Moments of GPDs from the OPE of nonlocal quark bilinears

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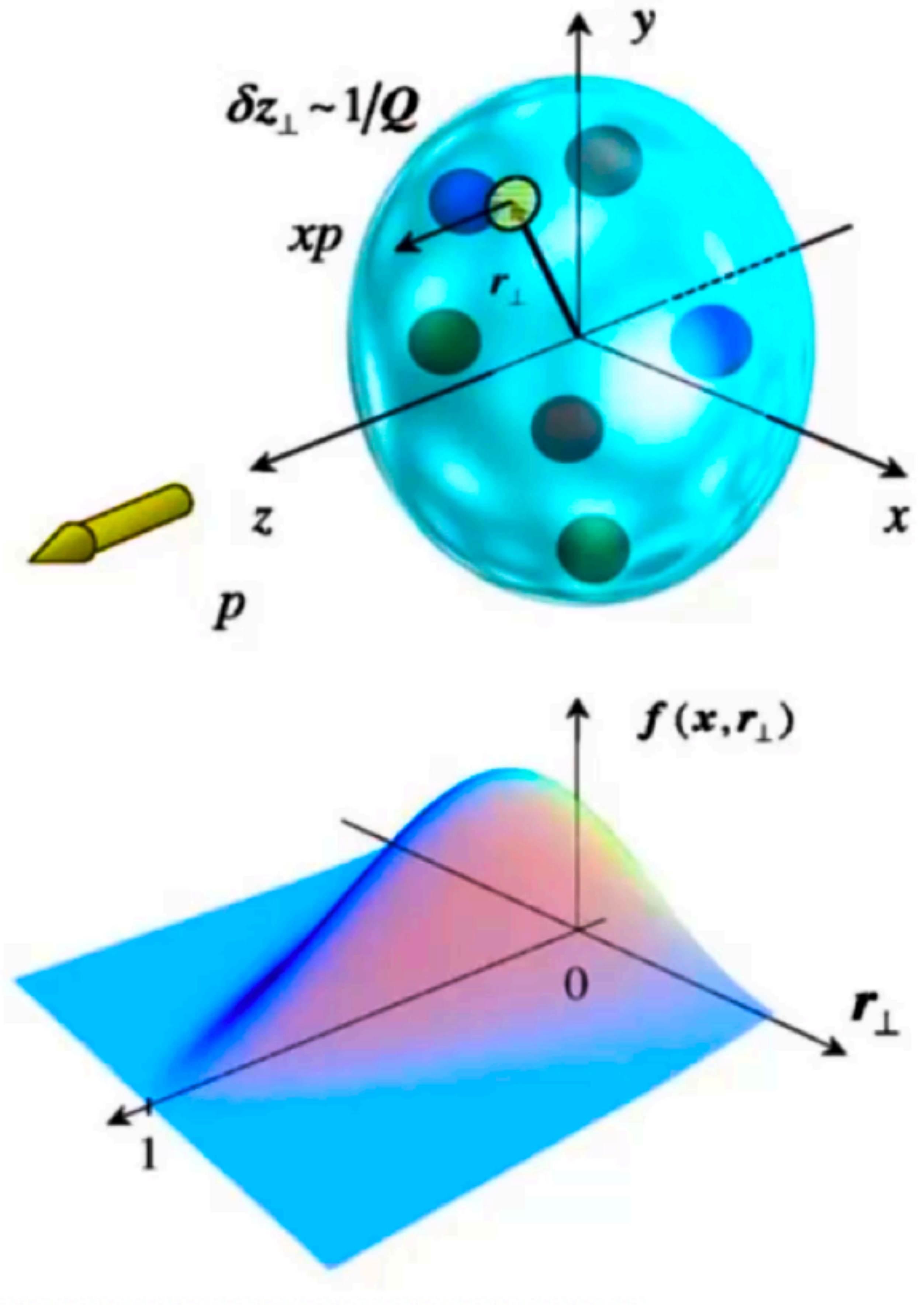
In collaboration with: S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, F. Steffens, and Y. Zhao

Towards improved hadron femtography with hard exclusive reactions

Jefferson Lab, Aug 7–11, 2023

Generalized parton distributions

GPDs goes far beyond the 1D PDFs and the transverse structure encoded in the form factors.



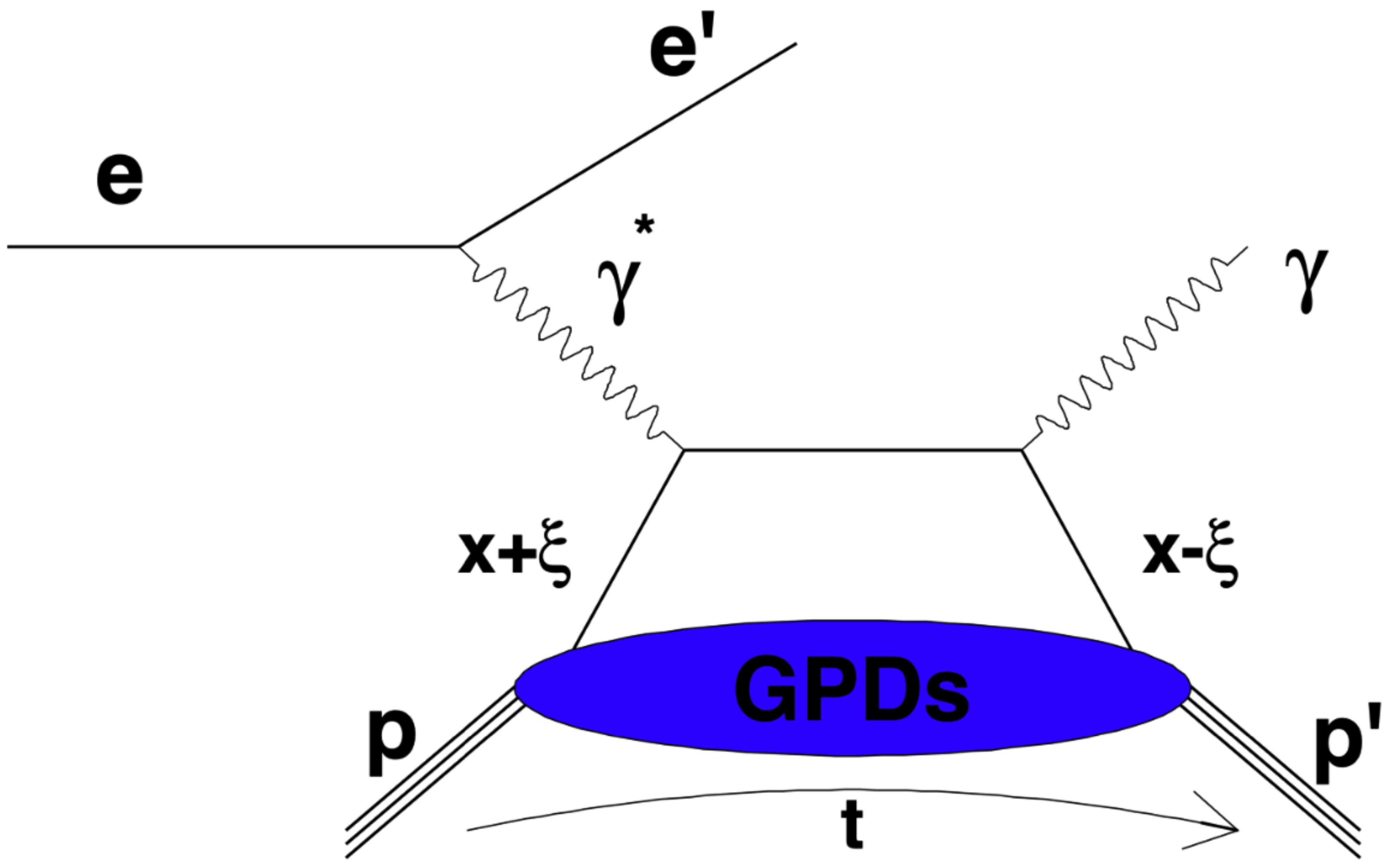
$$F_q(x,\xi,t)=\int\frac{dz^-}{4\pi}e^{-ixP^+z^-}\langle p_f|\,\bar{q}(-\tfrac{z}{2})\gamma^\mu\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})\,|p_i\rangle$$

- Offer insights into the 3D image of hadrons.
- Give access to the orbital motion and spin of partons.
- Have a relation to pressure and shear forces inside hadrons.

- Ji, PRL 78 (1997)
- Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

Generalized parton distributions

DVCS



The golden process to study the quark GPDs is DVCS

Challenging:

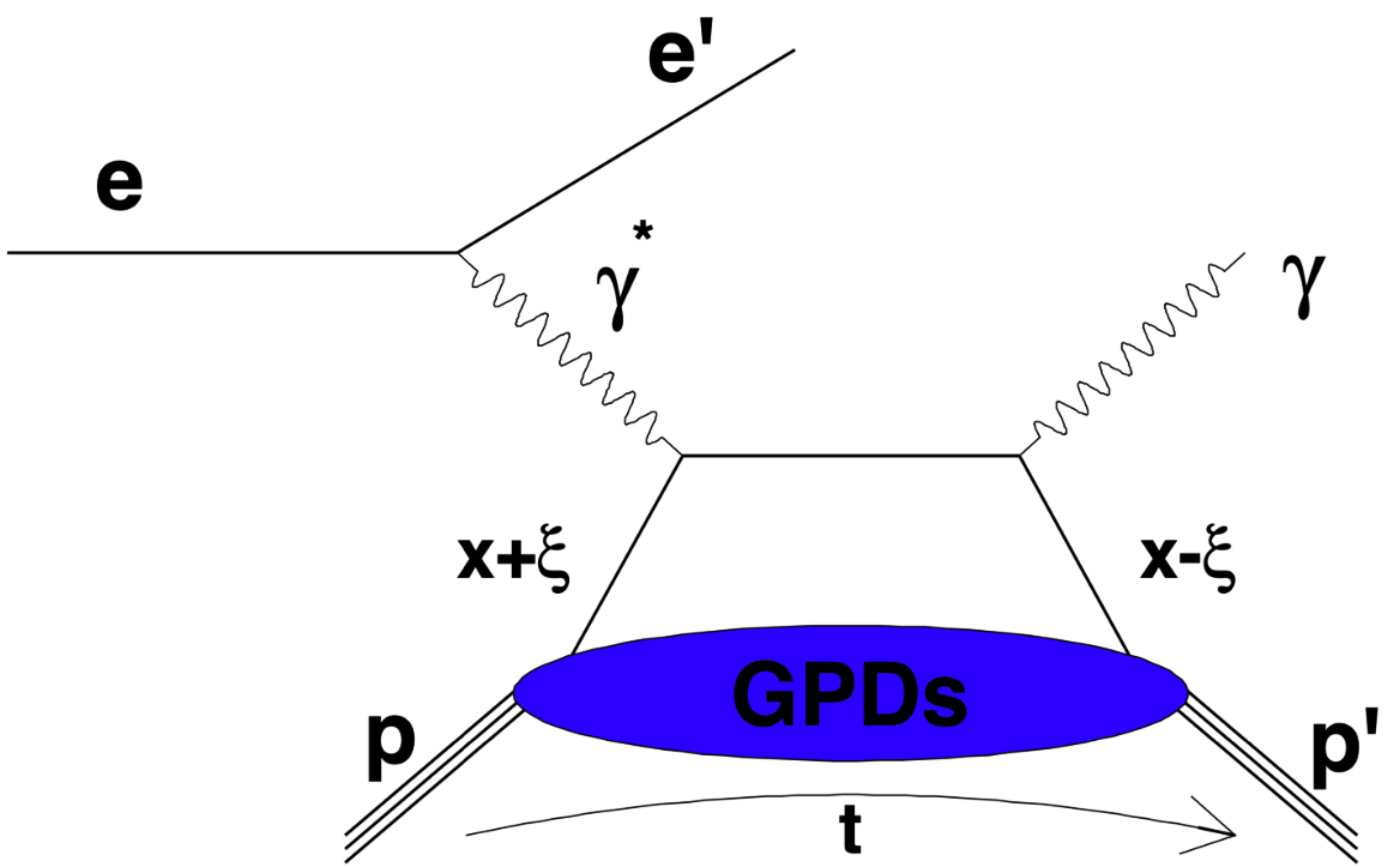
- Observables appear at the amplitude level.
- Multi-dimensionality (x, ξ, t).
- The momentum fraction x is integrated over (Compton Form Factors).

$$\mathcal{F}(\xi, t; Q^2) = \int_{-1}^{1} \mathrm{d}x \left[\frac{1}{\xi - x - i\epsilon} \pm \frac{1}{\xi + x - i\epsilon} \right] F(x, \xi, t; Q^2)$$

- Ji, PRL 78 (1997)

Generalized parton distributions

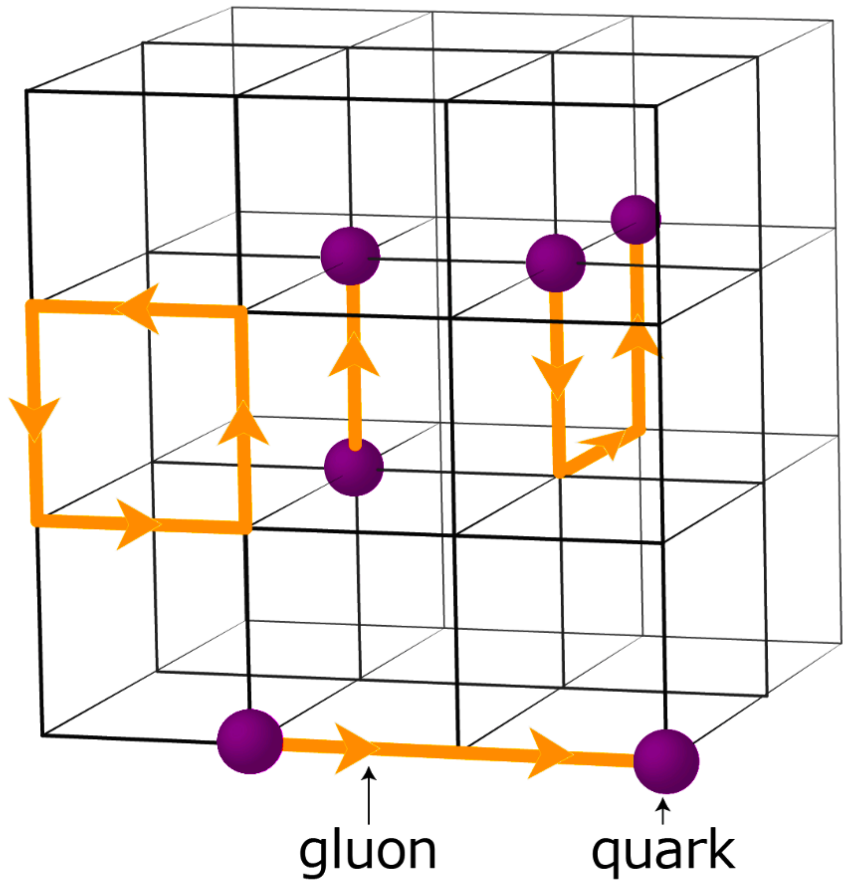
DVCS



Challenging:

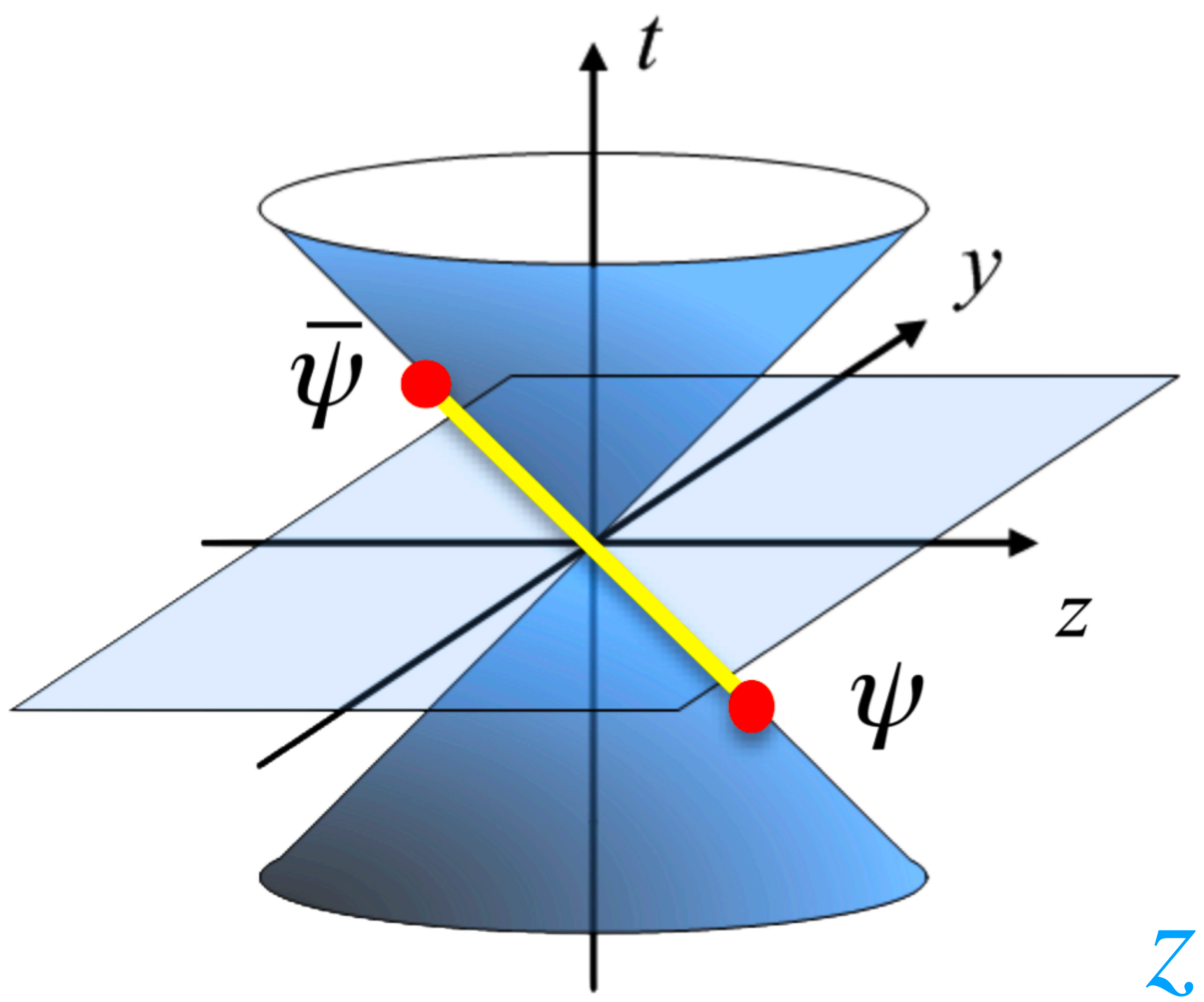
- Observables appear at the amplitude level.
- Multi-dimensionality (x, ξ, t).
- The momentum fraction x is integrated over (Compton Form Factors).

Complementary knowledge from lattice QCD is essential.



• Ji, PRL 78 (1997)

GPDs from Lattice QCD: local operator



$z^2 = 0$

$$\langle p_f | \bar{q}(-\frac{z^-}{2})\gamma^\mu \mathscr{W}(-\frac{z^-}{2}, \frac{z^-}{2}) q(\frac{z^-}{2}) | p_i \rangle$$

Light-cone correlation: Cannot be calculated on the lattice

OPE of the light-cone operator

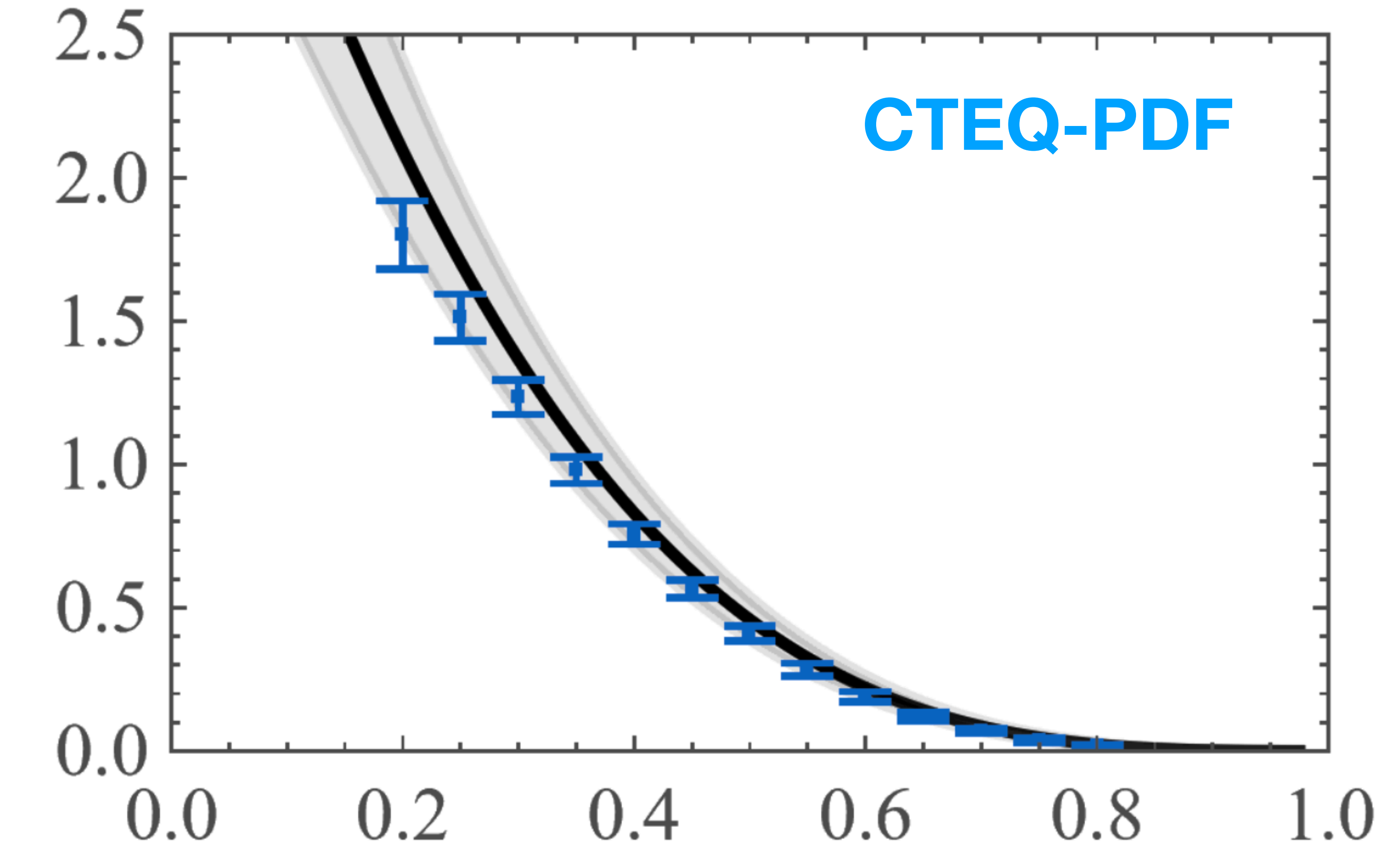
$$\bar{q}(-\frac{z^-}{2})\gamma^+ \mathscr{W}(-\frac{z^-}{2}, \frac{z^-}{2}) q(\frac{z^-}{2})$$

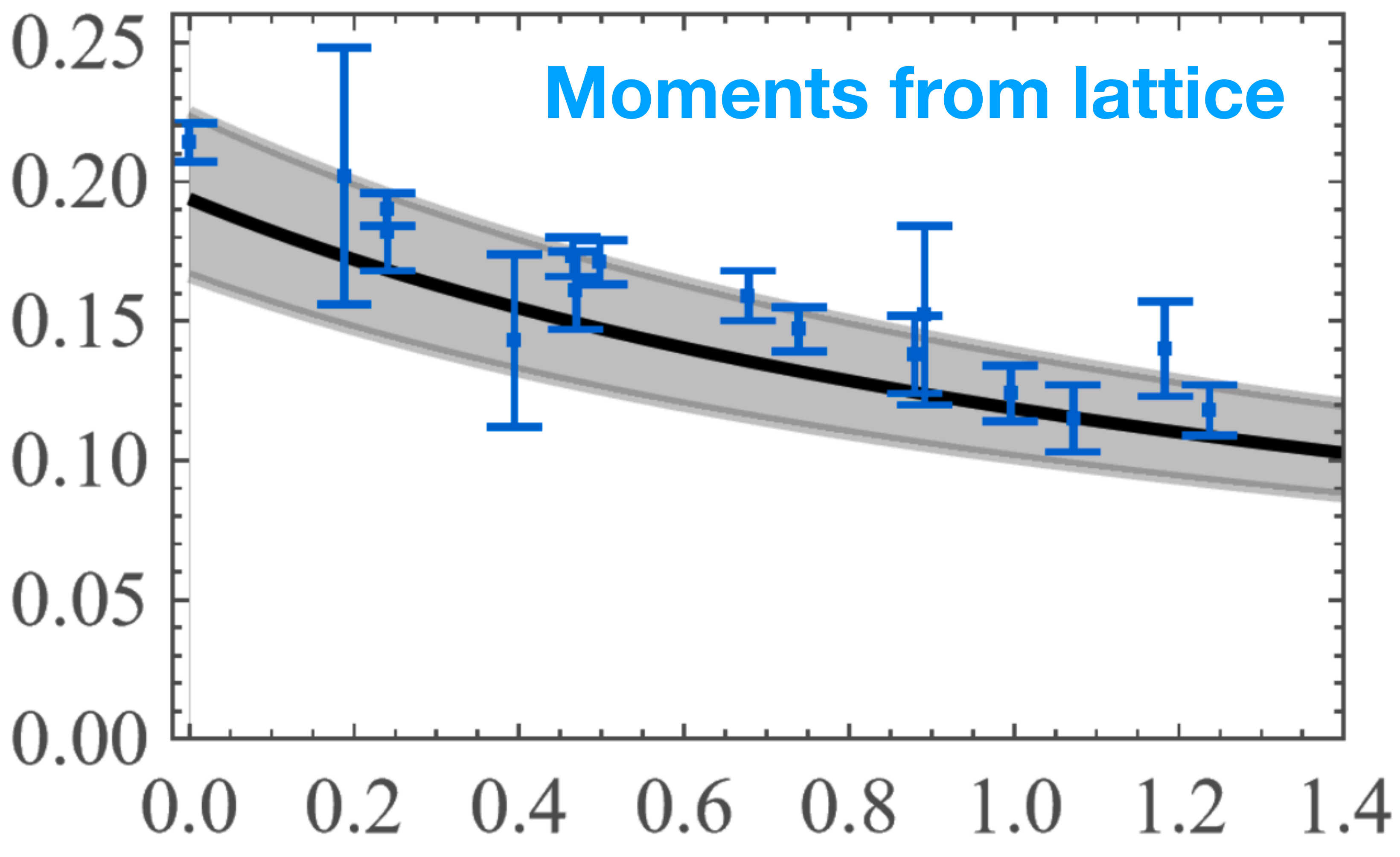
$$= \sum_{n=0}^{\infty} \frac{(-iz^-)^n}{n!} O^{++\ldots+}(\mu)$$

- Moments from Local operator

$$\bar{q}\gamma^{\{\mu_0} iD^{\mu_1} \ldots iD^{\mu_n\}} q$$

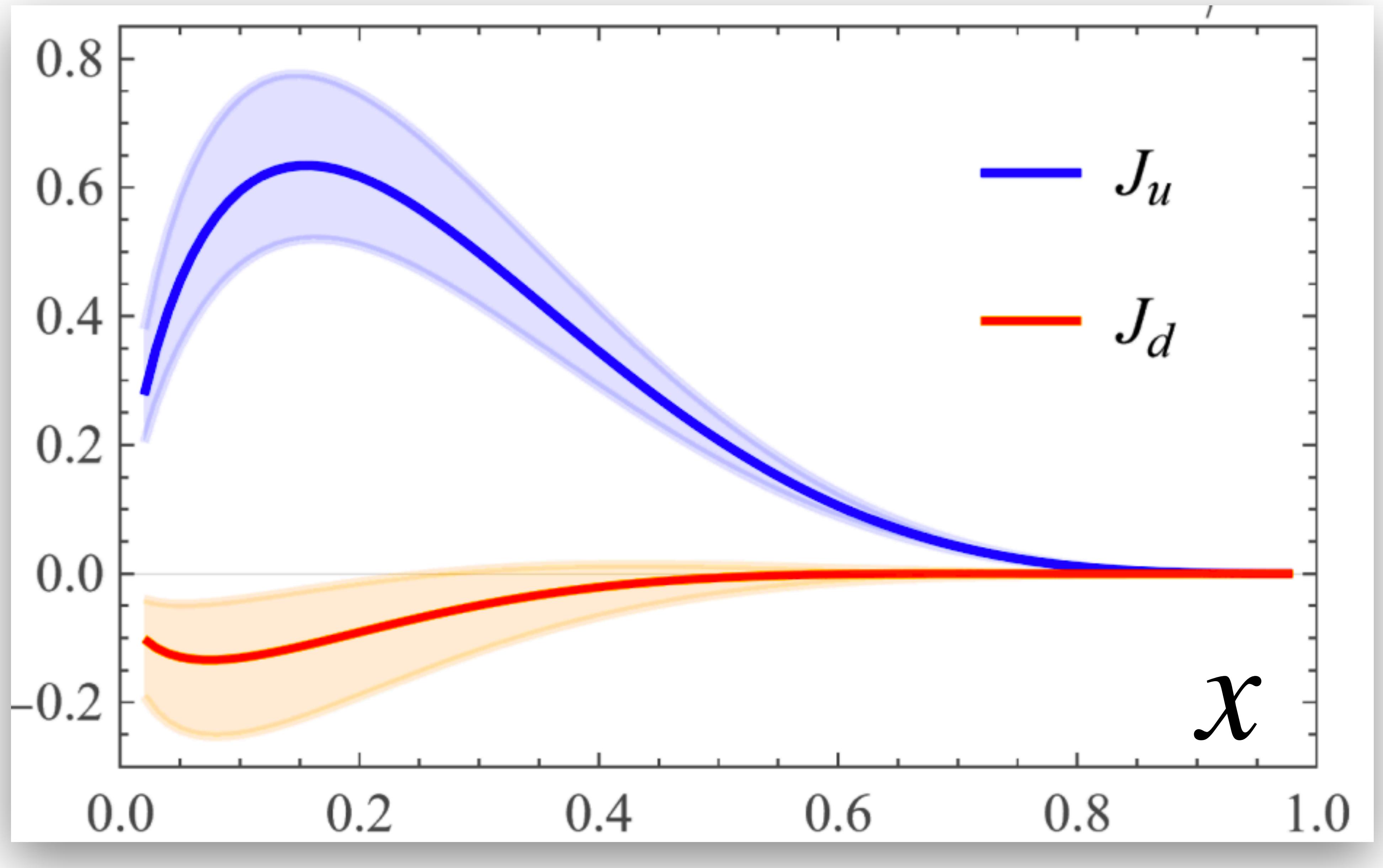
GPDs from global analysis and lattice





simultaneous fit

Transverse angular momentum density $J_{u/d}(x)$



$$J_{u/d}(x,t) = \frac{1}{2} x \left(H_{u/d}(x,t) + E_{u/d}(x,t) \right)$$

- Y. Guo, X. Ji, K. Shiells, JHEP 09 (2022) 215

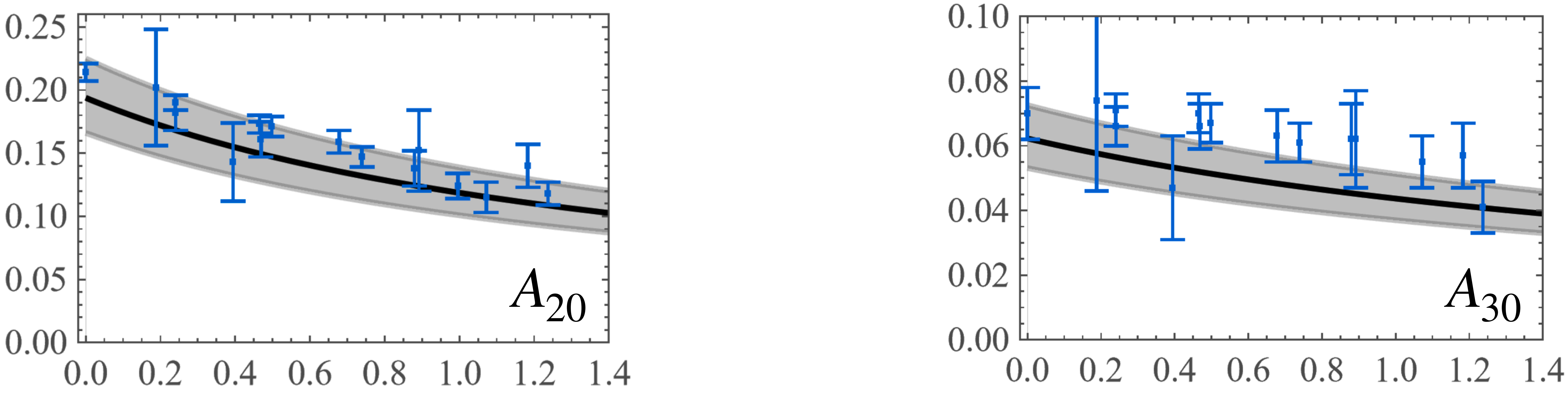
GPDs from Lattice QCD: local operator

- Moments from Local operator

$$\bar{q}\gamma^{\{\mu_0}iD^{\mu_1}\ldots iD^{\mu_n\}}q$$

High dimensional operator

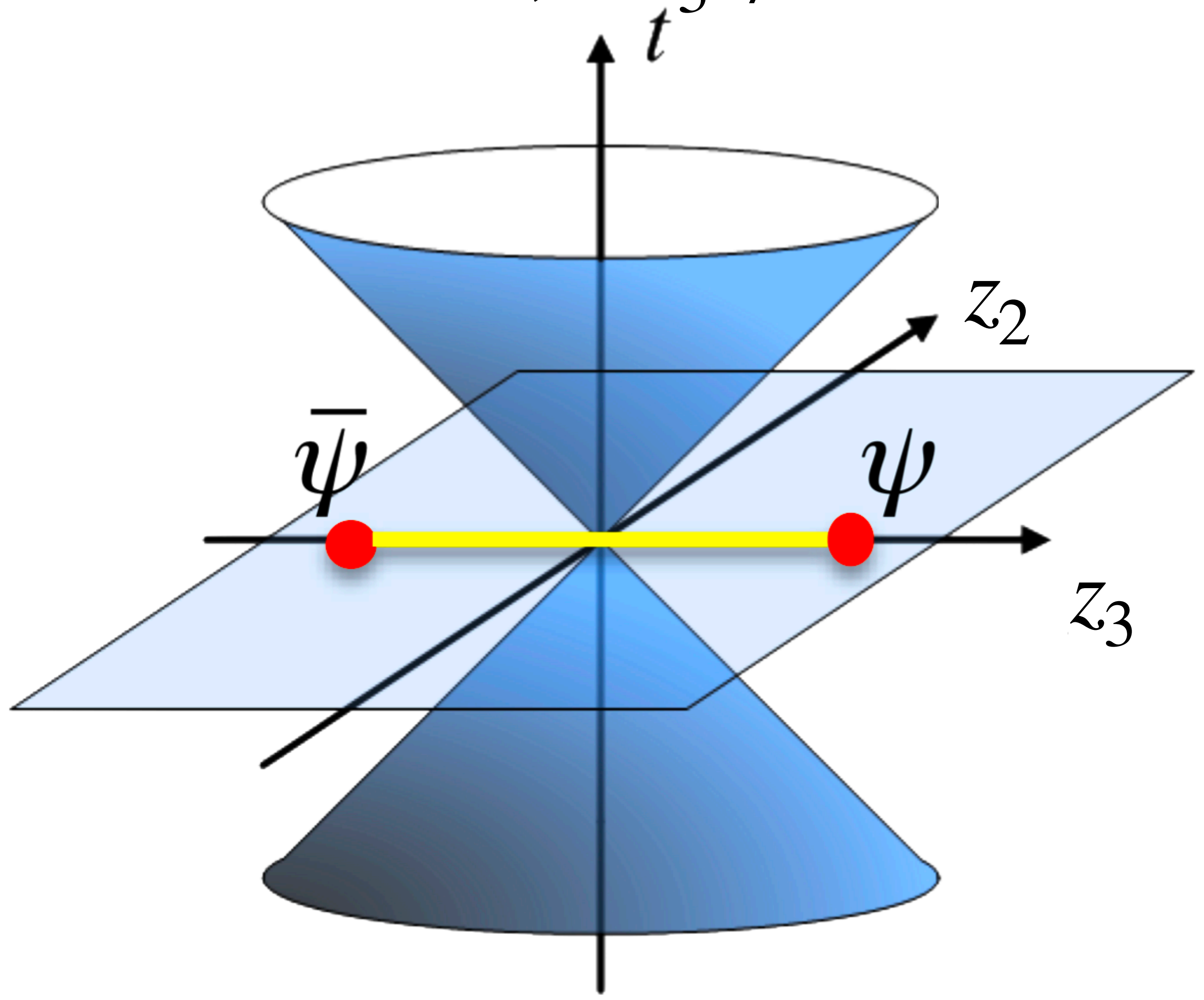
- Limited up to $\langle x^3 \rangle$ due to signal decay and power-divergent mixing under renormalization.



- LHPC, Phys.Rev.D 77 (2008) 094502

GPDs from Lattice QCD: non-local operator

$t = 0, \quad z_3 \neq 0$



$$F^\mu(z, P, \Delta) = \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^\mu \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

$$z = (0,0,0,z_3), \; z^2 = z_3^2$$

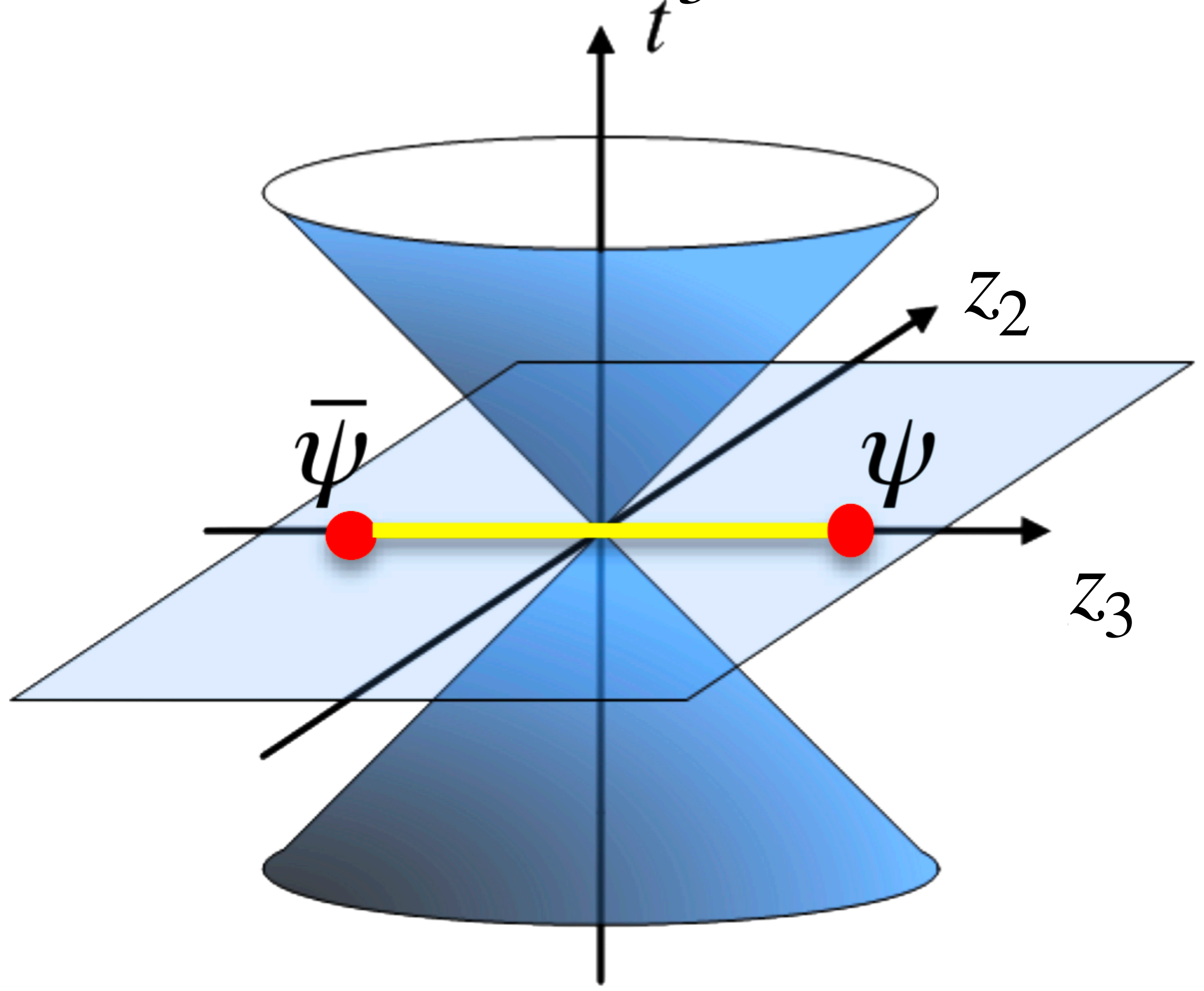
- **Large-momentum effective theory**: x-space matching of quasi-PDF.
  - X. Ji, PRL 2013
  - X. Ji, et al, RevModPhys 2021

  Y. Zhao's talk on Thu.

- **Short distance factorization** of the quasi-PDF matrix elements or the pseudo-PDF approach.
  - A. Radyushkin, PRD 100 (2019)
  - A. Radyushkin, Int.J.Mod.Phys.A 2020

GPDs from short distance factorization

$t = 0, \quad z_3 \neq 0$



$$F^\mu(z, P, \Delta) = \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^\mu \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

$$z = (0,0,0,z_3), \; z^2 = z_3^2$$

OPE of the equal-time operator

$$\bar{q}(-\tfrac{z}{2})\gamma^\mu \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) = \sum_{n=0}^{\infty} C_n(\mu^2 z^2) \frac{(-iz)^n}{n!} e_{\mu_1} \ldots e_{\mu_n} O^{\mu_0 \mu_1 \ldots \mu_n}(\mu)$$

+Higher twist operators

GPDs from short distance factorization

SDF/OPE of the quasi-GPD matrix elements:
zero skewness case

$$F^R(z,P,\Delta) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2)\langle x^n\rangle(t;\mu) + \mathcal{O}(z^2\Lambda_{\mathrm{QCD}}^2)$$

Perturbative coefficients

E.g.

$$\int_{-1}^{1} dx\, x^n H^q(x,\xi=0,t) = A^q_{n+1,0}(t)$$

$$\int_{-1}^{1} dx\, x^n E^q(x,\xi=0,t) = B^q_{n+1,0}(t)$$

- The perturbative matching is valid in short range of z^2.
- The information is limited to the first moments by the range of finite $\lambda = zP$.
- Free of power divergent mixing so that can be systematically improved.

quasi-GPD matrix elements

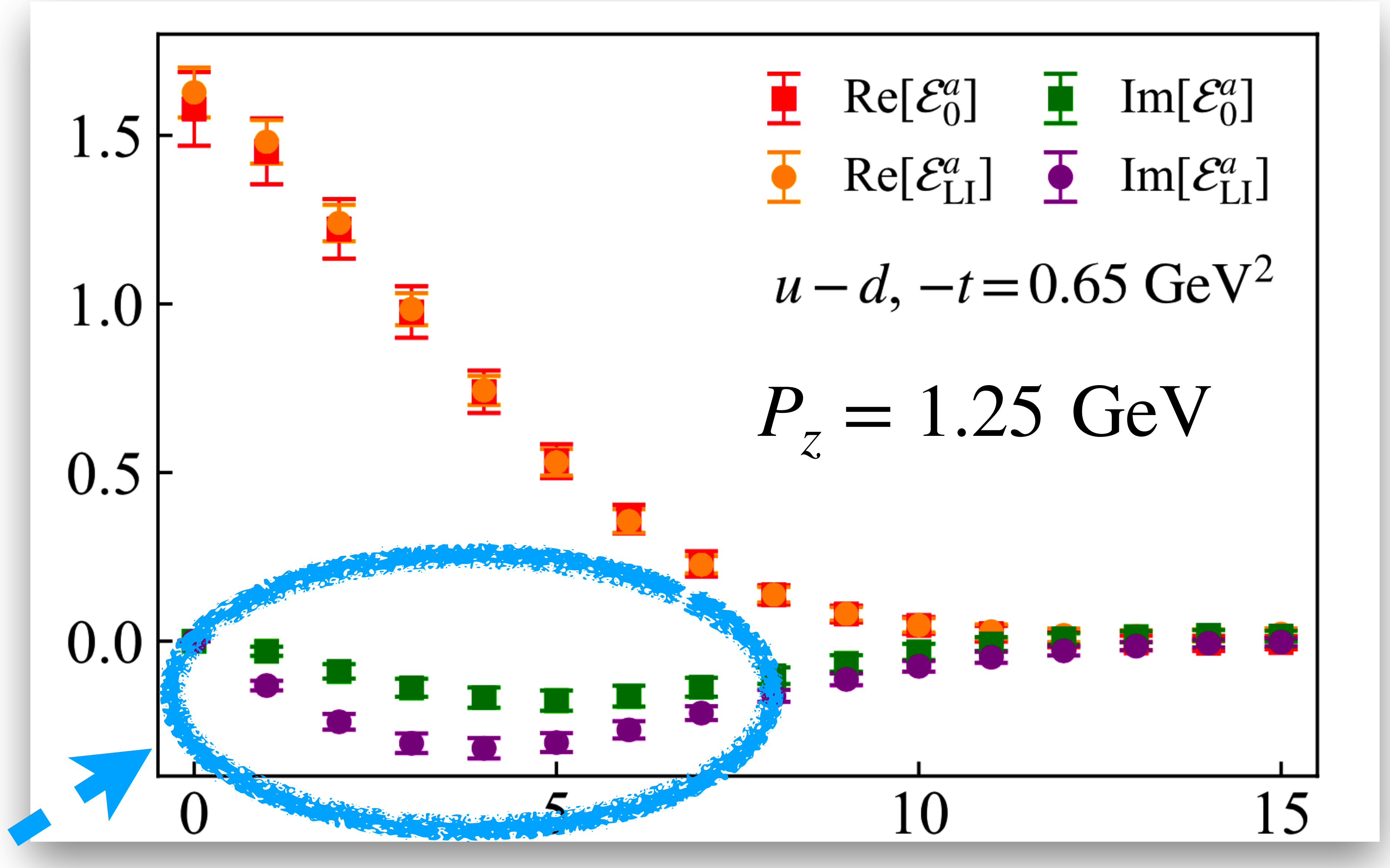
The unpolarized qGPD matrix elements in γ_0 definition:

$$F^0(z,P,\Delta) = \langle p_f |\, \bar{q}(-\tfrac{z}{2})\gamma^0 \mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2}) q(\tfrac{z}{2})\, | p_i\rangle$$

$$= \bar{u}(p_f,\lambda')\left[\gamma^0 \mathcal{H}_0(z,P,\Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m}\mathcal{E}_0(z,P,\Delta)\right] u(p_i,\lambda)$$

Problem:

- The qGPDs are frame dependent though light-cone GPDs are Lorentz invariant.
- Computationally expensive for multiple $-t = Q^2$ in symmetric frame.

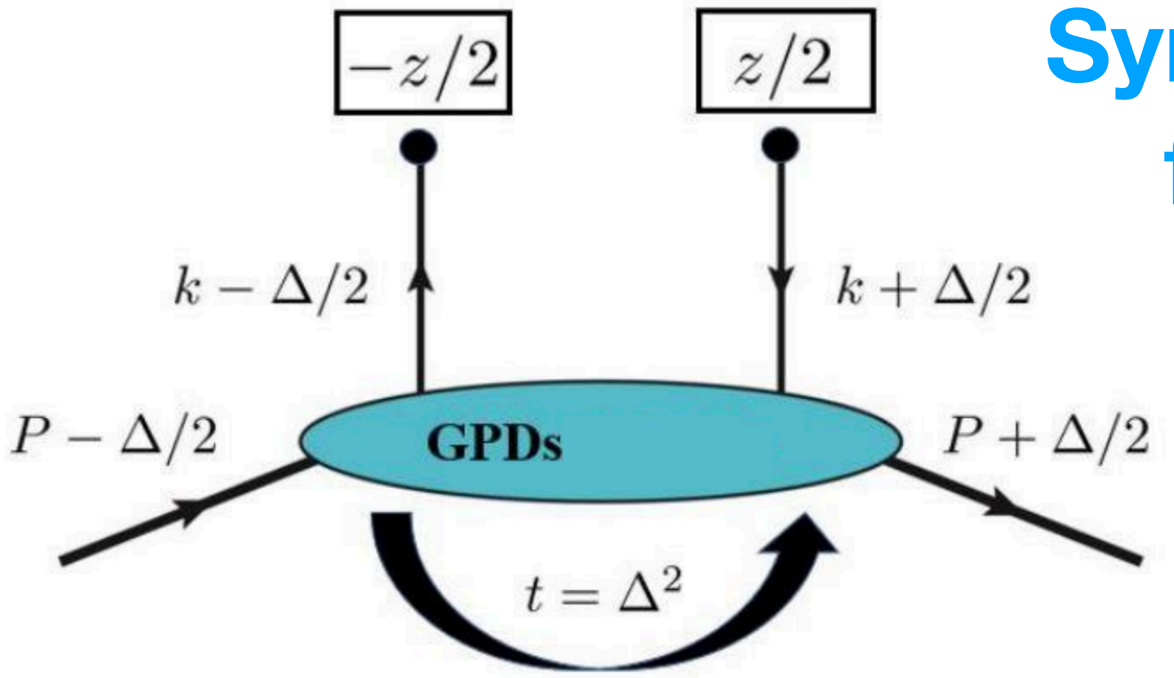


frame-dependent power corrections $\sim \Delta/P$ at the tree level

quasi-GPD matrix elements

The matrix elements can be parametrized in terms of Lorentz invariant amplitudes $A_i\left(z\cdot P, z\cdot\Delta, \Delta^2, z^2\right)$:

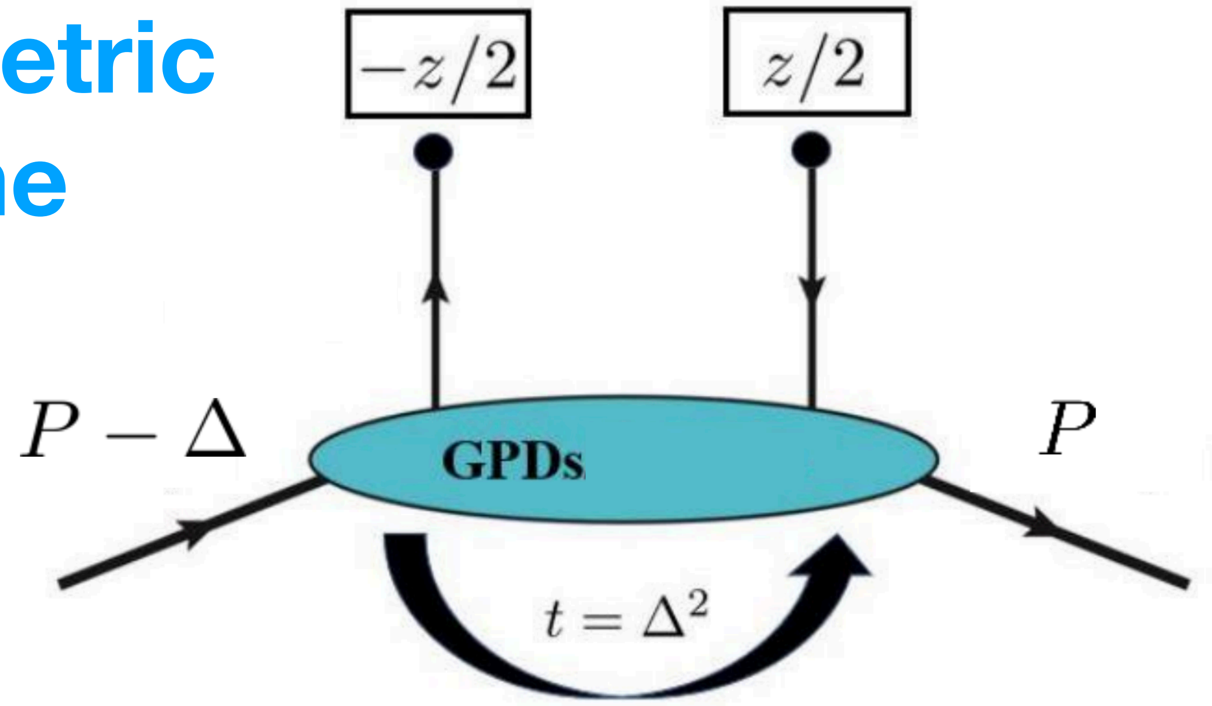
$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$



Symmetric frame

Lorentz transform

Asymmetric frame



New development:

- S. Bhattacharya, XG, et al., Phys.Rev.D 106 (2022), 114512

- Construct qGPD from asymmetric-frame calculation.
- Computational much cheaper for multiple $-t$, and possibly reducing the power corrections with proper construction.

quasi-GPD matrix elements

The matrix elements can be parametrized in terms of Lorentz invariant amplitudes $A_i\left(z\cdot P, z\cdot\Delta, \Delta^2, z^2\right)$:

$$F^{\mu}(z,P,\Delta)=\bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+mz^{\mu}i\sigma^{z\Delta}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$

A Lorentz invariant (LI) choice analogous to the light-cone GPD:

$$\mathscr{H}(z,P,\Delta)=A_1+\frac{\Delta\cdot z}{P\cdot z}A_3$$

$$\mathscr{E}(z,P,\Delta)=-A_1-\frac{\Delta\cdot z}{P\cdot z}A_3+2A_5+2P\cdot zA_6+2\Delta\cdot zA_8$$

- Differ from Light-cone GPD only by $z^2\neq 0$

Renormalization

- The operator can be **multiplicatively renormalized**

• X. Ji, J. H. Zhang and Y. Zhao, PRL120.112001
• J. Green, K. Jansen and F. Steffens, PRL.121.022004

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m(a)|z|} Z(a)[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

- Short distance factorization with **ratio scheme renormalization**

• A. V. Radyushkin et al., PRD 96 (2017)
• BNL, PRD 102 (2020)

$$\mathscr{M}(z^2, zP, \Delta^2) = \frac{\mathscr{H}^R(z, P, \Delta; \mu)}{\mathscr{H}^R(z, P=0, \Delta=0; \mu)} = \frac{\mathscr{H}^B(z, P, \Delta; a)}{\mathscr{H}^B(z, P=0, \Delta=0; a)}$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle (\Delta^2; \mu) + \mathscr{O}(z^2 \Lambda^2_{\mathrm{QCD}})$$

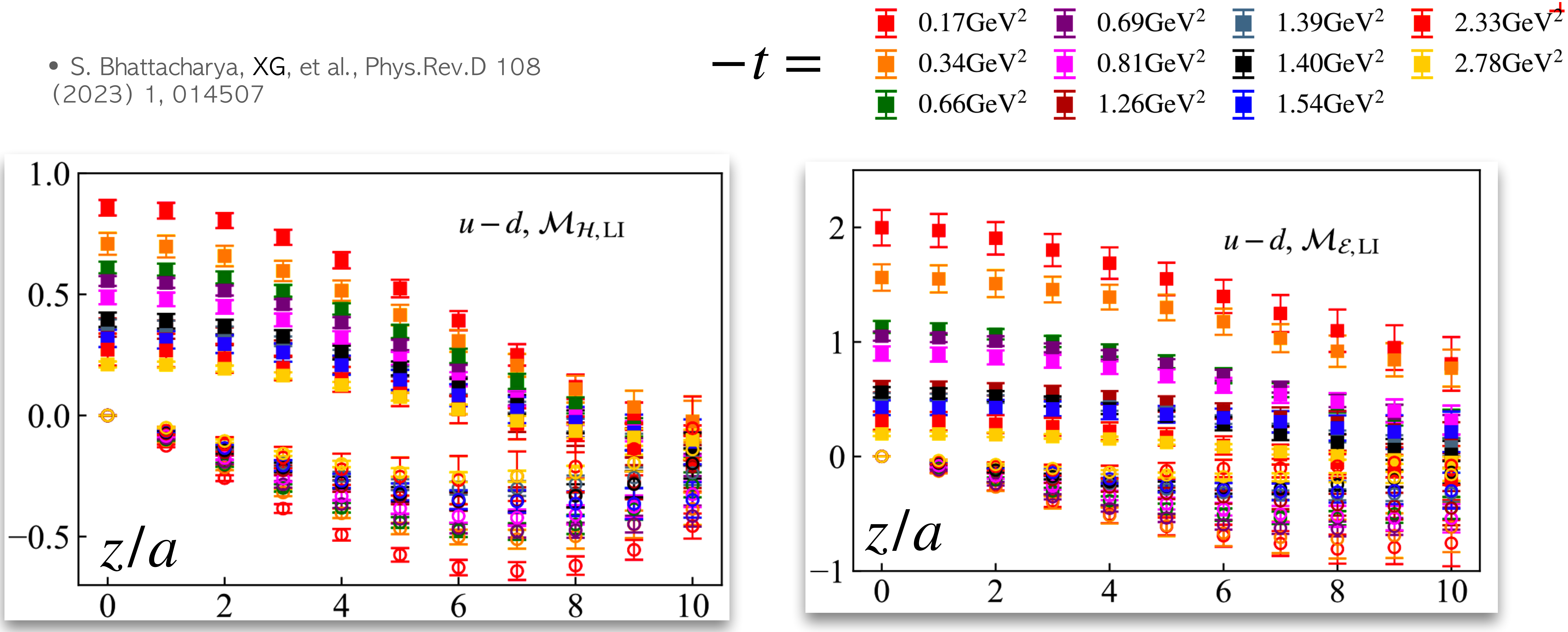
$C_n^{\overline{MS}}(\mu^2 z^2) = 1 + \alpha_s C^{(1)}(\mu^2 z^2) + \ldots$ up to NNLO

- **Lattice setup**

$m_\pi = 260$ MeV, $a = 0.093$ fm, $32^3 \times 64$, $N_f = 2+1+1$ twisted mass fermions

Renormalized matrix elements

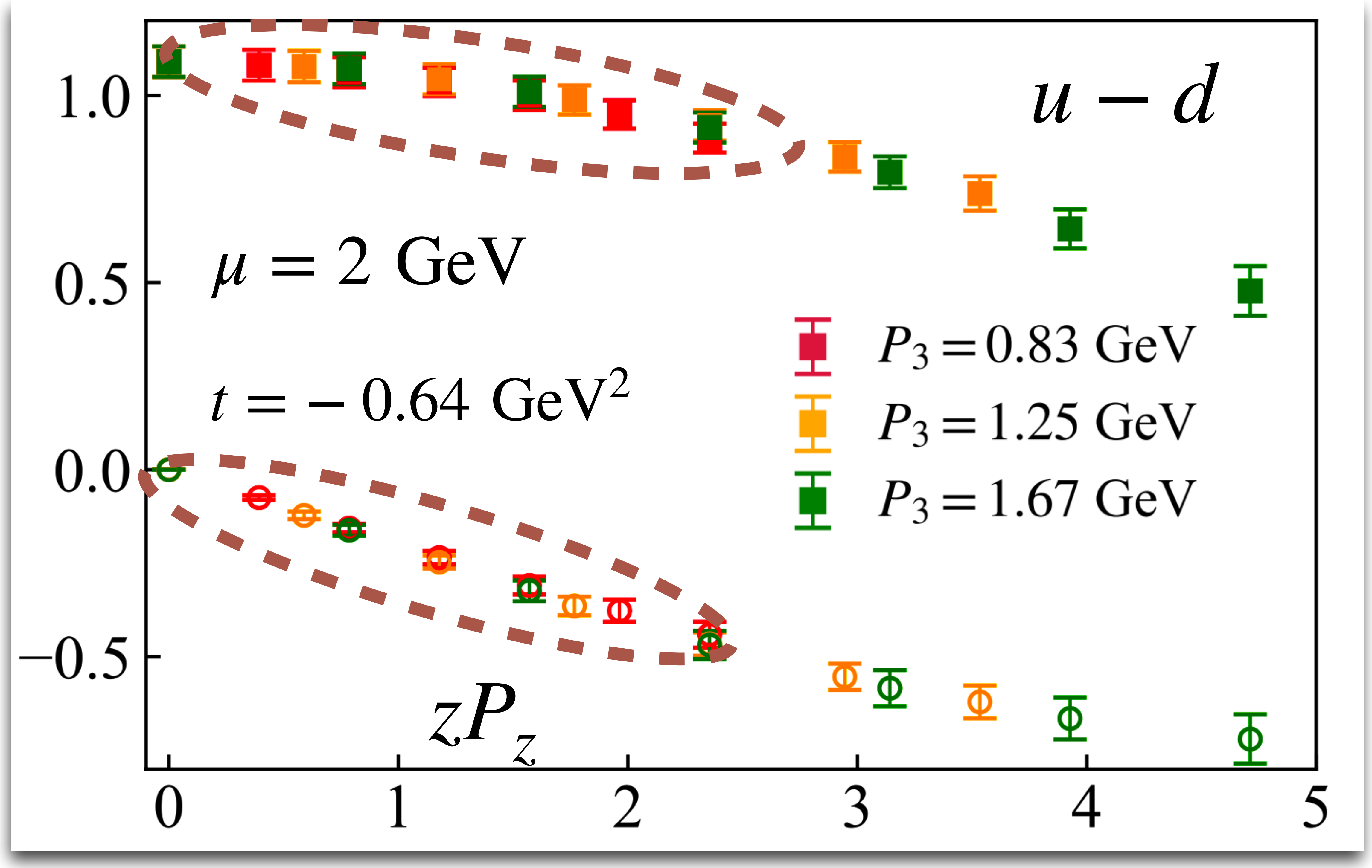
- S. Bhattacharya, XG, et al., Phys.Rev.D 108 (2023) 1, 014507

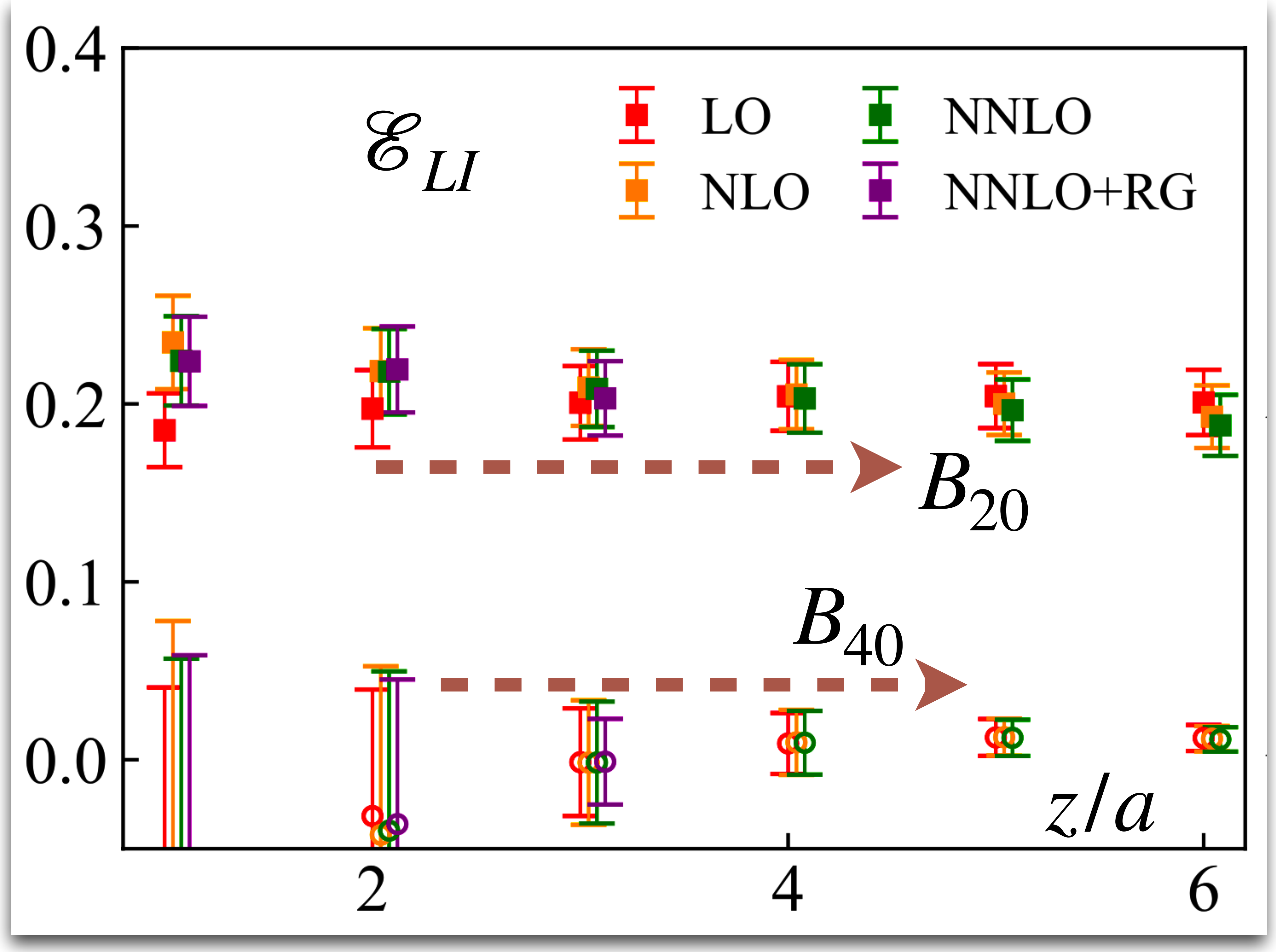


- filled symbols: real part, sensitive to even moments
- unfilled symbols: imaginary part, sensitive to odd moments

$P_z = 1.25$ GeV, $a = 0.093$ fm

SDF of qGPDs: LI definition

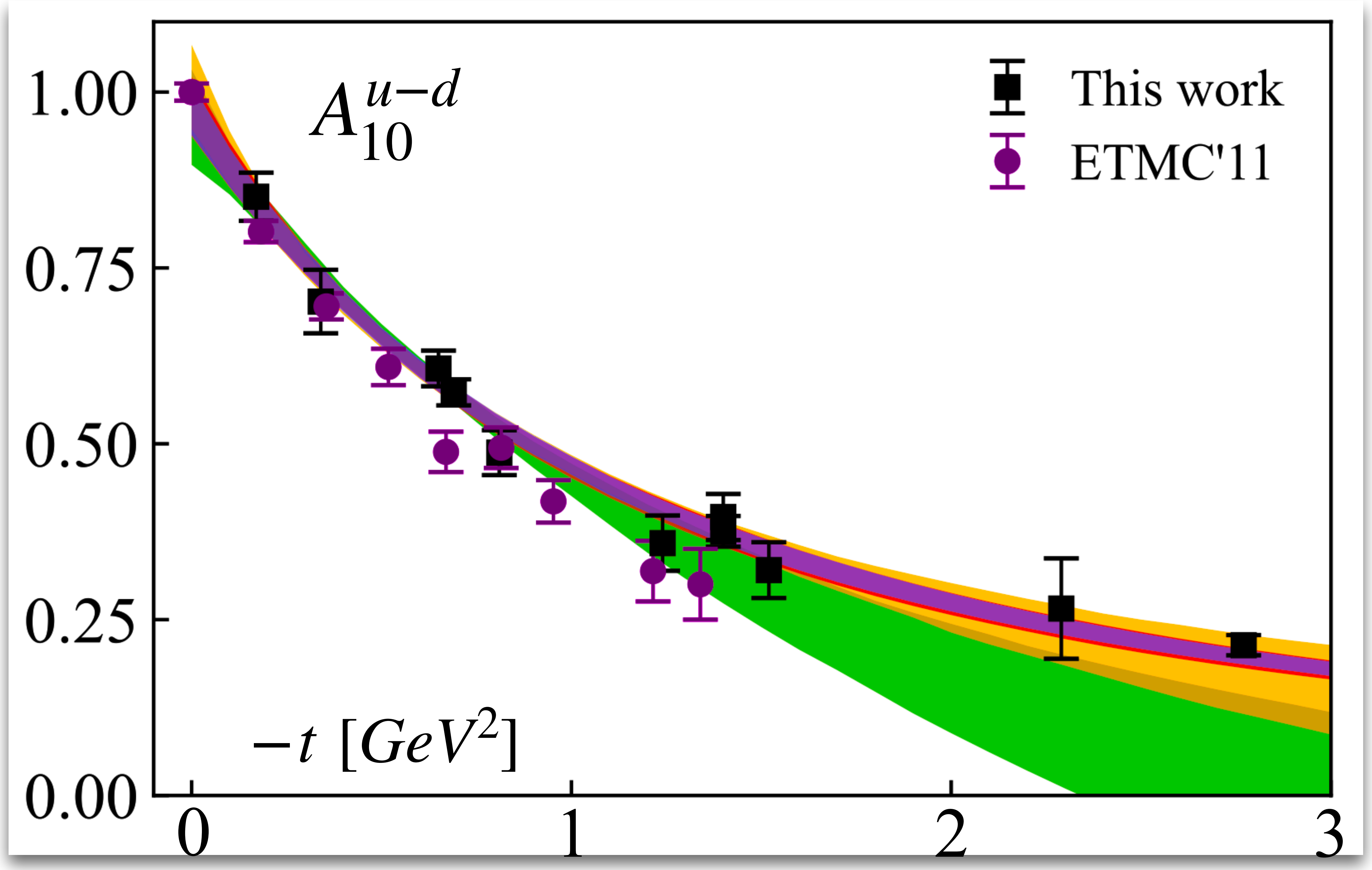


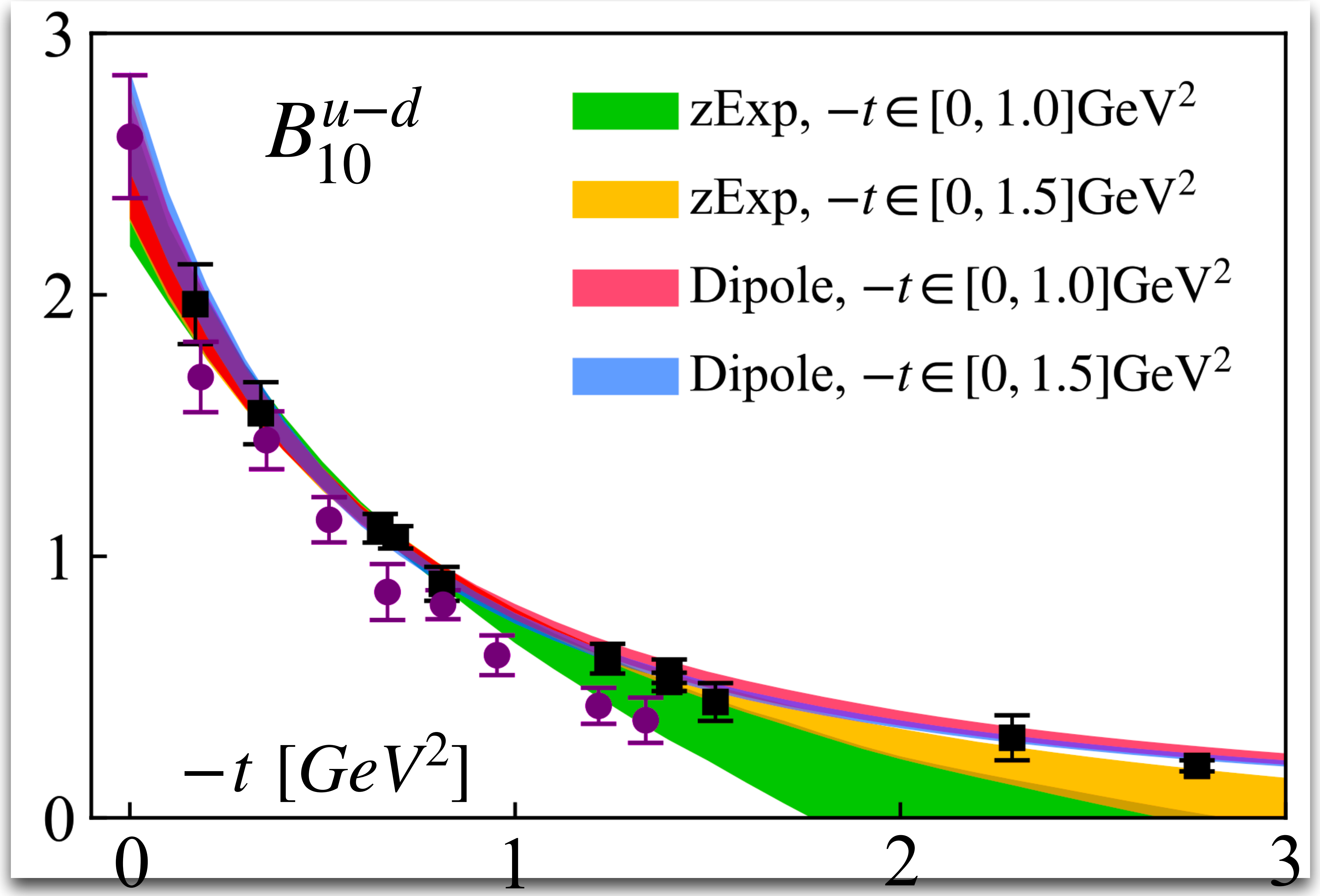


- Perturbative corrections $C_n(z^2\mu^2) = 1 + \mathcal{O}(\alpha_s)$
- Stable moments $\langle x^n \rangle(\mu)$

$$\mathcal{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$

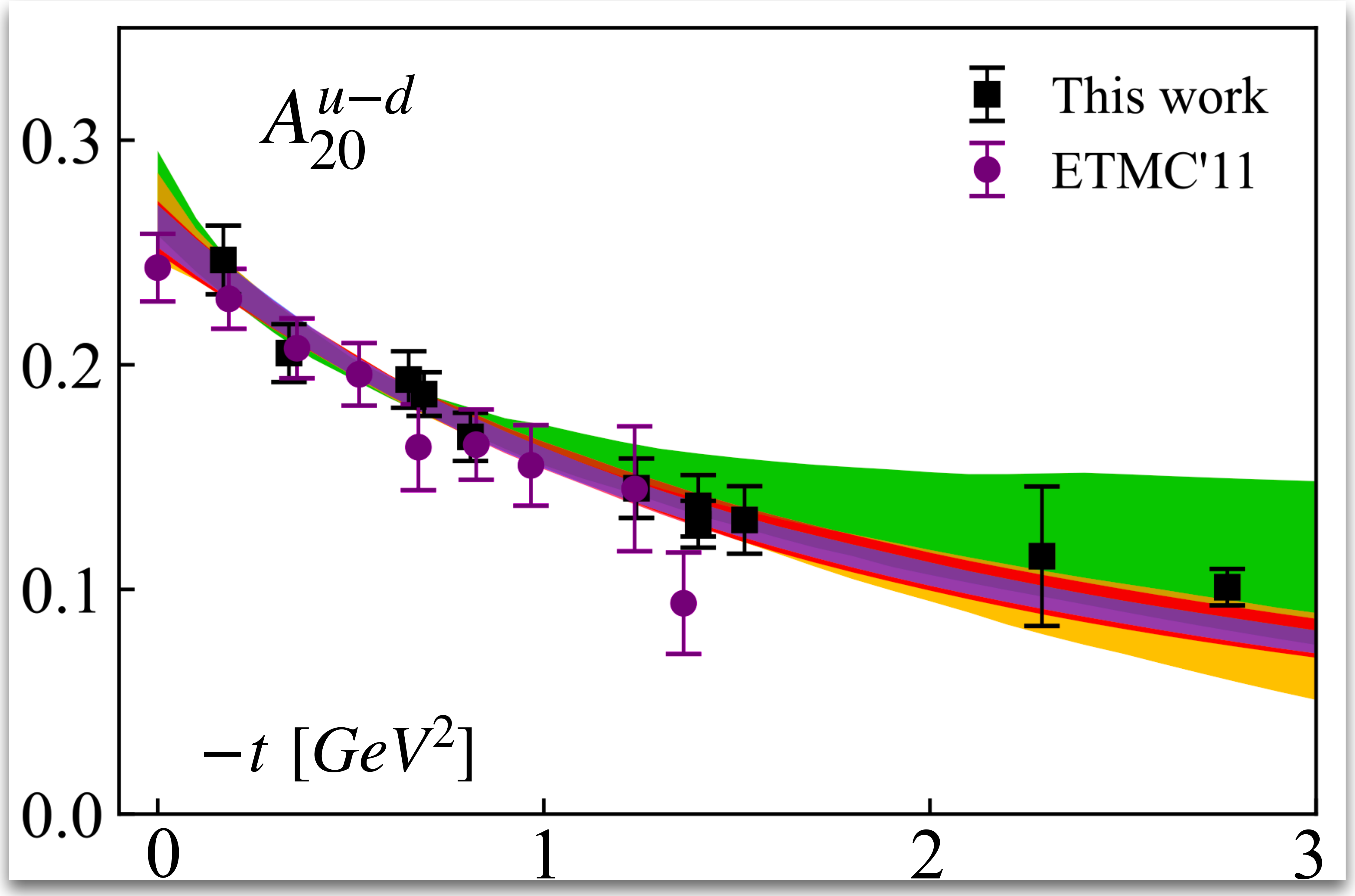
Mellin moments of GPDs

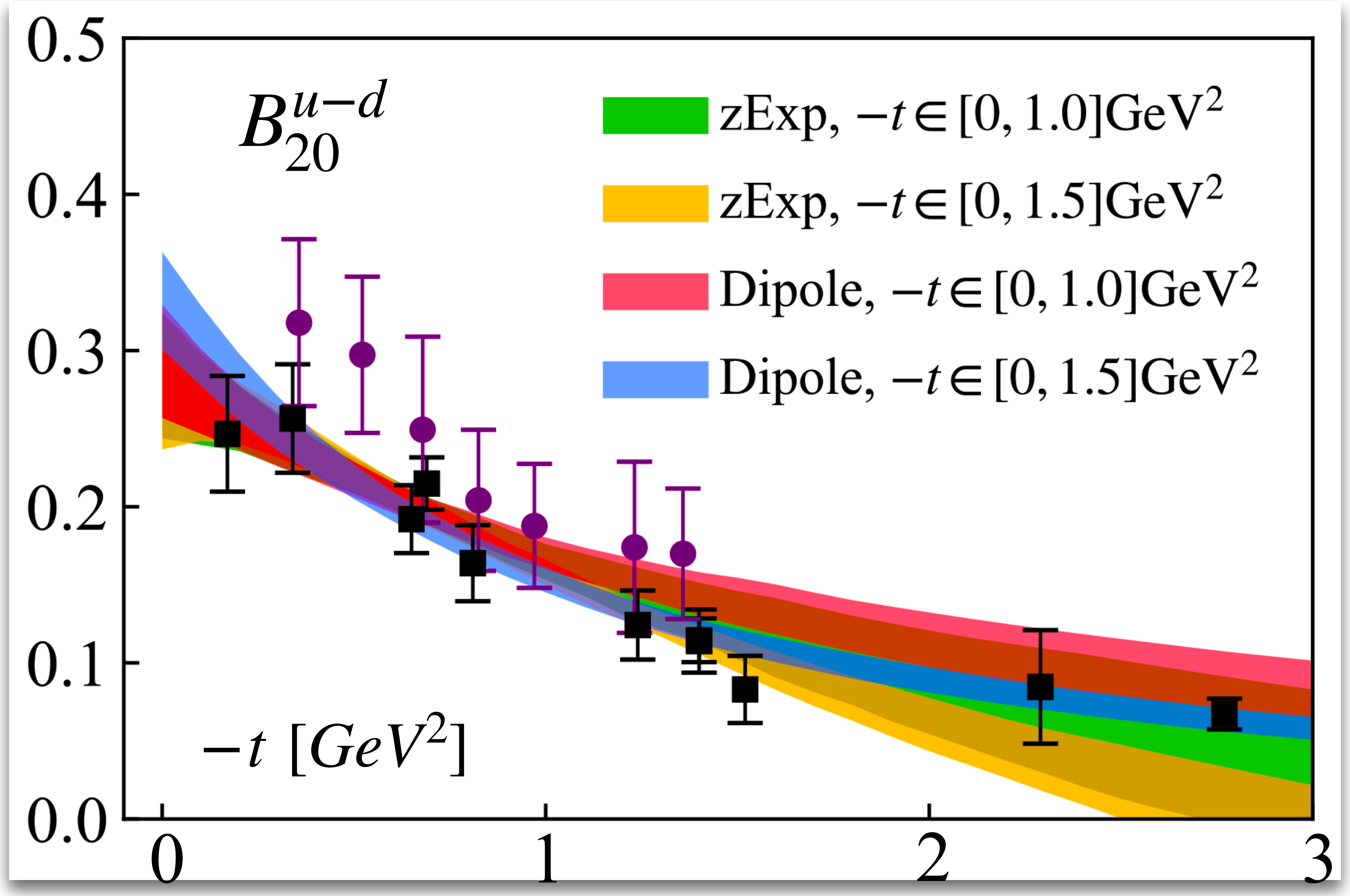




- Good agreement with available traditional lattice QCD calculations of GPD moments using local operators (ETMC'11)

Mellin moments of GPDs





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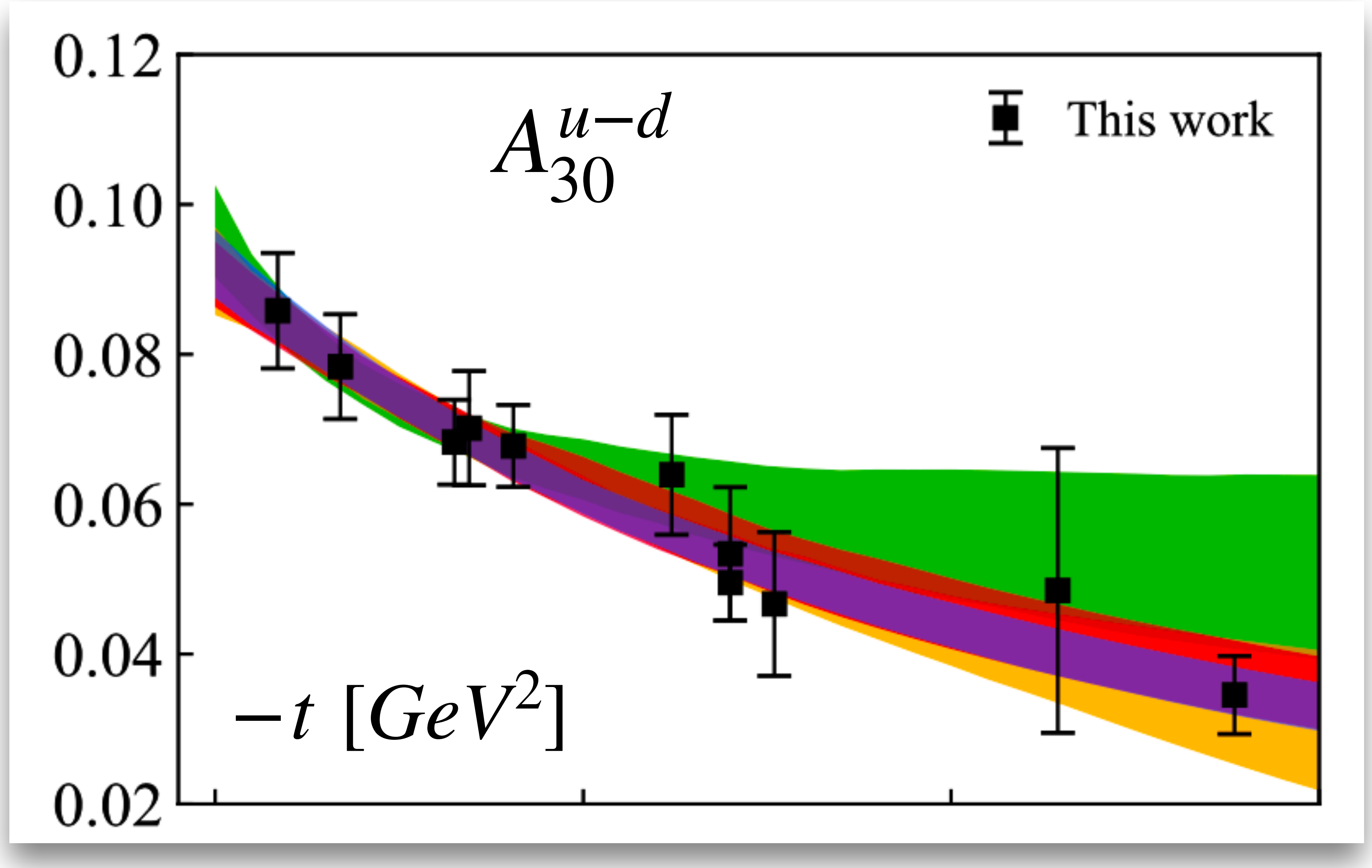
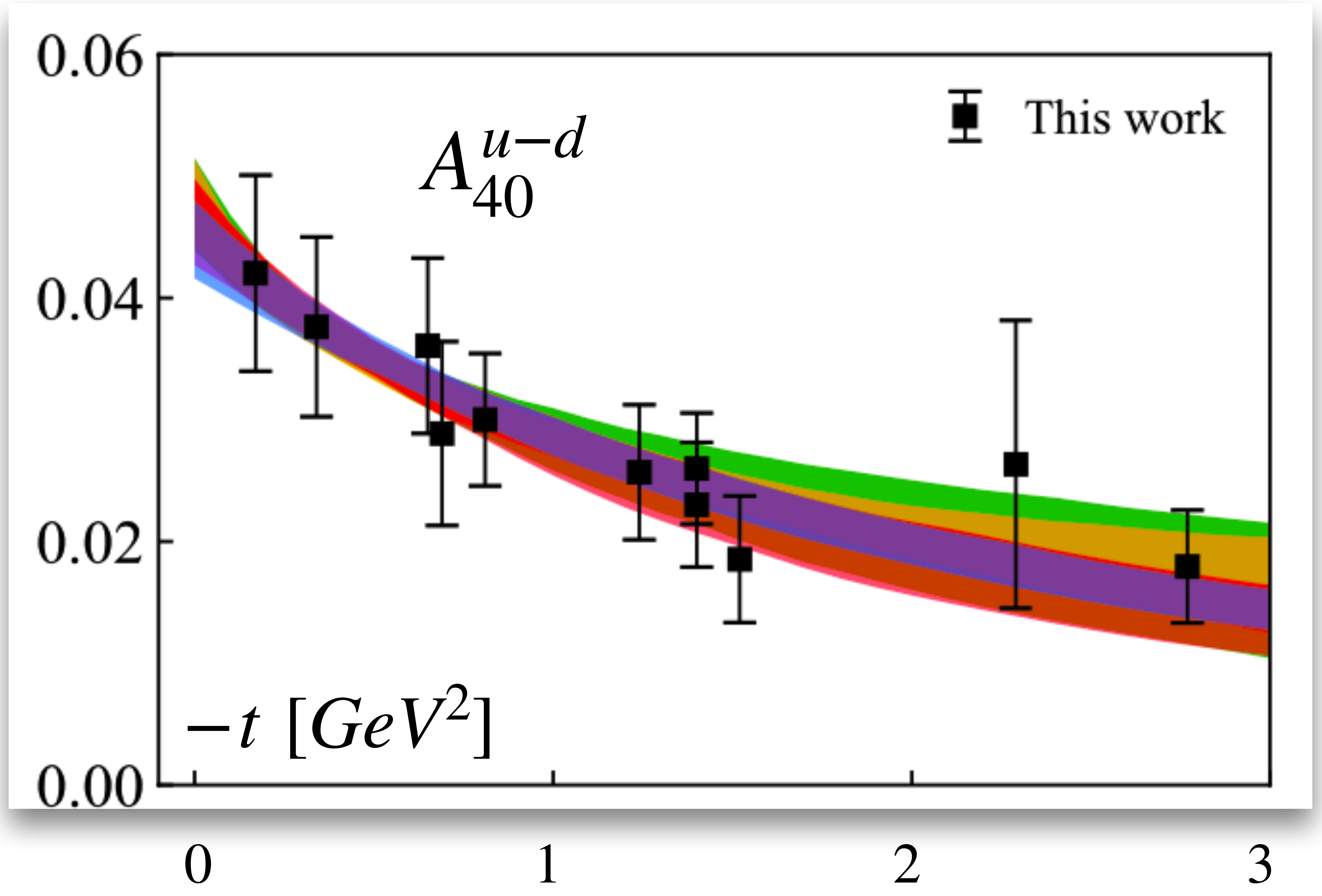
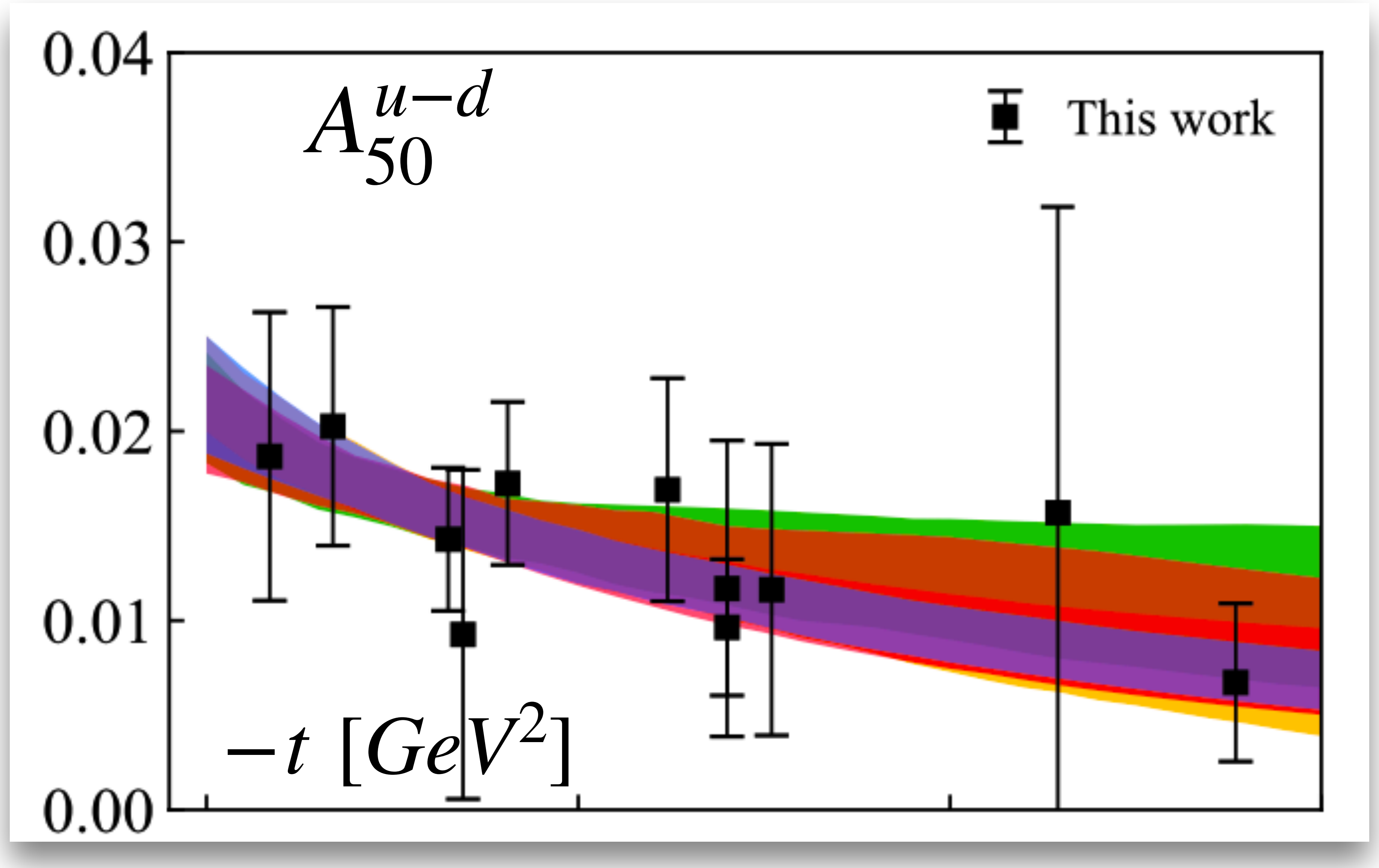
Mellin moments of GPDs

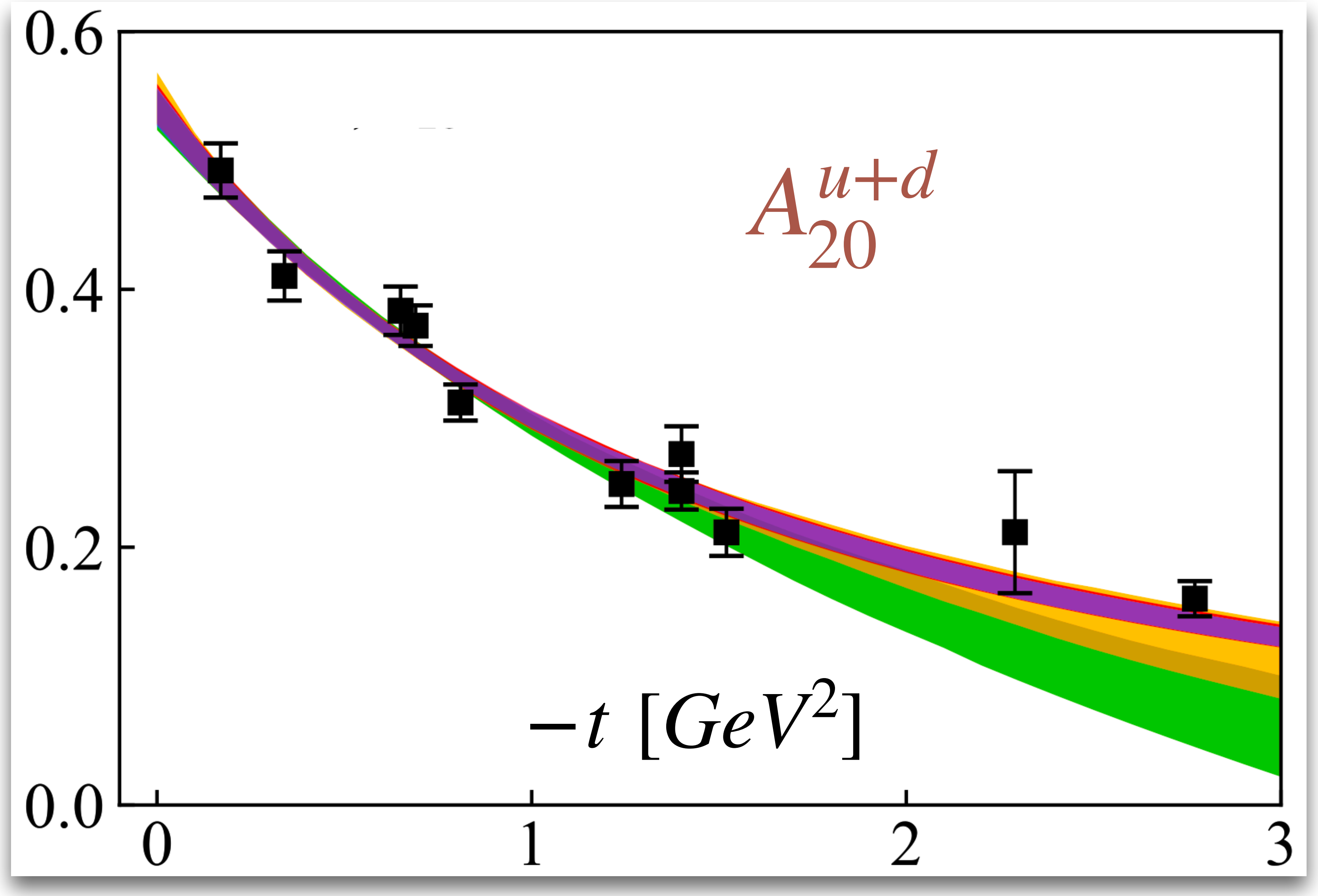


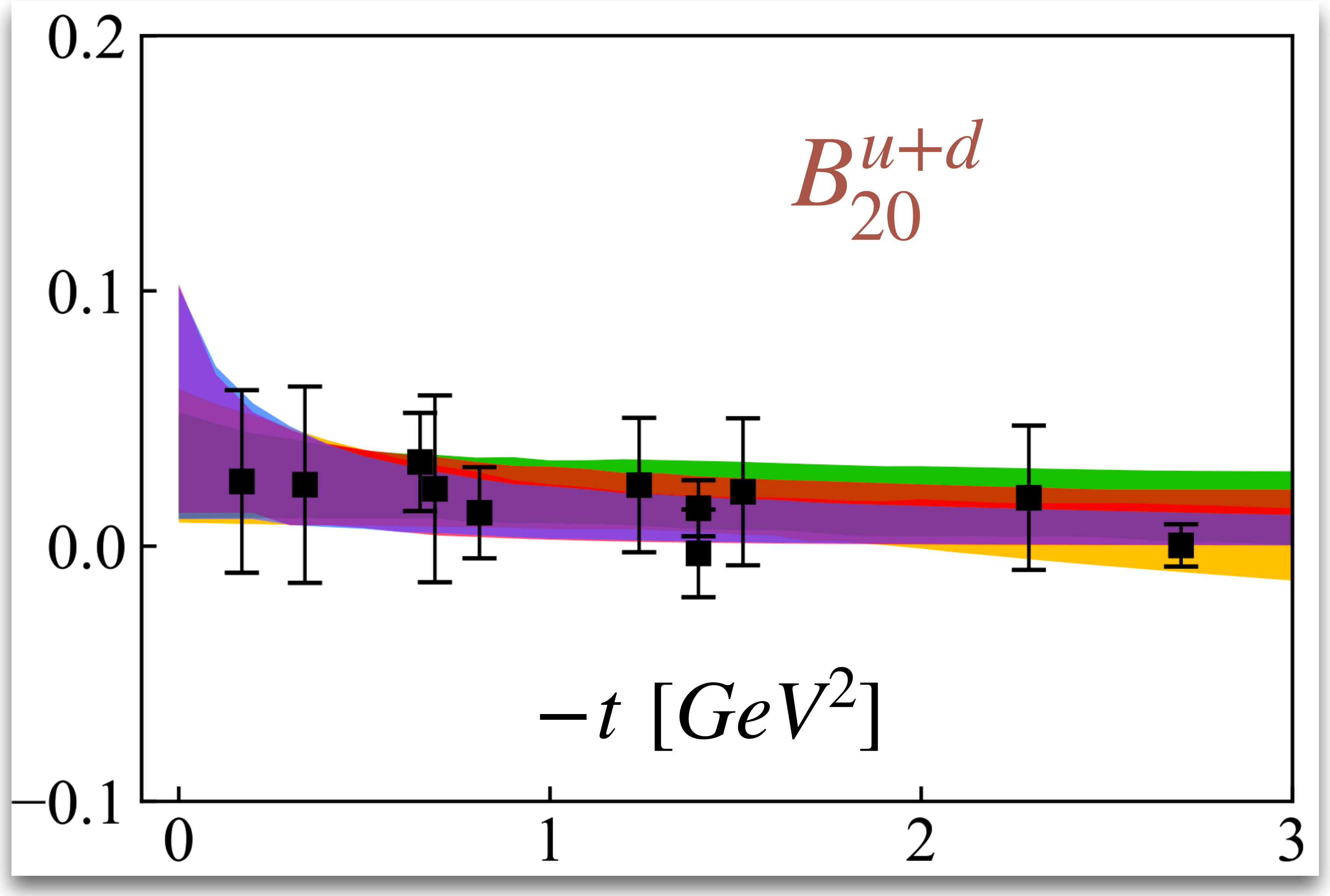




- Up to 5th moments of GPDs show reasonable signals and smooth $-t$ dependence.
- Higher moments can be constrained by increasing the hadron momentum.

Mellin moments of GPDs





- 2nd moments: Gravitational form factors

Ji sum rule: $$J^q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

$$J^{u-d} = 0.281(21)(11)$$

$$J^{u+d} = 0.296(22)(33)$$

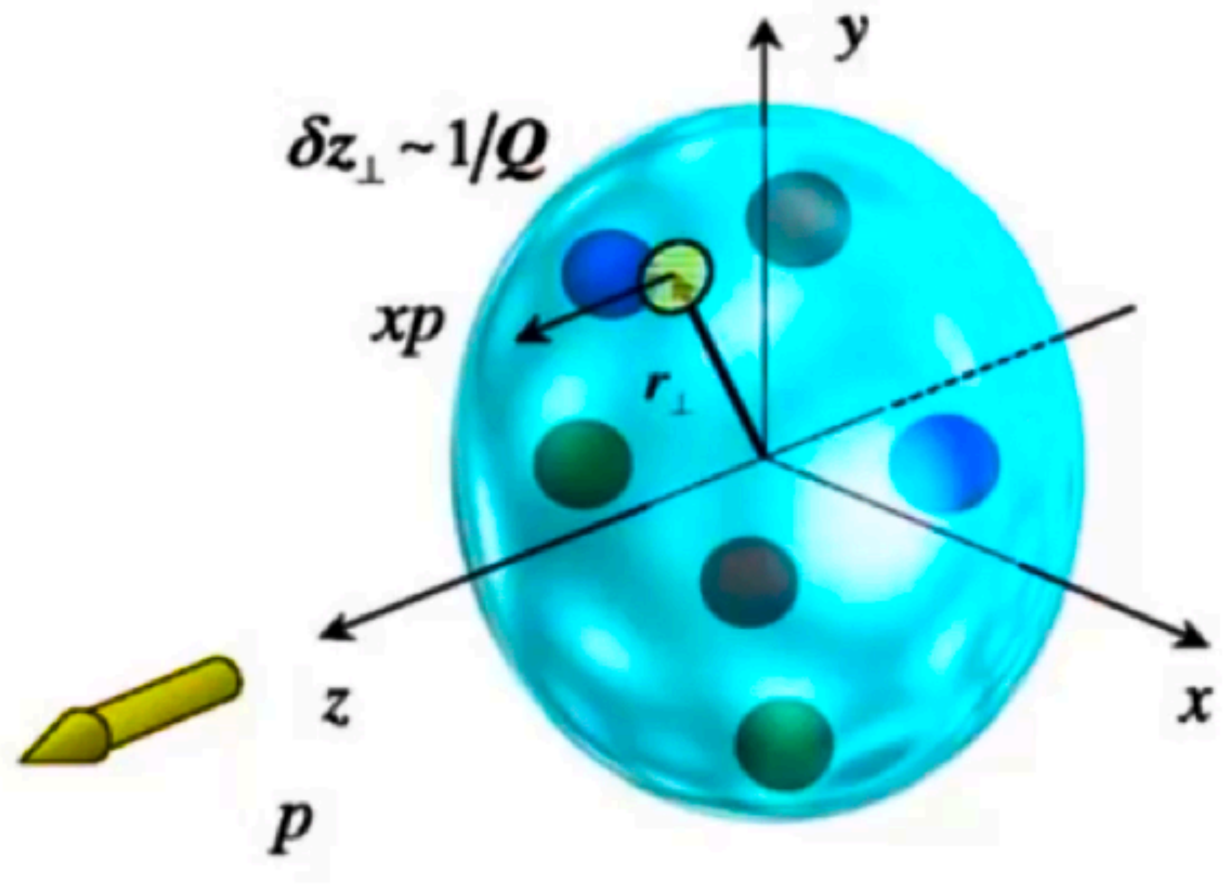
- $m_\pi = 260$ MeV, $a = 0.093$ fm
- Disconnected diagrams neglected

Impact parameter space interpretation

- Unpolarized quark inside unpolarized nucleon

$$q(x,\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} H(x,-\overrightarrow{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

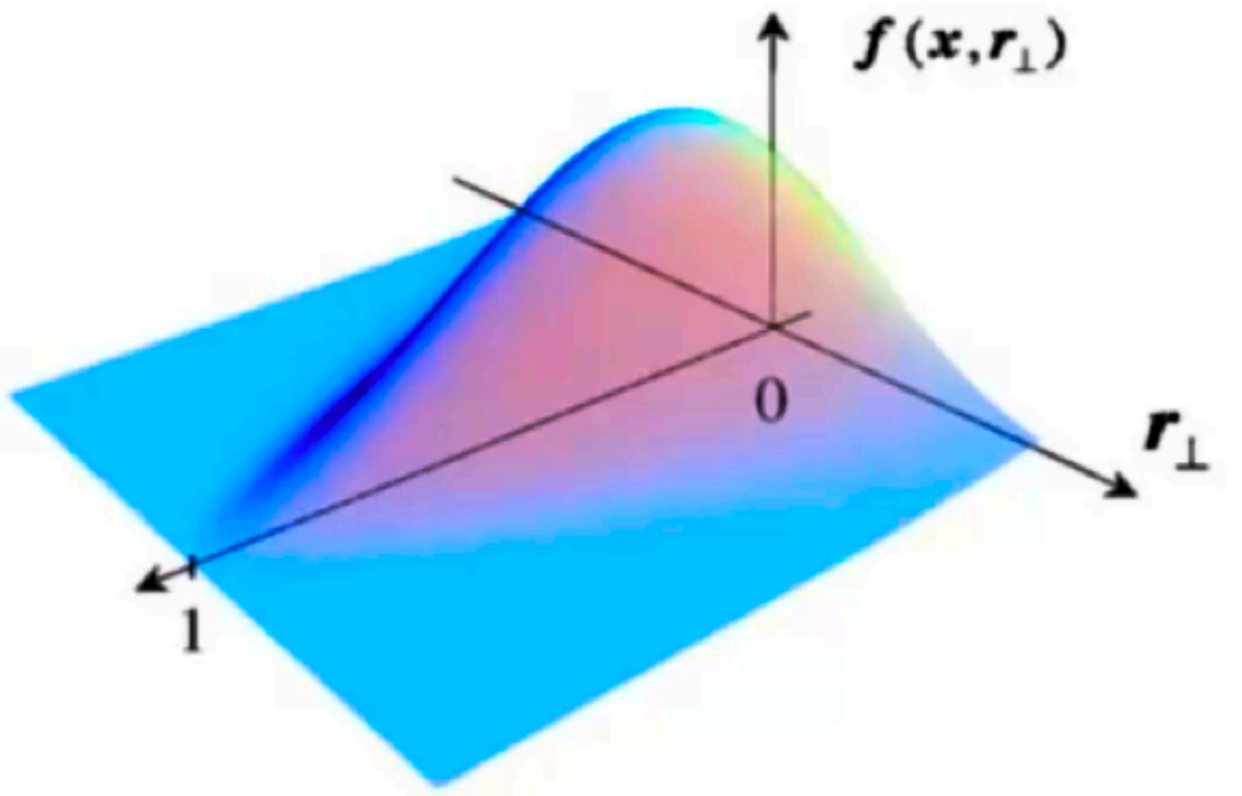
$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\overrightarrow{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$



- Unpolarized quark inside transversely polarized nucleon

$$q^T(x,\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} \left[H(x,-\overrightarrow{\Delta}_\perp^2) + \frac{i\Delta_y}{2M} E(x,-\overrightarrow{\Delta}_\perp^2) \right] e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

$$\rho^T_{n+1}(\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} \left[A_{n+1,0}(-\overrightarrow{\Delta}_\perp^2) + \frac{i\Delta_y}{2M} B_{n+1,0}(-\overrightarrow{\Delta}_\perp^2) \right] e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

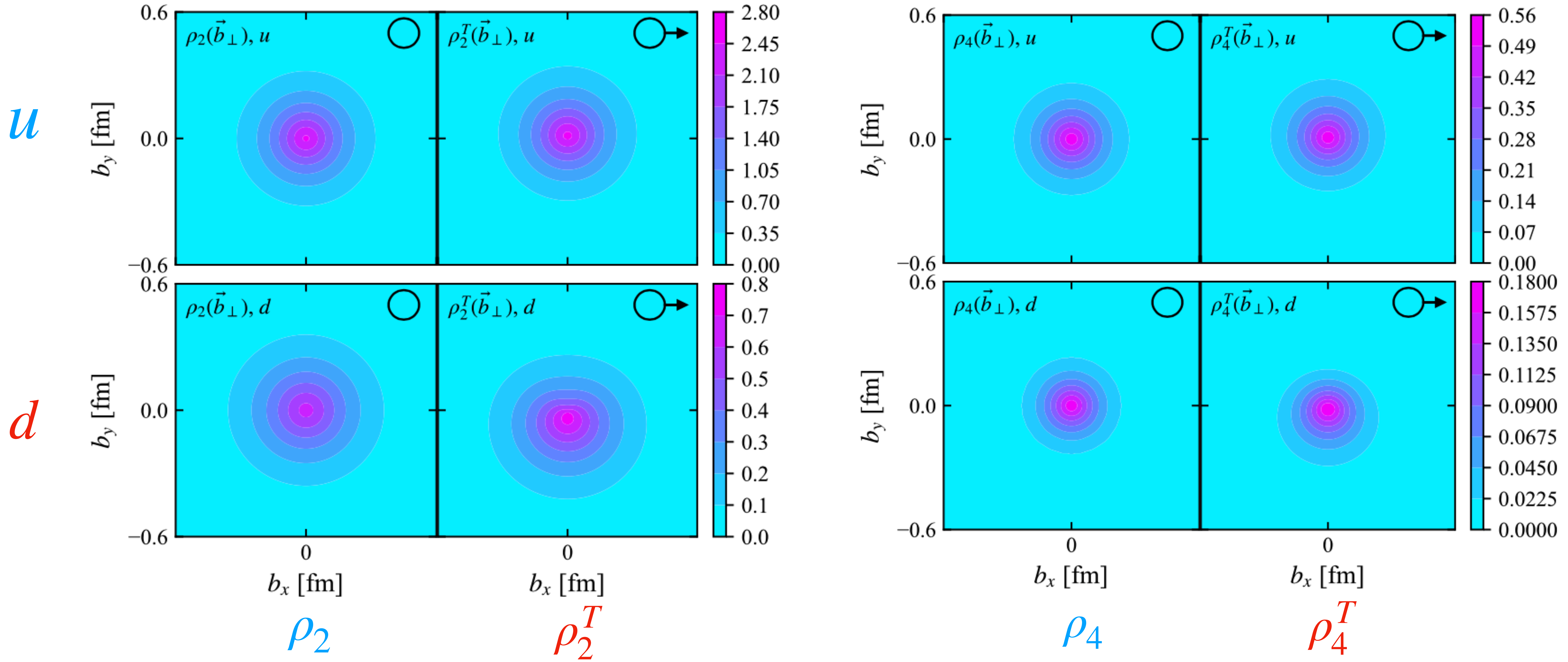


- Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

- Matthias Burkardt, Int.J.Mod.Phys.A 18 (2003) 173-208

Impact parameter space interpretation

- x^n weighted momentum distribution in the impact parameter plane.

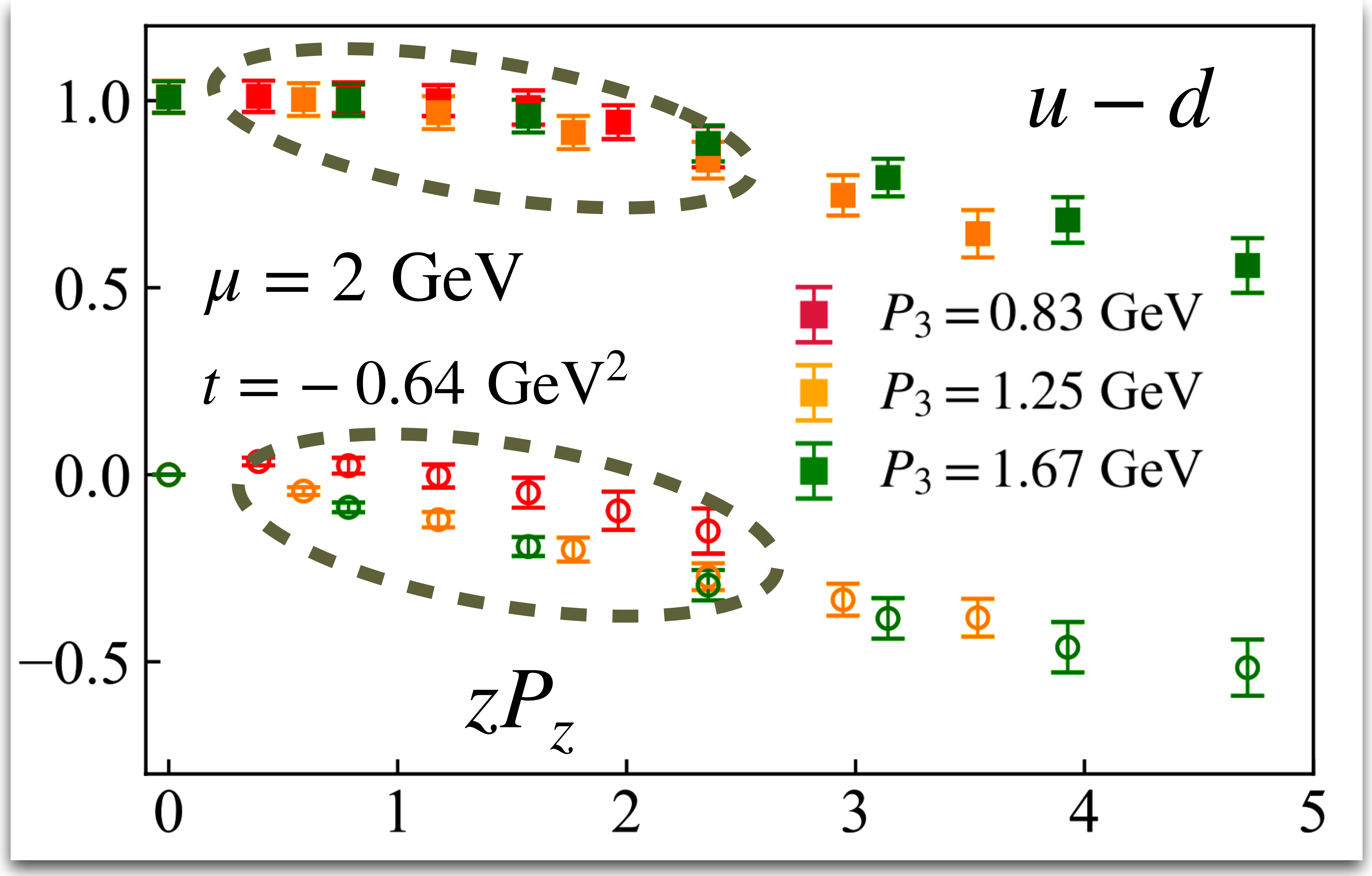


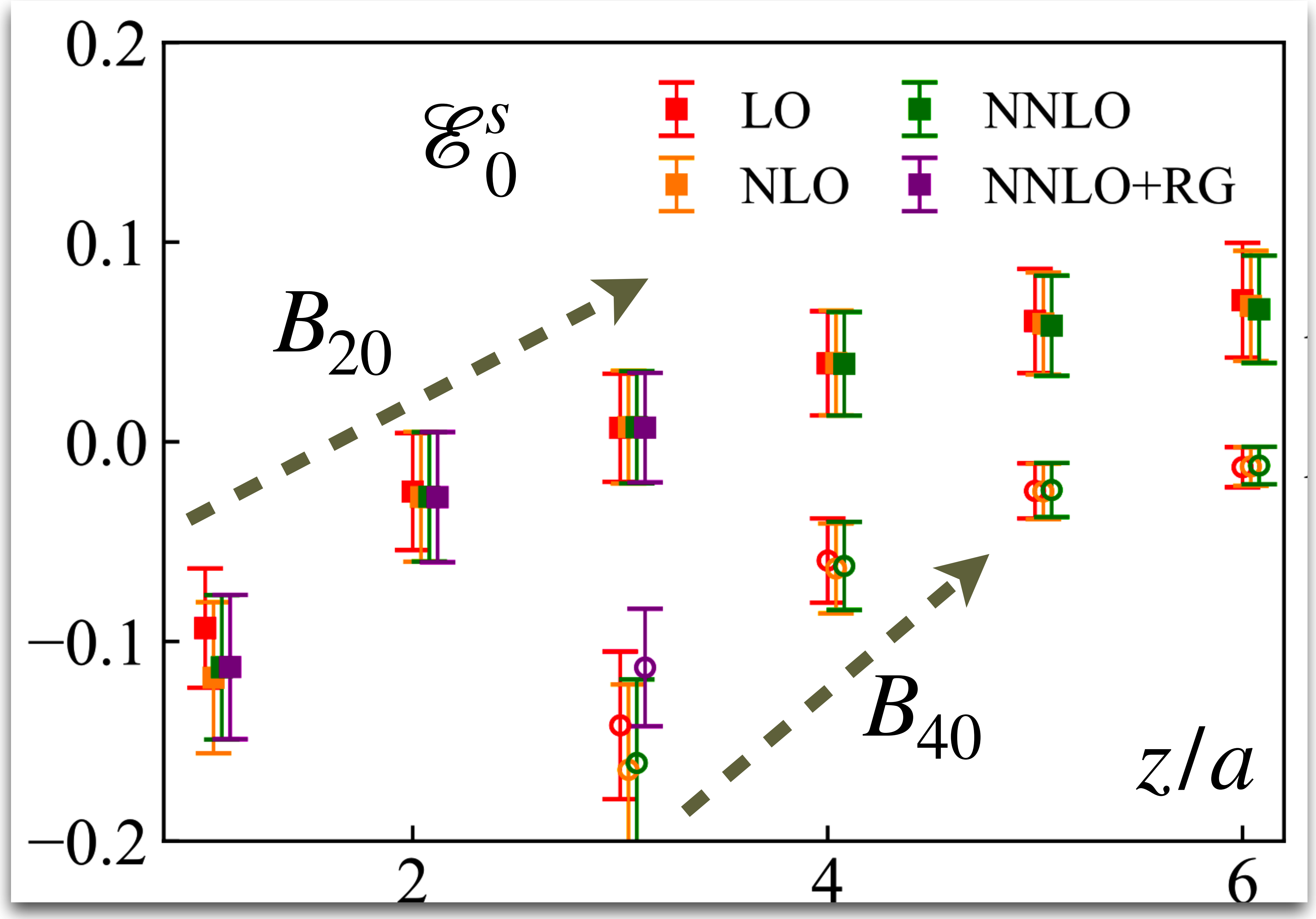
Summary and outlook

- We carried out lattice calculation of the quasi-GPD matrix elements of proton using the Lorentz invariant amplitudes.
- The matrix elements are renormalized in ratio scheme and the Mellin moments up to the 5th ones were extracted using the leading-twist short distance factorization.
- ‣ The methods can be extended to other kind of GPDs and non-zero skewness.
- ‣ Using hybrid renormalization and LaMET matching for x dependence.

Thanks for your attention!

SDF of qGPDs: γ_0 definition





- no scaling with zP_z
- not constant in z

$$\mathcal{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$