

Muon-induced tracks in minerals

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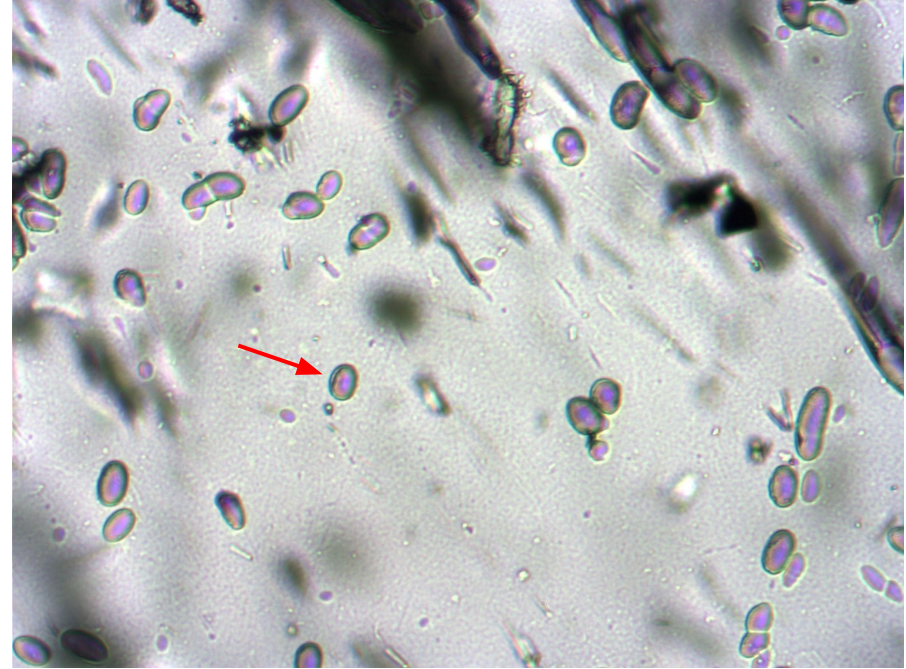
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Introduction

Fission tracks are regularly observed to date minerals (e.g. obsidians from volcanic eruptions).

On the right there is an image of tracks found after etching in a sample of obsidian coming from Lipari, in Italy. The image has been taken with an optic microscope (20x).

The sample has been studied in the University of Milan.



Introduction

Tracks are formed after the spontaneous fission of heavy nuclei:

- almost all from ^{238}U $\longrightarrow \tau_{1/2}^{238} = 4.5 \times 10^9 \text{ yr}$ $\text{BR}_{\text{SF}}^{238} = 5.4 \times 10^{-7}$
- small amount from ^{232}Th $\longrightarrow \tau_{1/2}^{232} = 14 \times 10^9 \text{ yr}$ $\text{BR}_{\text{SF}}^{\text{Th}} = 1.1 \times 10^{-11}$

The fission of heavy nuclei can be also induced by muons stopped inside the mineral.

Since muons are part of EAS induced by primary cosmic rays, we can exploit this information to obtain useful information on the past flux of cosmic rays.

Goal of the analysis: use ancient mineral to observe the tracks induced by muons and obtain information on the past flux of cosmic rays (e.g. transient events inside of our galaxy, see Lorenzo Caccianiga's talk).

Starting point for a first calculation

Finding a mineral rich in Uranium and Thorium.

Zircons (ZrSiO_4) are rich in both elements (typical concentration of 0.001-0.005 g/g [1]) and can reach a concentration in Thorium of O(0.01) g/g, as in [2]. We take these zircons, formed around 5 kyr ago, as case of study for the analysis.



[1] Van Schmus, W.R. (1995). Natural Radioactivity of the Crust and Mantle. In *Global Earth Physics*, T.J. Ahrens (Ed.). <https://doi.org/10.1029/RF001p0283>

[2] Sun, Y., Schmitt, A.K., Häger, T. *et al.* Natural blue zircon from Vesuvius. *Miner Petrol* **115**, 21–36 (2021). <https://doi.org/10.1007/s00710-020-00727-7>

Spontaneous Fission (SF)

Define rate of spontaneous fission of Th and U per mass and time.

$$R_{\text{SF}}^{\text{X}}(Z_1, Z_2) = \text{BR}_{\text{SF}}^{\text{X}} \frac{N_{\text{A}}}{A} \text{pdf}(Z_1, Z_2) \frac{f^{\text{X}}}{\tau^{\text{X}}}$$

$\text{BR}_{\text{SF}}^{\text{X}}$ = branching ratio for SF

$$\text{BR}_{\text{SF}}^{\text{U}} = 5.4 \times 10^{-7} \quad \text{BR}_{\text{SF}}^{\text{Th}} = 1.1 \times 10^{-11}$$

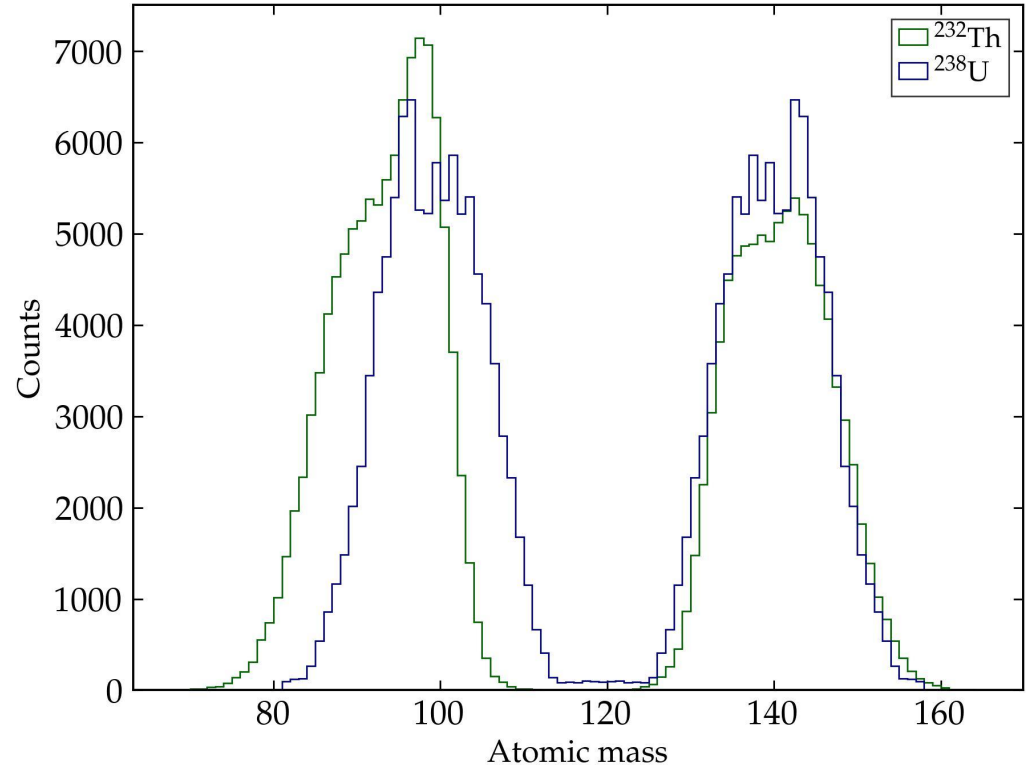
N_{A} = Avogadro number

A = mass number

Z_1, Z_2 = atomic numbers

$\text{pdf}(Z_1, Z_2)$ = probability density function of decaying in Z_1 and Z_2

f^{X} = concentration of X nucleus

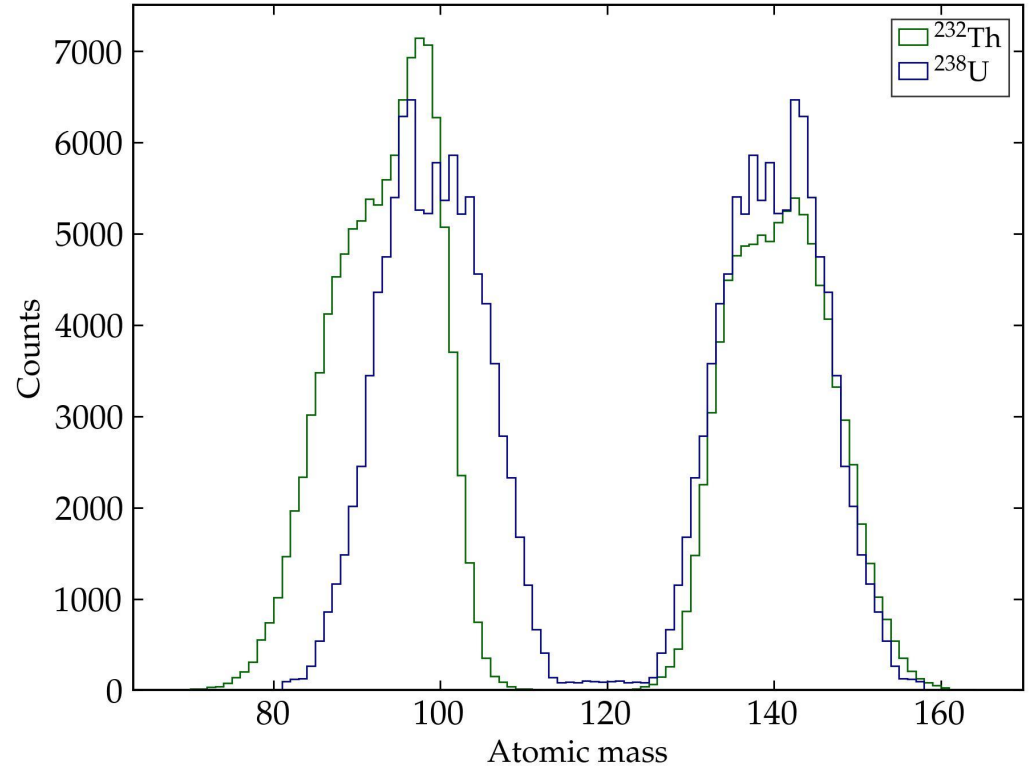


Spontaneous Fission (SF)

Define rate of spontaneous fission of Th and U per mass and time.

$$R_{\text{SF}}^X(Z_1, Z_2) = \text{BR}_{\text{SF}}^X \frac{N_A}{A} \text{pdf}(Z_1, Z_2) \frac{f^X}{\tau^X}$$

Computing $x(E)$ with SRIM and evaluating the fission fragments energy, we can find the rate as a function of the track length x ($R(x)$).



Muon-Induced Fission (MIF)

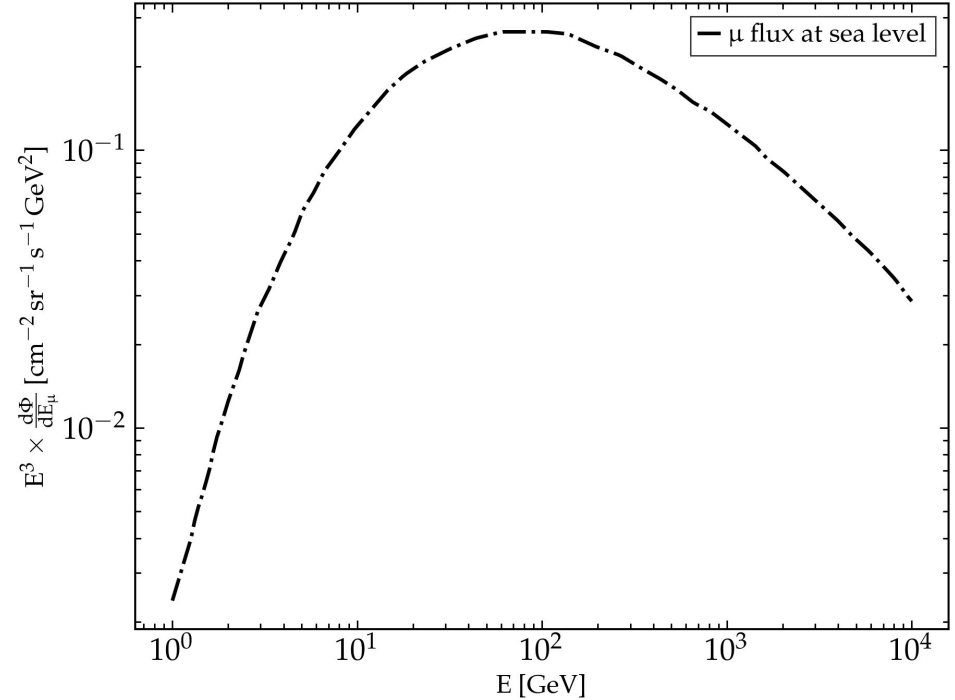
From [3] we find the muon induced fission in ^{232}Th and ^{238}U per number of muon stopped (n_μ).

$$R_{\text{MIF}}^{\text{X}}(Z_1, Z_2) = n_\mu^{\text{X}} \frac{N_\mu(T, m)}{T \times m} \text{pdf}(Z_1, Z_2) f^{\text{X}}$$

$$n_\mu^{\text{U}} = 0.14 \quad n_\mu^{\text{Th}} = 0.02$$

N_μ is the number of muons hitting the sample in time period T considering a sample of mass m .

Using SRIM we find $R_{\text{MIF}}(\text{x})$.



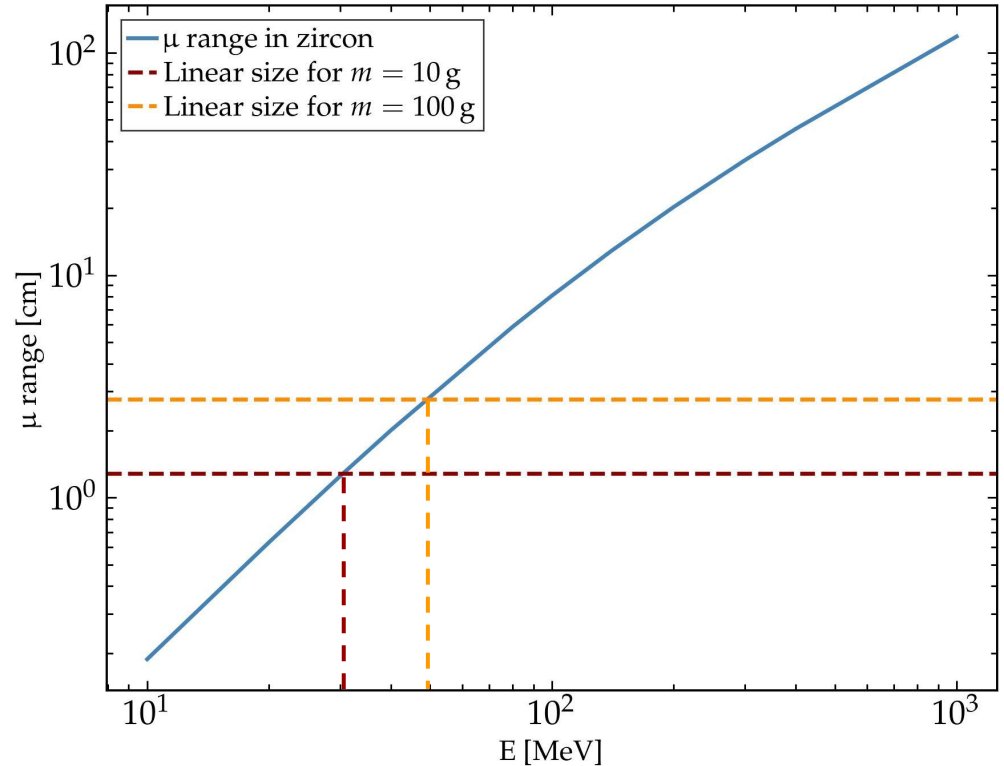
[3] D.F. Measday, The nuclear physics of muon capture, Physics Reports, Volume 354, Issues 4–5, 2001, Pages 243-409, ISSN 0370-1573, [https://doi.org/10.1016/S0370-1573\(01\)00012-6](https://doi.org/10.1016/S0370-1573(01)00012-6)

Muon-Induced Fission (MIF)

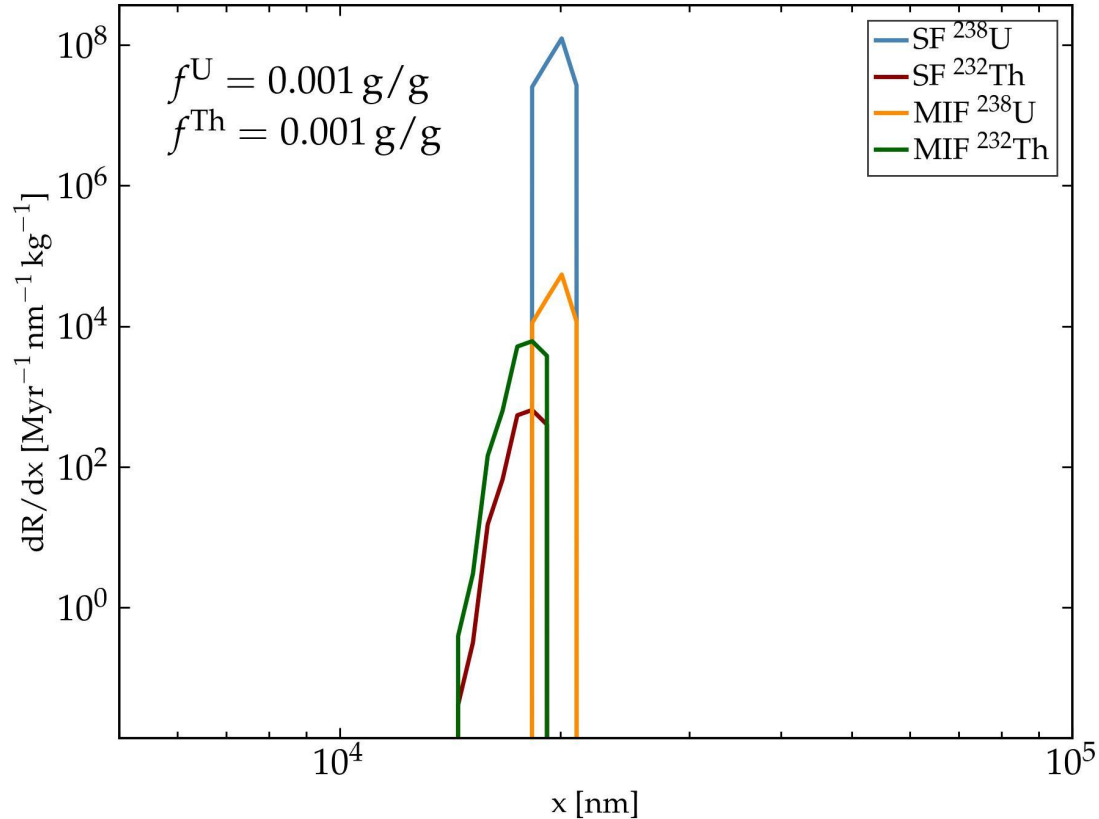
To find N_μ , we integrated the muon flux at the surface.

N_μ is related to the time period considered and the dimension of the sample selected.

In this case of study we consider cubic sample.



Differential track rate



Analysis

Now we can evaluate the number of tracks formed after a time period t .

$$N = (f^U \tilde{R}_U^{SF} + f^U \tilde{R}_U^{MIF} + f^{Th} \tilde{R}_{Th}^{SF} + f^{Th} \tilde{R}_{Th}^{MIF}) \times t \times m = \left(\sum_{i,j} f^i \tilde{R}_i^j \right) \times t \times m$$

$$\tilde{R}_X = \frac{R_X}{f^X} \quad \begin{array}{l} i = \{U, Th\} \\ j = \{SF, MIF\} \end{array}$$

PROBLEM: the number of tracks depends on the time period t , which is difficult to measure precisely.

Analysis

To avoid the time dependency we can consider two samples, produced in the same event (i.e. having the same age), but having two different concentrations in U and Th.

$$N_1 = (f_1^U \tilde{R}_U^{\text{SF}} + f_1^U \tilde{R}_U^{\text{MIF}} + f_1^{\text{Th}} \tilde{R}_{\text{Th}}^{\text{SF}} + f_1^{\text{Th}} \tilde{R}_{\text{Th}}^{\text{MIF}}) \times t \times m = \left(\sum_{i,j} f_1^i \tilde{R}_i^j \right) \times t \times m$$
$$N_2 = (f_2^U \tilde{R}_U^{\text{SF}} + f_2^U \tilde{R}_U^{\text{MIF}} + f_2^{\text{Th}} \tilde{R}_{\text{Th}}^{\text{SF}} + f_2^{\text{Th}} \tilde{R}_{\text{Th}}^{\text{MIF}}) \times t \times m = \left(\sum_{i,j} f_2^i \tilde{R}_i^j \right) \times t \times m$$

And now we take the ratio between these two values

$$\rho = \frac{N_1}{N_2} = \frac{f_1^U \tilde{R}_U^{\text{SF}} + f_1^U \tilde{R}_U^{\text{MIF}} + f_1^{\text{Th}} \tilde{R}_{\text{Th}}^{\text{SF}} + f_1^{\text{Th}} \tilde{R}_{\text{Th}}^{\text{MIF}}}{f_2^U \tilde{R}_U^{\text{SF}} + f_2^U \tilde{R}_U^{\text{MIF}} + f_2^{\text{Th}} \tilde{R}_{\text{Th}}^{\text{SF}} + f_2^{\text{Th}} \tilde{R}_{\text{Th}}^{\text{MIF}}} = \frac{\sum_{i,j} f_1^i \tilde{R}_i^j}{\sum_{i,j} f_2^i \tilde{R}_i^j}$$

Uncertainties

This technique could be useful to unveil transient events happened in the past. However, we need to consider the uncertainties involved to understand its discovery potential.

$$\rho(N_1, N_2; f_1^U, f_2^U, f_1^{\text{Th}}, f_2^{\text{Th}}) \quad \longrightarrow \quad \begin{array}{ll} N_1 \pm \sqrt{N_1} & f_1^i \pm \sigma_{f_1^i} \\ N_2 \pm \sqrt{N_2} & f_2^i \pm \sigma_{f_2^i} \end{array}$$

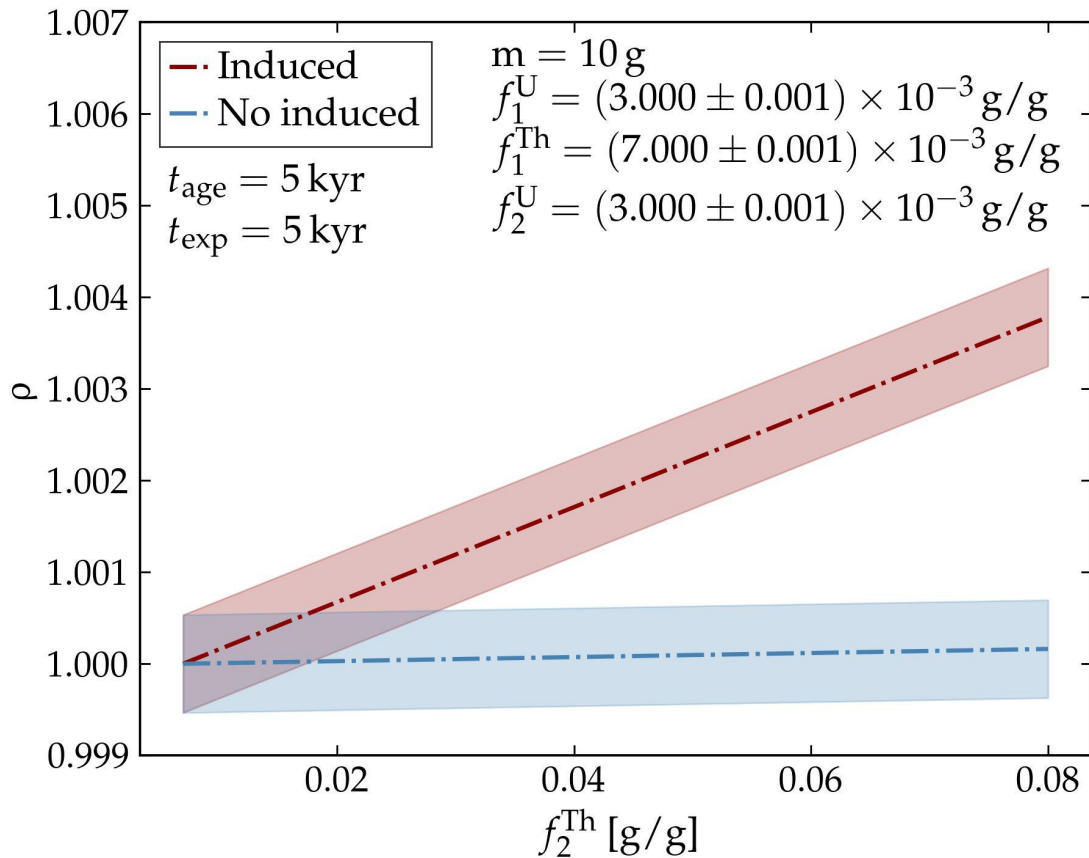
$$\sigma_\rho = \left[\sum_k \left(\frac{\partial \rho}{\partial N_k} \sigma_{N_k} \right)^2 + \sum_{k,i} \left(\frac{\partial \rho}{\partial N_k} \frac{\partial N_k}{\partial f_k^i} \sigma_{f_k^i} \right)^2 \right]^{1/2} \quad k = \{1, 2\}$$

Results

Since the MIF tracks have never been observed, first, we consider the chance to distinguish the case in which the MIF tracks are formed with respect to the case which they are not formed.

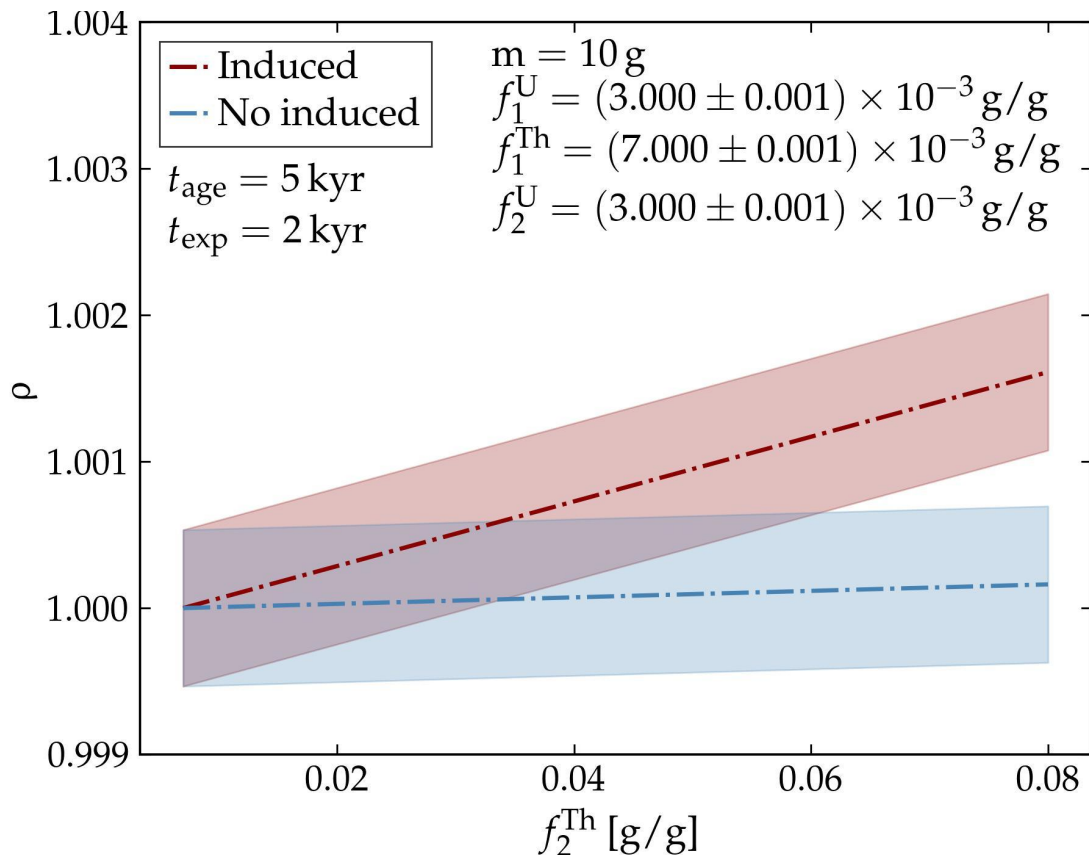
t_{age} = sample age

t_{exp} = exposure time to muons



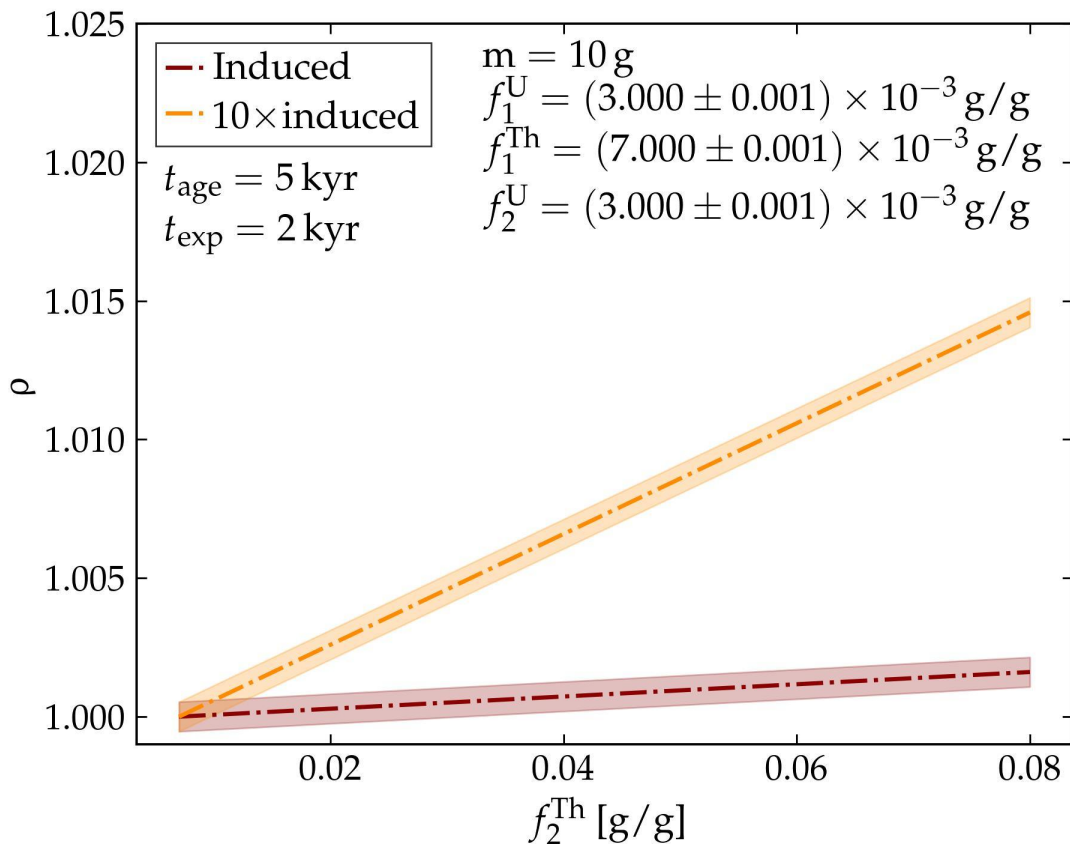
Results

We consider a case in which the exposure time is not equal to the sample age (i.e. the sample has been covered by some layers of material for a certain amount of time).



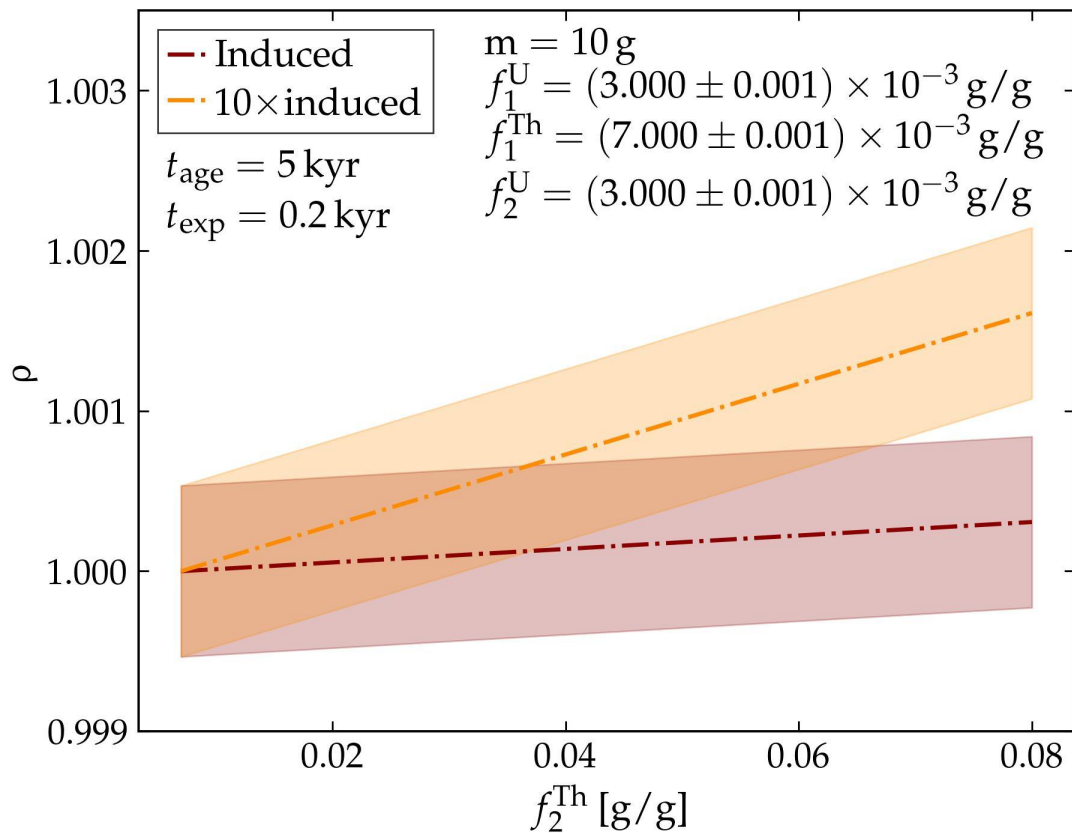
Results

We consider if this method is optimal to distinguish a transient event, during which the muon flux increased.



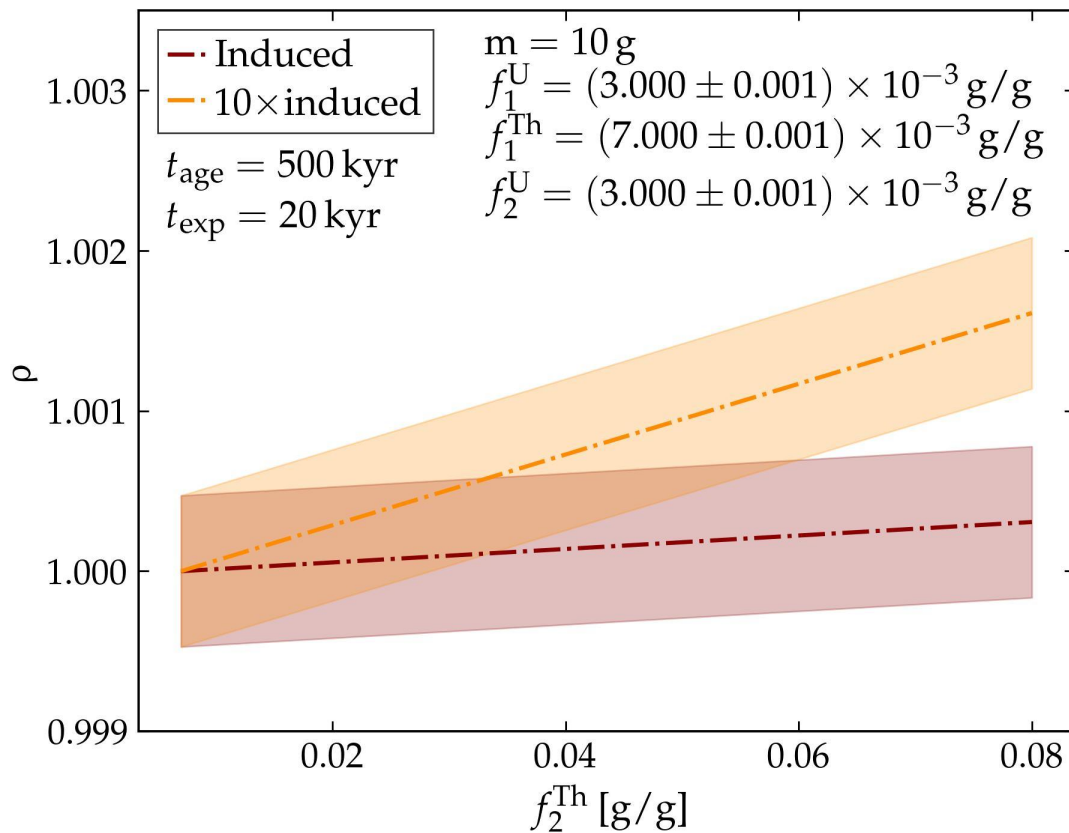
Results

We consider if this method is optimal to distinguish a transient event, during which the muon flux increased.



Results

We scale this method to samples with large age.



Conclusion

We propose a new technique which considers the Muon-Induced Fission (MIF) tracks to evaluate the presence of past transient events in cosmic rays, using minerals rich in Th and U.

- This technique exploits the fission tracks, which are seen on daily basis to date ancient minerals.
- Defining the quantity ρ we can get rid of the uncertainties linked to the sample age, leaving just the uncertainties of the concentration.
- To obtain relevant results we need to find minerals with high concentration of Th ($O(10^{-3}-10^{-2})$ g/g) measured with a precision of $O(10^{-6})$ g/g.