Modeling Meson Structure: Form Factors to GPDs

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Towards improved hadron femtography with hard exclusive reactions

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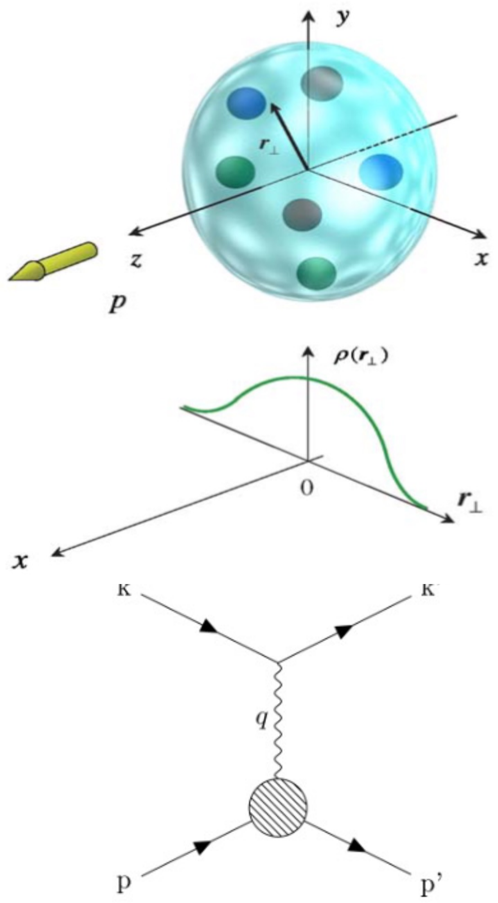
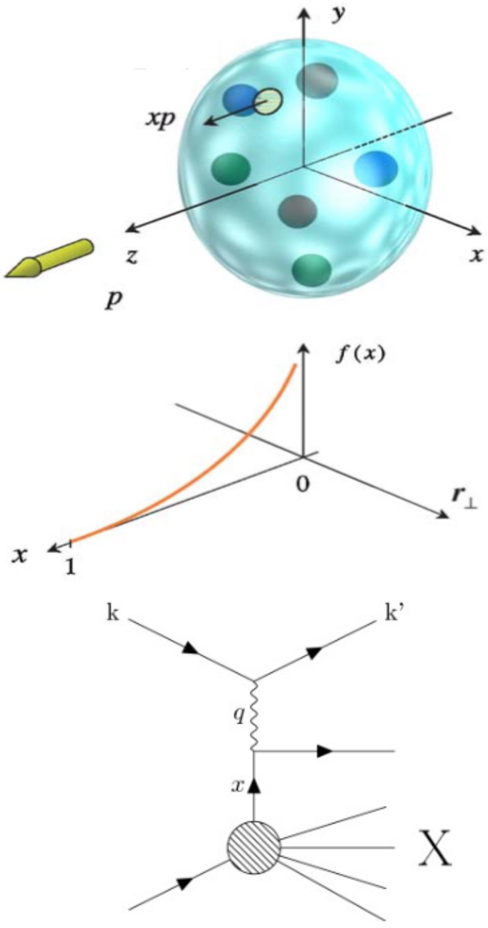
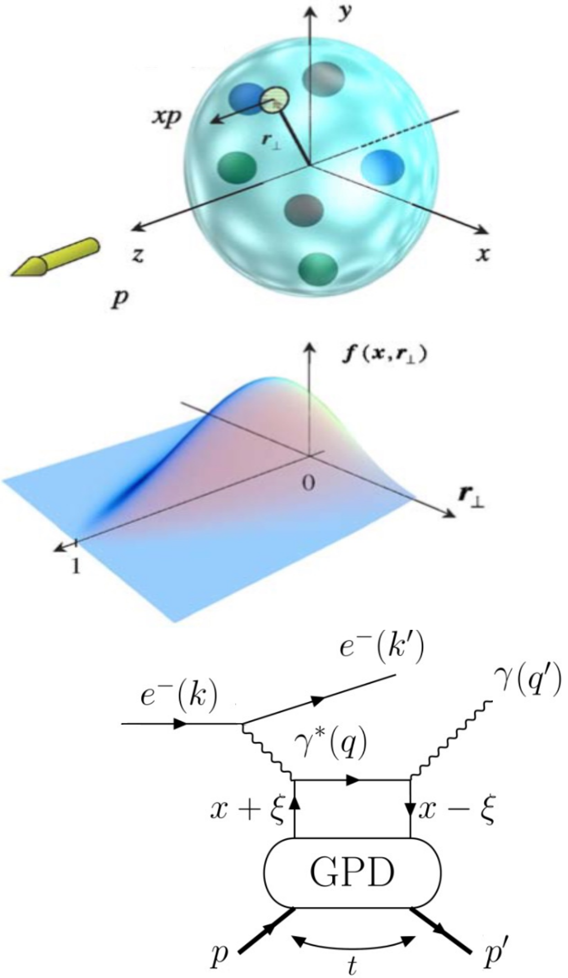
Generalized Parton Distributions (GPDs)



Hofstadter, et. al., Phys. Rev. 91, 422 (1953)
Hofstadter, Rev. Mod. Phys. 28, 214 (1956)



Feynman, Phys. Rev. Lett. 23, 1415 (1969)



GPDs: Footprints of QCD

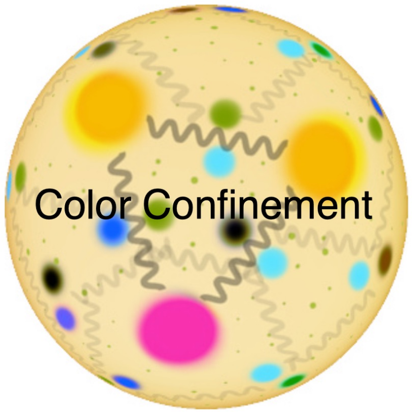
3D Structure of Hadrons: **Global analysis** of **experimental data**, discrete **lattice studies**, **effective field theories** and continuum **Schwinger-Dyson equations**.

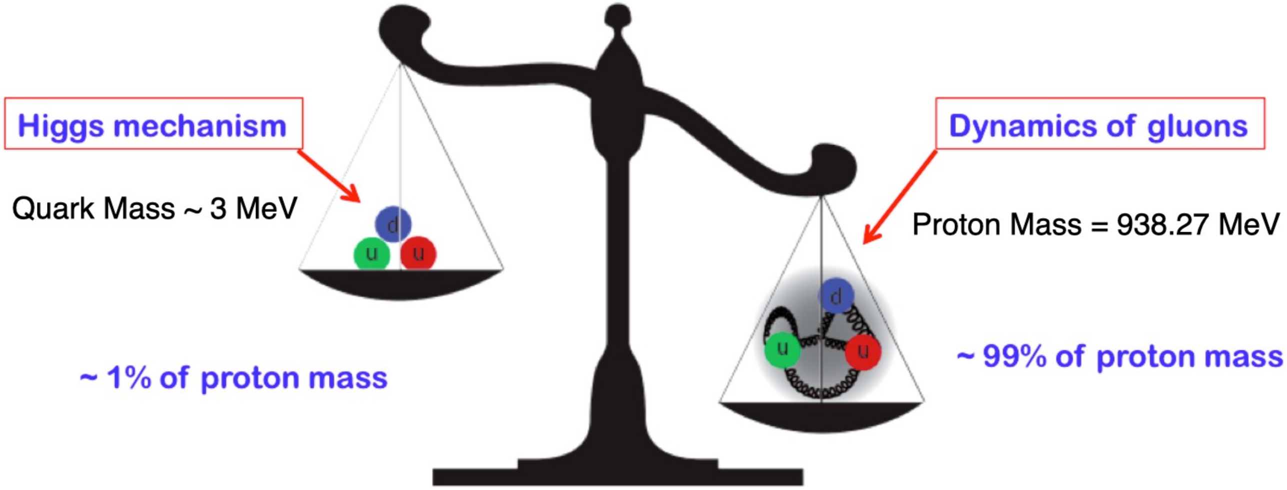
$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

QCD is characterized by two **emergent** phenomena: **confinement** and **emergent hadron mass** (**EHM**).

Emergence of hadron masses **(EHM)** from QCD **dynamics**

- Formation of color-singlet bound states: "**Hadrons**"



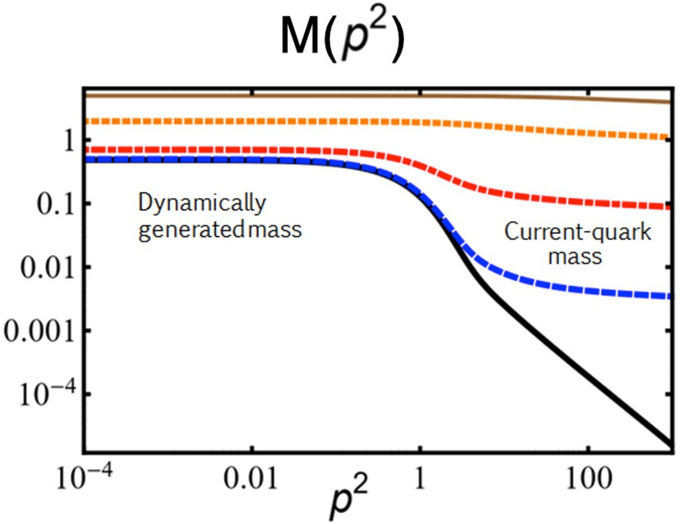


From QCD to the Form Factors and GPDs

Origins of **confinement** and **dynamical chiral symmetry breaking (DCSB)** can be traced back to the Green functions of **quarks** and **gluons**.

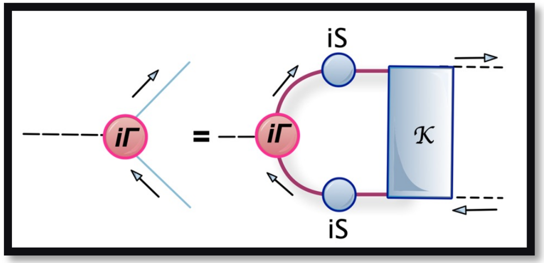
These emergent phenomena of **QCD**, non-existent in perturbation theory are naturally linked to its **non-abelian nature** and infrared enhancement of the **strong running coupling**.

The effects of of the pattern of **DCSB** are traceable in the **Q^2 evolution** of the π and **K Bethe-Salpeter Amplitudes (BSAs)** and **form factors** explored and planned in the **JLab** and the **EIC**.

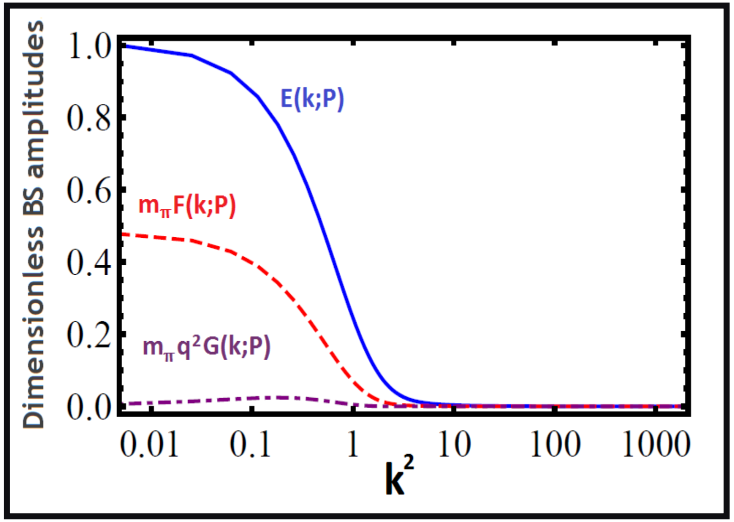


$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma \cdot p + \mathbf{M_f(p^2)})$$

PS-mesons have **4 BSAs**
V-Mesons have **8 BSAs**

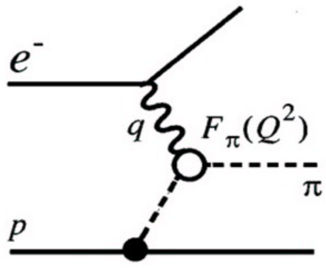


Bethe-Salpeter Equation

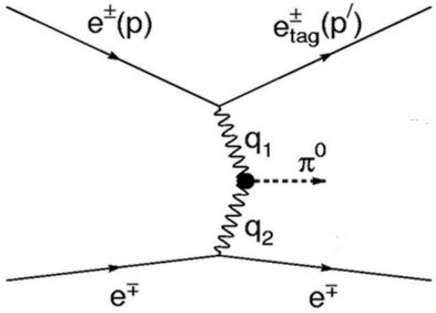


Mesons: Probing Quarks with Photons

In studying the **elastic** or **transition form factors**, it is the **photon** which probes the **dressed quarks** inside the **bound states**, highlighting the importance of the **quark-photon vertex**.



Pion Elastic Form Factor



Pion Transition Form Factor

$$\Gamma^L_\mu(p,k,q) = \sum_{i=1}^{4} \lambda_i \, L^i_\mu(p,k)$$

$$L^1_\mu = \gamma_\mu$$

$$L^2_\mu(p,k) = (\not p + \not k)(p+k)_\mu$$

$$L^3_\mu(p,k) = -(p+k)_\mu$$

$$L^4_\mu(p,k) = -\sigma_{\mu\nu}\,(p+k)^\nu$$

$$\Gamma^T_\mu(p,k,q) = \sum_{i=1}^{8} \tau_i(p^2,k^2,q^2) T^i_\mu(p,k)$$

$$T^1_\mu = p_\mu(k\cdot q) - k_\mu(p\cdot q)\,,$$

$$T^2_\mu = [p_\mu(k\cdot q) - k_\mu(p\cdot q)]\,(\not p + \not k)\,,$$

$$T^3_\mu = q^2\gamma_\mu - q_\mu\,\not q\,,$$

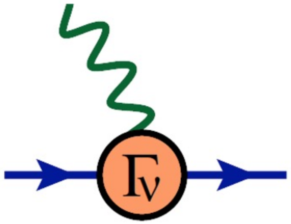
$$T^4_\mu = q^2\left[\gamma^\mu\,(\not k + \not p) - (k+p)^\mu\right] + 2(k-p)^\mu\sigma_{\nu\lambda}\,p^\nu k^\lambda\,,$$

$$T^5_\mu = -\sigma_{\mu\nu}\,q^\nu\,,$$

$$T^6_\mu = \gamma_\mu(p^2-k^2) + (p+k)_\mu\,\not q\,,$$

$$T^7_\mu = \tfrac{1}{2}(p^2-k^2)\left[\gamma_\mu(\not p + \not k) - (p+k)_\mu\right] - (p+k)_\mu\sigma_{\nu\lambda}\,p^\nu k^\lambda\,,$$

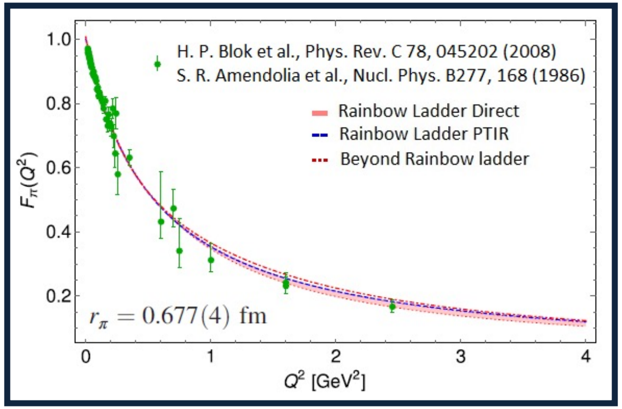
$$T^8_\mu = \gamma_\mu\sigma_{\nu\lambda}\,p^\nu k^\lambda - p_\mu\,\not k + k_\mu\,\not p\,.$$

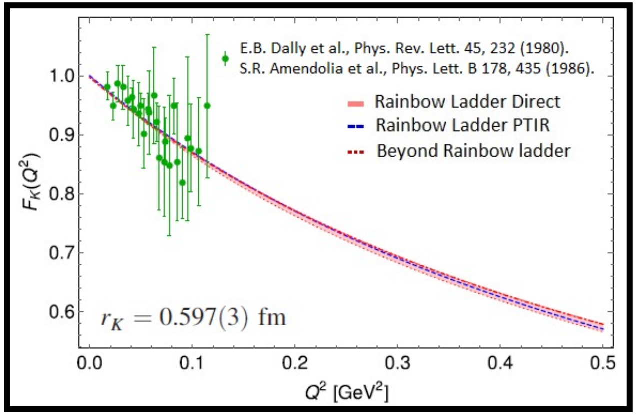


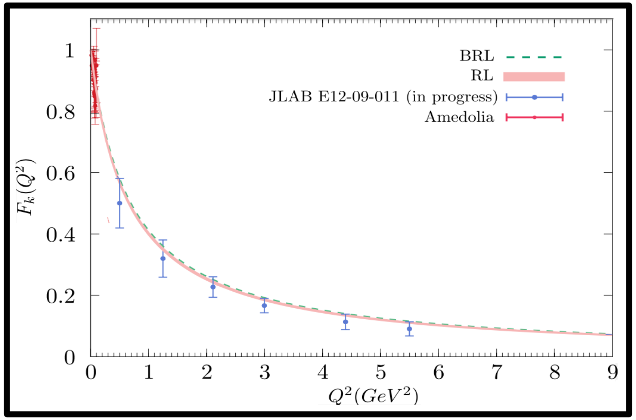
Gauge covariance (WTI,TTI, LKFT), kinematic singularities, perturbation theory, multiplicative renormalizability

AB, M.R. Pennington, Phys. Rev. D50 7679 (1994)
R. Bermudez et. al., Phys. Rev. C85, 045205 (2012)
V. Banda, AB, Phys. Rev. D107 073008 (2023)

Meson Form Factors: Probing the Standard Model

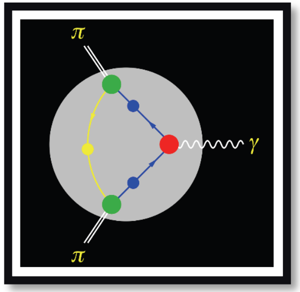


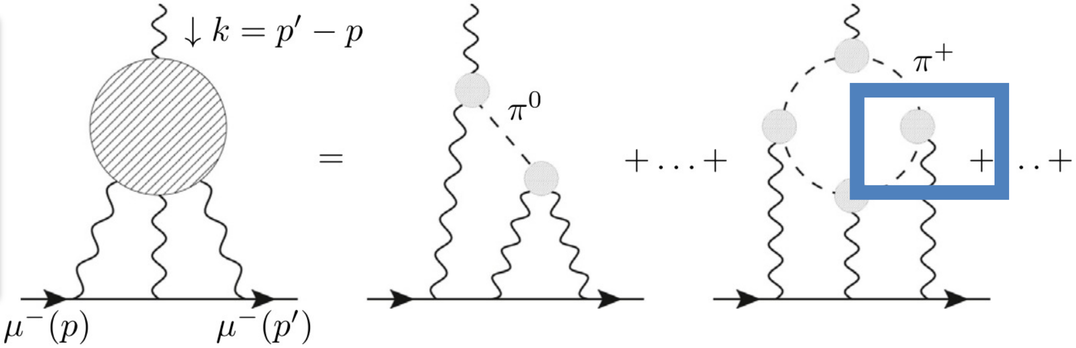




A. Miramontes, AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013

$$a_\mu^{\pi^\pm\text{-box}} = -(15.6 \pm 0.2) \times 10^{-11}$$
$$a_\mu^{K^\pm\text{-box}} = -(0.48 \pm 0.02) \times 10^{-11}$$





Dispersive methods:

$$a_\mu^{\pi\text{-box}} = -15.9(2) \times 10^{-11}$$
$$a_\mu^{K^+\text{-box,VMD}} = -0.50 \times 10^{-11}$$

$$a_\mu^{K^+\text{-box,DSE}} = -0.48(2) \times 10^{-11}$$

Eichmann, et. al.
Phys.Rev.D 101 (2020) 5, 054015

Radial excitations π^1, K^1:

A. Miramontes, et. al. in preparation

$$a_\mu^{\pi_1\text{-box}} = (-3.2 \pm 0.6) \times 10^{-13}$$
$$a_\mu^{K_1\text{-box}} = -6.8 \times 10^{-14}$$

Towards Algebraic Models

Sophisticated Truncations

Numerically Demanding
"QCD Predictions"

QCD Green Functions
Mesons – Static, FFs, PDAs, PDFs
Baryons – Static, FF

Schwinger-Dyson Equations

Most Observables
Mesons, Baryons

Contact Interaction

Simple Implementation
Insightful - Mindful

Moderate Simplicity
QCD-like Predictions

Algebraic Models

LFWFs, GPDs, TMDs,
PDFs, FFs

The Algebraic Model (AM)

The quark propagator:

$$S_{q(\bar{h})}(k) = [-i\gamma \cdot k + M_{q(\bar{h})}]\Delta(k^2, M^2_{q(\bar{h})})$$

$$\Delta(s,t) = (s+t)^{-1}$$

Bethe-Salpeter Amplitude:

$$n_{\mathrm{M}}\Gamma_{\mathrm{M}}(k,P) = i\gamma_5 \int_{-1}^{1} dw\, \rho_{\mathrm{M}}(w)[\hat{\Delta}(k_w^2, \Lambda_w^2)]^{\nu}$$

$$\hat{\Delta}(s,t) = t\Delta(s,t), \quad k_w = k + (w/2)P$$

$$\Lambda^2(w) = M_q^2 - \frac{1}{4}(1-w^2)m_{\mathrm{M}}^2 + \frac{1}{2}(1-w)(M_{\bar{h}}^2 - M_q^2)$$

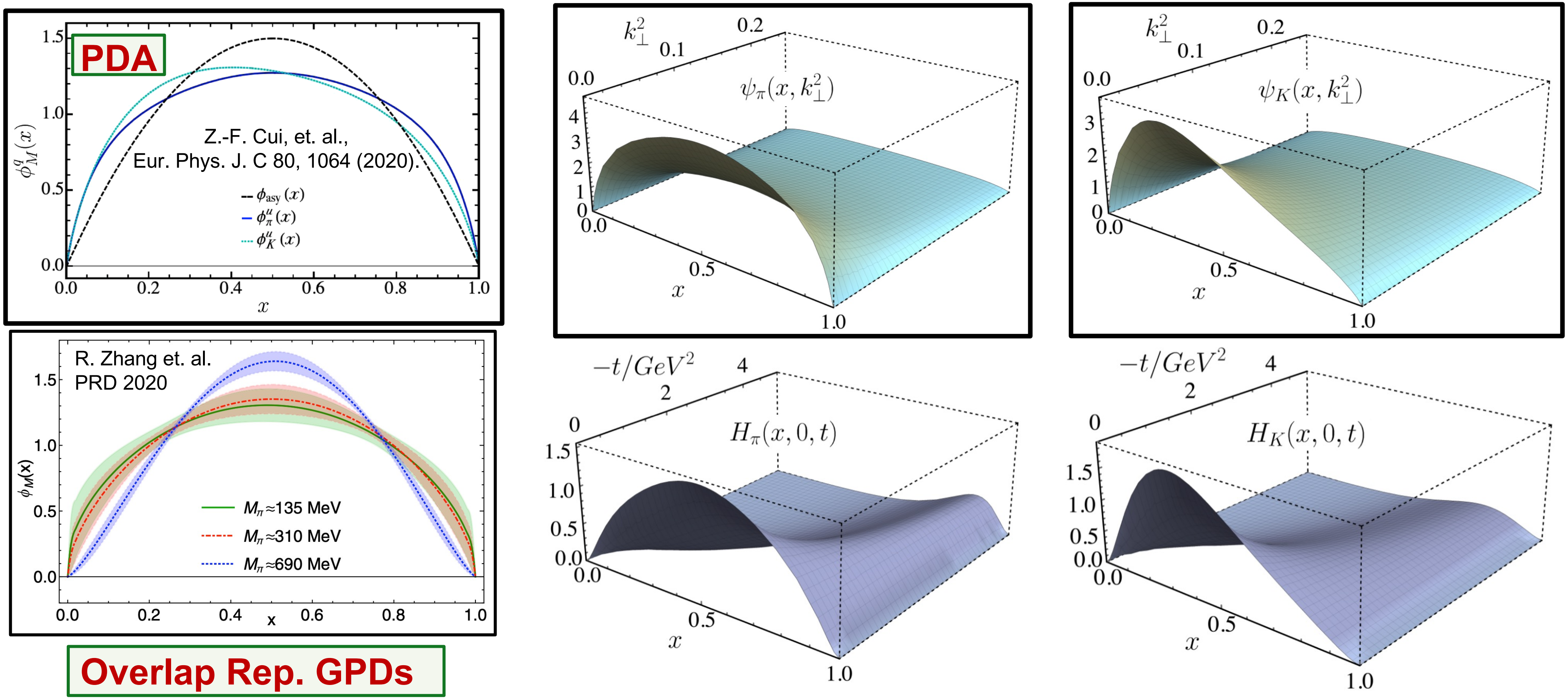
The Algebraic Model:

L. Albino, M. Higuera, K. Raya, AB, Phys. Rev. D 106 (2022) 3, 034003

$$\psi_{\mathrm{M}}^q(x, k_\perp^2) = 16\pi^2 f_{\mathrm{M}} \frac{\nu \Lambda_{1-2x}^{2\nu}}{(k_\perp^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi_{\mathrm{M}}^q(x)$$

For a quark in pseudo-scalar meson **M**, the **leading twist** (2-particle) **light front wave function**, ψ_{M}, can be obtained via the light front projection of the meson's **BSWF**.

From the PDAs to the GPDs

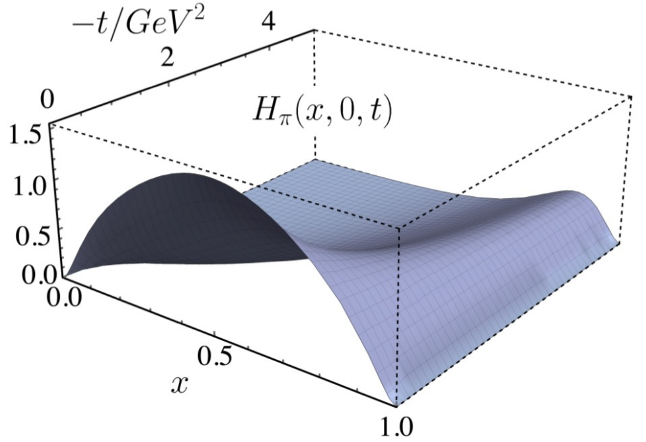


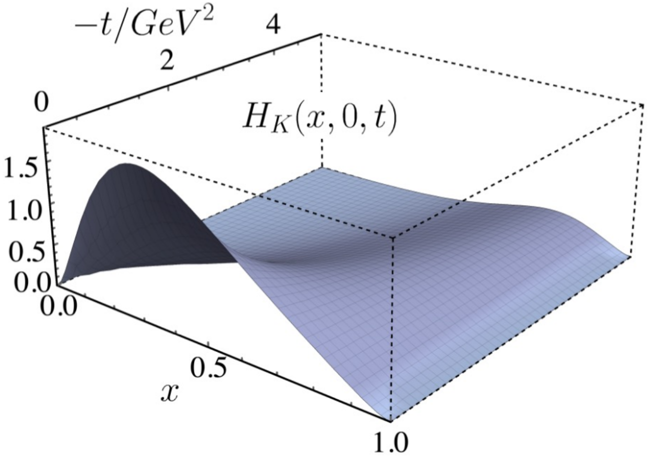
Overlap Rep. GPDs

M. Diehl, et. al., Nucl. Phys. B 596 (2001)

L. Albino, M. Higuera, K. Raya, AB, Phys. Rev. D 106 (2022) 3, 034003

From the GPDs to the PDFs



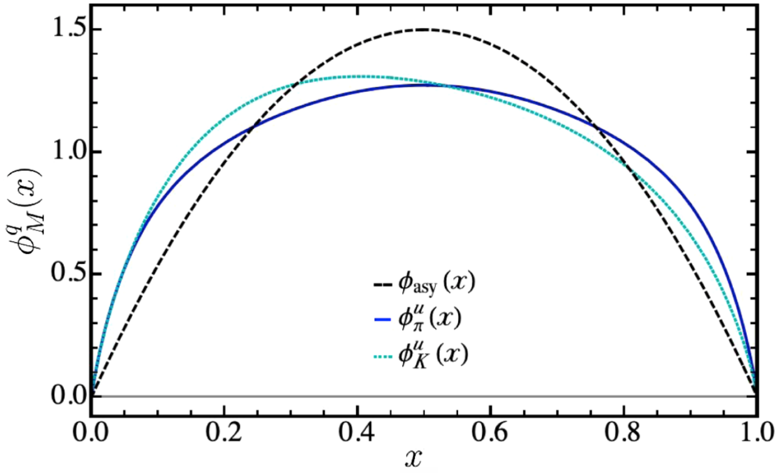


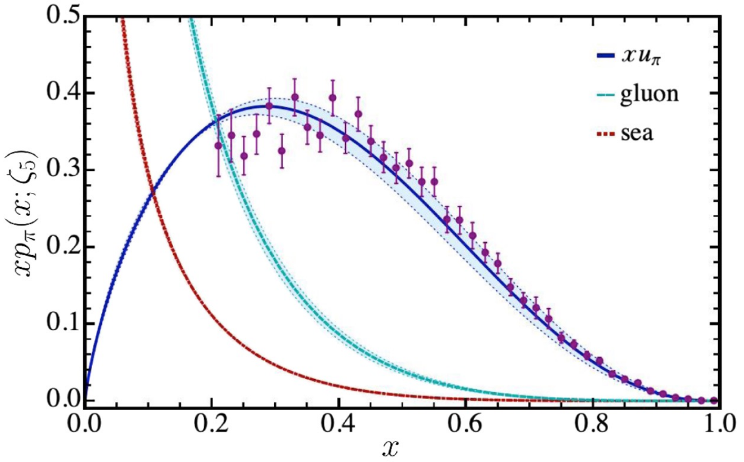
$$q_{\rm M}(x) \equiv H^q_{\rm M}(x,0,0)$$

DGLAP Evolution Equations

L. Albino, M. Higuera, K. Raya, AB
Phys. Rev. D 106 (2022) 3, 034003

Aicher et. al.
Phys. Rev. Lett. 105, 252003 (2010)

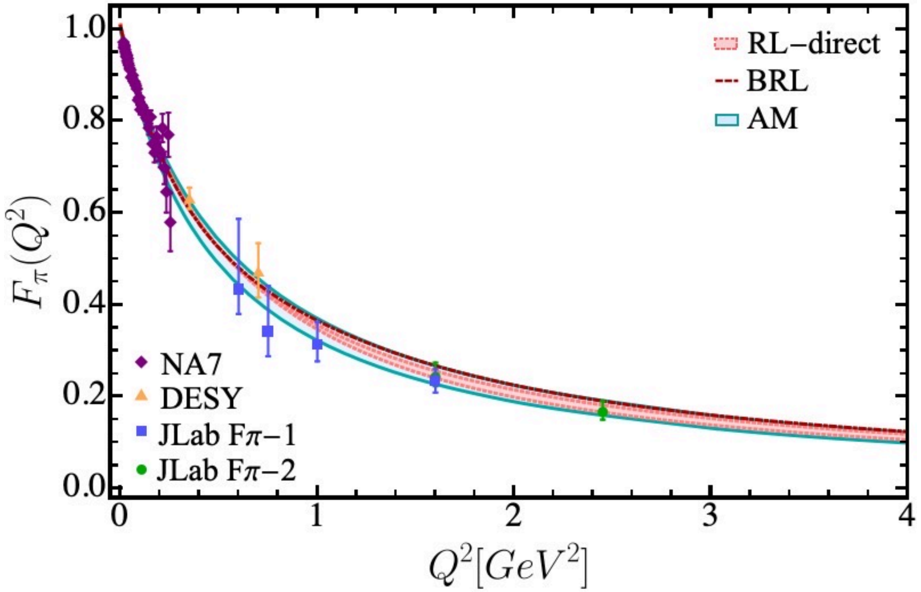


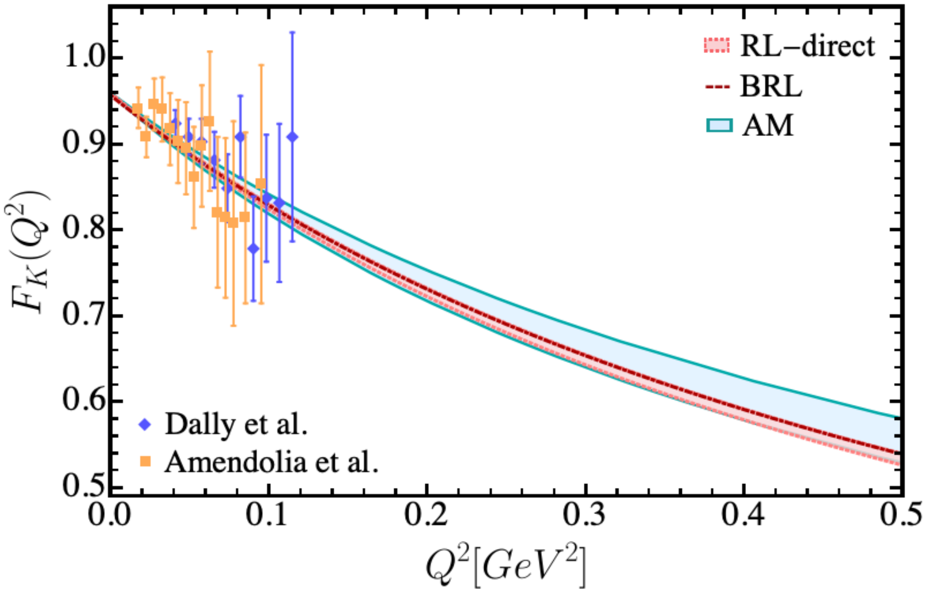


Completing the Cycle – Back to Form Factors

The **electromagnetic form factors** using our **algebraic model** can be obtained either through the knowledge of the **GPDs** or the direct evaluation of the **triangle diagram**.

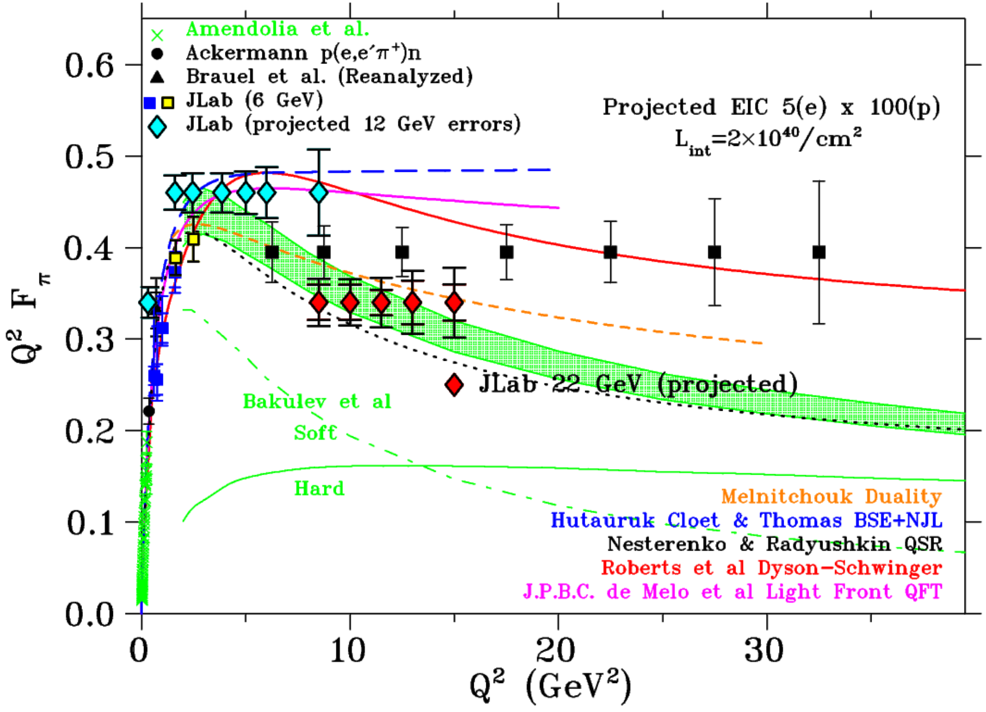
Such an exercise provides stringent constraints on the efficacy of the **algebraic model** we have constructed by direct comparison with the refined calculation of these **form factors.**

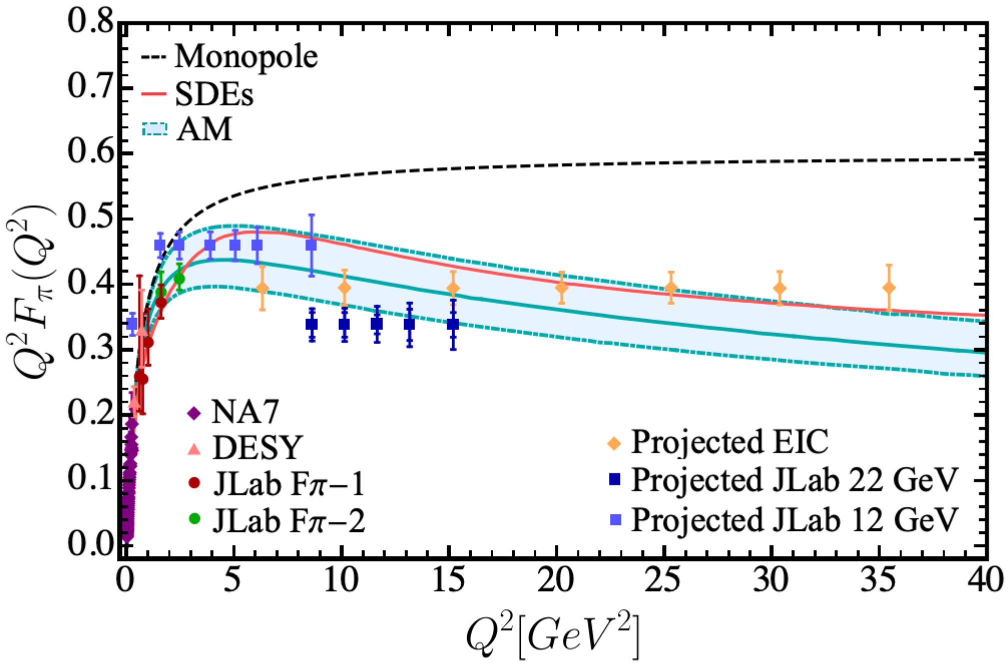




Completing the Cycle – Back to Form Factors

We can extend this analysis of the **Algebraic Model** to compute the **pion electromagnetic form factors** to larger **Q^2** range: **0-40 GeV^2** which would likely cover the photon virtualities accessible to the **JLab12**, **JLab22** and **EIC programs**:

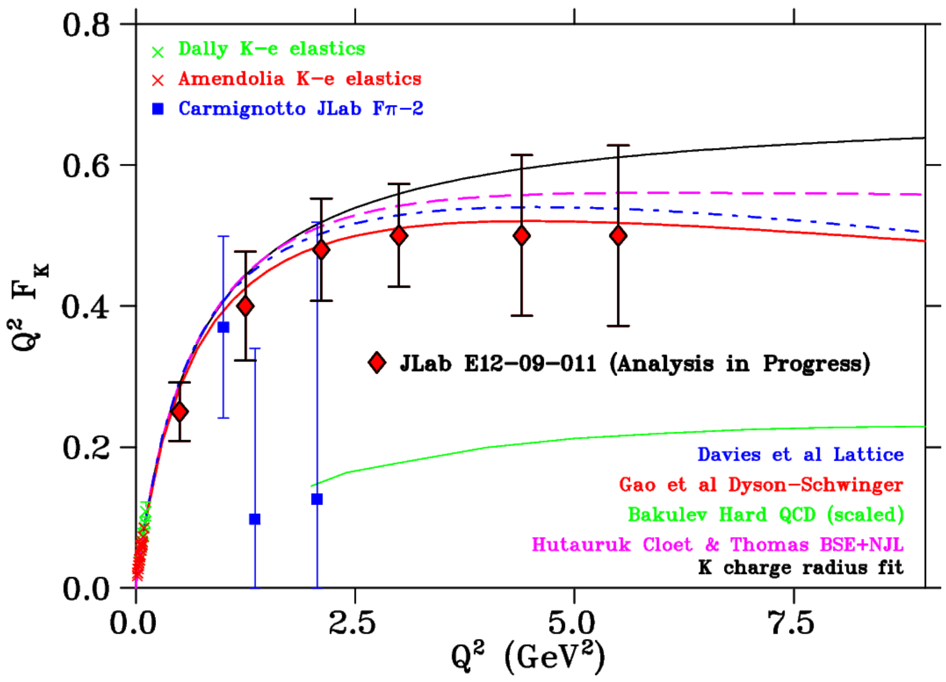




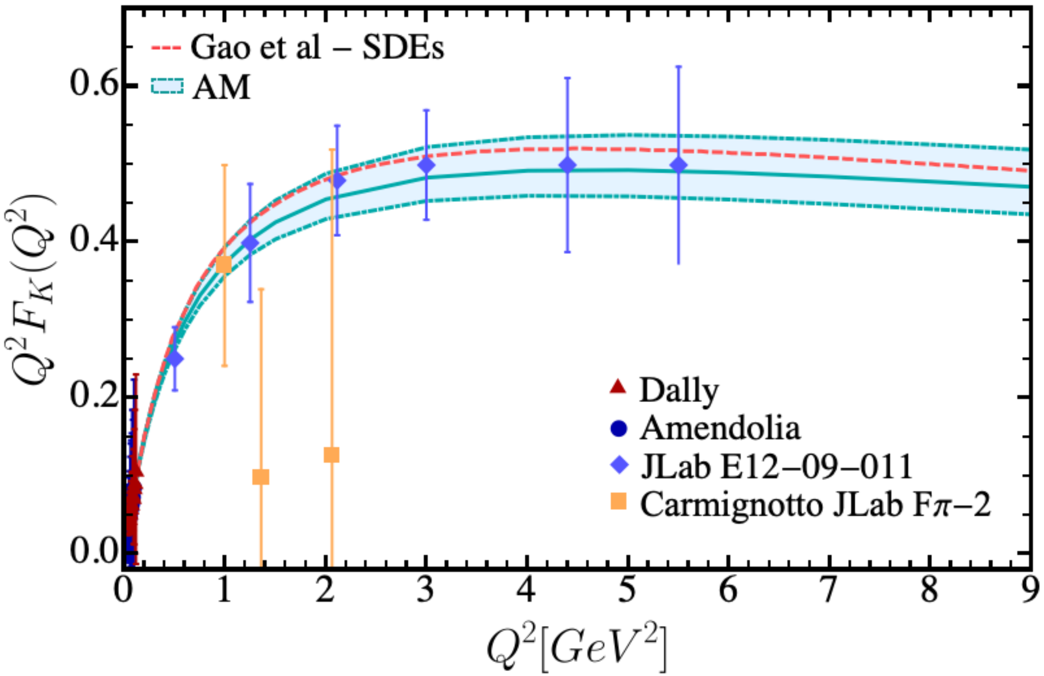
Completing the Cycle – Back to Form Factors

There is an analysis underway of the **kaon electromagnetic form factor** till **5.5 GeV²** of the data obtained in **JLab E12-09-011** experiment.

Courtesy Garth Huber



Algebraic Model results



Summary and Outlook

- The interplay of **QCD akin** truncations of **Schwinger-Dyson equations** and **algebraic model** based upon these studies shed important light on the **internal structure** of **pion** and **kaon.**

- **QCD akin** refined computation of **pion** and **kaon electromagnetic form factors** at low and intermediate virtualities of the probing photon in electroproduction processes:

A. Miramontes AB, K. Raya, P. Roig, Phys. Rev. D 105 (2022) 7, 074013
L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802

- Results for the **pion electromagnetic form factor** at large photon virtualities accessible to the potential **22GeV upgrade** of the **JLab** and **EIC** are also available:

L. Chang, I.C. Cloët, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 14, 141802
J. Arrington, et al. (Feb 23, 2021, J.Phys. G 48 (2021) 7, 075106

- More recently, **pion** and **kaon form factors** have been computed in the the **time-like region**

A.S. Miramontes, H. Sanchis Alepuz, R. Alkofer, Phys. Rev. D 103 (2021) 11, 116006
A.S. Miramontes, AB, Phys. Rev. D 107 (2023) 1, 014016

Summary and Outlook

- Carefully constructed **Algebraic Models** can enable computation of the **GPDs**, **PDFs** and **EFF** with relative ease while mimicking the reliability of **QCD akin** refined truncations of **Schwinger-Dyson equations**.

L. Albino, M. Higuera, K. Raya, AB Phys. Rev. D 106 (2022) 3, 034003

- Despite these encouraging results and synergy with experimental endeavors at **JLab** and **EIC**, further improvements and extensions in the **continuum QCD approach** are desirable.
- More work into the theoretical foundations of the truncations involved at the level of the **Green functions** of the fundamental degrees of freedom, i.e., **quarks**, **gluons**, as well as **quark-gluon** and **gluon-gluon** interactions continues vigorously.
- **Schwinger-Dyson equations** have also been of substantial success in the studies of **baryons** such as the **transition form factors** of **nucleon** to its **excited states** which is a hallmark of **CLAS**, **CLAS12** and **CLAS22** programs at **JLab** and hold the promise to offer a reliable tool for the future **JLab** and **EIC era** research into the heart of **hadronic matter**.

"Strong Interaction Physics at the Luminosity Frontier with 22 GeV Electrons at Jefferson Lab", A. Accardi, et. al., e-Print: 2306.09360 [nucl-ex]

Thank you for your attention