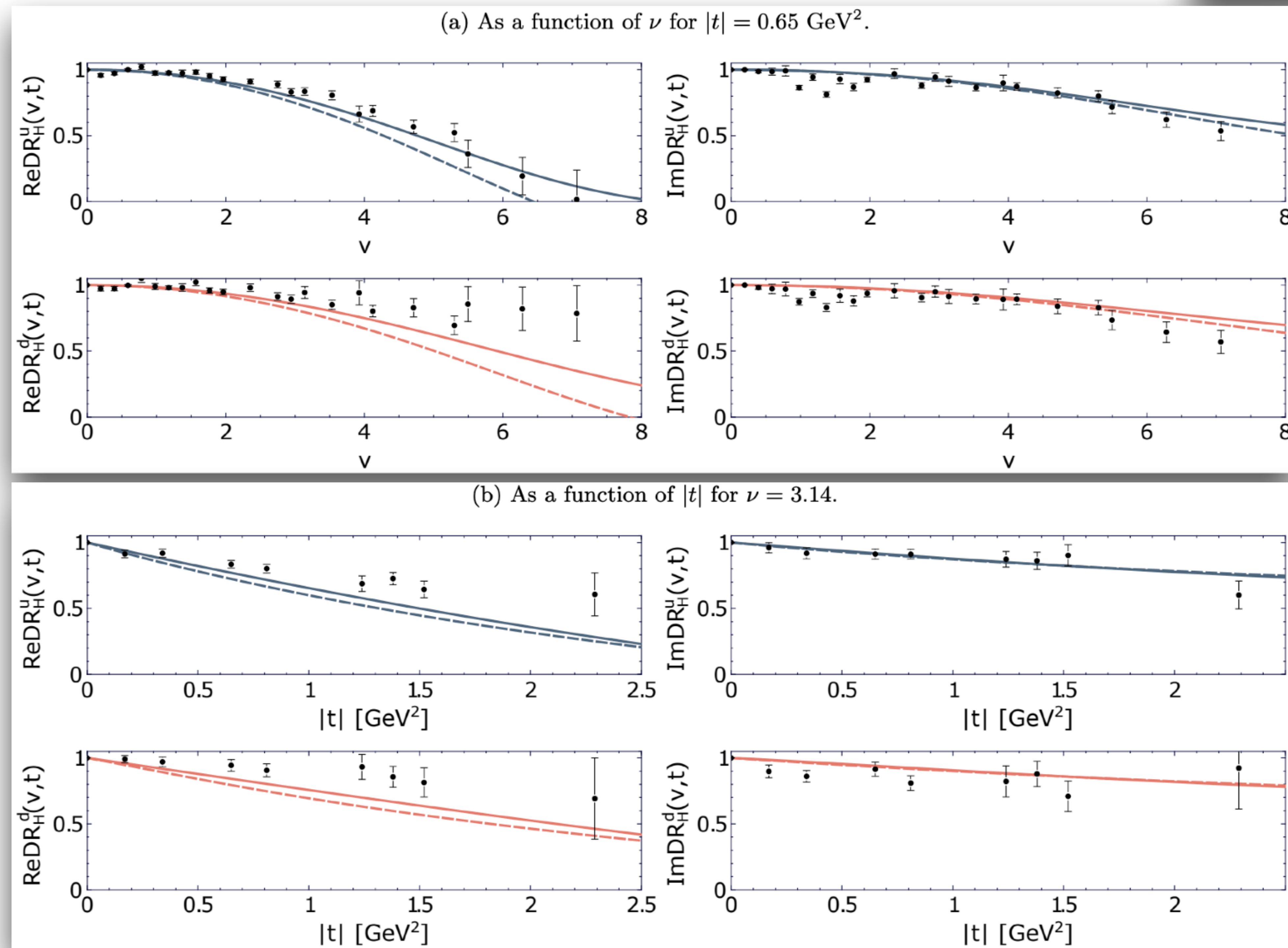
Toward synergy for GPDs

★ Forming ratios of GPDs seems to suppress systematic uncertainties

[K. Cichy et al., PRD 110 (2024) 11, 114025]

$$\mathrm{DR}^{\hat{H}^q}_{\mathrm{Re}}(\nu,t) = \frac{\mathrm{Re}\hat{H}^q(\nu,t)}{\mathrm{Re}\hat{H}^q(\nu,0)}\,\frac{\mathrm{Re}\hat{H}^q(0,0)}{\mathrm{Re}\hat{H}^q(0,t)}\,,$$

$$\mathrm{DR}^{\hat{H}^q}_{\mathrm{Im}}(\nu,t) = \lim_{\nu'\to 0}\frac{\mathrm{Im}\hat{H}^q(\nu,t)}{\mathrm{Im}\hat{H}^q(\nu,0)}\,\frac{\mathrm{Im}\hat{H}^q(\nu',0)}{\mathrm{Im}\hat{H}^q(\nu',t)}$$



- GK (solid curve)
- VGG (dashed curve)

- Good agreement for up quark
- Reasonable agreement for down quark

- Further study needed on how to combine lattice results with data