(G)TMD Observables from Lattice QCD

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What do we compute – and why

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, \Delta_T, S, \ldots) \equiv \frac{1}{2}\langle P + \Delta_T/2, S|\ \bar{q}(-b/2)\ \Gamma\ \mathcal{U}[-b/2, b/2]\ q(b/2)\ |P - \Delta_T/2, S\rangle$$

- Operator separation b Fourier conjugate to quark momentum k.
  - Transverse component $b_T \longleftrightarrow k_T$
  - Longitudinal component $b \cdot P \longleftrightarrow x$ (momentum fraction)
- (Transverse) momentum transfer Δ_T Fourier conjugate to impact parameter $r_T \longrightarrow$ GTMDs
- Dirac structures $\Gamma \longrightarrow$ spin physics
- Now, what about the gauge link $\mathcal{U}\ldots$

What do we compute – and why

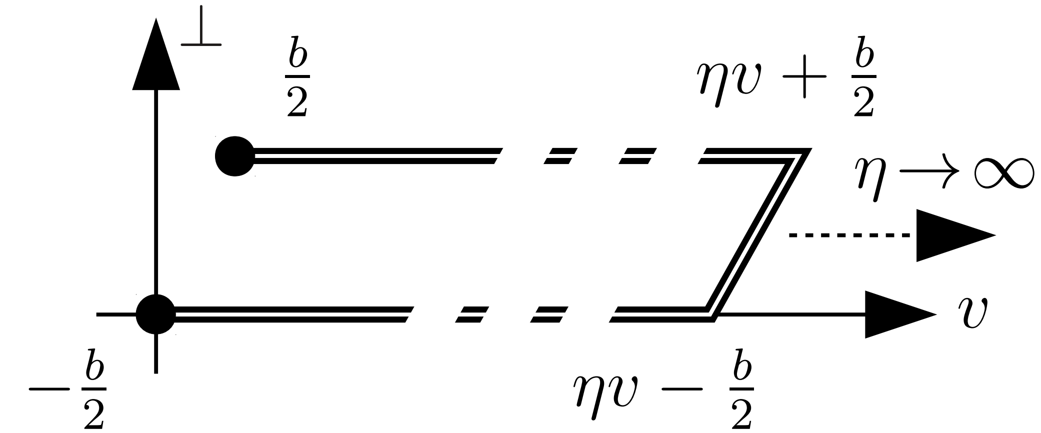
$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, \Delta_T, S, \ldots) \equiv \frac{1}{2}\langle P + \Delta_T/2, S|\ \bar{q}(-b/2)\ \Gamma\ \mathcal{U}[-b/2, b/2]\ q(b/2)\ |P - \Delta_T/2, S\rangle$$

Role of the gauge link $\mathcal{U}$:

Staple-shaped $\mathcal{U}[-b/2, b/2] \longrightarrow$ final state interactions

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\quad \hat{\zeta} \equiv \dfrac{P \cdot v}{|P||v|}$



Are interested in $\hat{\zeta} \longrightarrow \infty$

Systematic decomposition of $\widetilde{\Phi}$ into invariant amplitudes (forward limit only)

$$\frac{1}{2}\widetilde{\Phi}^{[\gamma^\mu]}_{\text{unsubtr.}} = P^\mu \widetilde{A}_2 - i m_N^2 b^\mu \widetilde{A}_3 - i m_N \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha S_\beta \widetilde{A}_{12} + \frac{m_N^2}{v\cdot P} v^\mu \widetilde{B}_1 + \frac{m_N}{v\cdot P}\epsilon^{\mu\nu\alpha\beta} P_\nu v_\alpha S_\beta \widetilde{B}_7$$
$$-\frac{i m_N^3}{v\cdot P}\epsilon^{\mu\nu\alpha\beta} b_\nu v_\alpha S_\beta \widetilde{B}_8 - \frac{m_N^3}{v\cdot P}(b\cdot S)\epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha v_\beta \widetilde{B}_9 - \frac{i m_N^3}{(v\cdot P)^2}(v\cdot S)\epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha v_\beta \widetilde{B}_{10}$$

and similarly for other Γ structures (32 amplitudes altogether for TMDs)

- Essential use of Lorentz covariance in this calculational scheme
- Frames in which TMDs are defined phenomenologically and in which lattice calculations are performed differ
- Use of invariant amplitudes permits direct translation of results
- For (G)TMDs, all separations (b, v) are spacelike $\Rightarrow$ can find a frame in which they are purely spatial

Generalized shifts

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

Boer-Mulders shift:

$$\langle k_y \rangle_{UT} \equiv m_H \frac{\tilde{h}_1^{\perp [1](1)}}{\tilde{f}_1^{[1](0)}} = m_H \frac{\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Average transverse momentum of quarks polarized in the orthogonal transverse ("T") direction in an unpolarized ("U") hadron; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Analogously, Sivers shift (in a polarized hadron):

$$\langle k_y \rangle_{TU} = -m_N \frac{\widetilde{A}_{12B}}{\widetilde{A}_{2B}}$$

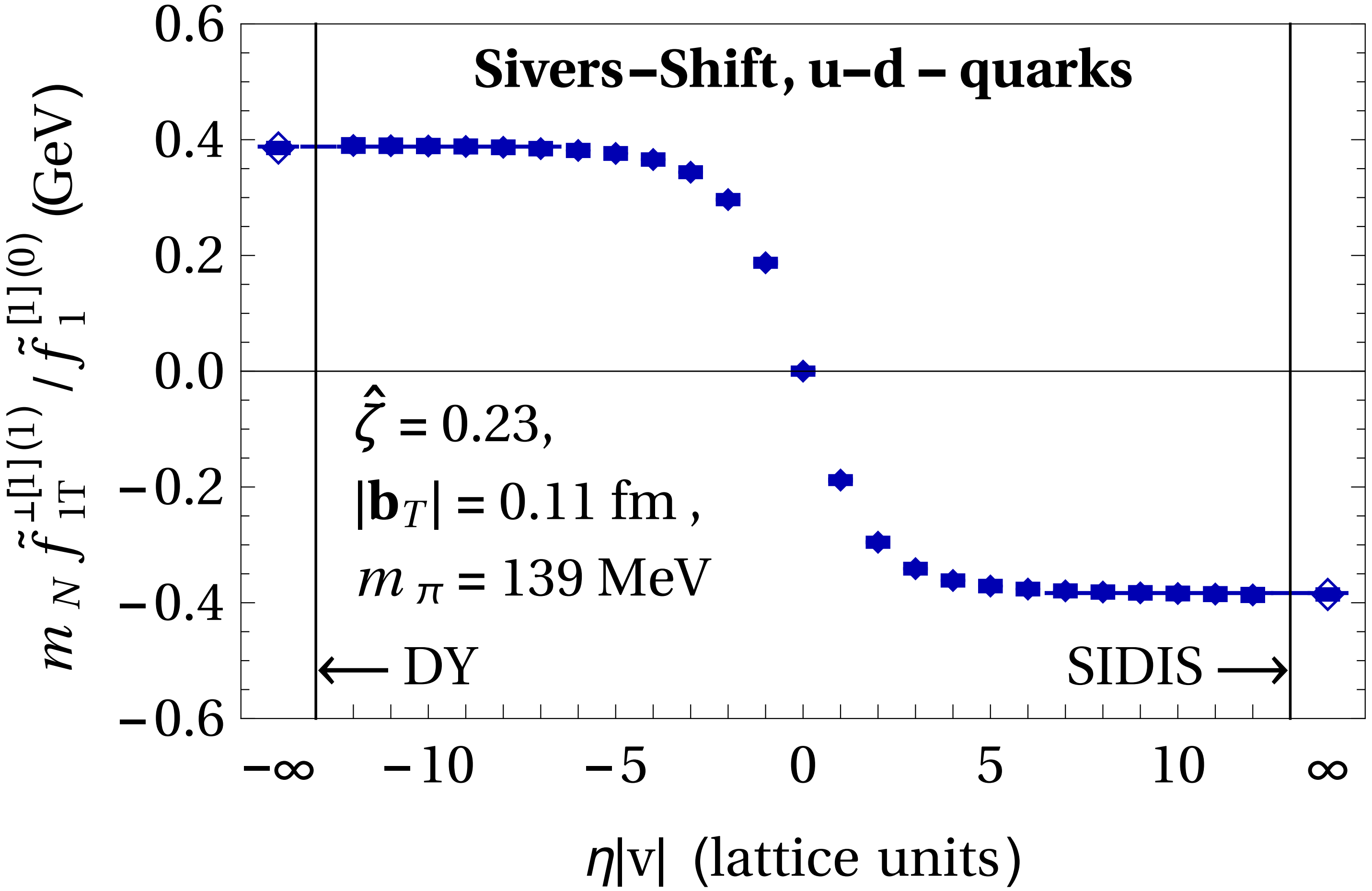
$h_{1L}^{\perp}$ worm gear shift:

$$\langle k_x \rangle_{LT} = -m_N \frac{\widetilde{A}_{10B}}{\widetilde{A}_{2B}}$$

. . . and more.

Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



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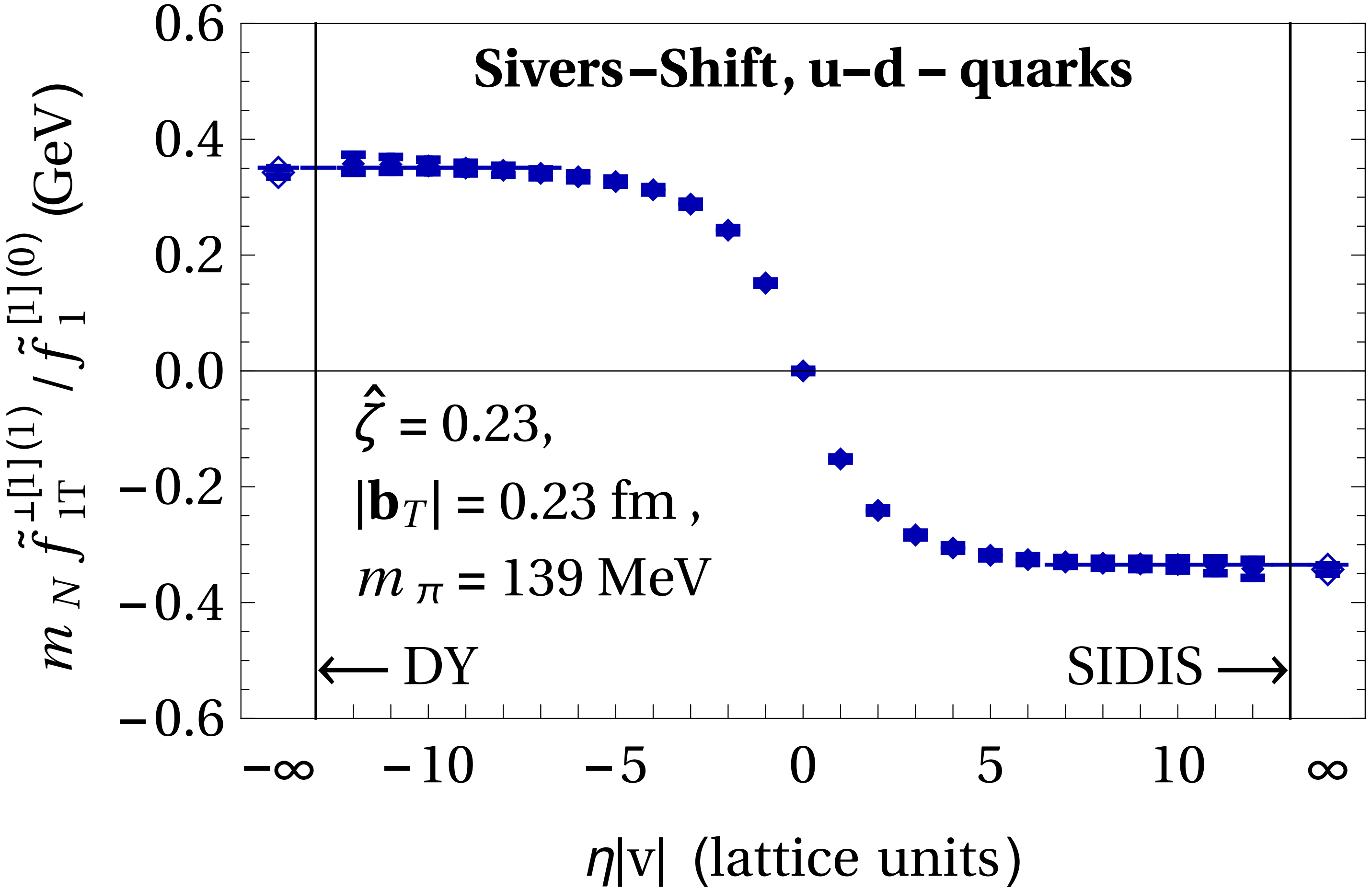

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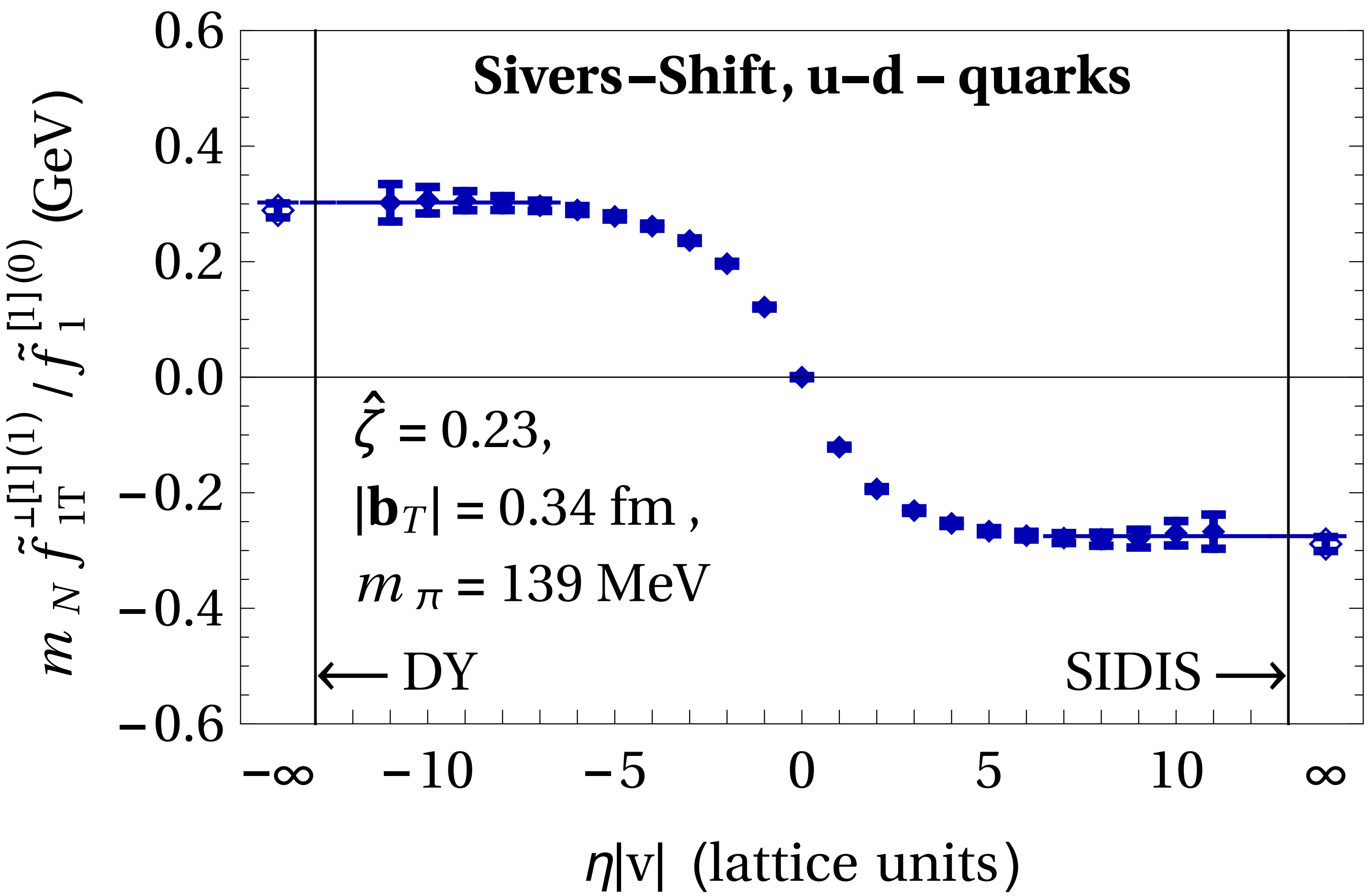

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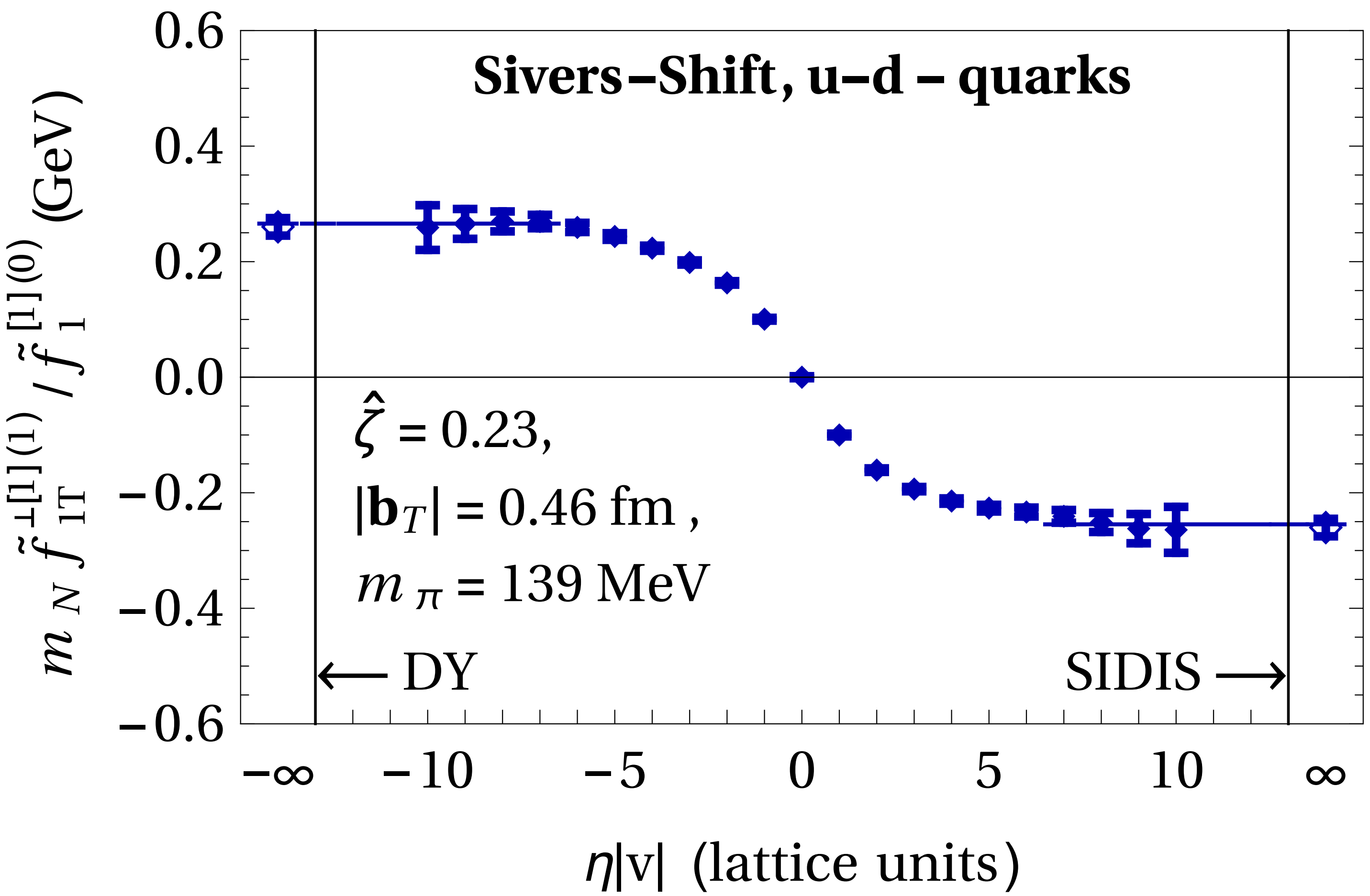
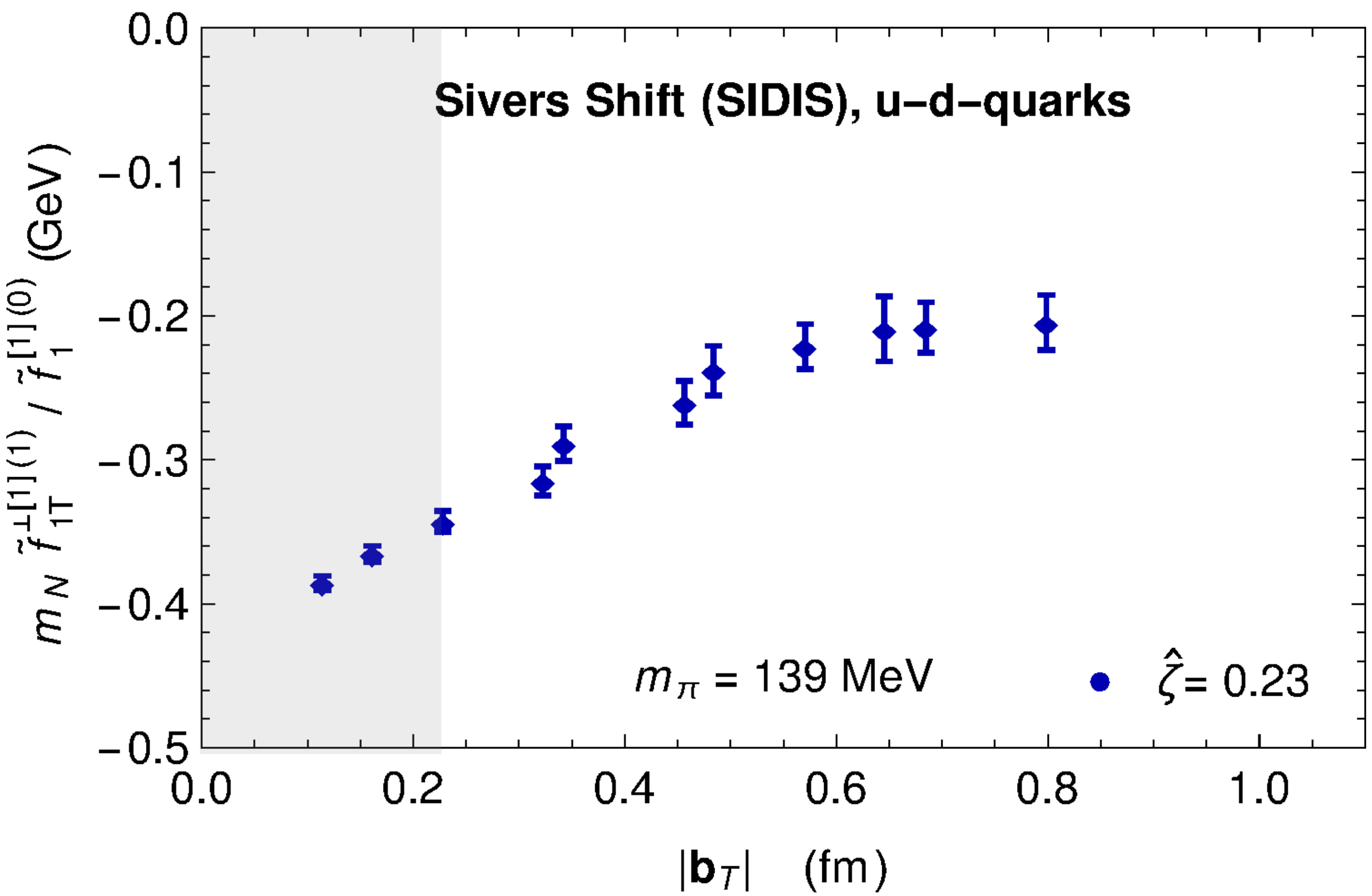


Results: Sivers shift

Dependence of SIDIS limit on $|b_T|$



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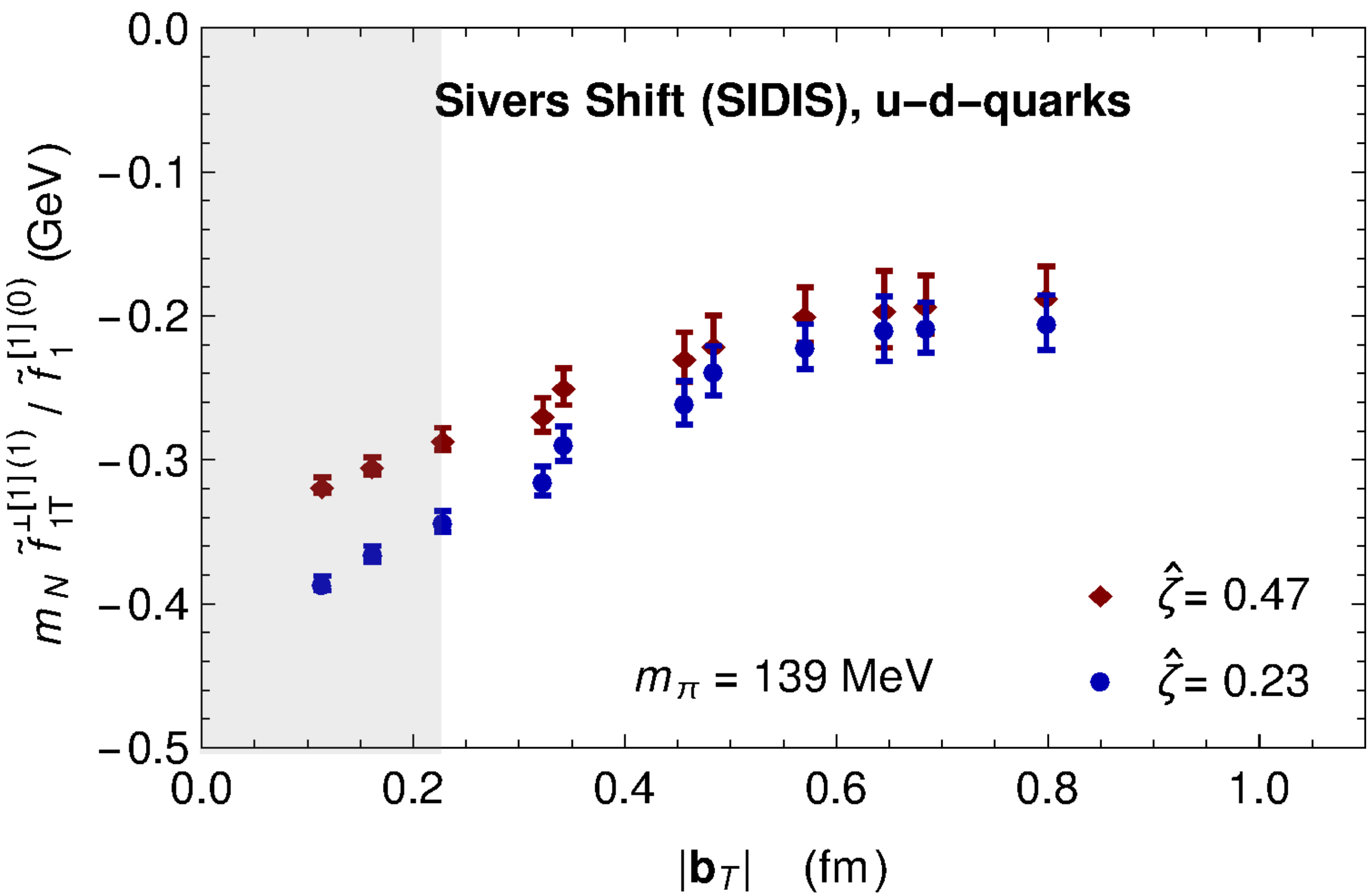
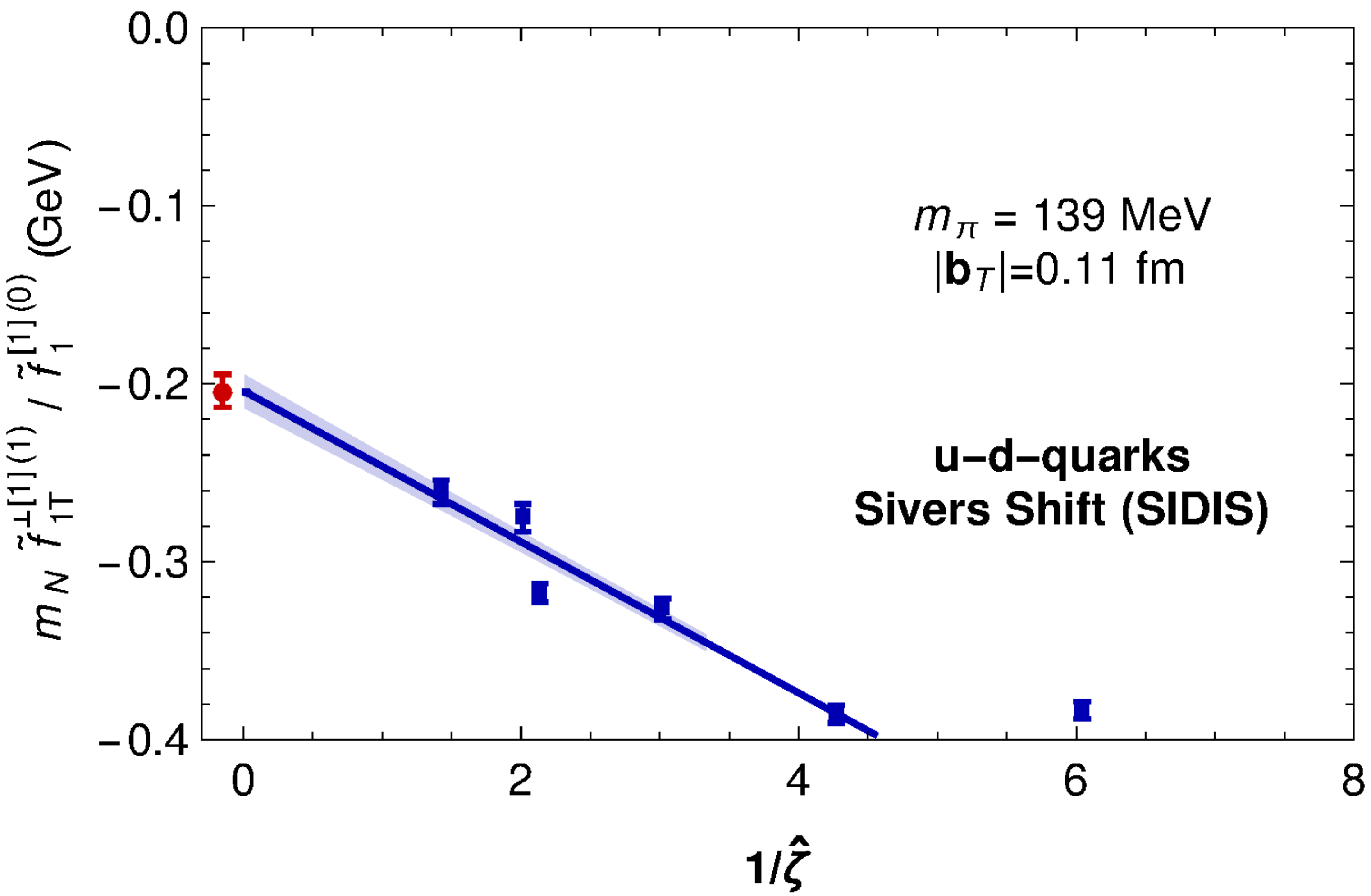


Results: Sivers shift

Extrapolation in $\hat{\zeta}$ for given $|b_T|$ – SIDIS limit



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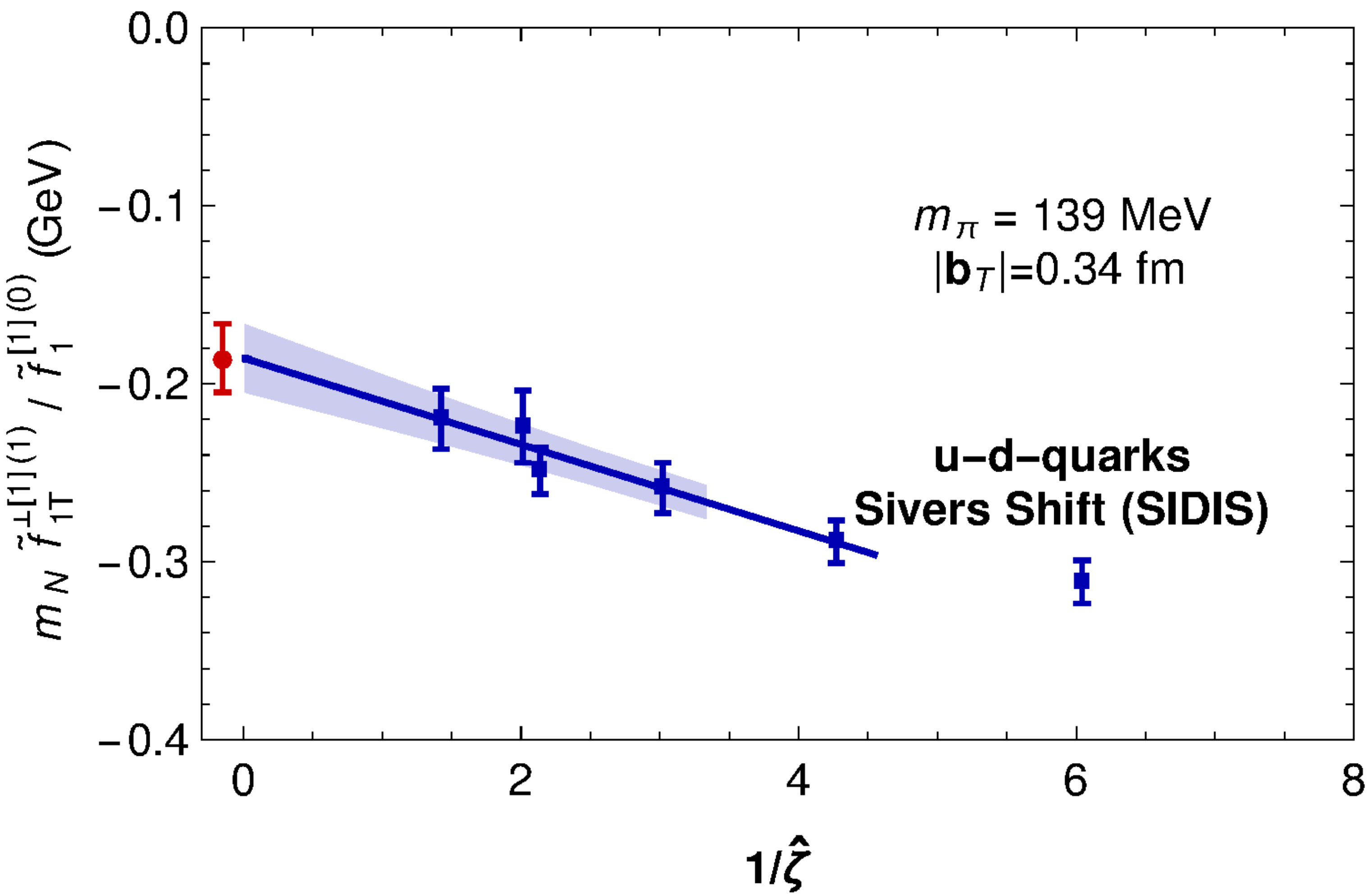

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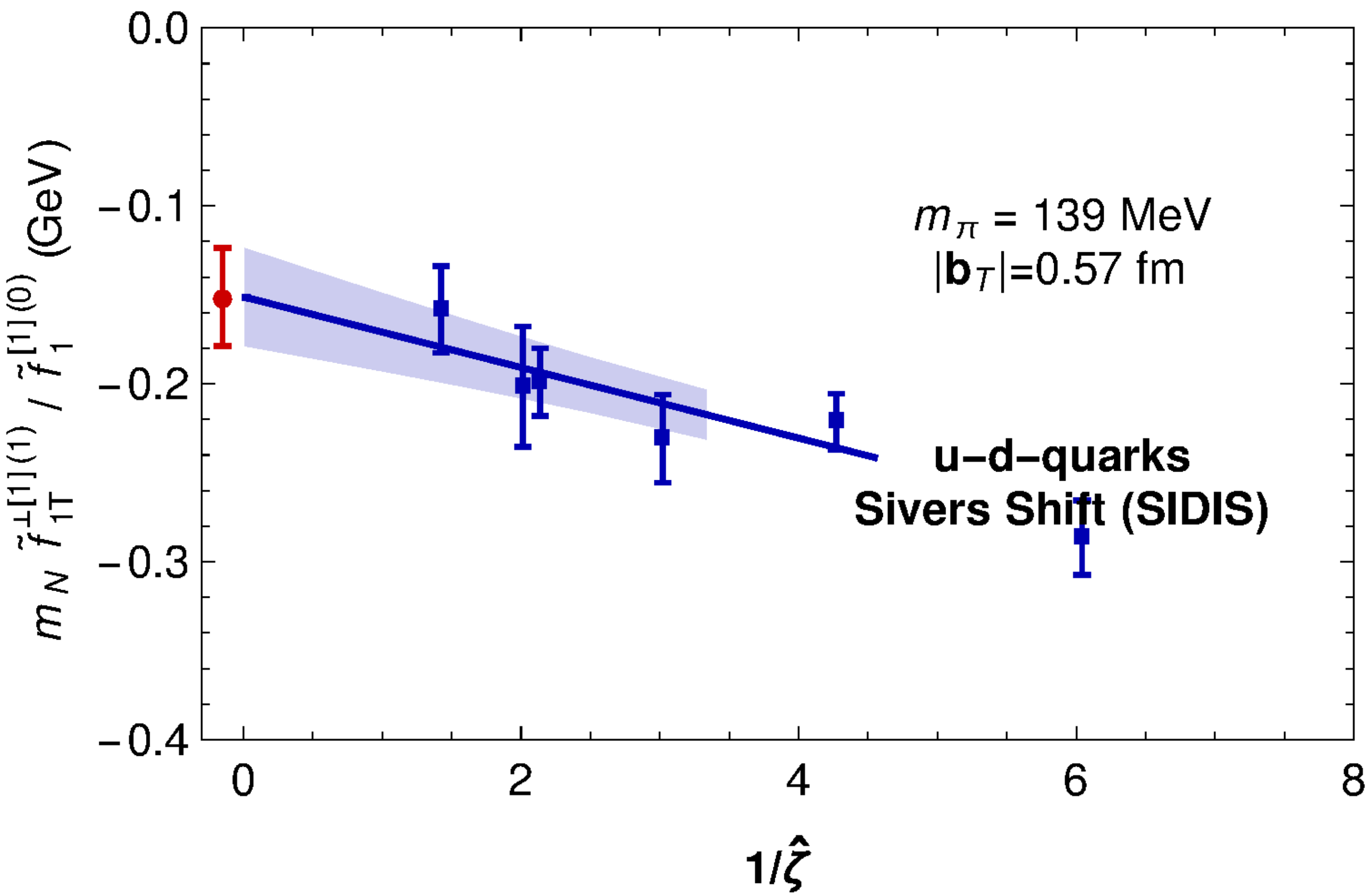

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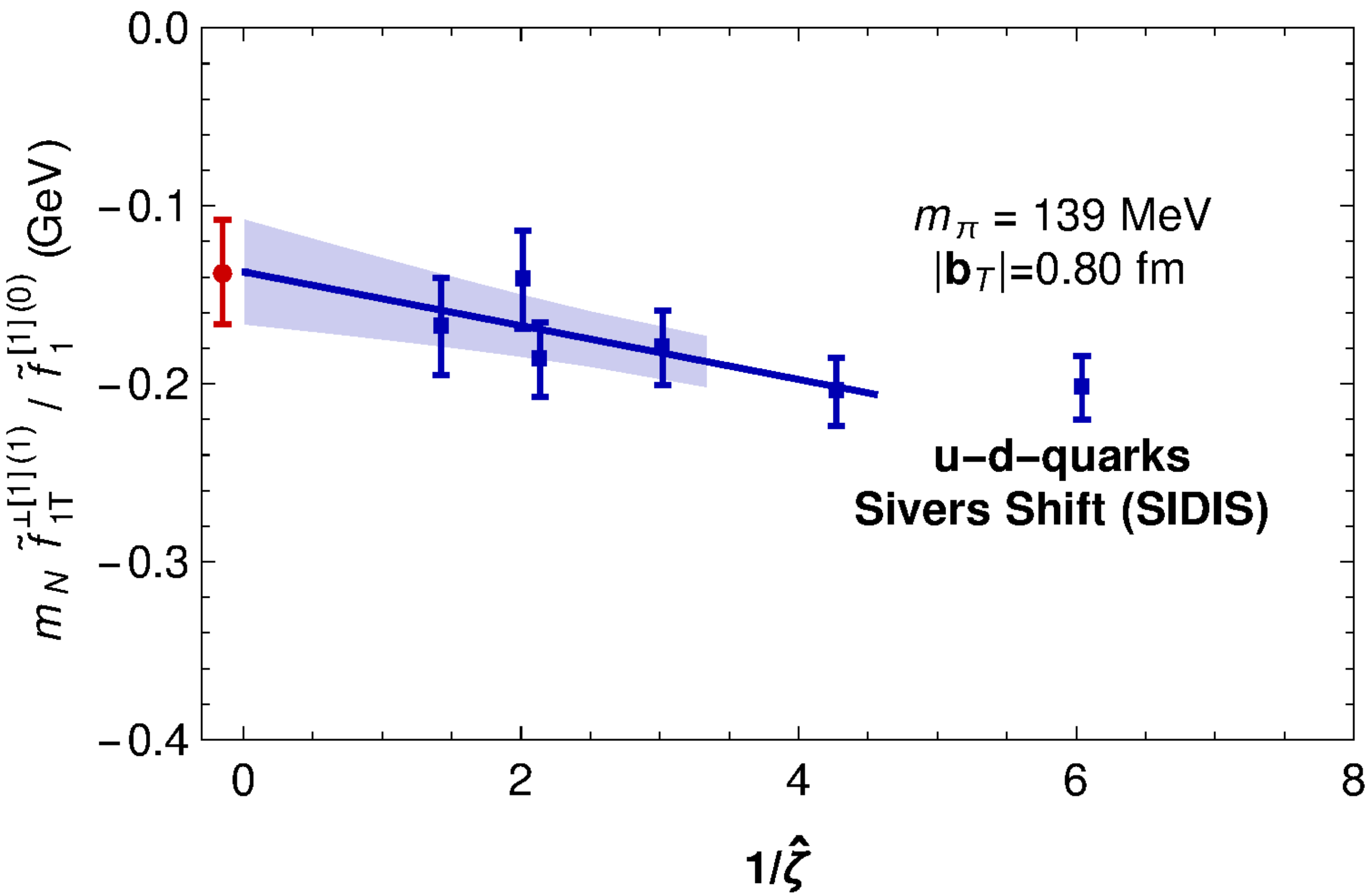

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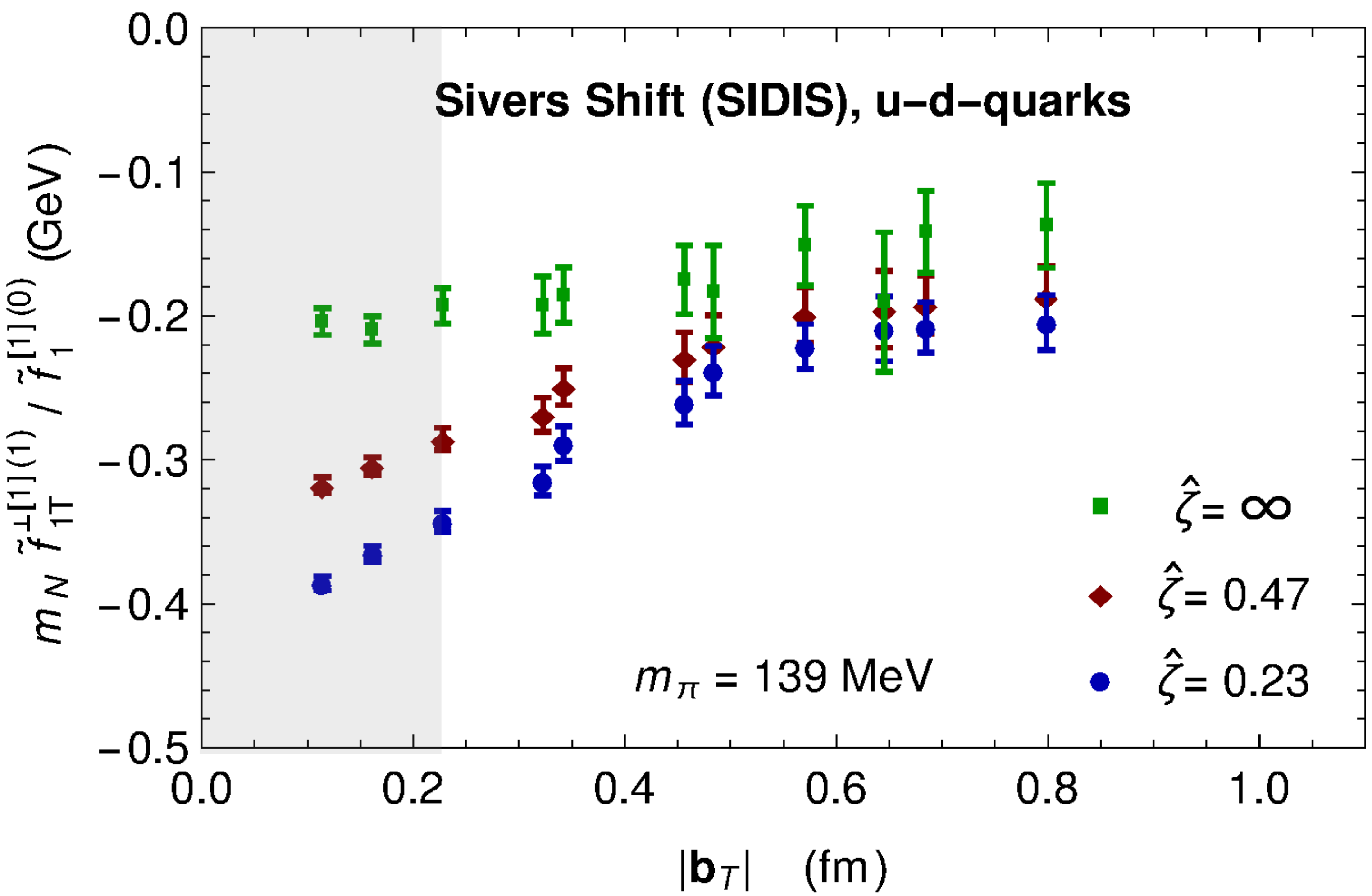
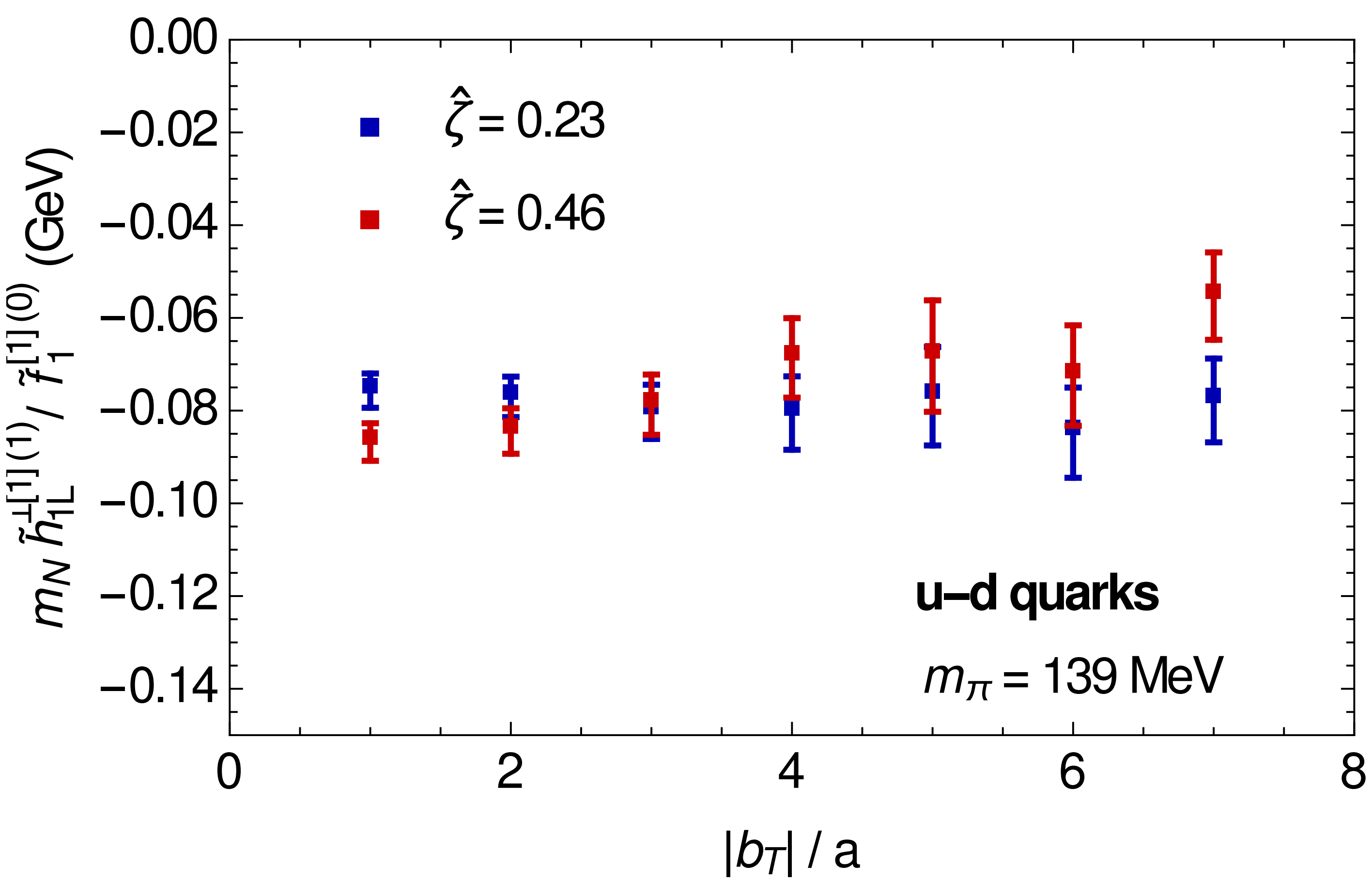
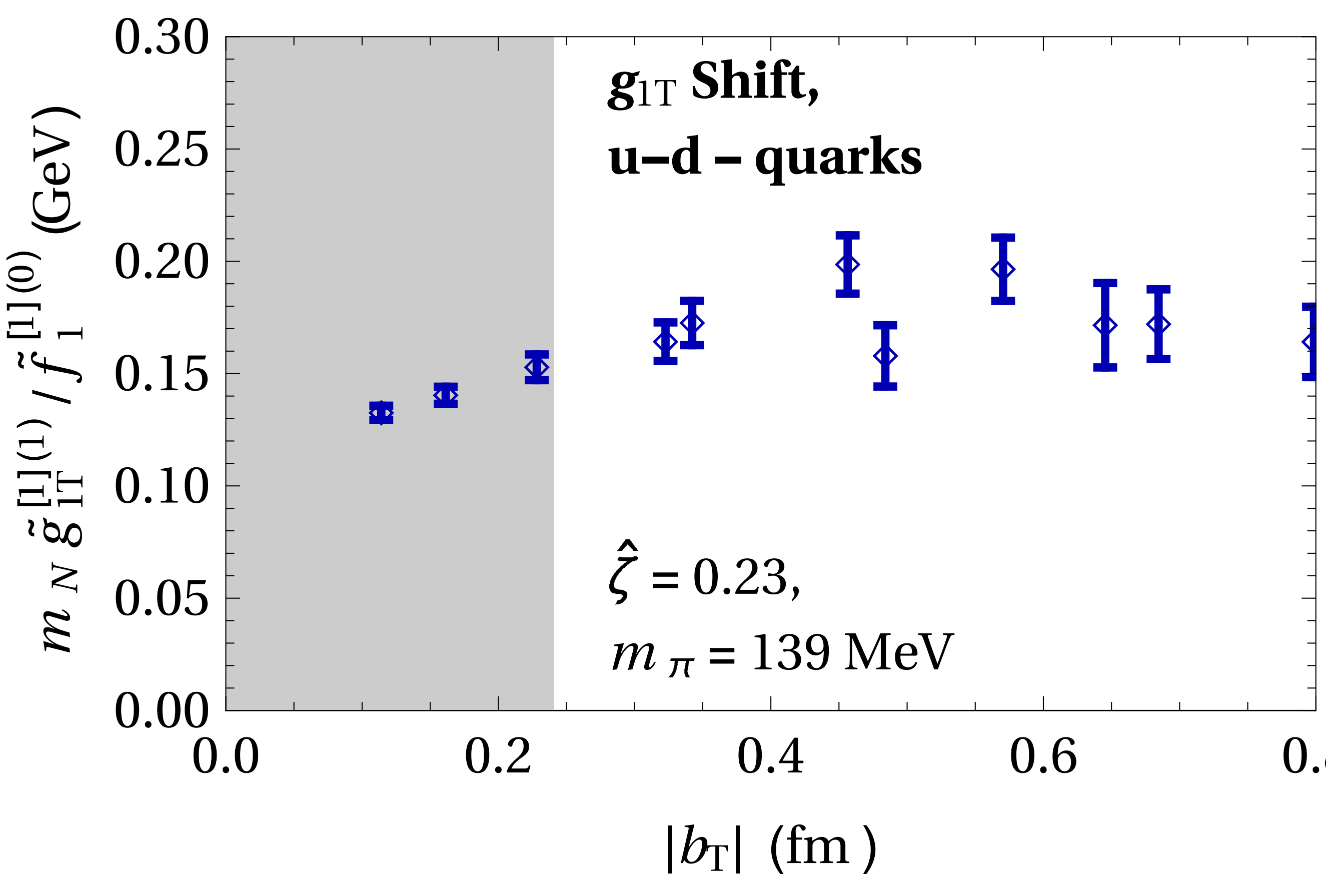


Experimental value from global fit to HERMES, COMPASS and JLab data, evaluated at $b_T \sim 0.35\,\text{fm}$, M. Echevarria, A. Idilbi, Z.-B. Kang and I. Vitev, Phys. Rev. D 89 (2014) 074013:

$$\langle k_y \rangle_{TU} = -0.146(49)$$

Worm-gear shift comparison





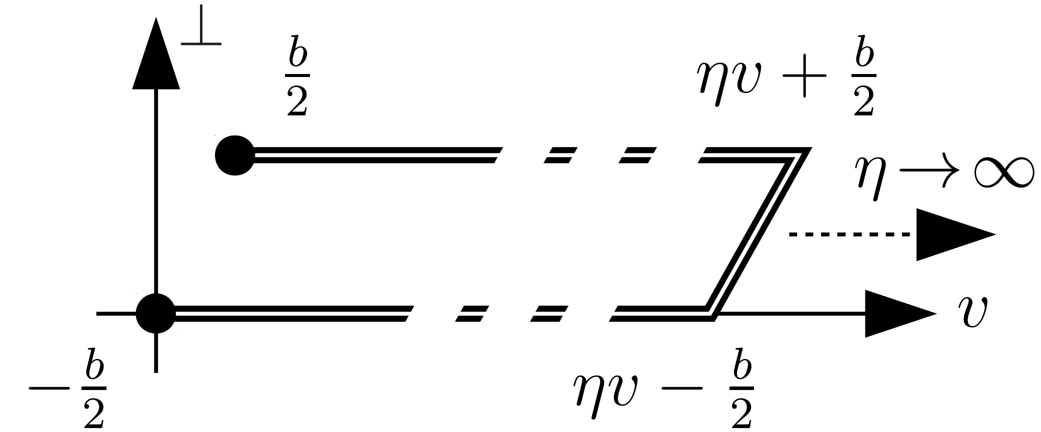
A wide variety of models predicts $h_{1L}^{\perp}$ and g_{1T} to have the same magnitude (and opposite sign):
Spectator model, light-front constituent quark model, covariant parton model, bag model, light-front quark-diquark model, light-front version of the chiral quark-soliton model, nonrelativistic quark model …
Significant QCD effects not captured by models – nontrivial QCD prediction.

GTMD observables: Direct evaluation of quark orbital angular momentum

$$\frac{L_3}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial b_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle P+\Delta_T/2,3 \mid \overline{\psi}(-b/2)\gamma^+ \, \mathcal{U}[-b/2,b/2]\psi(b/2) \mid P-\Delta_T/2,3 \,\rangle\big|_{b^+=b^-=0\,,\ \Delta_T=0\,,\ b_T\to 0}}{\langle P+\Delta_T/2,3 \mid \overline{\psi}(-b/2)\gamma^+\mathcal{U}[-b/2,b/2]\psi(b/2) \mid P-\Delta_T/2,3 \,\rangle\big|_{b^+=b^-=0\,,\ \Delta_T=0\,,\ b_T\to 0}}$$

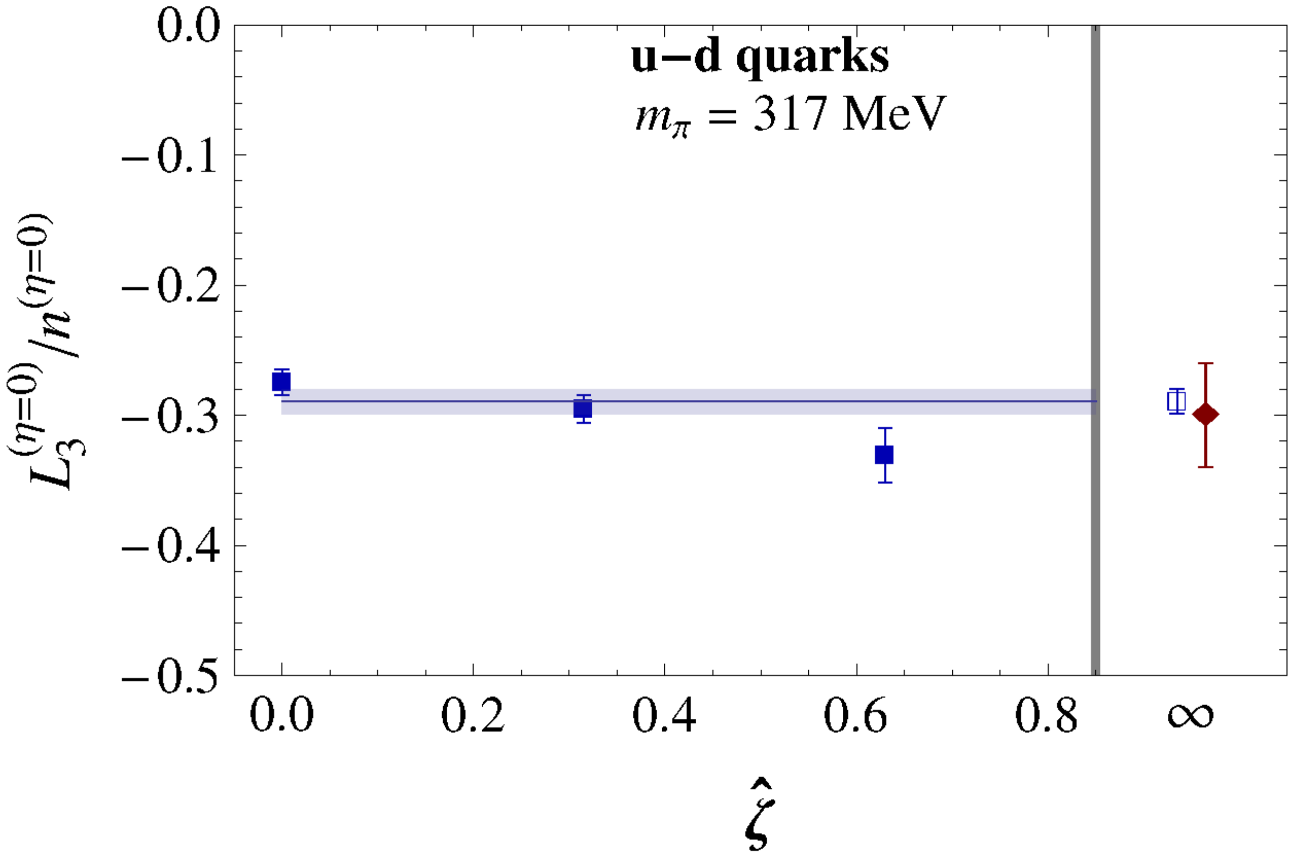
Role of the gauge link $\mathcal{U}$:

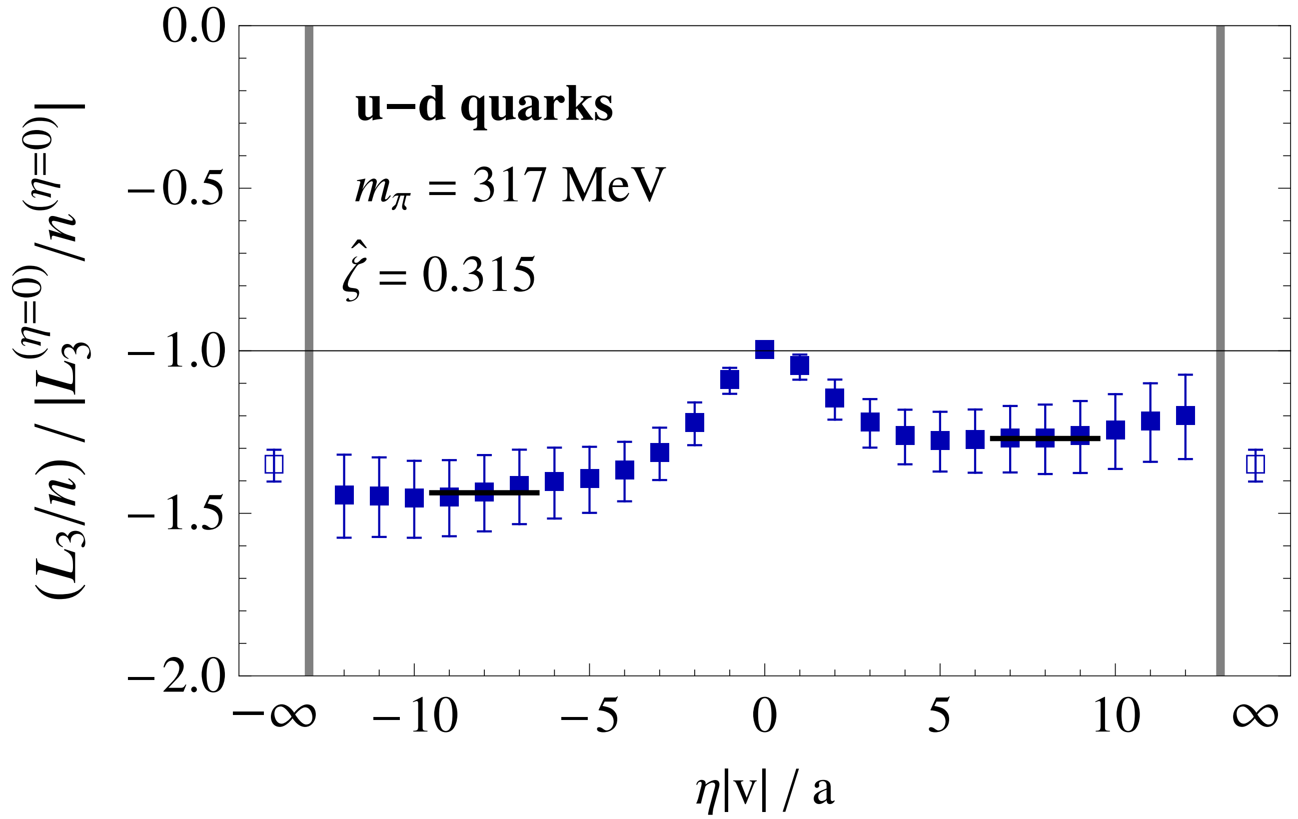
- Straight $\mathcal{U}[-b/2,b/2] \longrightarrow$ Ji OAM
- Staple-shaped $\mathcal{U}[-b/2,b/2] \longrightarrow$ Jaffe-Manohar OAM
- Difference is torque accumulated due to final state interaction



$$L_3 = -\int dx \int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14}$$

GTMD observables: Direct evaluation of quark orbital angular momentum





$\longrightarrow$ Careful evaluation of $\partial f / \partial \Delta_T$ using direct derivative method

Lorentz invariance relations: Another evaluation of quark orbital angular momentum

$L_3 + 2S_3 =$

$$\frac{1}{2}\epsilon_{ij}\frac{\partial}{\partial(b\cdot P)}\frac{\partial}{\partial\Delta_{T,i}}\langle P+\Delta_T/2,+\mid \overline{\psi}(-b/2)\gamma^j\,\mathcal{U}[-b/2,b/2]\psi(b/2)\mid P-\Delta_T/2,+\rangle\Big|_{b^+=b^-=0\,,\;\Delta_T=0\,,\;b_T\to 0}$$

Renormalize using number of valence quarks n,

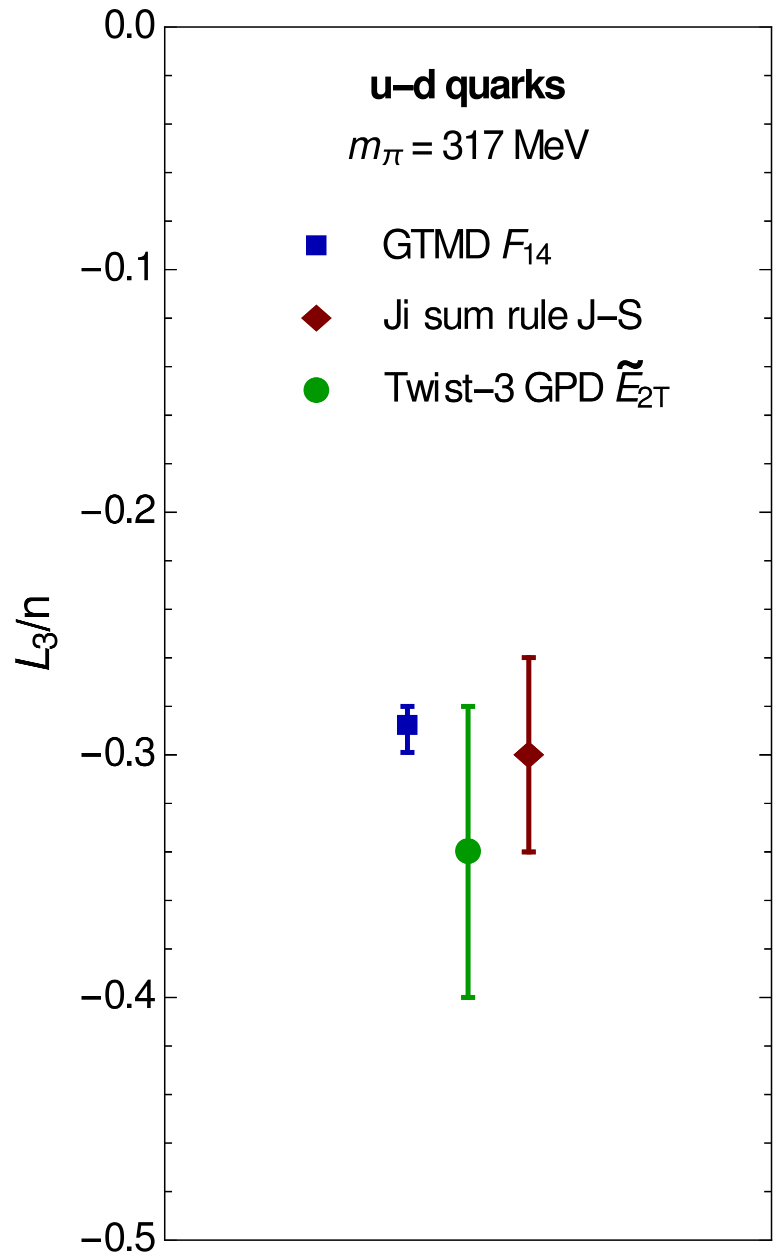
$$2\;P_j\;n = \langle P,+\mid \overline{\psi}(-b/2)\gamma^j\,\mathcal{U}[-b/2,b/2]\psi(b/2)\mid P,+\rangle\Big|_{b^+=b^-=0\,,\;b_T\to 0}$$

i.e., ultimately determine ratio $(L_3 + 2S_3)/n$

Note only straight gauge link $\mathcal{U}$ ($\to$ Ji OAM) accessed here due to collapse of staple link in limit $b_T \to 0$.

$$L_3 + 2S_3 = -\int dx\, x\, \widetilde{E}_{2T}$$

Result for L_3 and comparison of approaches



- Combine with $2S_3$ previously obtained on same ensemble, same parameters (arXiv:1703.06703): $2S_3 = 1.18(2)$ (extrapolate $\widetilde{E}_{2T}$ to $P = 0$ to match).
- Different systematics within the various approaches appear to remain smaller than statistical uncertainty.
- Larger uncertainty of twist-3 GPD approach: Difference of large numbers, incomplete cancellation of fluctuations due to use of rotational symmetry in normalization.
- It is feasible to extract OAM from twist-3 GPDs, but higher effort compared to other approaches for comparable accuracy.

At the end of this tour, you do not need to fill out a survey!