Light meson production in holographic QCD

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References

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Generalized Parton Distributions (GPDs)

- GPDs are the matrix elements for quark and gluon non-local operators $\mathcal{O}^{qq/gg}(w_1, w_2)$

$$\langle p_2|\mathcal{O}^{qq}(-w^-, w^-)|p_1\rangle = \int_{-1}^{1} dx\ \mathrm{e}^{-ixp^+w^-}\left\{h^+ H^q(x,\eta,\Delta^2) + e^+ E^q(x,\eta,\Delta^2)\right\}$$

$$\langle p_2|\widetilde{\mathcal{O}}^{qq}(-w^-, w^-)|p_1\rangle = \int_{-1}^{1} dx\ \mathrm{e}^{-ixp^+w^-}\left\{\tilde{h}^+ \widetilde{H}^q(x,\eta,\Delta^2) + \tilde{e}^+ \widetilde{E}^q(x,\eta,\Delta^2)\right\}$$

$$\langle p_2|\mathcal{O}^{gg}(-w^-, w^-)|p_1\rangle = \frac{1}{4}p^+ \int_{-1}^{1} dx\ \mathrm{e}^{-ixp^+w^-}\left\{h^+ H^g(x,\eta,\Delta^2) + e^+ E^g(x,\eta,\Delta^2)\right\}$$

$$\langle p_2|\widetilde{\mathcal{O}}^{gg}(-w^-, w^-)|p_1\rangle = \frac{1}{4}p^+ \int_{-1}^{1} dx\ \mathrm{e}^{-ixp^+w^-}\left\{\tilde{h}^+ \widetilde{H}^g(x,\eta,\Delta^2) + \tilde{e}^+ \widetilde{E}^g(x,\eta,\Delta^2)\right\}$$

Conformal (Gegenbauer) Moments of GPDs

- Conformal (Gegenbauer) moments of GPDs are the matrix elements of twist-2 and spin-j quark and gluon local operators $\mathcal{O}_j^{qq/gg}(w)$

$$\begin{aligned}
\mathbb{F}^q_{j(valence/singlet)}(\eta,t;\mu_0^2) &= \eta^{j-1}\int_0^1 dx\; C_{j-1}^{3/2}\left(\frac{x}{\eta}\right) H^q_{valence/singlet}(x,\eta,t;\mu_0^2) \\
&= (p^+)^{-(j-1)-1}\langle p_2|\mathbb{O}_j^{qq}(0)|p_1\rangle \\
\mathbb{F}^g_j(\eta,t;\mu_0^2) &= 2\eta^{j-2}\int_0^1 dx\; C_{j-2}^{5/2}\left(\frac{x}{\eta}\right) H^g(x,\eta,t;\mu_0^2) \\
&= 4(p^+)^{-(j-1)-1}\langle p_2|\mathbb{O}_j^{gg}(0)|p_1\rangle
\end{aligned}$$

for odd (valence) $j = 1, 3, ...$, and even (singlet or gluon) $j = 2, 4, ...$ with

$$H^q_{valence/singlet}(x,\eta;\mu^2) = H^q(x,\eta;\mu^2) \pm H^q(-x,\eta;\mu^2)$$

Reconstruction of Quark and Gluon GPDs from their Conformal Moments

- The quark and gluon GPDs can be reconstructed from their conformal moments as

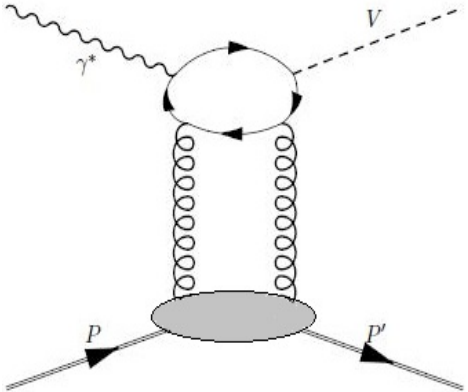
$$H^q_{valence/singlet}(x,\eta,t;\mu_0^2) = \frac{1}{\eta}\left(1-\frac{x^2}{\eta^2}\right)\sum_{j=1\text{or}2}^{\infty}\frac{1}{\eta^{j-1}}\frac{1}{N_{j-1}(\frac{3}{2})}C_{j-1}^{3/2}\left(\frac{x}{\eta}\right)\mathbb{F}^q_{j(valence/singlet)}$$

$$H^g(x,\eta,t;\mu_0^2) = \frac{1}{\eta}\left(1-\frac{x^2}{\eta^2}\right)^2\sum_{j=2}^{\infty}\frac{1}{\eta^{j-2}}\frac{1}{N_{j-2}(\frac{5}{2})}C_{j-2}^{5/2}\left(\frac{x}{\eta}\right)\times\mathbb{F}^g_j(\eta,t;\mu_0^2)$$

for odd (valence) or even (singlet or gluon) j

Conformal moments of GPDs from t-channel string exchange in AdS/QCD

- we match t-channel string exchange amplitudes in holographic QCD to electroproduction amplitudes in QCD



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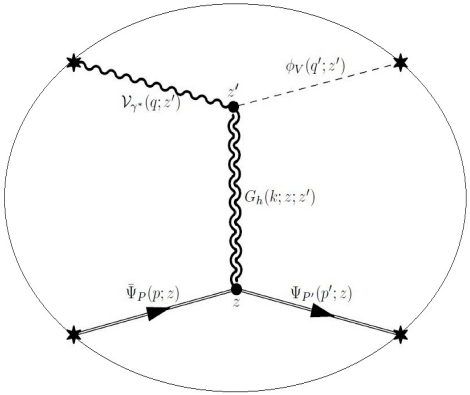

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electroproduction of a vector meson probing the **gluon GPD**: (a) leading QCD contribution in the Regge limit; (b) leading Witten diagram in the large-N_c limit.

Conformal moments of GPDs from t-channel string exchange in AdS/QCD

- the QCD amplitude

$$\mathcal{A}^{LL}_{\gamma^* p\to Vp}(s,t,Q_0,\epsilon_L,\epsilon'_L) \approx e\times\frac{\lambda_s(\mu^2)}{4N_c^2}\frac{1}{Q_0}\left[\int_0^1 dz\sum_q e_q\frac{\Phi^q(z)}{z(1-z)}\right]\frac{1}{m_N}\times\bar{u}(p_2)u(p_1)$$
$$\times \sum_{j=2}^{\infty}\frac{1}{\xi^j}\frac{1}{N_{j-2}(\frac{5}{2})}\left[\int_0^{\xi}\frac{dx}{\xi}\left(1-\frac{x^2}{\xi^2}\right)C^{5/2}_{j-2}\left(\frac{x}{\xi}\right)\right]\mathbb{F}^g_j(\xi,t;\mu_0^2)$$

matches the holographic amplitude

$$\mathcal{A}^{LL}_{\gamma^* p\to Vp}(s,t,Q_0,\epsilon_L,\epsilon'_L) \sim e\times\frac{1}{g_5}\times g_5\times 2\kappa^2\times\frac{1}{2}\times\frac{1}{Q_0}\times[f_V]\times\frac{1}{m_N}\times\bar{u}(p_2)u(p_1)$$
$$\times \sum_{j=2}^{\infty}\frac{1}{\xi^j}\left[\frac{1}{\Gamma(\Delta_g(j)-2)}\left[\mathcal{A}(j,\tau,\Delta_g(j),t)+\mathcal{D}_\eta(j,\tau,\Delta_g(j),t)\right]\right]$$

Conformal moments of GPDs from t-channel string exchange in AdS/QCD

- we extract the conformal (Gegenbauer) moments of the gluon GPDs as

$$\mathbb{F}_j^g(\eta \sim \xi, t; \mu_0^2) \propto \frac{1}{\Gamma(\Delta_g(j)-2)} \times [\mathcal{A}(j,\tau,\Delta_g(j),t) + \mathcal{D}_\eta(j,\tau,\Delta_g(j),t)]$$

- the skewness or η-dependent spin-j $\mathcal{D}_\eta$-terms are given by

$$\mathcal{D}_\eta(j,\tau,\Delta_g(j),-t=K^2) \;=\; \left(\hat{d}_j(\eta,-K^2)-1\right) \times [\mathcal{A}(j,\tau,\Delta_g(j),K,\tilde{\kappa}_T) - \mathcal{A}_S(j,\tau,\Delta_g(j),K,\tilde{\kappa}_S)]$$

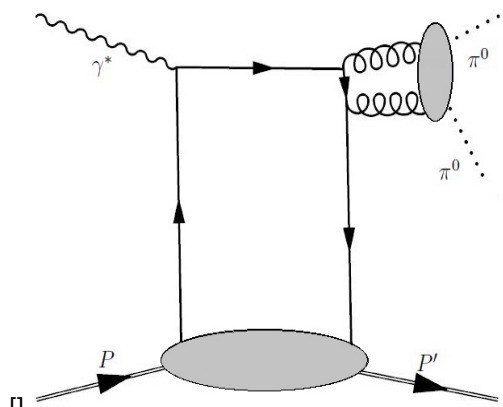
where

$$\mathcal{A}_S(j,\tau,\Delta_g(j),K,\tilde{\kappa}_S) \;\equiv\; \mathcal{A}(j,\tau,\Delta_g(j),K,\tilde{\kappa}_T \to \tilde{\kappa}_S)$$

and

$$\hat{d}_j(\eta,-K^2) = {}_2F_1\left(-\frac{j}{2}, \frac{1-j}{2}; \frac{1}{2}-j; \frac{4m_N^2}{K^2} \times \eta^2\right)$$

Conformal moments of GPDs from t-channel string exchange in AdS/QCD



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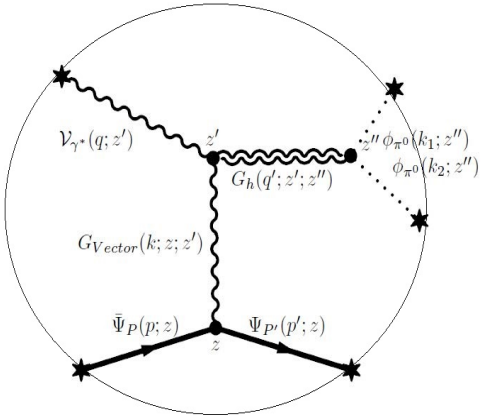

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electroproduction of double pions probing the **non − singlet** (**valence**) **vector quark GPDs** : (a) leading QCD contribution in the Regge limit; (b) leading Witten diagram in the large-N_c limit.

Conformal moments of GPDs from t-channel string exchange in AdS/QCD

- we identify the conformal moments of valence quark GPDs as

$$\mathbb{F}^{q}_{j(valence)}(\eta \sim \xi, t; \mu_0^2) \quad \propto \quad \frac{1}{\Gamma(\Delta_q(j) - 2)} \times [\mathcal{F}_1(j, \tau, \Delta_q(j), t) + \mathcal{D}_{q\eta}(j, \tau, \Delta_q(j), t)]$$

with

$$\mathcal{F}_1(j, \tau, \Delta_q(j), -t = K^2) \quad \propto \quad \mathcal{A}(j \to j + 1, \tau, \Delta_g \to \Delta_q, t)$$

and the valence quark spin-j skewness dependent $\mathcal{D}_{q\eta}$-terms are

$$\mathcal{D}_{q\eta}(j, \tau, \Delta_q(j), t) \quad \propto \quad \mathcal{D}_{\eta}(j \to j + 1, \tau, \Delta_g \to \Delta_q, t; \tilde{\kappa}_T \to \frac{1}{2} \times \tilde{\kappa}_V)$$

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment

- the electroproduction of neutral rho meson (ρ^0) in terms of singlet quark DAs and quark GPDs is given by [Goeke:2001]

$$\mathcal{A}^{LL(quark)}_{\gamma^* p\to\rho^0 p}(s,t,Q,\epsilon_L,\epsilon'_L) =$$
$$-e\times\frac{C_F}{N_c}\times\frac{1}{2}\times(4\pi\alpha_s(\mu^2))\times\frac{1}{Q}\times\left[\int_0^1 dz\frac{\Phi^q_\rho(z)}{z}\right]\times\frac{1}{p^+}\left\{A_{\rho^0 p}\,h^+ + B_{\rho^0 p}\,e^+\right\}$$

with and

$$A_{\rho^0 p} = \int_{-1}^{1} dx\,\frac{-1}{\sqrt{2}}\left(Q_u\,H^u(x,\eta,t;\mu^2) - Q_d\,H^d(x,\eta,t;\mu^2)\right)\left\{\frac{1}{\xi-x-i\epsilon}-\frac{1}{\xi+x-i\epsilon}\right\}$$
$$B_{\rho^0 p} = \int_{-1}^{1} dx\,\frac{-1}{\sqrt{2}}\left(Q_u\,E^u(x,\eta,t;\mu^2) - Q_d\,E^d(x,\eta,t;\mu^2)\right)\left\{\frac{1}{\xi-x-i\epsilon}-\frac{1}{\xi+x-i\epsilon}\right\}$$

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment

- the electroproduction of neutral rho meson (ρ^0) in terms of the conformal moments of singlet quark GPDs is

$$\mathcal{A}^{LL(quark)}_{\gamma^* p\to\rho^0 p}(s,t,Q,\epsilon_L,\epsilon'_L) \approx e\times f_V\times\alpha_s(\mu)\times\frac{1}{Q}$$
$$\times\sum_{j=2}^{\infty}\frac{1}{\xi^j}\times\mathcal{N}_q(j)\left(Q_u\ \mathbb{F}^u_{j(singlet)}(\xi,t;\mu^2)\ -\ Q_d\ \mathbb{F}^d_{j(singlet)}(\xi,t;\mu^2)\right)\frac{1}{2\sqrt{2}m_N}\bar{u}(p_2)u(p_1)$$

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment

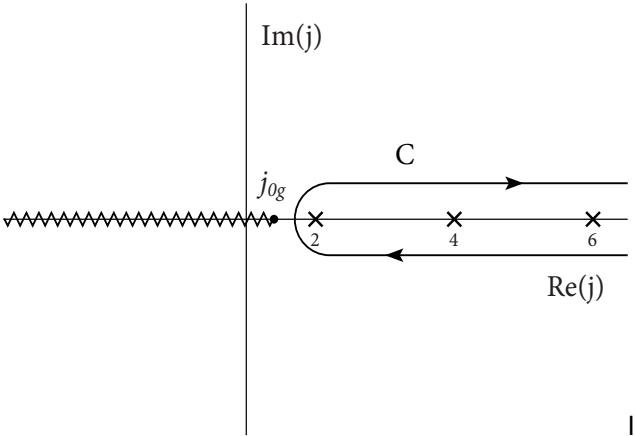


Illustration of the integration contour $\mathbb{C}$ in the complex j-plane used for computing the contour integral with a branch cut displayed for Re(j) $\leq j_{0g} = 2 - 2/\sqrt{\lambda}$.

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment

- we rewrite the sum over even $j = 2, 4, ...$ as a contour integral in the complex j-plane

$$\begin{aligned}
\mathcal{A}^{LL(quark)}_{\gamma^* p \to \rho^0 p}(s, t, Q, \epsilon_L, \epsilon'_L) &\approx -\int_{\mathbb{C}} \frac{dj}{4i} \frac{1 + e^{-i\pi j}}{\sin \pi j} \times \frac{1}{\xi^j} \times \frac{1}{\Gamma(\Delta_g(j) - 2)} \times \mathcal{N}_q(j) \\
&\times \left(Q_u \, \widehat{\mathbb{F}}^u_{j(singlet)}(\xi, t; \mu^2) - Q_d \, \widehat{\mathbb{F}}^d_{j(singlet)}(\xi, t; \mu^2) \right) \\
&\times e \times f_V \times \alpha_s(\mu) \times \frac{1}{Q} \times \frac{1}{2\sqrt{2} m_N} \times \bar{u}(p_2) u(p_1)
\end{aligned}$$

where the contour $\mathbb{C}$ is at the right most of the branch point of $\Delta_g(j) - 2 = \sqrt{j - j_{0g}}$ and enclosing the poles at even $j = 2, 4, ...$

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment

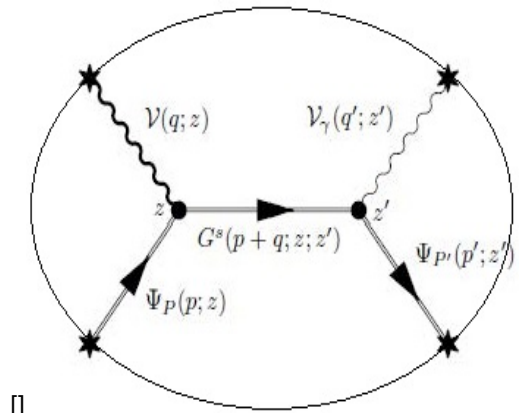
- the singlet quark GPD contribution to the ρ^0 amplitude is given by

$$\begin{aligned}\mathcal{A}^{LL(quark)}_{\gamma^* p\to\rho^0 p}(s,t,Q,\epsilon_L,\epsilon'_L) &\propto \left(Q_u \widehat{\widehat{\mathbb{F}}}^{u}_{j_{0g}(singlet)}(\xi,t;\mu^2) - Q_d \widehat{\widehat{\mathbb{F}}}^{d}_{j_{0g}(singlet)}(\xi,t;\mu^2)\right)\\ &\times f_V\,\alpha_s(\mu)\,\frac{1}{Q}\,\frac{1}{2\sqrt{2}m_N}\,\bar{u}(p_2)u(p_1)\times\frac{1}{\xi^{j_{0g}}}\times\left[(\sqrt{\lambda}/\pi)+i\right]\end{aligned}$$

- we have also found the gluon GPD contribution to the ρ^0 amplitude to be

$$\begin{aligned}\mathcal{A}^{LL(gluon)}_{\gamma^* p\to\rho^0 p}(s,t,Q,\epsilon_L,\epsilon'_L) &\propto \widehat{\widehat{\mathbb{F}}}^{g}_{j_{0g}}(\xi,t;\mu^2)\times f'_V\,\alpha_s(\mu)\,\frac{1}{Q}\,\frac{1}{2\sqrt{2}m_N}\,\bar{u}(p_2)u(p_1)\\ &\times \frac{1}{\xi^{j_{0g}}}\times\left[(\sqrt{\lambda}/\pi)+i\right]\end{aligned}$$

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment



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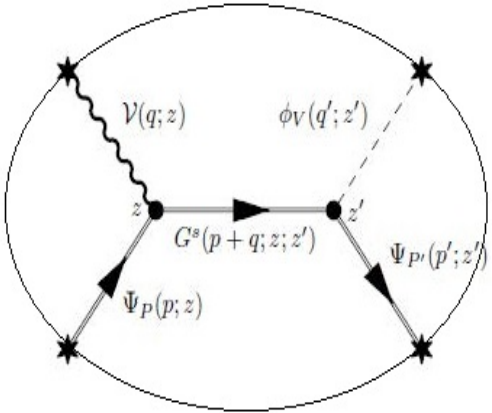


[] (a)

Holographic **s-channel** contribution to the electroproduction of a photon (**DVCS**). (b) Holographic **s-channel** contribution to the electroproduction of a **vector meson**.

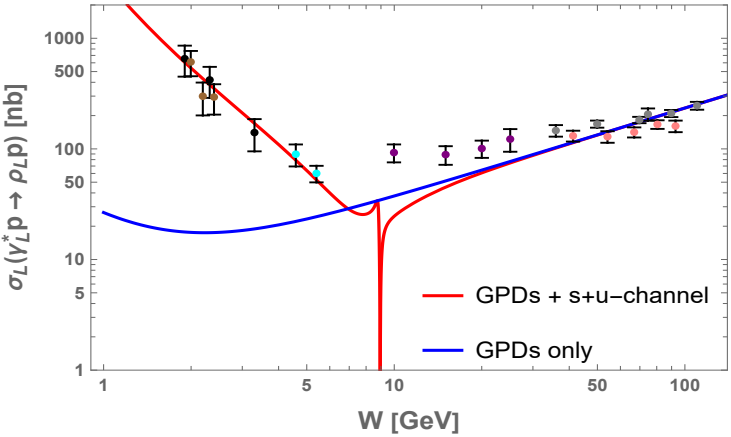
Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment

- the holographic s+u-channel contribution to the electroproduction of ρ^0 is

$$\mathcal{A}^{LL(s+u-channel)}_{\gamma^* p\to\rho^0 p}(s,t,Q,\epsilon_L,\epsilon'_L) \propto e\times g_5\times\left[\epsilon_{L\mu}\epsilon'^{*}_{L\nu}\,T^{\mu\nu}_{s+u}(s,t;Q^2)\right]\times g_5\times\frac{f_V}{M_V}$$

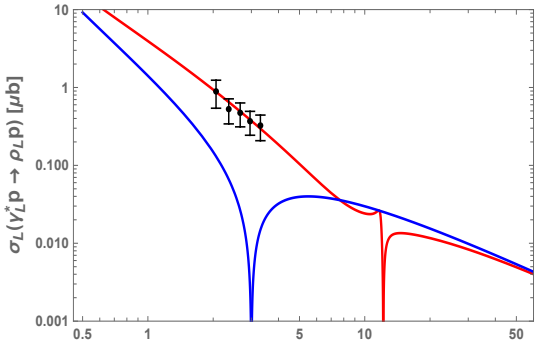
$$\propto \frac{Q}{M_V}\times F_1^P(Q)\times\frac{f_V}{M_V}\times\frac{1}{2\sqrt{2}m_N}\,\bar{u}(p_2)u(p_1)\times\frac{1}{1-\xi^2}$$

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment



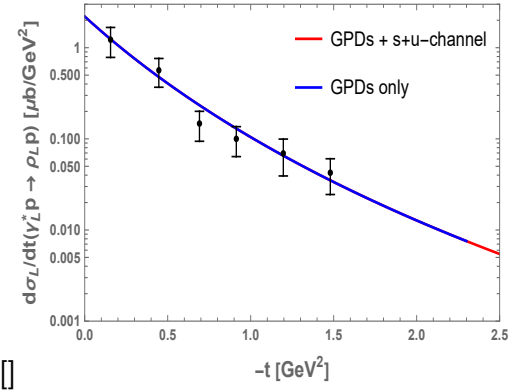
[] Longitudinal cross section for the electroproduction of neutral rho mesons versus $W = \sqrt{s}$ in GeV, for $Q^2 = 4$ GeV^2.

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment



[] Longitudinal cross section for the electroproduction of neutral rho mesons versus Q^2 in GeV^2, for Bjorken $x_B = 0.34 - 0.40$. The black data points are from 5.754 GeV [CLAS:2008].

Electroproduction of longitudinal ρ^0 meson with evolved singlet quark and gluon GPDs: a comparison to experiment



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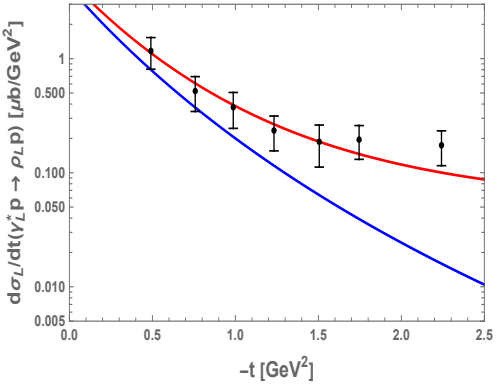

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Longitudinal differential cross section for the electroproduction of neutral rho mesons versus $-t$ in GeV^2, for $Q^2 = 0 - 1.9$ GeV^2, $\mathbf{x_B = 0.16 - 0.22}$ (a), and $Q^2 = 1.9 - 2.2$ GeV^2, $\mathbf{x_B = 0.34 - 0.40}$ (b). The black data points are from 5.754 GeV [CLAS:2008].

Electroproduction of longitudinal ρ^+ meson with evolved valence quark GPDs: a comparison to experiment

- The electroproduction of longitudinal charged rho meson (ρ^+) in terms of quark DAs and quark GPDs is given by [Goeke:2001]

$$\mathcal{A}^{LL}_{\gamma^* p \to \rho^+ n}(s,t,Q,\epsilon_L,\epsilon'_L) =$$
$$-e \times \frac{C_F}{N_c} \times \frac{1}{2} \times (4\pi\alpha_s(\mu^2)) \times \frac{1}{Q} \times \left[\int_0^1 dz \frac{\Phi_{\rho^+}(z)}{z}\right] \times \frac{1}{p^+}\left\{A_{\rho^+ n}\, h^+ \; + \; B_{\rho^+ n}\, e^+\right\}$$

where $\Phi_{\rho^+}(z)$ is the charged rho meson (ρ^+) distribution amplitude (DA), and

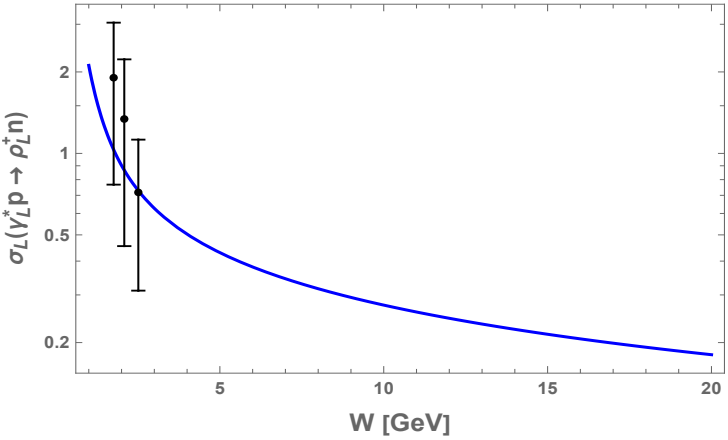
$$A_{\rho^+ n} \;=\; \int_{-1}^{1} dx \left(H^u(x,\eta,t;\mu^2) \; - \; H^d(x,\eta,t;\mu^2)\right) \left\{\frac{Q_u}{\xi - x - i\epsilon} - \frac{Q_d}{\xi + x - i\epsilon}\right\}$$
$$B_{\rho^+ n} \;=\; \int_{-1}^{1} dx \left(E^u(x,\eta,t;\mu^2) \; - \; E^d(x,\eta,t;\mu^2)\right) \left\{\frac{Q_u}{\xi - x - i\epsilon} - \frac{Q_d}{\xi + x - i\epsilon}\right\}$$

Electroproduction of longitudinal ρ^+ meson with evolved valence quark GPDs: a comparison to experiment

- the electroproduction of charged rho meson (ρ^+) in terms of the conformal moments of valence quark GPDs is

$$\mathcal{A}^{LL(valence)}_{\gamma^* p\to\rho^+ n}(s,t,Q,\epsilon_L,\epsilon'_L) \approx -e\times f_V^+ \times \alpha_s(\mu)\times\frac{1}{Q}$$
$$\times\sum_{j=1}^{\infty}\frac{1}{\xi^j}\times\mathcal{N}_{q(valence)}(j)\times\left(\mathbb{F}^u_{j(valence)}(\xi,t;\mu^2) - \mathbb{F}^d_{j(valence)}(\xi,t;\mu^2)\right)\frac{1}{2m_N}\bar{u}(p_2)u(p_1)$$

Electroproduction of longitudinal ρ^+ meson with evolved valence quark GPDs: a comparison to experiment



[] Longitudinal cross section for the electroproduction of charged rho mesons versus $W = \sqrt{s}$ in GeV, for $Q^2 = 3.0 - 3.5$ GeV^2. The black data points are from 5.776 GeV CLAS [Fradi:2010]

Thank You!