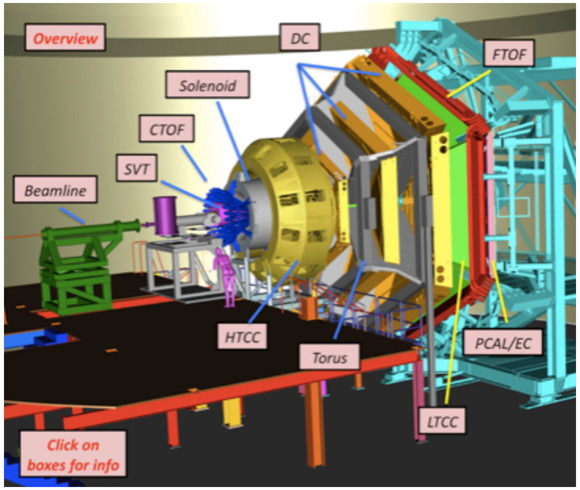
Nuclear Femtography - what Lattice QCD can do for you.

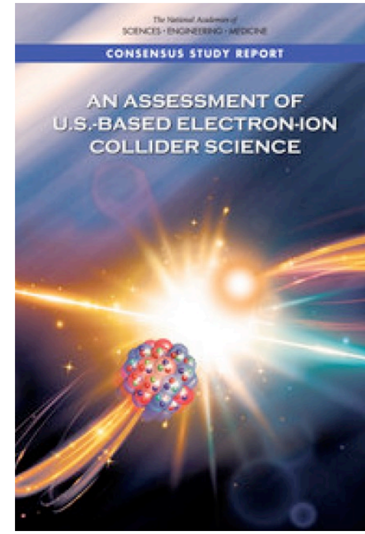
David Richards
Jefferson Lab
For Hadstruc Collaboration

Femtography with Hard Exclusive Reactions, August 2023

A New Opportunity in Hadron Structure



Lattice QCD

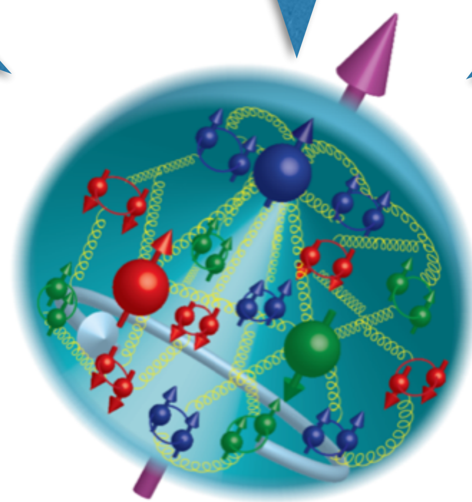


JLab@12GeV

Future Electron-Ion Collider

CENTER *for* NUCLEAR FEMTOGRAPHY

LATTICE QCD



3D Image of nucleon and nuclei at the femtoscale

HadStruc Collaboration

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Jefferson Lab

Carl Carlson, Herve Dutrieux, Tanjib Khan, Christopher Monahan,

Kostas Orginos

William and Mary

Wayne Morris, Anatoly Radyushkin

Old Dominion University

Savvas Zafeiropoulos

Aix Marseille Univ, Marseille, France

Yan-Qing Ma

Peking University, Beijing, China

Balint Joo

ORNL

Graduate students, and now post-docs + faculty.

Outline

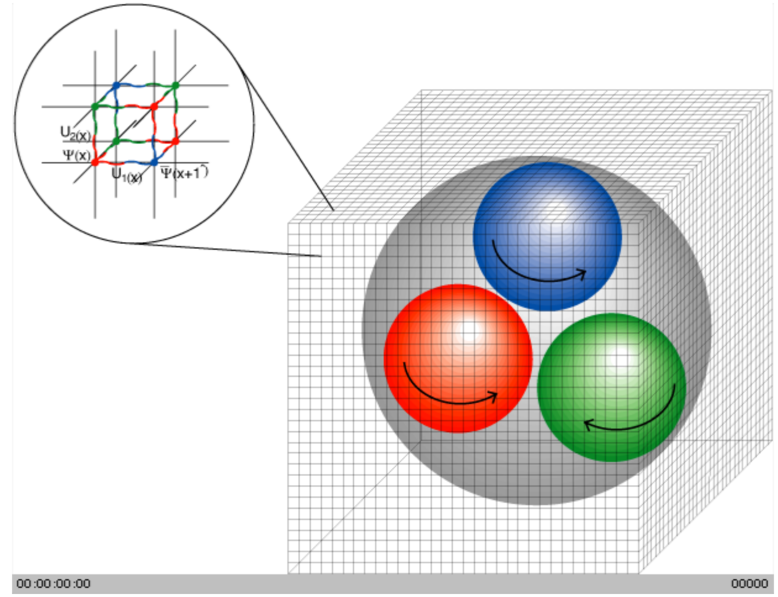
- Lattice QCD
- Hadron Structure on Euclidean Lattice
- Short-distance factorization and pseudo-PDFs
- Understanding systematic effects
  - Distillation + momentum smearing to reach high momenta
- Precision calculations of isovector PDFs
- Isoscalar structure of the nucleon - *gluon distribution*
- Lattice QCD + Expt $\longrightarrow$ *global analysis of pion*
- Onto 3D Imaging…..
- Summary

Lattice QCD

- Continuum Euclidean space time replaced by four-dimensional lattice, or grid, of "spacing" a
- Gauge fields are represented at SU(3) matrices on the links of the lattice - work with the elements rather than algebra

$$U_\mu(n) = e^{iaT^a A^a_\mu(n)}$$

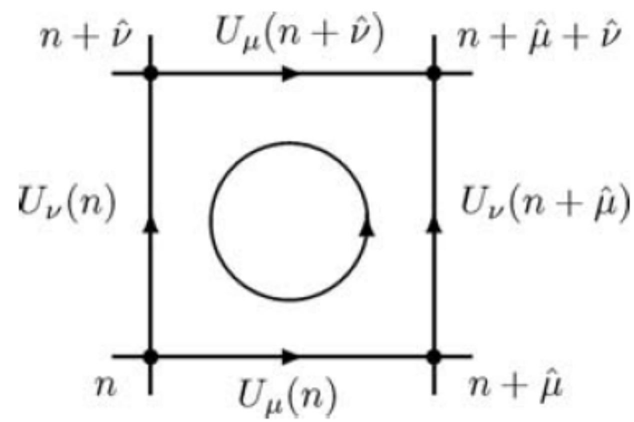
Quarks ψ, ψ are Grassmann Variables, associated with the sites of the lattice



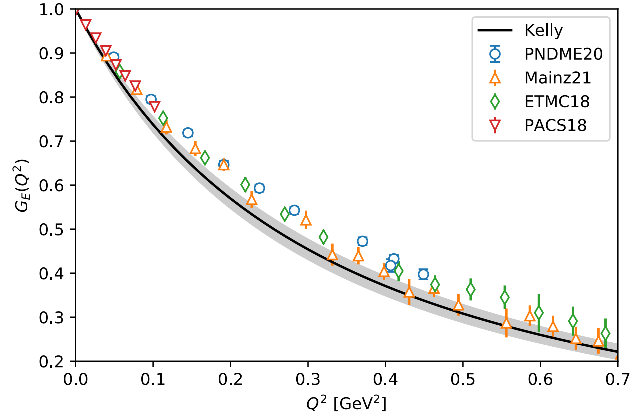
Work in a finite 4D space-time volume

- Volume V sufficiently big to contain, e.g. proton
- Spacing a sufficiently fine to resolve its structure

$$V \simeq (6\ \mathrm{fm})^4$$
$$a \leq 0.1\mathrm{fm}$$



Rich Menu of calculations….

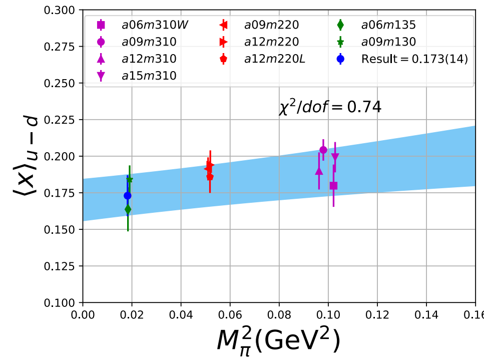


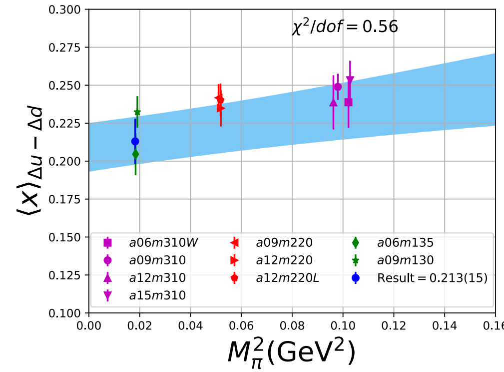
Isovector Sach's Form Factor

D.Djukanovic, Lattice 2022

Momentum and spin fractions of nucleon

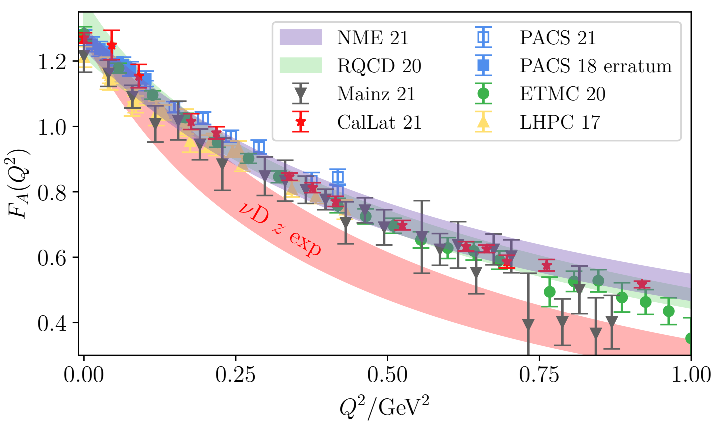
S.Mondal *et al., Phys. Rev. D 102, 054512 (2020)*





Axial-vector form factors - neutrino program

A>S. Meyer, A. Walker-Loud, C.Wilkinson, arXiv:2201.01839



Each characterized by matrix element of local operator → *calculable on Euclidean lattice.*

PDFs, GPDs, TMDs?

Parton Distribution Functions (PDFs)

Describe the longitudinal momentum distribution of the partons (quarks and gluons) within the pion

Hadron Structure: No-go Theorem?

- First Challenge:
  - Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

PDFs, GPDs, TMDs

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)}\psi(0) \mid P\rangle$$

So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

$$\longrightarrow \quad \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\ldots D_{\mu_n}\psi \mid P\rangle \to P_{\mu_1}\ldots P_{\mu_n} a^{(n)}$$

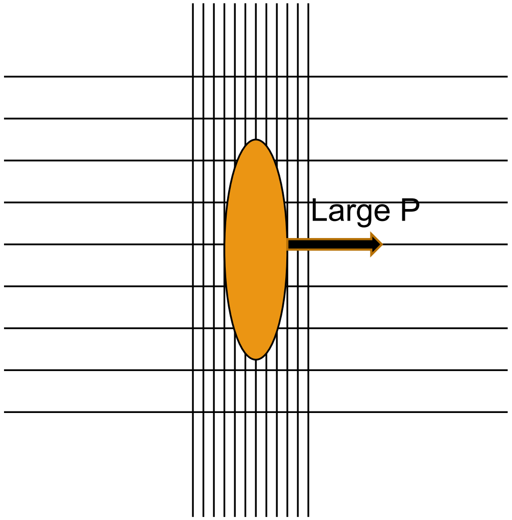
- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

Moment Methods

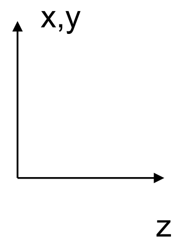
Recent work by ETMC

- Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
- Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

PDFs from Euclidean Lattice



Large-Momentum Effective Theory (LaMET)



"Equal time" correlator

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

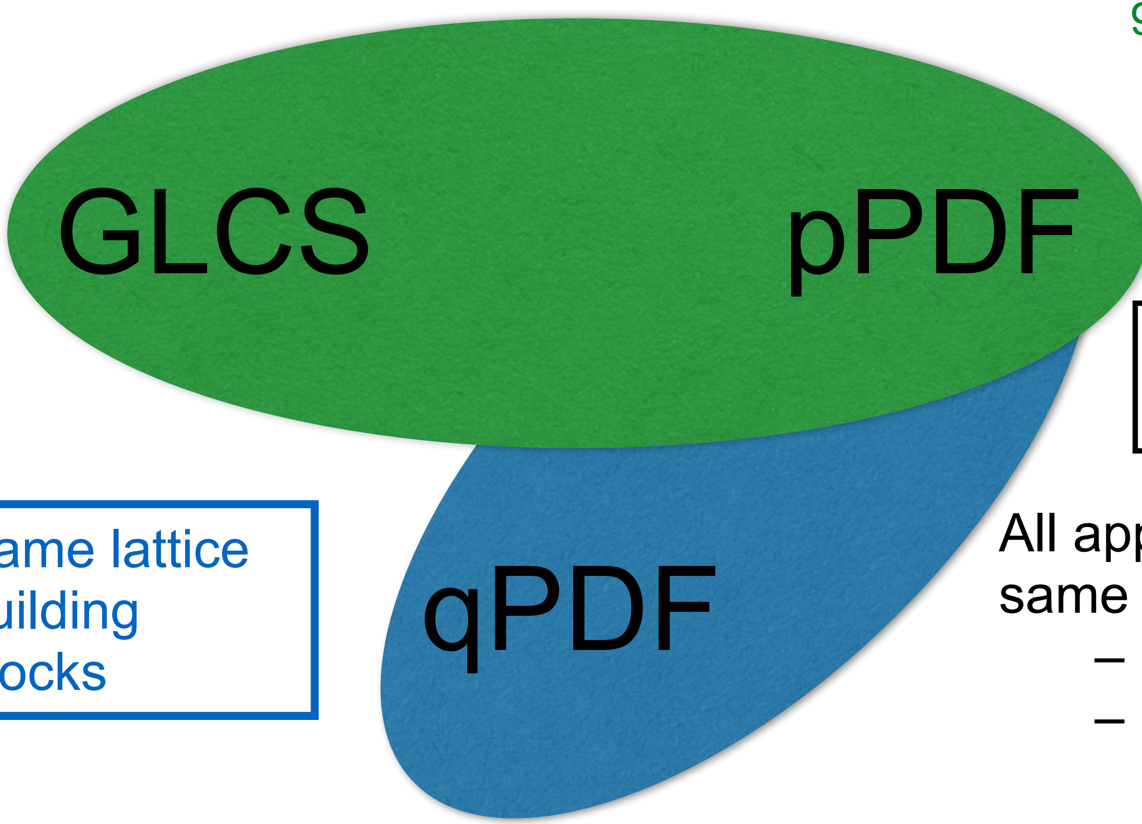
"quasi-PDF Approach"

PDFs, GPDs and TMDs

Ma and Qiu, Phys. Rev. Lett. 120 022003

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

Light cone reduces to a point



Characterized by *short-distance factorization*

Same lattice building blocks

All approaches should give same after:

- Finite volume
- Discretization Uncertainties
- *Infinite momentum*

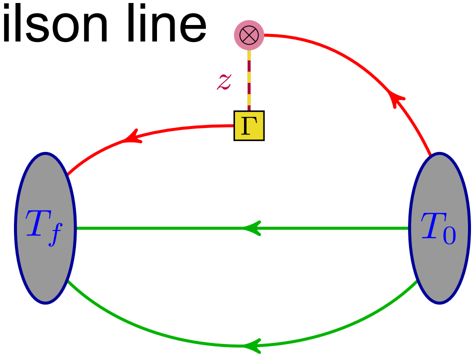
X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Pseudo-PDFs

Lattice "building blocks" that of quasi-PDF approach.

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Wilson line



A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $\quad$ $z = (0, z_-, 0_T)$

$$\Downarrow$$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

$$\mathfrak{M} = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)} \equiv \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)}\right) / \left(\frac{\mathcal{M}(0, z^2)}{\mathcal{M}(0, 0)}\right)$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu, \mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

⇓

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Need data for all ν, *or additional physics input*

Inverse problem

ITD ↔ PDF

Ioffe-Time Distribution to PDF

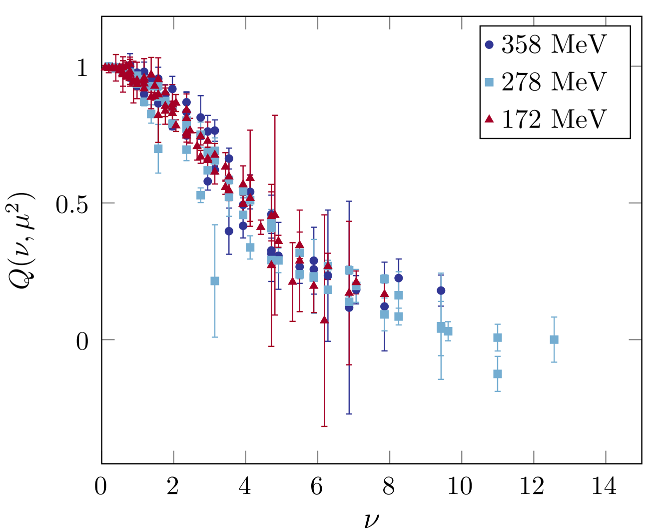
J.Karpie, K.Orginos, A.Radyushkin, S.Zafeiropoulos, Phys.Rev.D 96 (2017)

B.Joo *et al.*, HEP 12 (2019) 081, J.Karpie et al., *Phys.Rev.Lett.* 125 (2020) 23, 232003

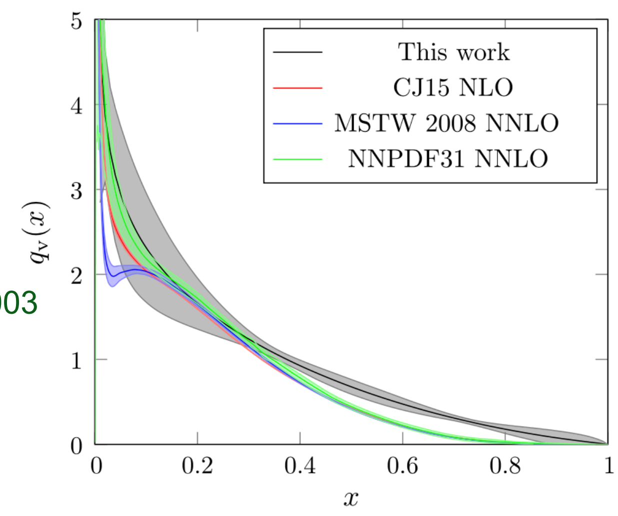
To extract PDF requires additional information - *use a phenomenologically motivated parametrization*

$$f(x) = x^a(1-x)^b P(x)$$ MSTW, CJ

$$P(x) = \frac{1 + c\sqrt{x} + dx}{B(a+a, b+1) + cB(a+1.5, b+1) + dB(a+2, b+1)}$$



B.Joo *et al.*,PRL 125 (2020) 23, 232003

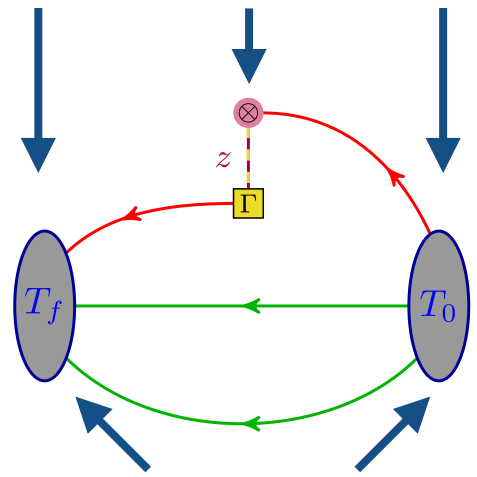


Distillation and Hadron Structure

To control systematic uncertainties, need precise computations over a wide range of momentum.

- Use a low-mode projector to capture states of interest "distillation" M.Peardon *et al* (Hadspec), Phys.Rev.D 80 (2009) 054506
- Enables momentum projection at each temporal point.

Momentum projection

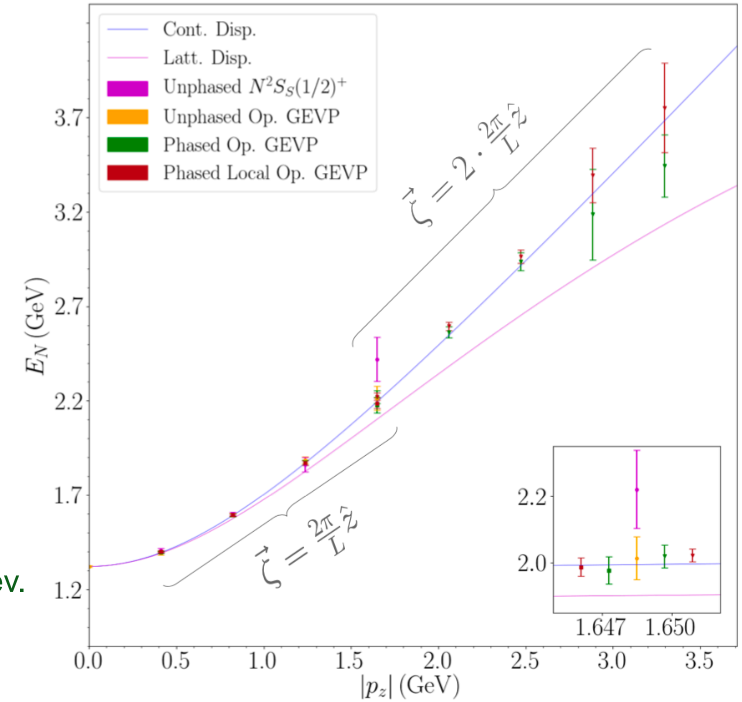


Variational basis

+ momentum smearing

G.Bali *et al*, Phys.Rev.D 93 (2016) 9, 094515

C.Egerer *et al* (Hadstruc), Phys. Rev. D 103, 034502 (2021)



Isovector PDF using Distillation

C.Egerer *et al.* (hadstruc), JHEP 11 (2021) 148

Expand the x-dependence in terms of (shifted) Jacobi Polynomials

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \mathfrak{Re}\int_0^1 dx\, \mathcal{K}_{\rm v}\left(x\nu, z^2\mu^2\right) x^\alpha (1-x)^\beta\, \Omega_n^{(\alpha,\beta)}(x)$$

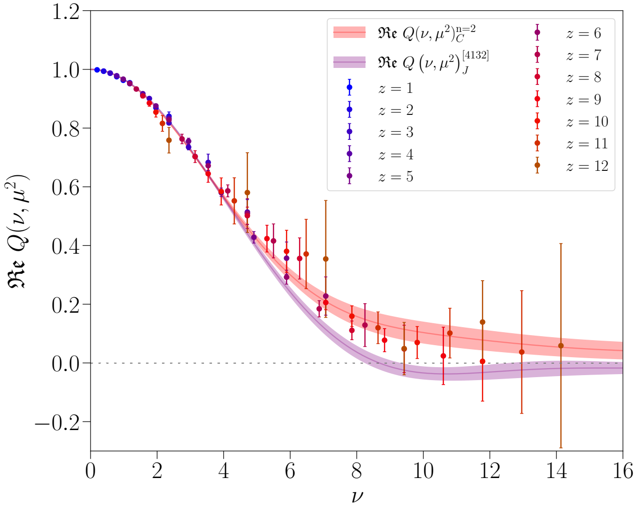
Matching kernel

J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos, arXiv:2105.13313

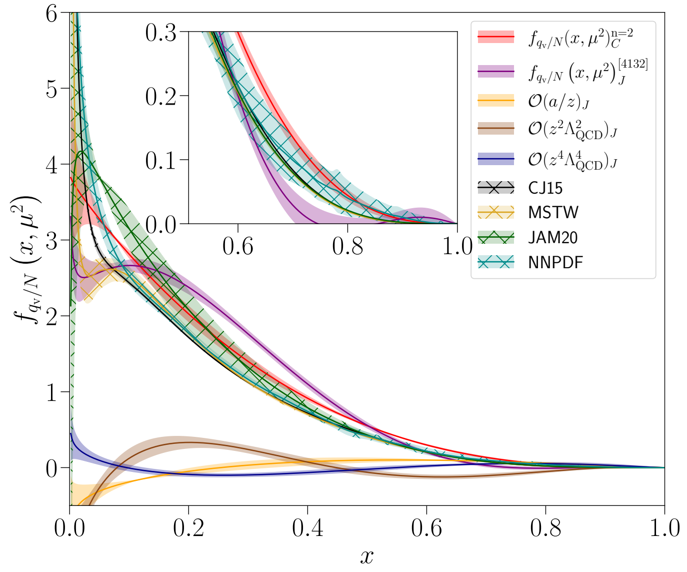
$$\mathfrak{Re}\,\mathfrak{M}_{\rm fit}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{{\rm v},n}^{lt\,(\alpha,\beta)} + \left(\frac{a}{z}\right) \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{az\,(\alpha,\beta)} + z^2\Lambda_{\rm QCD}^2 \sum_{n=1}^{\infty} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{t4\,(\alpha,\beta)}$$

Discretization

Higher twist

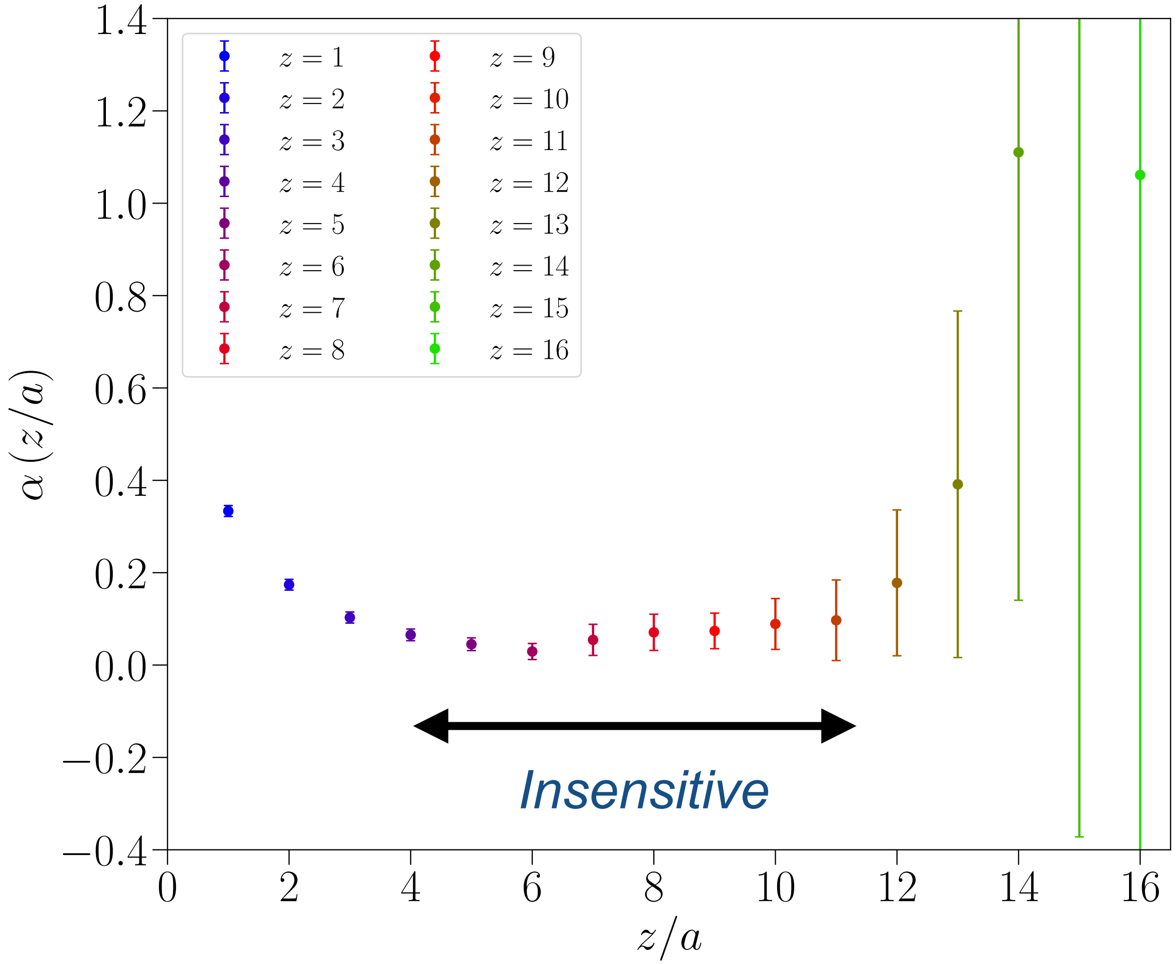


$m_\pi \simeq 358$ MeV



DGLAP Evolution

- Data demonstrate "precious scaling"…



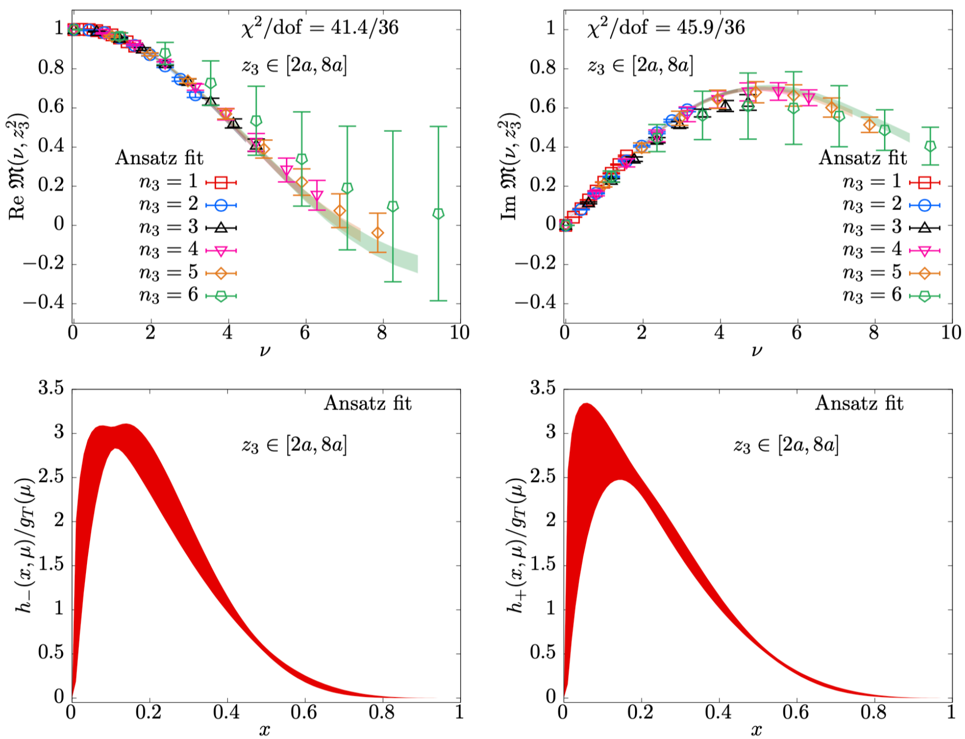
Transversity + Helicity

Phys.Rev.D 105 (2022) 3, 034507, Hadstruc Collaboration, (C.Egerer et al).

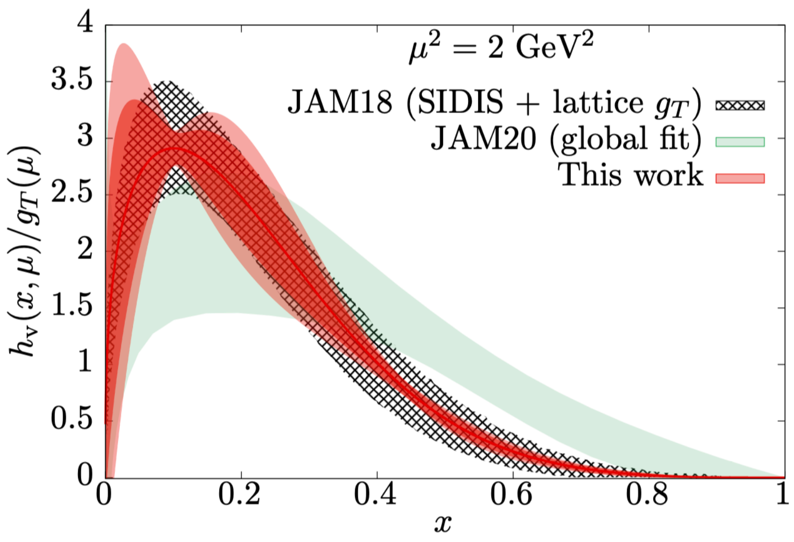
$$2P^+S^{\rho_\perp}\mathcal{I}(P^+z^-,\mu) = \langle P,S^{\rho_\perp}|\bar{\psi}(z^-)\gamma^+\gamma^{\rho_\perp}\gamma_5 W_+(z^-,0)\psi(0)|P,S^{\rho_\perp}\rangle$$

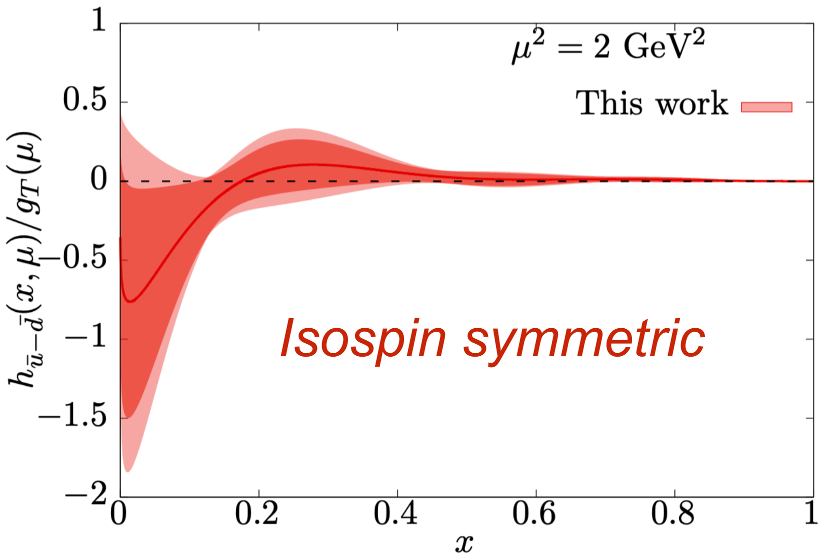
$$h(x,\mu) = \int_{-\infty}^{\infty}\frac{d\nu}{2\pi}e^{-ix\nu}\mathcal{I}(\nu,\mu)$$

In contrast to unpolarized PDF, there is no conserved current - so express in terms of the (renormalized) tensor charge.



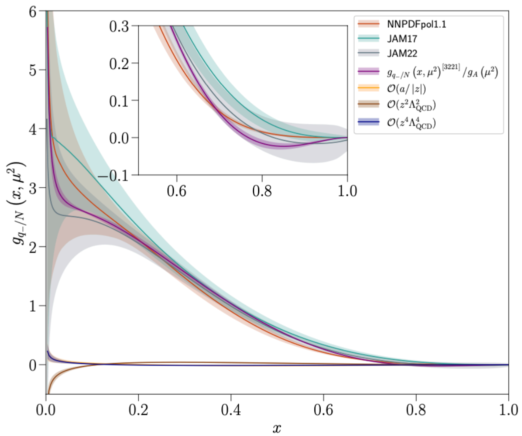
Transversity Distribution



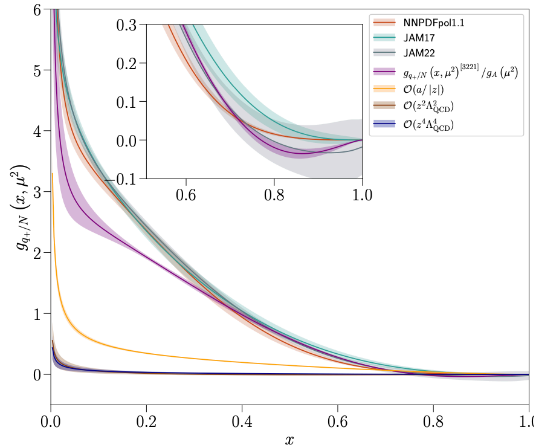


Helicity Distribution

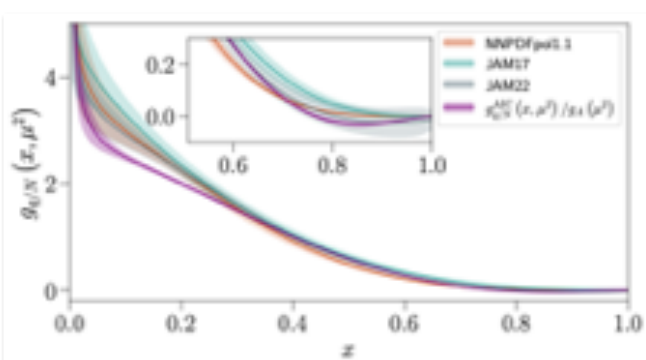
Valence quark helicity distribution, together with contamination terms

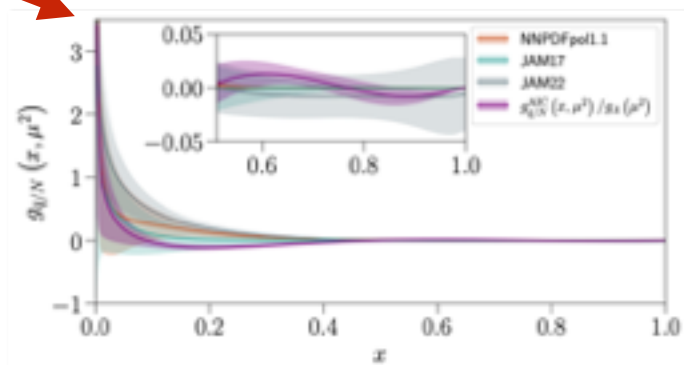


CP-odd helicity distribution, together with contamination terms



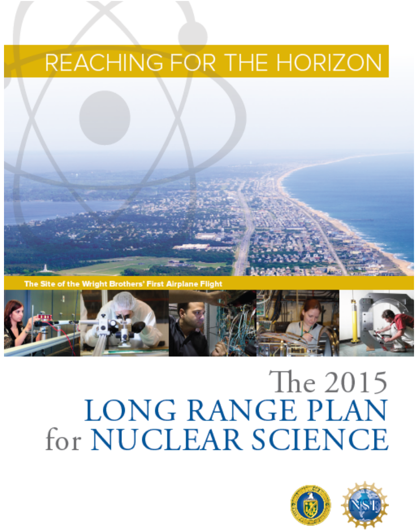
Small NS anti-quark helicity





Unpolarized and Polarized Gluon

“Understanding the Glue That Binds Us All: The Next QCD Frontier in Nuclear Physics”

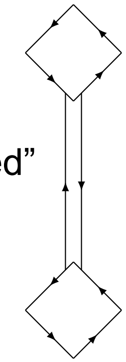


Gluon Contribution to unpolarized PDF

c.f. Z.Fan, H-W-Lin, arXiv:2104.06372,arXiv:2007.16113

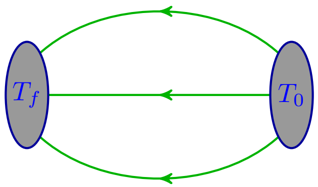
T.Khan *et al.* (Hadstruc), *hys.Rev.D* 104 (2021) 9, 094516

“disconnected”



$$M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$$

$$O_g(z) = G_{ji}(z)\, U(z,0)\, G_{ij}(0)\, U(0,z) - \; G_{ti}(z)\, U(z,0)\, G_{it}(0)\, U(0,z).$$



Two-point functions as in isovector case

Reduced matrix element: $\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}}\right) / \left(\frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0,z=0}}\right)$

Flavor-singlet quantities are subject to severe signal-to-noise problems compared with isovector measures:

- Use distillation and many more measurements per configuration - *sampling of lattice*
- Use of summed Generalized Eigenvalue Problem (sGEVP) - *better control over excited state contributions*
- Use of *Gradient Flow - smoothing of short-distance fluctuations*

ITD to PDF

Matching: I.Balitsky,W.Morris,A.Radyushkin,Phys.Lett.B 808 (2020) 135621

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{I}_g(\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} - \frac{\alpha_s N_c}{2\pi}\int_0^1 du\, \frac{\mathcal{I}_g(u\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} \left\{ \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4\left[\frac{u+\ln(\bar{u})}{\bar{u}}\right]_+ + \frac{2}{3}\left[1-u^3\right]_+ \right\}$$

N.B neglecting quark-gluon mixing

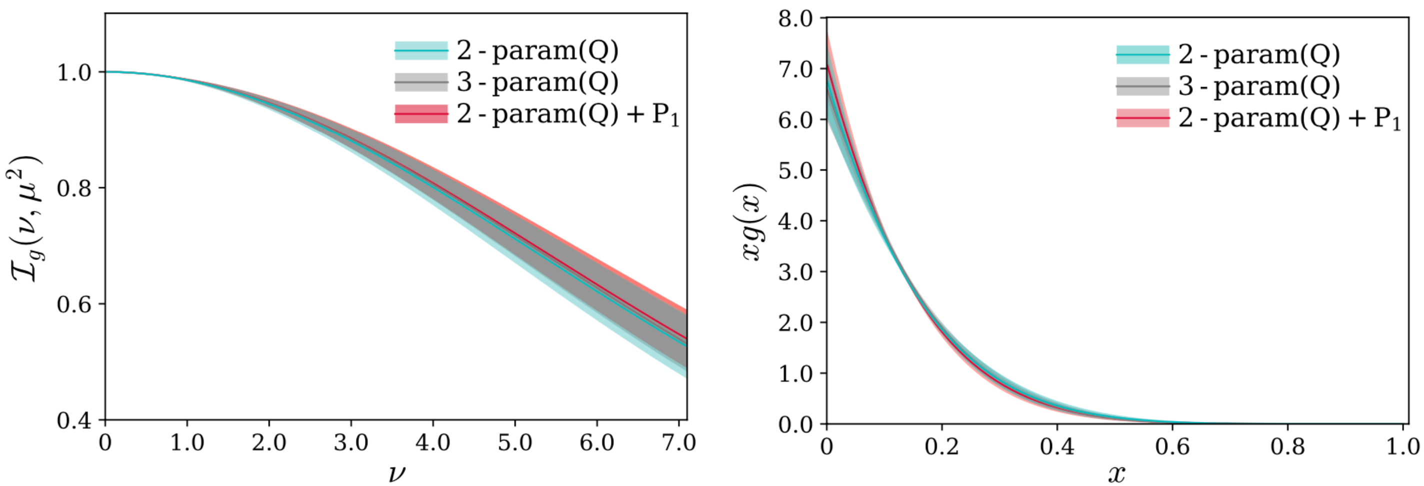
Implementation for obtaining the PDFs follows that of the isovector distribution

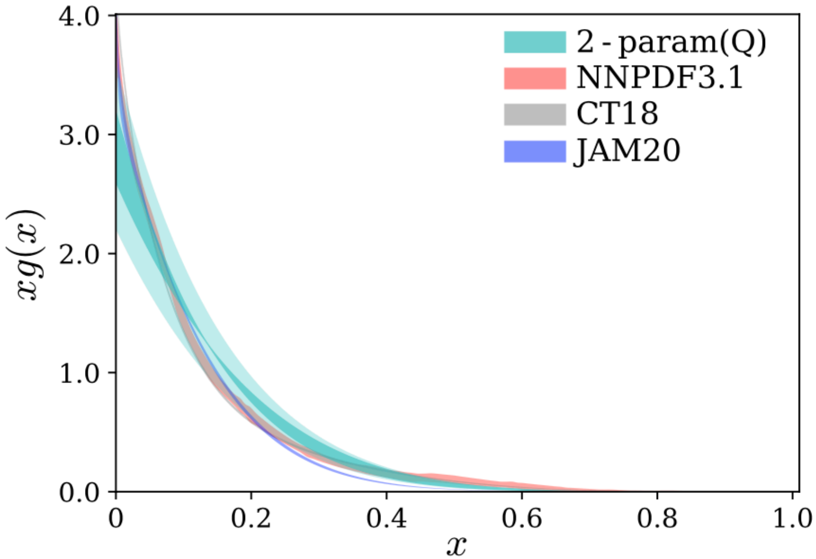
– *Expand in Jacobi Polynomials*

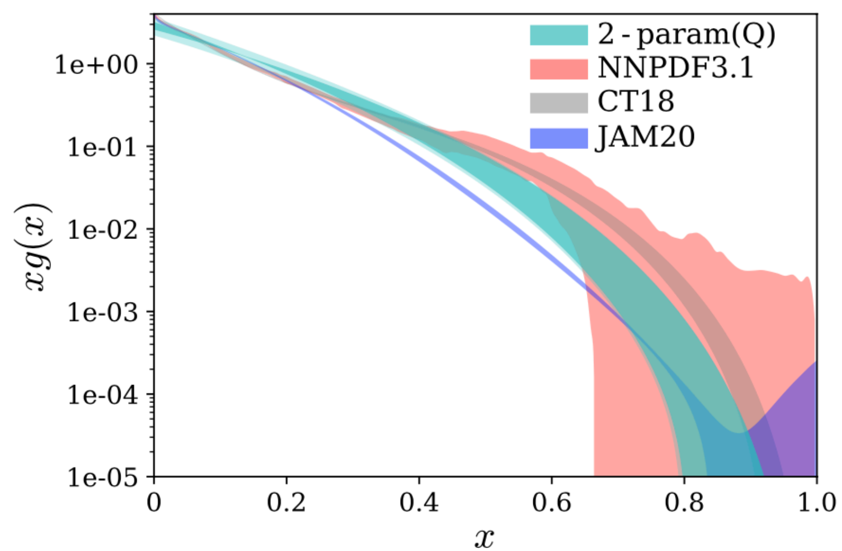
$$x^{\alpha}(1-x)^{\beta}$$

$$+J_1^{\alpha,\beta}$$

$$+a/|z|$$







Require normalization of $xg(x)$ $\langle x \rangle_g^{\overline{\text{MS}}}(\mu = 2\,\text{GeV}) = 0.427(92)$

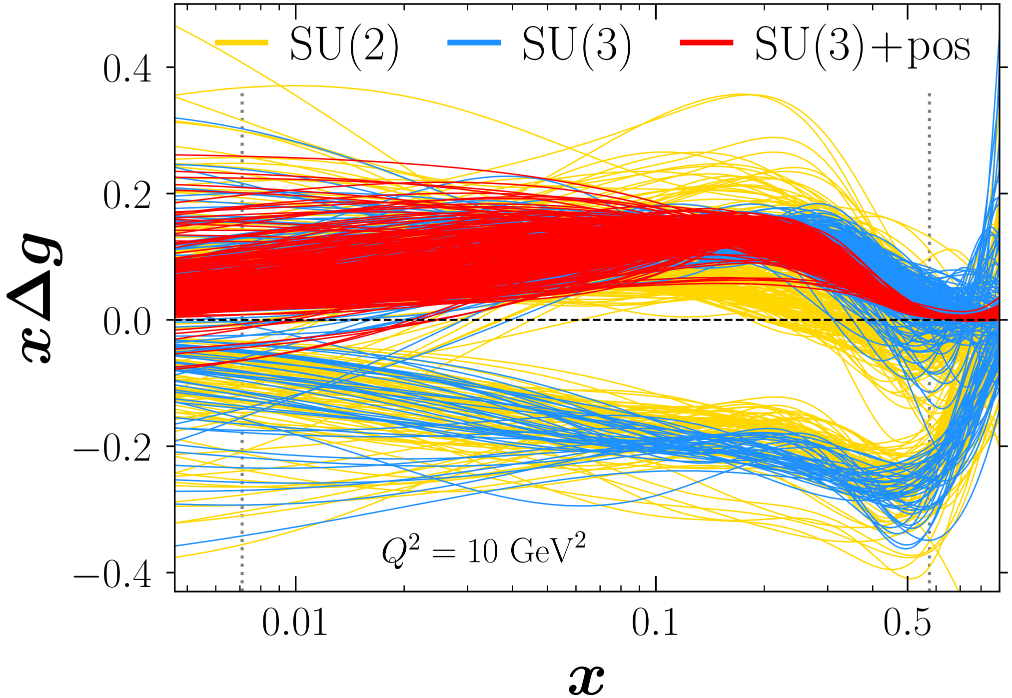
C.Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017)

Gluon Helicity Distribution

- Crucial questions in global analysis - do we need to apply positivity constraint:

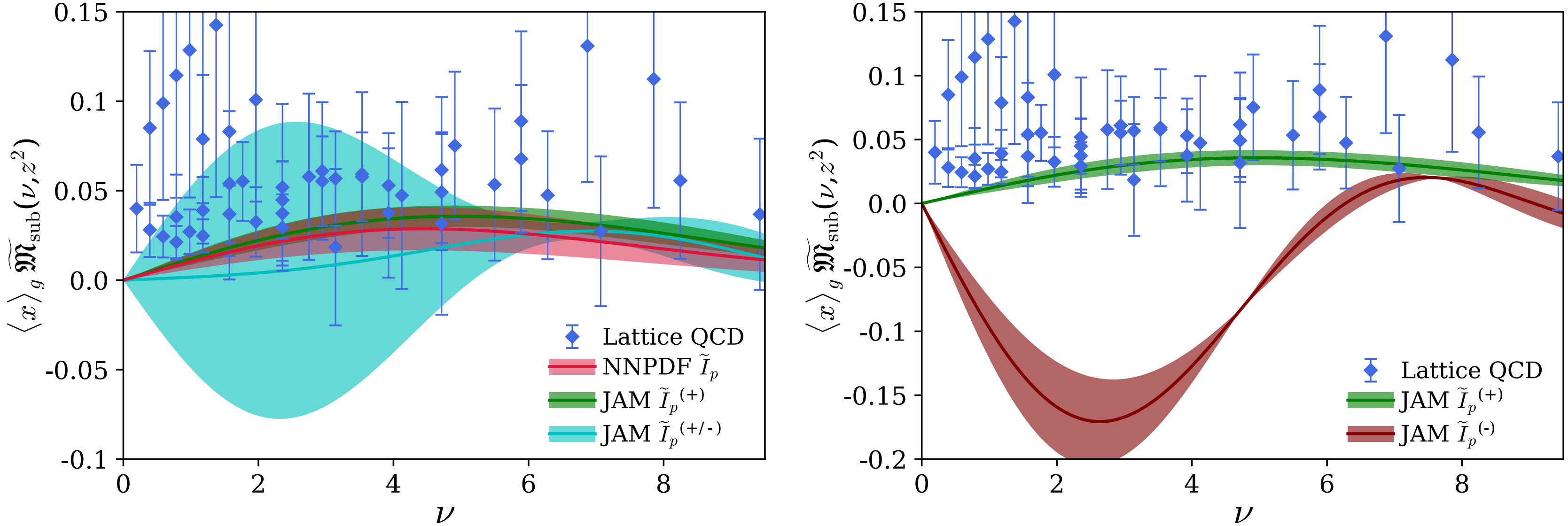
$$|\Delta g(x)| \leq g(x) \forall x$$

Relaxing constraint leads to new “replicas” in global analysis:



Zhou, Sato and Melnitchouk, Phys. Rev. D 105, 074022 (2022)

Recall ITD ↔ PDF



C.Egerer *et al. (HadStruc)*, Phys.Rev.D 106 (2022) 9, 094511

LQCD Calculation of gluon helicity distribution compared with global analyses

LQCD can inform in advance of EIC!

Lattice QCD + Experiment: Greater than their parts

Pion PDF

Pion PDF has high level of uncertainty - *no free-pion targets*

$$q_v^\pi(x) = \frac{x^\alpha(1-x)^\beta(1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$

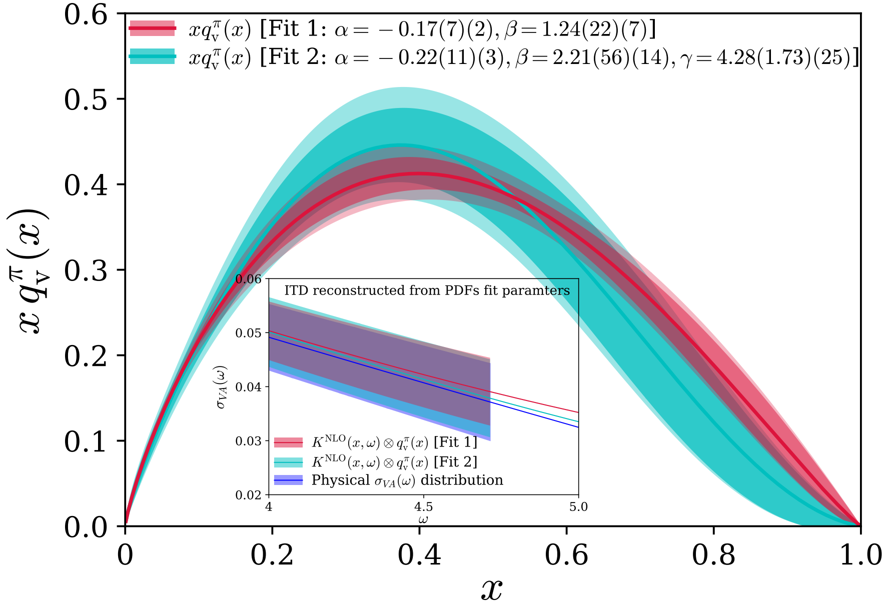
"Good Lattice Cross Sections"

Ma and Qiu, Phys. Rev. Lett. 120 022003

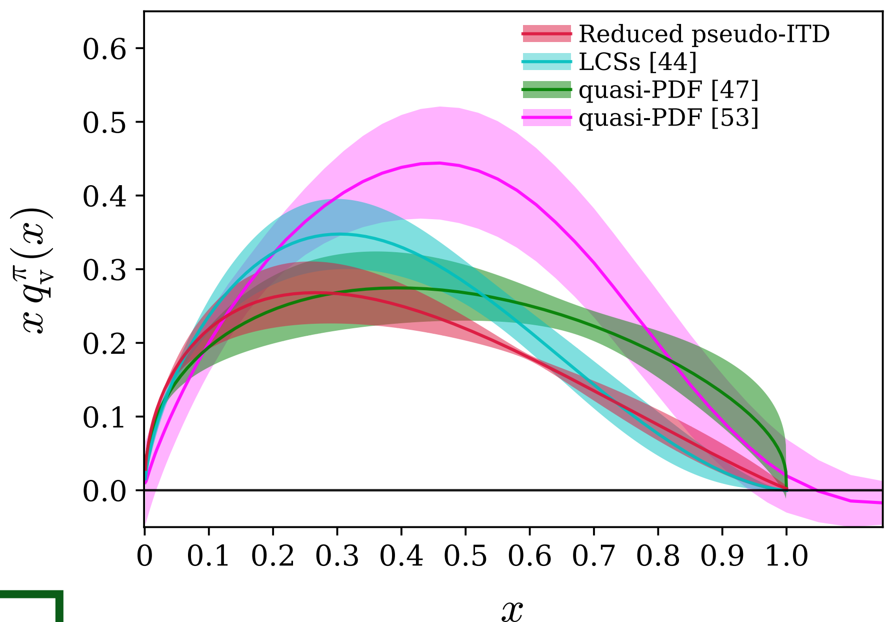
$$\mathcal{O}_S(\xi) = \xi^4 Z_S^2[\bar{\psi}_q\psi_q](\xi)[\bar{\psi}_q\psi](0)$$
$$\mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2[\bar{\psi}_q\xi\cdot\gamma\psi_{q'}](\xi)[\bar{\psi}_{q'}\xi\cdot\gamma\psi](0)$$

T.Izubuchi et al., Phys. Rev. D 100, 034516

J-H Zhang et al., Phys. Rev. D 100, 034505



Sufian *et al.*, Phys. Rev. D102, 05408 (2020)



Back to expt........

PHYSICAL REVIEW D **105**, 114051 (2022)

Complementarity of experimental and lattice QCD data on pion parton distributions

P. C. Barry,[1] C. Egerer,[1] J. Karpie,[2] W. Melnitchouk,[1] C. Monahan,[1,3] K. Orginos,[1,3] Jian-Wei Qiu,[1,3] D. Richards,[1] N. Sato,[1] R. S. Sufian,[1,3] and S. Zafeiropoulos[4]

(Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations)

Can we use LQCD + expt in global analysis: what is the impact?

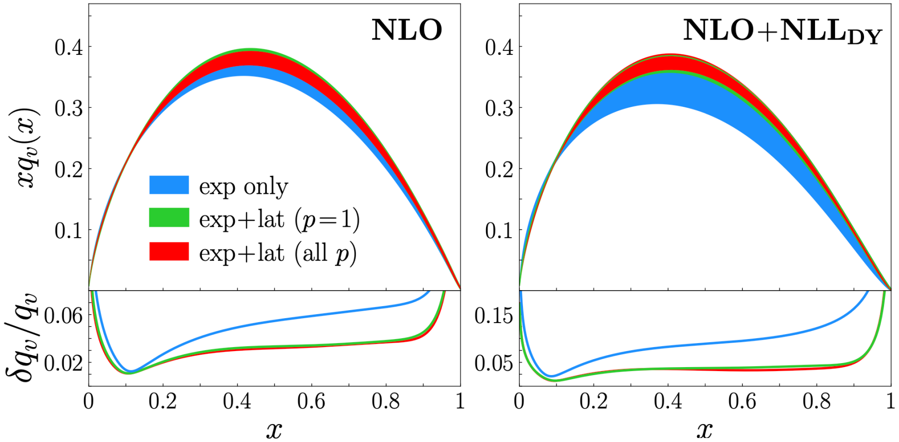
$$\frac{d\sigma}{dx_F d\sqrt{\tau}} = \frac{4\pi\alpha^2}{9Q^2 S}\sum_{ij}\int_{x_\pi^0}^1 dx_\pi \int_{x_A^0}^1 dx_A\, f_i^\pi(x_\pi,\mu)\, f_j^A(x_A,\mu)\, C_{ij}^{\mathrm{DY}}(x_\pi, x_\pi^0, x_A, x_A^0, Q, \mu),$$

Measured Cross Section

PDF

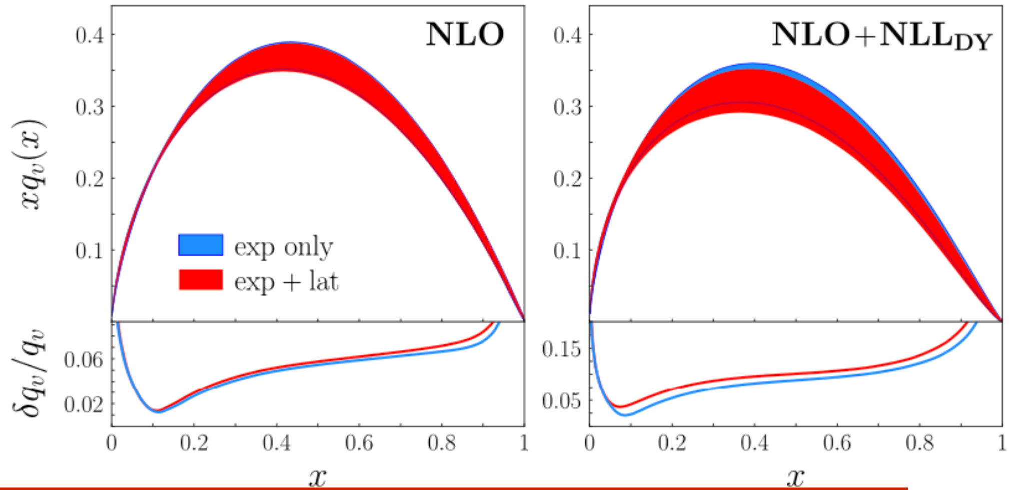
Hard Process

$$f(x,\mu_0^2) = \frac{N_f\, x^{\alpha_f}(1-x)^{\beta_f}(1+\gamma_f x^2)}{B(\alpha_f+2,\beta_f+1)+\gamma_f B(\alpha_f+4,\beta_f+1)}$$



From *pseudo-PDF* data

From *Good Lattice Cross Section* data

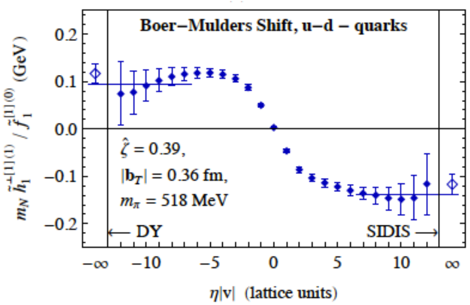


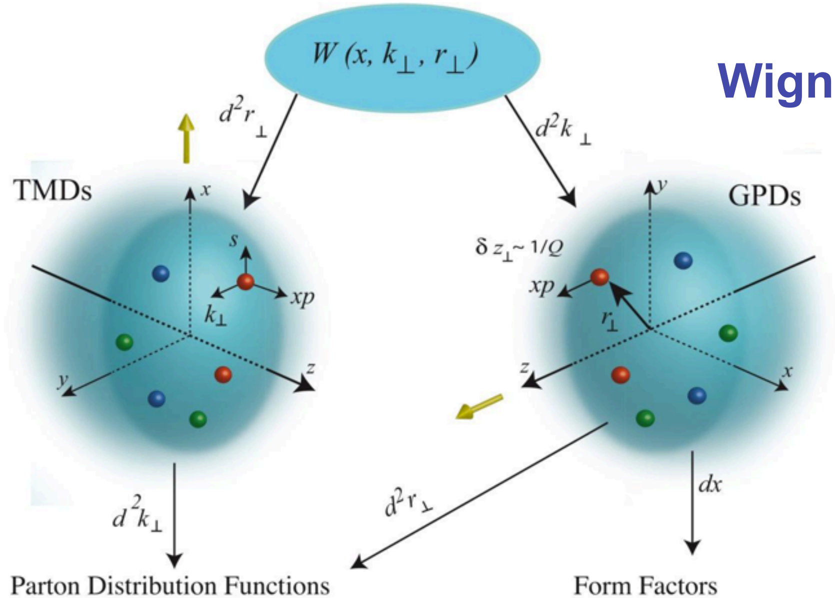
Combined analysis for gluon helicity distribution in progress

3D Imaging + GPDs

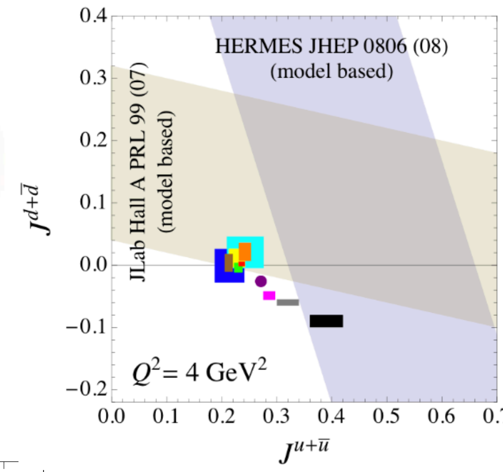
The Structure of Hadrons

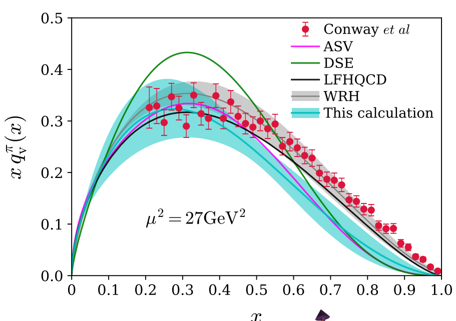
B.Musch, M.Englehardt et al

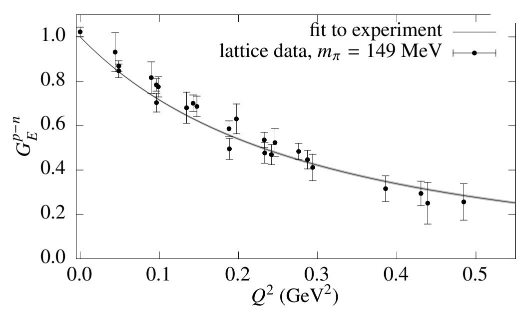


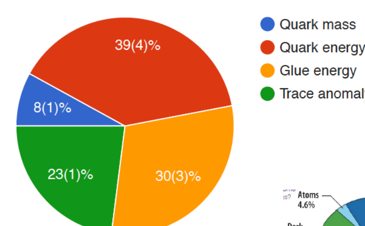


Wigner distributions



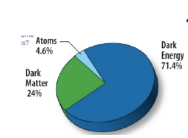






Next Frontier

χQCD



GPDs in pseudo-PDF approach

Thanks to Joe Karpie, Lattice 2023

- GPDs correspond to off-forward matrix elements. In pseudo-PDF framework, our starting point is the Generalized Ioffe Time Distributions

$$I_\mu(p', p, s = s - , \mu^2) = \langle p' \mid \bar{q}(-z^-/2)\gamma_\mu W(-z^-/s, z^-/2) q(z^-/2) \mid p \rangle_{\mu^2}$$

Where Ioffe time $\nu = (p + p')/2$, $t = (p - p')^2$ and skewness $\xi = q \cdot z / P \cdot z$

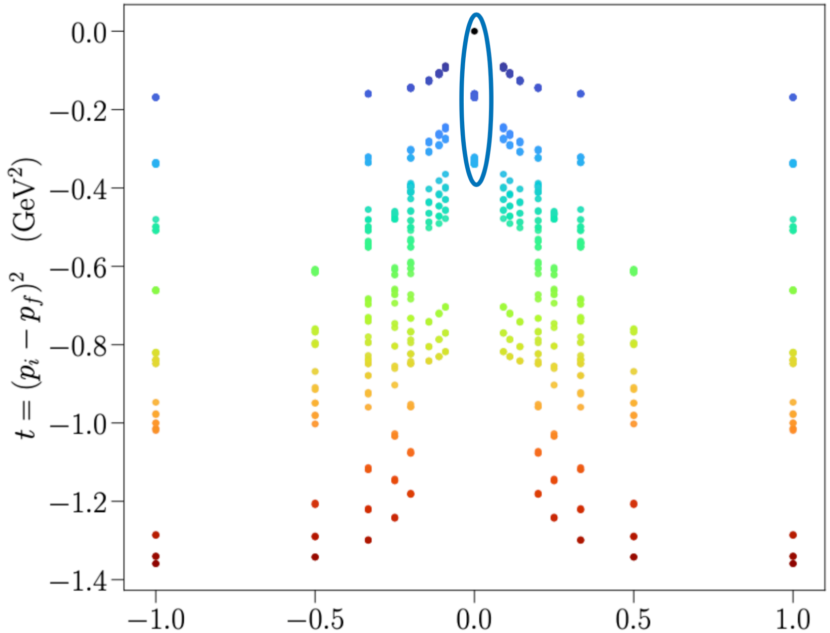
Extends to generalized pseudo-ITD in manner of pseudo ITD.

↓ GPDs

Requires solution of inverse problem

Allows us to obtain 3D GITDs/GPDs at discrete values of momentum transfer and skewness, in contrast to $x = \xi$ in DVCS.

GPDs - II



C. Egerer et al., JHEP 11 (2021) 148

Accessible values on our "paradigm" lattice

ID	a_s (fm)	m_π (MeV)	$L_s^3 \times N_t$	$N_{\rm cfg}$	$N_{\rm srcs}$	$R_{\mathcal{D}}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	4	64

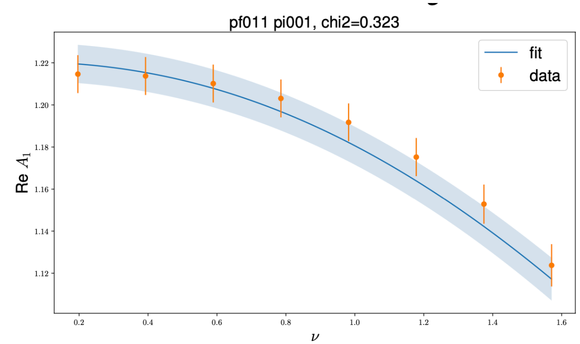
Introduce double distributions

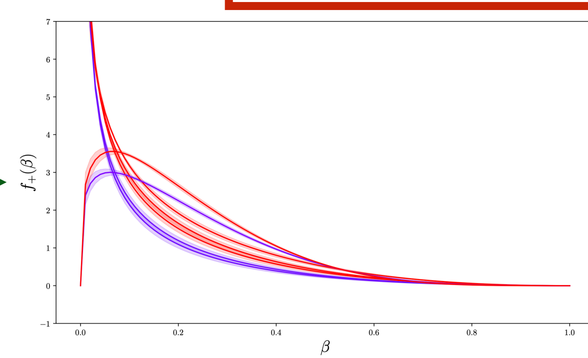
$$f(x,\xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \xi\alpha)\tilde{f}(\alpha,\beta)$$

A.Radyushkin, PLB380 (1996), 417; M.Polyakov,C.Weiss PRD60 (1999) 114017

$$A_1(\nu, t, \xi = 0, z^2) = N \int^{\xi} d\beta e^{i\nu\beta} \beta^a (1-\beta)^b$$

Thanks to Joe Karpie, Lattice 2023





Summary

- Realistic calculation of light-cone distributions from LQCD now available
- Focus on understanding systematic contributions in pseudo-PDF framework
- Distillation + boosting enables both far increased reach in momentum, and improved sampling of lattice
  - *Essential in calculations of gluon contributions*
- Are able to isolate leading twist from higher-twist and discretization contamination
- *Calculation of GPDs Underway*
- Lattice QCD + Expt - global analysis

Quark-Gluon Tomography (QGT)

Focus areas

Tom Mehen

Theory

- Theoretical studies of high-momentum transfer processes using perturbative QCD
- Study the properties of GPDs using non-perturbative methods

Lattice QCD

- Non-perturbative calculations of Euclidean correlation functions relevant to GPDs

Phenomenology

- Global analysis of GPDs based on experimental data, theoretical constraints, and lattice QCD input, using modern data analysis techniques for inference and uncertainty quantification

Support and training

- Support 12 postdocs and 6 graduate students
- Provide summer schools and workshops
- Create three bridge positions in nuclear theory

Thanks to Chris Monahan

Focus areas: Lattice QCD

Expertise in:

- numerical methods in lattice gauge theories
- simulations of QCD
- non-perturbative renormalization
- numerical calculations for hadron structure

Studying:

- quark and gluon PDFs and GPDs
- gravitational form factors
- nucleon spin, momentum & angular momentum
- quark charge & renormalization
- electric dipole moments
- x-dependence of PDFs & GPDs
- neutrino-nucleus scattering cross-sections
- structure of light nuclei

Pseudo-PDF in Precision Era

FLAG 2019 Flavour Lattice Averaging Group

What do we need?

ID	a(fm)	M_π(MeV)	β	c_{SW}	κ	$L^3 \times T$	N_{cfg}
$\tilde{A}5$	0.0749(8)	446(1)	5.2	2.01715	0.13585	$32^3 \times 64$	1904
E5	0.0652(6)	440(5)	5.3	1.90952	0.13625	$32^3 \times 64$	999
N5	0.0483(4)	443(4)	5.5	1.75150	0.13660	$48^3 \times 96$	477

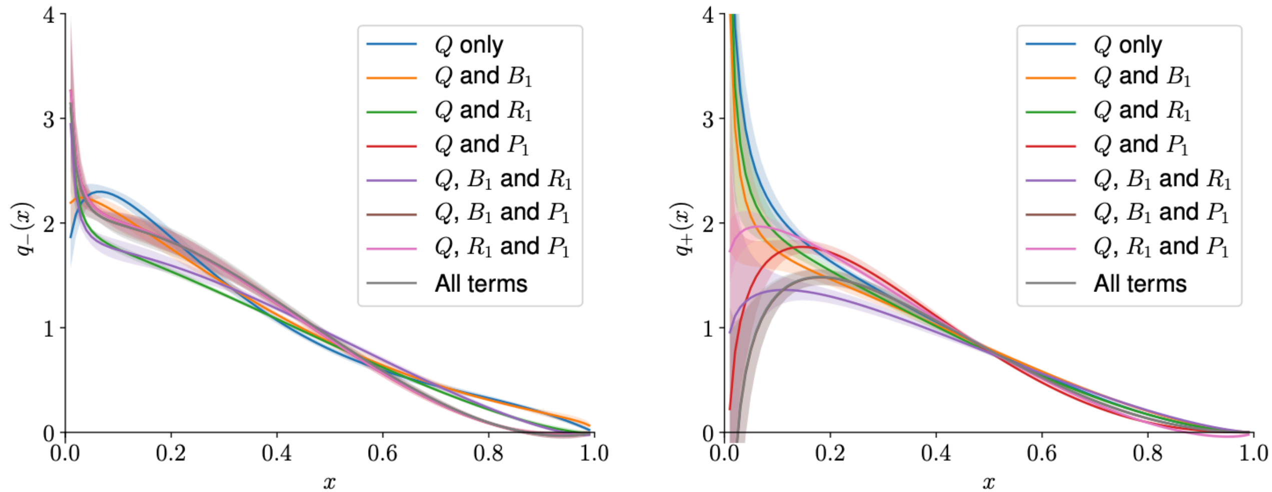
J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos, arXiv:2105.13313

$$\mathfrak{M}(p,z,a) = \mathfrak{M}_{\rm cont}(\nu,z^2) + \sum \left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\rm QCD})^n R_n(\nu)\,.$$

Lattice spacing

$$\mathfrak{M}_{\rm cont}(\nu,z^2) = \mathfrak{M}_{\rm lt}(\nu,z^2) + \sum_{n=1} (z^2\Lambda^2_{\rm QCD})^n B_n(\nu)\,.$$

Higher twist



Distillation

Low-rank approximation to (typically) Jacobi-smearing kernel

$$-\nabla^2(t)\xi^{(k)}(t) = \lambda^{(k)}(t)\xi^{(k)}(t)$$

M. Peardon *et al.*, PRD80,054506 (2009)

Rank →

$$\Box(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{R_{\mathcal{D}}} \xi_a^{(k)}(\vec{x},t)\,\xi_b^{(k)\dagger}(\vec{y},t)\,,$$

Components of distillation:

$$\tau_{\alpha\beta}^{(l,k)}(t',t) = \xi^{(l)\dagger}(t')\,M_{\alpha\beta}^{-1}(t',t)\,\xi^{(k)}(t)$$

Perambulators → quark propagation

$$\Phi_{\alpha\beta\gamma}^{(i,j,k)}(t) = \epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a\left(\mathcal{D}_2\xi^{(j)}\right)^b\left(\mathcal{D}_3\xi^{(k)}\right)^c(t)\,S_{\alpha\beta\gamma}$$

Elementals → (baryon) operators

Projection to irrep

$$C_{rs}(t) = \sum_{\vec{x},\vec{y}}\langle 0 \mid \mathcal{O}_r(t,\vec{x})\mathcal{O}_s^\dagger(0,\vec{y}) \mid 0\rangle \equiv \mathrm{Tr}\left[\Phi_r(t)\otimes\tau(t,0)\tau(t,0)\tau(t,0)\otimes\Phi_s(0)\right]$$

Matrix of correlators

Extension to 3pt functions straightforward

Discretization *Higher Twist*