Parton Distributions from Boosted Fields in the Coulomb Gauge

Towards Improved Hadron Femtography with Hard Exclusive Reactions

Jefferson Lab, Newport News, USA

Aug 7-11, 2023

YONG ZHAO

AUG 10, 2023

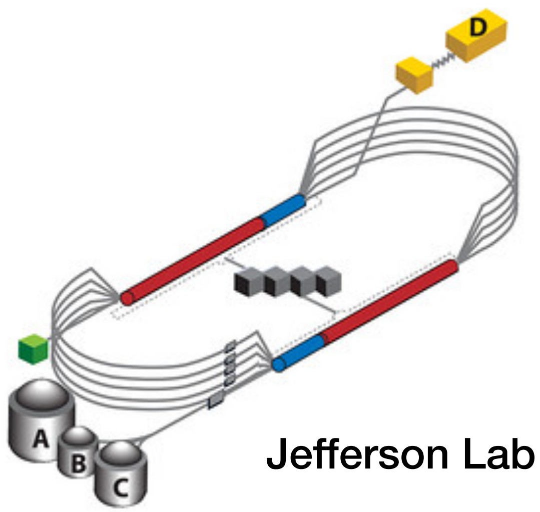
Xiang Gao, Wei-Yang Liu and Yong Zhao, arXiv: 2306.14960.

Argonne NATIONAL LABORATORY

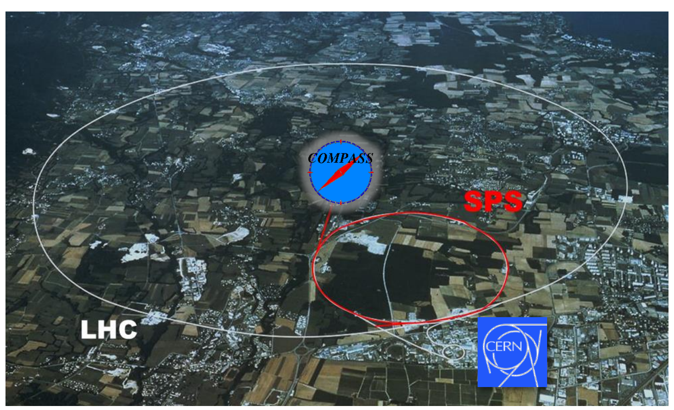
Outline

- **Methodology**
  - Large-Momentum Effective Theory
  - Universality class and quasi-PDF in the Coulomb gauge
  - Factorization
- **Lattice calculation**
  - Bare matrix elements at on- and off-axis momenta
  - Renormalization and matching
  - Comparison of final results
- **Outlook**

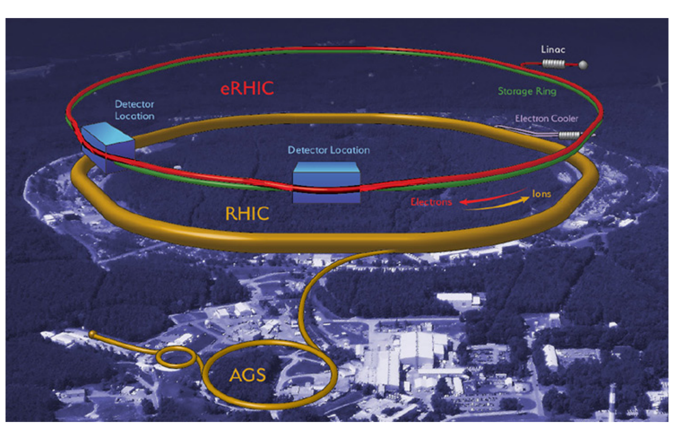
3D Imaging of the Nucleon



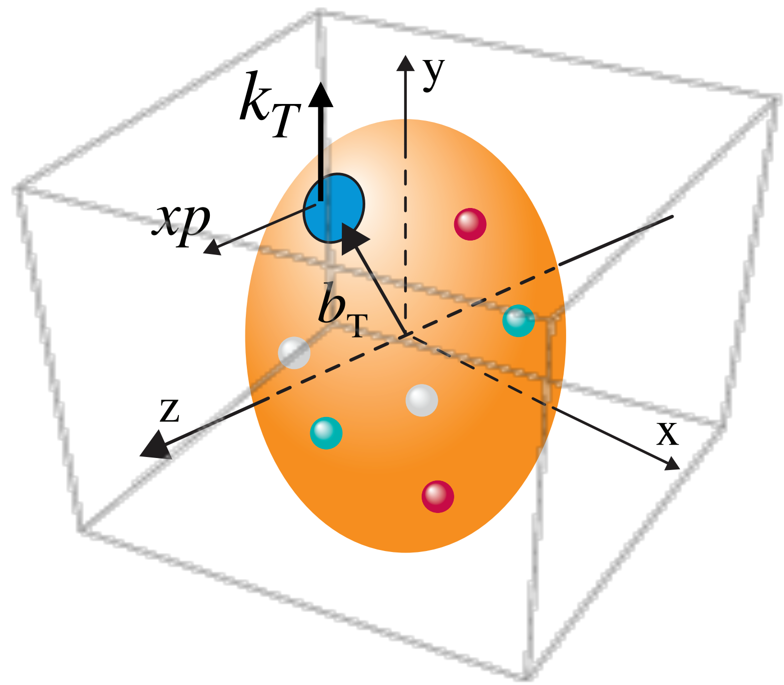
Jefferson Lab 12 GeV

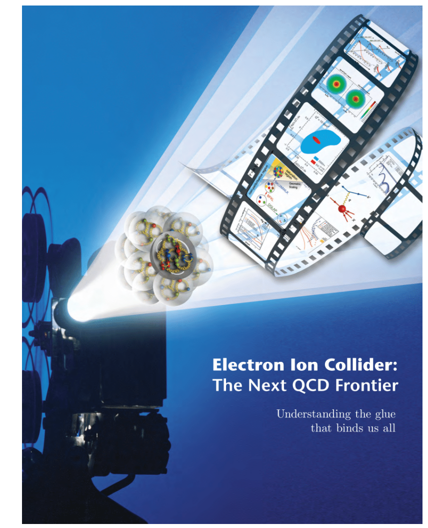


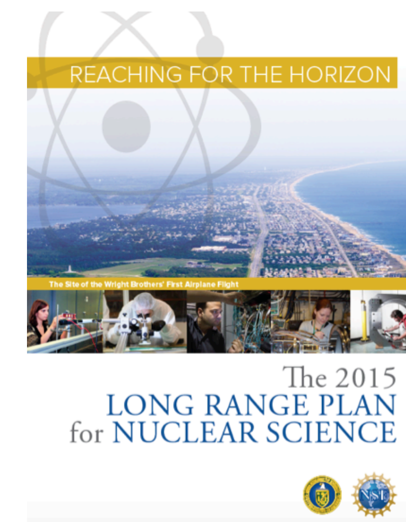
COMPASS, CERN

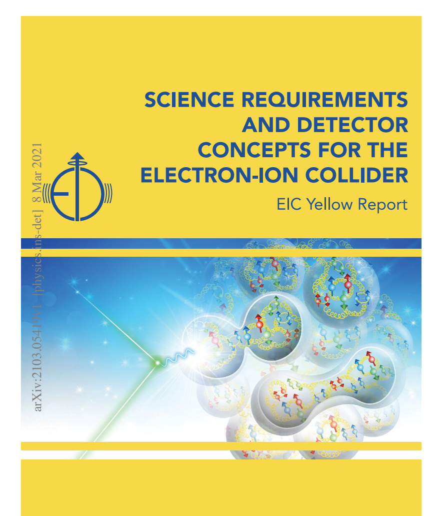


The Electron-Ion Collider, BNL

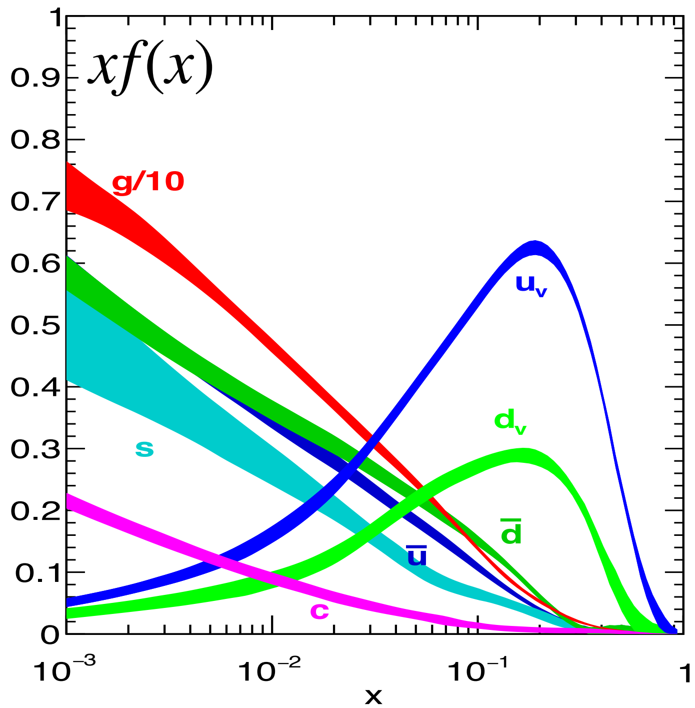






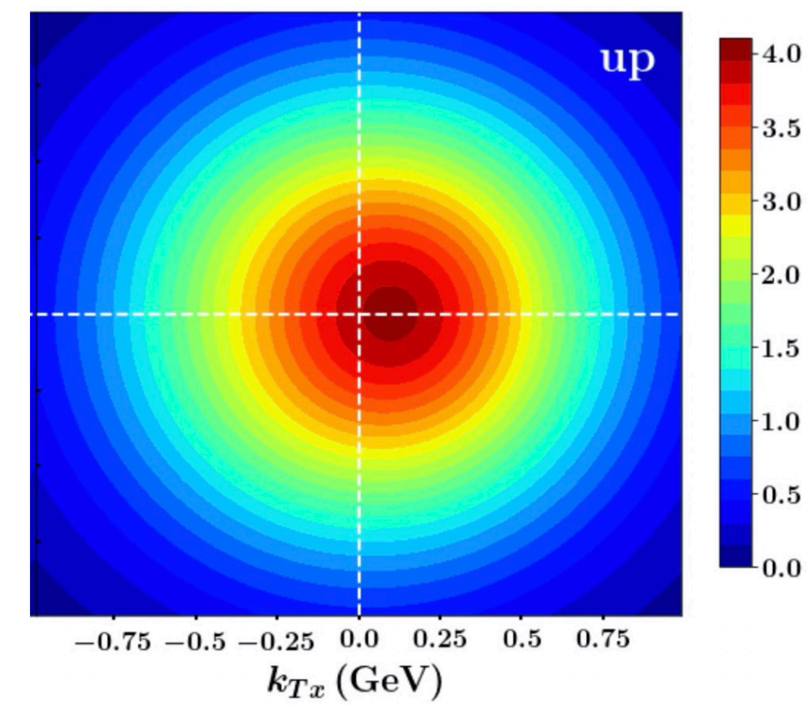


Parton Distribution Functions (PDFs)



NNPDF, EPJ C77 (2017)

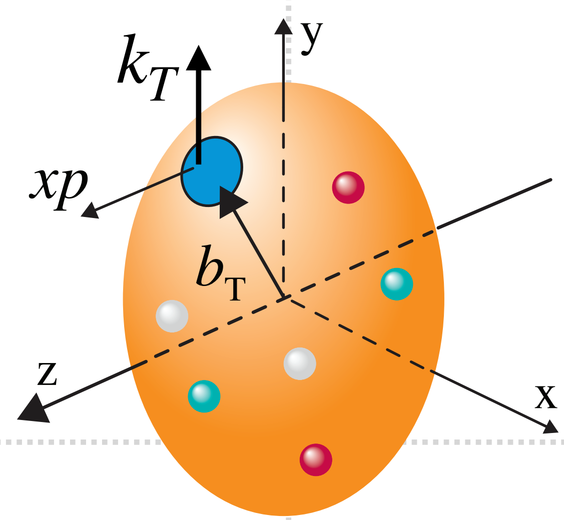
Transvers momentum distributions (TMDs)



Cammarota, et al. (JAM), PRD **102** (2020).

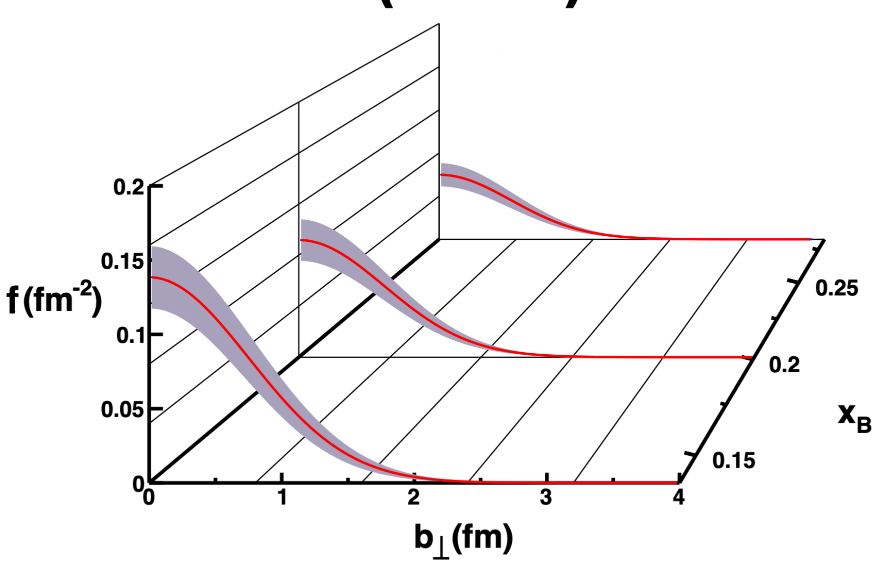
$\int d^2\vec{k}_T$

$\int d^2\vec{b}_T$



$\int d^2\vec{b}_T$

Generalized parton distributions (GPDs)



W. Armstrong et al., arXiv: 1708.00888.

$\int d^2\vec{k}_T$

Wigner distributions/ Generalized TMDs

$$W(x, \vec{k}_T, \vec{b}_T)$$

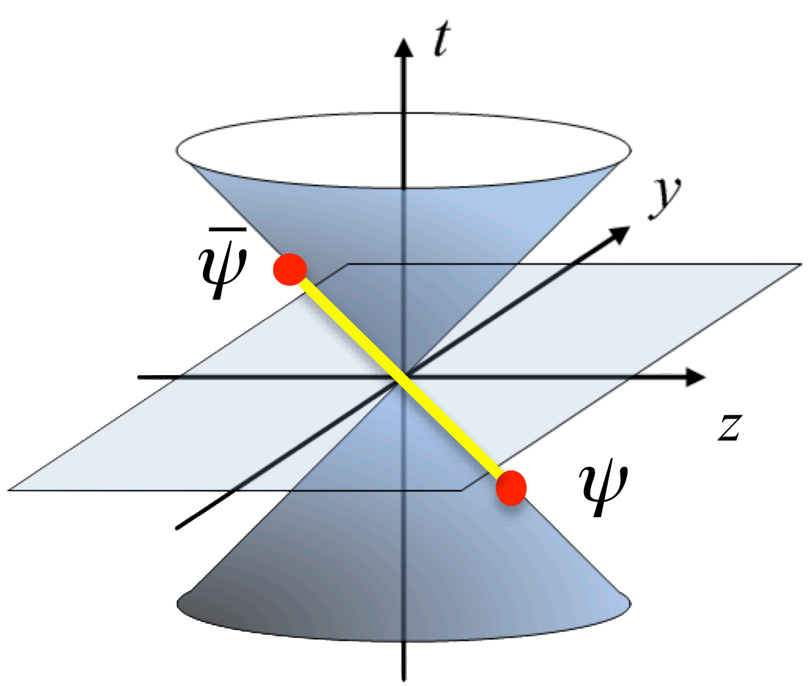
Can we calculate all of them in lattice QCD?

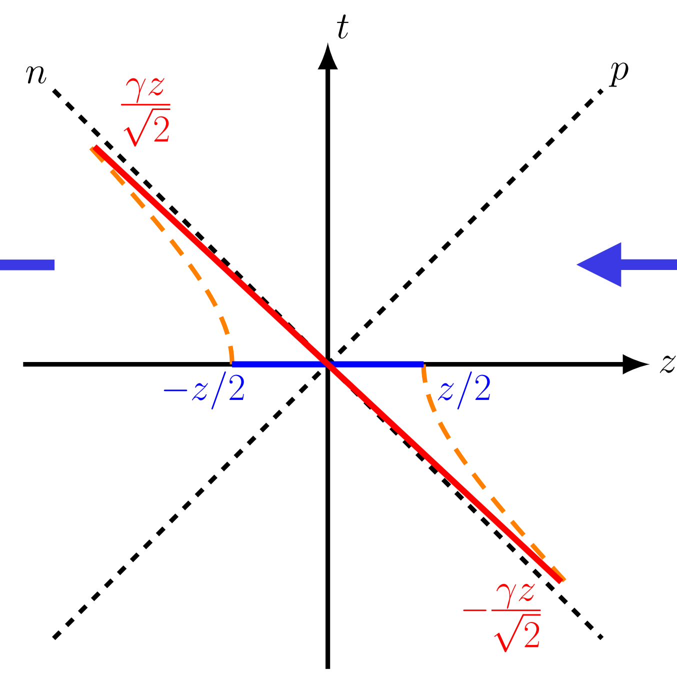
Large-Momentum Effective Theory (LaMET)

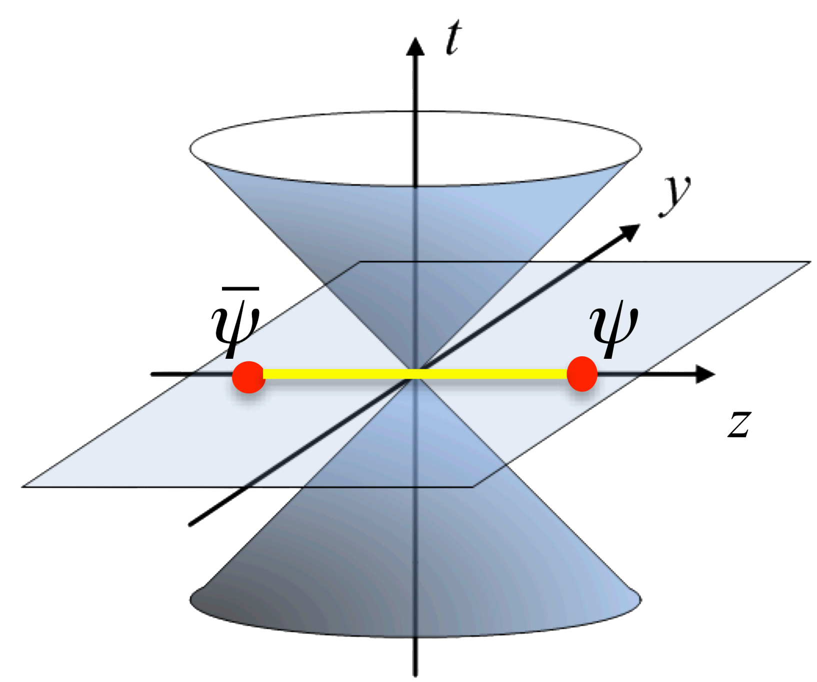
$z + ct = 0, \;\; z - ct \neq 0$

Related by Lorentz boost

$t = 0, \;\; z \neq 0$







PDF $f(x)$:
Cannot be calculated on the lattice

- X. Ji, PRL **110** (2013); SCPMA **57** (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

$$f(x) = \int \frac{dz^-}{2\pi} e^{-ib^-(xP^+)} \langle P | \bar{\psi}(z^-) \times \frac{\gamma^+}{2} W[z^-,0] \psi(0) | P \rangle$$

$$\tilde{f}(x, P^z) = \int \frac{dz}{2\pi} e^{iz(xP^z)} \langle P | \bar{\psi}(z) \times \frac{\gamma^z}{2} W[z,0] \psi(0) | P \rangle$$

Large-Momentum Effective Theory (LaMET)

Systematic calculation of *x*-dependence for $x \in [x_{\rm min}, x_{\rm max}]$:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \; + \; \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

Renormalization

Perturbative Matching

Large-Momentum Effective Theory (LaMET)

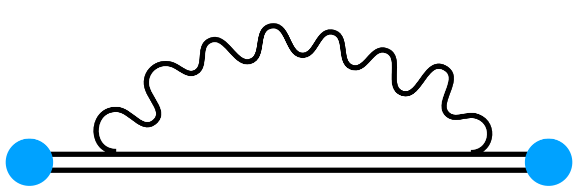
Systematic calculation of *x*-dependence for $x \in [x_{\min}, x_{\max}]$:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \;+\; \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

Renormalization

$$O_B^\Gamma(z,a) = \bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0)$$
$$= e^{-\delta m(a)|z|}\, Z_O(a) O_R^\Gamma(z)$$

Linear divergence in the Wilson line:



$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

- **Subtraction of linear divergence**

$$\delta m(a) = \frac{m_{-1}}{a} + \mathcal{O}(\Lambda_{\rm QCD})$$ Renormalon ambiguity

  - Self renormalization — Y. Huo, et al. (LPC), NPB **969** (2021).
  - Static potential — X. Gao, YZ, et al., PRL **128** (2022).
  - …

- **Subtraction of leading renormalon ambiguity**

Matching to the OPE of P^z=0 matrix element:

$$C_0^{\overline{\rm MS}}(\mu, z) = C_0^{\rm LRR}(\mu, z) e^{-m_0^{\overline{\rm MS}}|z|}$$

Leading-renormalon resummation (LRR)

- Holligan, Ji, Lin, Su and Zhang, NPB **993** (2023);
- Zhang, Ji, Holligan and Su (ZJHS23), PLB **844** (2023).

Large-Momentum Effective Theory (LaMET)

Systematic calculation of *x*-dependence for $x \in [x_{\min}, x_{\max}]$:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

Perturbative Matching

- **Next-to-next-to-leading order (NNLO) kernel**
  - Chen, Zhu and Wang, PRL **126** (2021);
  - Li, Ma and Qiu, PRL **126** (2021).
- **Resummation of small-*x* logarithms** $\alpha_s \ln \frac{\mu}{2xP^z}$ **(DGLAP evolution)**
  - X. Gao, K. Lee, and YZ et al., PRD **103** (2021);
  - Y. Su, J. Holligan et al., NPB **991** (2023).
- **Subtraction of leading renormalon** $C(x/y) \to C^{\rm LRR}(x/y)$
  - Holligan, Ji, Lin, Su and Zhang, NPB **993** (2023);
  - Zhang, Ji, Holligan and Su (ZJHS23), PLB **844** (2023).
- **Resummation of large-*x* (threshold) logarithms** $\frac{\alpha_s \ln(1-x/y)}{(1-x/y)}$
  - X. Gao, K. Lee, and YZ et al., PRD **103** (2021);
  - X. Ji, Y. Liu and Y. Su, arXiv:2305.04416.

Short-distance factorization in coordinate space

$$\tilde{h}(\lambda = zP^z, z^2\mu^2) = \sum_{n=0}^{\infty} C_n(z^2\mu^2)\frac{(-i\lambda)^n}{n!}a_n(\mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2),$$
$$= \int_0^1 d\alpha\, \mathscr{C}(\alpha, z^2\mu^2)\, h(\alpha\lambda, \mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2),$$

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi}\, e^{-ix\lambda}\, h(\lambda,\mu)$$

- A. Radyushkin, PRD **96** (2017);
- K. Orginos et al., PRD **96** (2017);
- T. Izubuchi, YZ, et al., PRD **98** (2018).

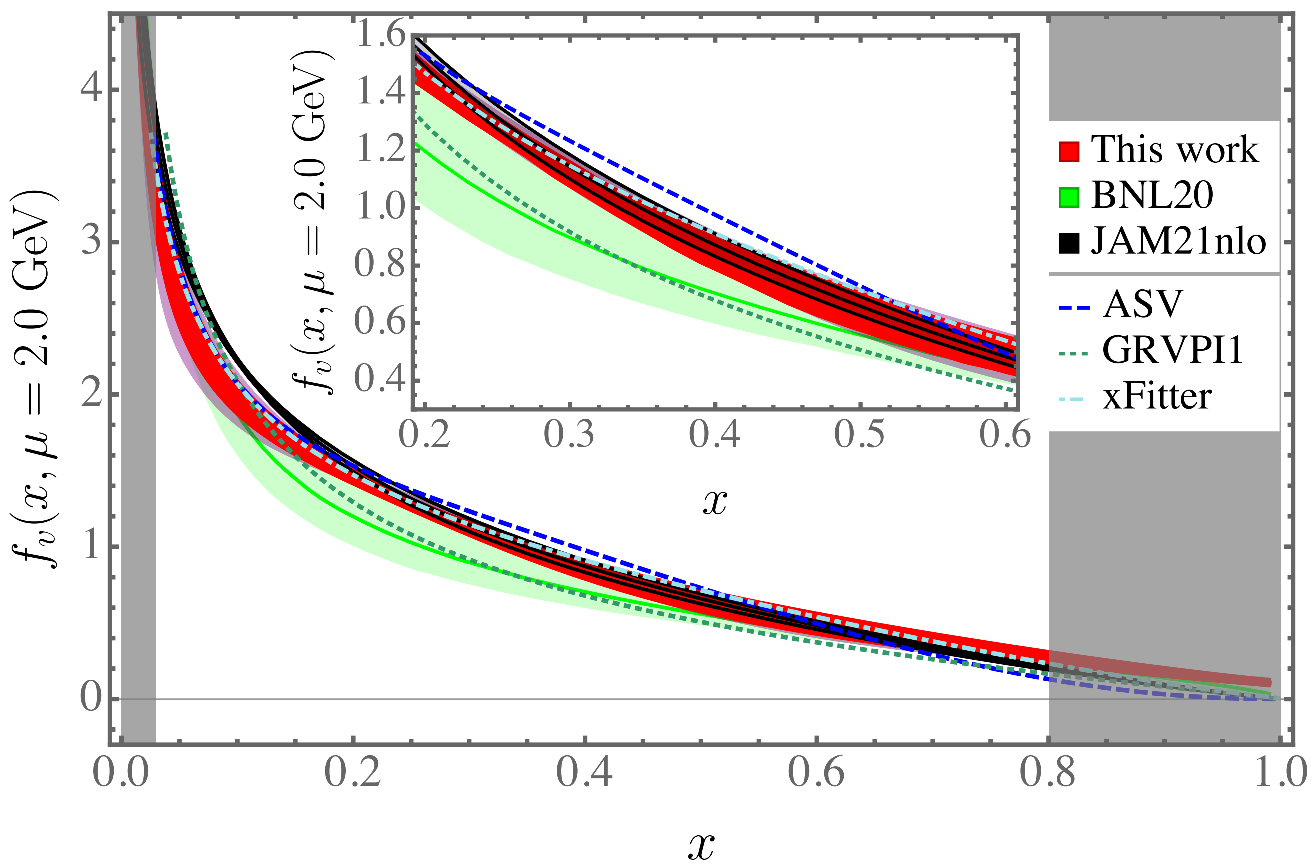
- Extraction of the lowest moments;
- Calculation of the light-cone correlation up to $\lambda_{\rm max} = z_{\rm max}P^z$;
- Fitting the *x*-dependence with model assumption of the PDF.

Towards systematic control:

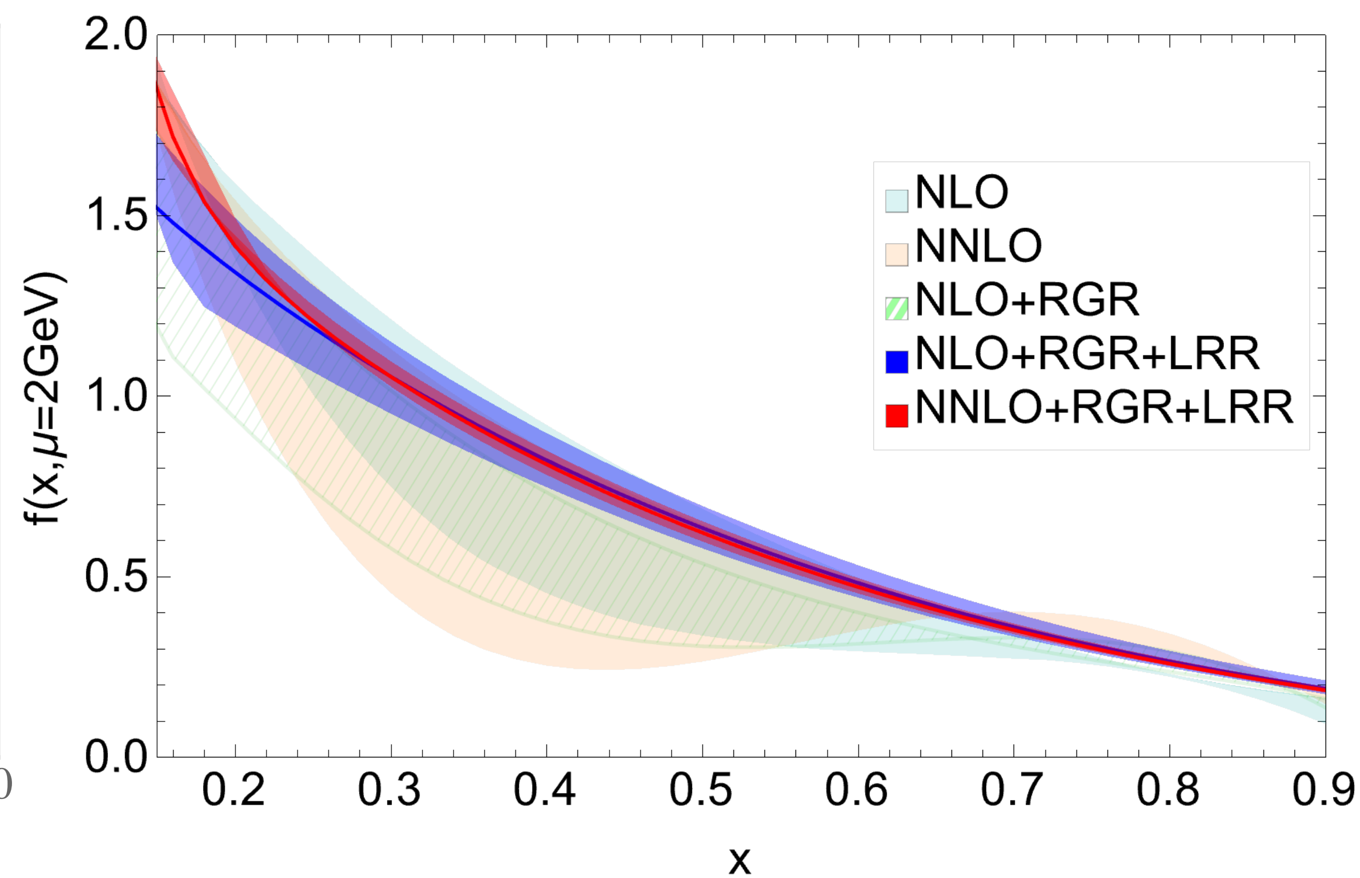
1) Higher-order matching (NNLO);
   - Chen, Zhu and Wang, PRL **126** (2021);
   - Li, Ma and Qiu, PRL **126** (2021).
2) Resummation of $\alpha_s \ln(z^2\mu^2)$;
   - X. Gao, K. Lee, and YZ et al., PRD **103** (2021);
   - X. Ji, Y. Liu and Y. Su, arXiv:2305.04416.
3) Threshold resummation of $\alpha_s^i \ln^j n$ or $\alpha_s \ln(1-\alpha)/(1-\alpha)$

State-of-the-art calculation of pion PDF

Gao, YZ et al. (BNL-ANL21), 128 (2022).



Zhang, Ji, Holligan and Su (ZJHS23), PLB 844 (2023)



	Subtraction of δm	Leading-renormalon resummation (LRR)	NNLO	Small-x resummation	Threshold resumation	Subleading renormalon	Discretization effects
BNL-ANL21	✔		✔				
ZJHS23	✔	✔	✔	✔			

Universality in LaMET

Gauge-invariant bilinear

$$\bar{\psi}(z)\Gamma W[z,0]\psi(0)$$

- Y. Hatta, X. Ji, and YZ, PRD **89** (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Current-current correlator

$$J^{\mu}(z)J^{\nu}(0)$$

- Liu and Dong, PRL **72** (1994);
- Detmold and Lin, PRD **73** (2006);
- Braun and Müller, EPJC **55** (2008);
- A Chambers et al. (QCDSF), PRL **118** (2017)
- Ma and Qiu, PRL **120** (2018).

Free bilinear in a physical gauge

$$\bar{\psi}(z)\Gamma\psi(0)\Big|_{G(A)=0} \qquad G(A) = A^0, A^z, \nabla\cdot\mathbf{A}$$

X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

$P \to \infty$

Light-cone bilinear

$$\bar{\psi}(\xi^-)\gamma^+ W[\xi^-,0]\psi(0)$$

Or

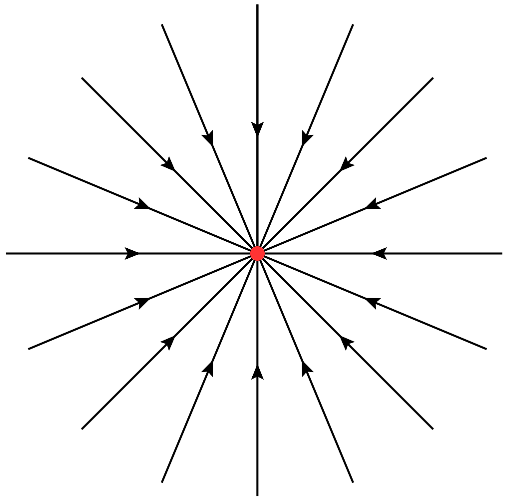
$$\bar{\psi}(\xi^-)\gamma^+\psi(0)\Big|_{A^+=0}$$

Quasi-PDF in the Coulomb gauge

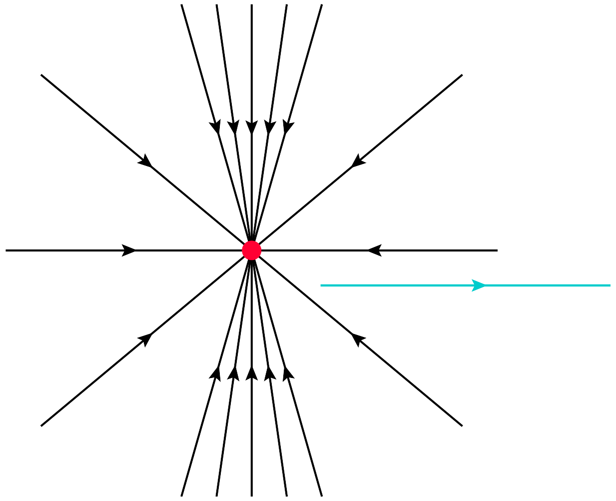
$$\tilde{h}(\vec{z},\vec{p},\mu) = \frac{1}{2p^t}\langle p\,|\,\bar{\psi}(\vec{z})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}\,|p\rangle\,, \quad \vec{z}\ //\ \vec{p}$$

$$\tilde{f}(x,|\vec{p}\,|,\mu) = |\vec{p}\,|\int_{-\infty}^{\infty}\frac{d|\vec{z}\,|}{2\pi}\,e^{ix\vec{p}\cdot\vec{z}}\,\tilde{h}(\vec{z},\vec{p},\mu)$$

Static charge



Moving charge



First proposed in the lattice calculation of gluon helicity

$$\Delta G = \langle P_\infty\,|\,(\mathbf{E}\times\mathbf{A})^3\,|_{\nabla\cdot\mathbf{A}=0}\,|\,P_\infty\rangle$$

- X. Ji, J.-H. Zhang and YZ, PRL **111** (2013);
- Y. Hatta, X. Ji, and YZ, PRD **89** (2014);
- X. Ji, J.-H. Zhang and YZ, PLB **743** (2015);
- Y.-B. Yang, R. Sufian, YZ, et al. PRL **118** (2017).

Factorization

- **Large-momentum factorization:**
  - X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD **90** (2014);
  - Y. Ma and J. Qiu, PRD **98** (2018);
  - T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD **98** (2018)

$$\tilde{f}(x, P^z, \mu) = \int \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{|y|P^z}\right) f(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{\rm QCD}^2}{(1-x)^2 P_z^2}\right)$$

$$C\left(\xi, \frac{\mu}{p^z}\right) = \delta(\xi - 1) + \frac{\alpha_s C_F}{2\pi} C^{(1)}\left(\xi, \frac{\mu}{p^z}\right) + \mathcal{O}(\alpha_s^2)$$

$$C^{(1)}\left(\xi, \frac{\mu}{p^z}\right) = C^{(1)}_{\rm ratio}\left(\xi, \frac{\mu}{p^z}\right) + \frac{1}{2|1-\xi|} + \delta(1-\xi)\left[-\frac{1}{2}\ln\frac{\mu^2}{4p_z^2} + \frac{1}{2} - \int_0^1 d\xi' \frac{1}{1-\xi'}\right]$$

$$C^{(1)}_{\rm ratio}\left(\xi, \frac{\mu}{p^z}\right) = \left[P_{qq}(\xi)\ln\frac{4p_z^2}{\mu^2} + \xi - 1\right]_{+(1)}^{[0,1]}$$

$$+\left\{P_{qq}(\xi)\left[\mathbf{sgn}(\xi)\ln|\xi| + \mathbf{sgn}(1-\xi)\ln|1-\xi|\right] + \mathbf{sgn}(\xi) + \frac{3\xi - 1}{\xi - 1}\frac{\tan^{-1}\left(\frac{\sqrt{1-2\xi}}{|\xi|}\right)}{\sqrt{1-2\xi}} - \frac{3}{2|1-\xi|}\right\}_{+(1)}^{(-\infty,\infty)}$$

$$\xrightarrow{\xi\to\infty} \frac{1}{\xi^2}$$

Factorization

- **Short-distance factorization:** A. Radyushkin, PRD **96** (2017).

$$\tilde{h}(z, P^z, \mu) = \int du\, \mathscr{C}(u, z^2\mu^2) h(u\tilde{\lambda}, \mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

$$\mathscr{C}\left(u, \frac{\mu}{p^z}\right) = \delta(u-1) + \frac{\alpha_s C_F}{2\pi}\mathscr{C}^{(1)}\left(u, \frac{\mu}{p^z}\right) + \mathcal{O}(\alpha_s^2)$$

$$\mathscr{C}^{(1)}(u, z^2\mu^2) = \mathscr{C}^{(1)}_{\rm ratio}(u, z^2\mu^2) + \frac{1}{2}\delta(1-u)\left(1 - \ln\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)$$

$$\mathscr{C}^{(1)}_{\rm ratio}(u, z^2\mu^2) = \left[-P_{qq}(u)\ln\frac{z^2\mu^2e^{2\gamma_E}}{4} - \frac{4\ln(1-u)}{1-u} + 1 - u\right]_{+(1)}^{[0,1]}$$

$$+\left[\frac{3u-1}{u-1}\frac{\tan^{-1}\left(\frac{\sqrt{1-2u}}{|u|}\right)}{\sqrt{1-2u}} - \frac{3}{|1-u|}\right]_{+(1)}^{(-\infty,\infty)} \quad \xrightarrow{u\to\infty} \frac{1}{u^2}$$

Lattice setup

Wilson-clover valence fermion on 2+1 flavor HISQ gauge configurations (HotQCD).

$\lvert\vec{p}\rvert$ (GeV)	$\vec{n}$	$\vec{k}$	t_s/a	(#ex,#sl)
0	(0,0,0)	(0,0,0)	8,10,12	(1, 16)
1.72	(0,0,4)	(0,0,3)	8	(1, 32)
			10	(3, 96)
			12	(8, 256)
2.15	(0,0,5)	(0,0,3)	8	(2, 64)
			10	(4, 128)
			12	(8, 256)
2.24	(3,3,3)	(2,2,2)	8	(1, 32)
			10	(2, 64)
			12	(4, 128)

$$a = 0.06 \text{ fm}$$

$$m_\pi = 300 \text{ MeV}$$

$$L_s^3 \times L_t = 48^3 \times 64$$

$$N_{\text{cfg}} = 109$$

- T. Izubuchi, L. Jin et al., PRD **100** (2019);
- X. Gao, N. Karthik, YZ et al., PRD **102** (2020).

#ex and #sl: numbers of exact and sloppy inversions per configuration

For n_z=(3,3,3): half the statistics for n_z=(0,0,5)

Coulomb gauge fixing

- Find the gauge transformation Ω of link variables $U_i(t,\vec{x})$ that minimizes:

$$F[U^\Omega] = \frac{1}{9V}\sum_{\vec{x}}\sum_{i=1,2,3}\left[-\textbf{re Tr } U_i^\Omega(t,\vec{x})\right]$$

Precision ~ 10^{-7}

- Gauge-variant correlations may differ in different Gribov copies.

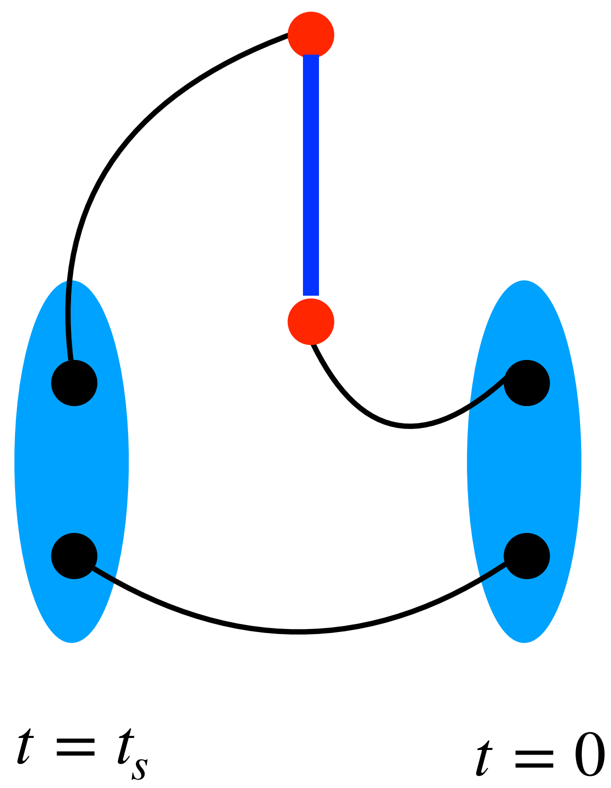
- In SU(2) Yang-Mills theory, different Gribov copies only affects the gluon propagator at far infrared region $|q| \lesssim 0.2$ GeV, though the ghost propagator are more sensitive to them.

A. Mass, Annals. Phys. **387** (2017).

- 🧐: Gribov copies should only affect large $|z|$ correlations in physical states, or PDF at small *x* where LaMET does not work.

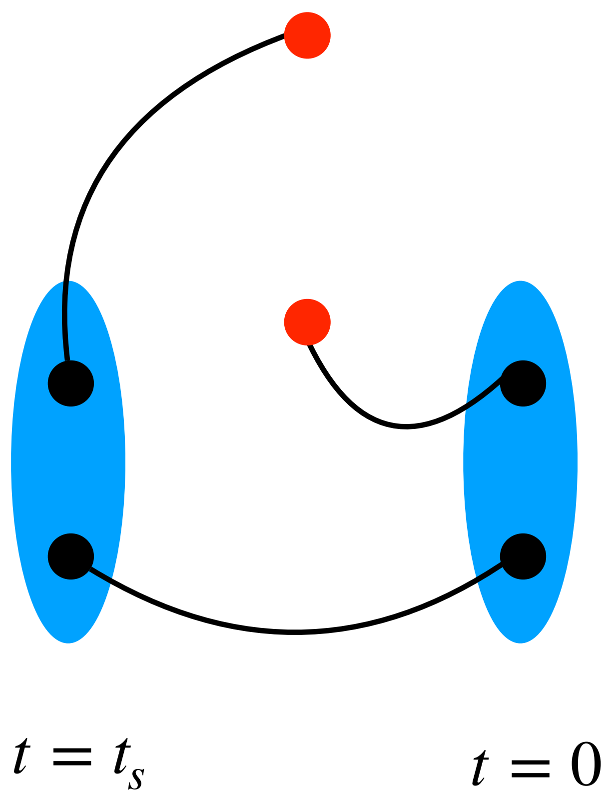
Bare matrix elements

GI



1-step hypercubic smeared Wilson line

CG

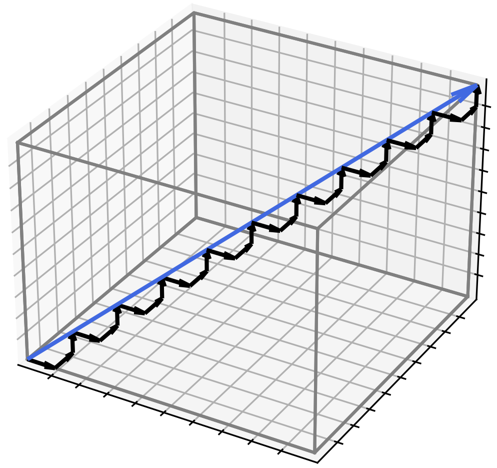


No Wilson line

Same quark propagators, free to calculate both!

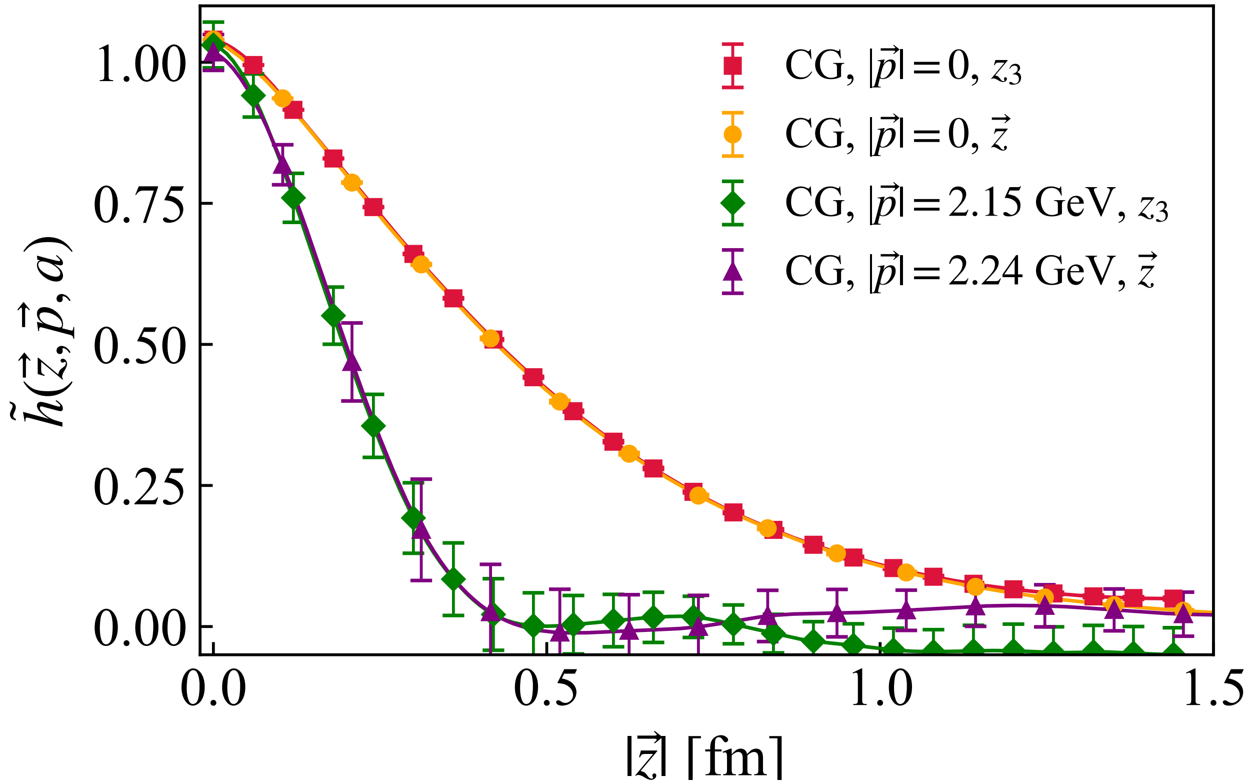
Bare matrix elements

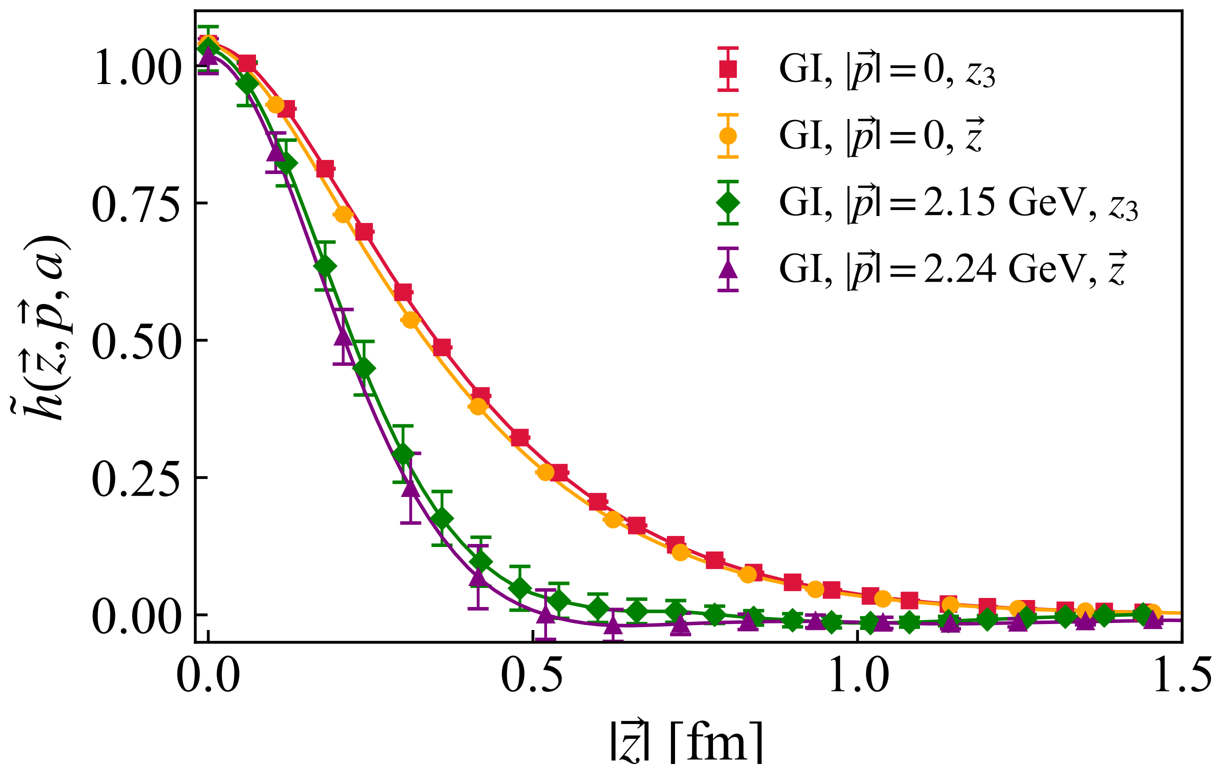
Rotational symmetry: on-axis V.S. off-axis momenta



Zig-zagged Wilson line for GI bilinear

B. Musch et al., PRD **83** (2011).





CG matrix elements precisely preserve the 3D rotational symmetry, which is broken for GI matrix elements with a zig-zagged Wilson line

Renormalizability

GI $\Leftrightarrow A^z = 0$

CG: $\nabla \cdot \mathbf{A} = 0$

$$\bar{\psi}_0(z)\Gamma\psi_0(0) = Z_\psi(a)\left[\bar{\psi}(z)\Gamma\psi(0)\right]_R \Rightarrow \lim_{a\to 0}\frac{\tilde{h}(z,0,a)}{\tilde{h}(z_s,0,a)} = \text{finite}$$

Wave function renormalization

- D. Zwanziger, NPB **518** (1998);
- Baulieu and Zwanziger, NPB **548** (1999);
- A. Niegawa, PRD **74** (2006);
- Niegawa, Inui and Kohyama, PRD **74** (2006).

Comparison with a finer lattice with

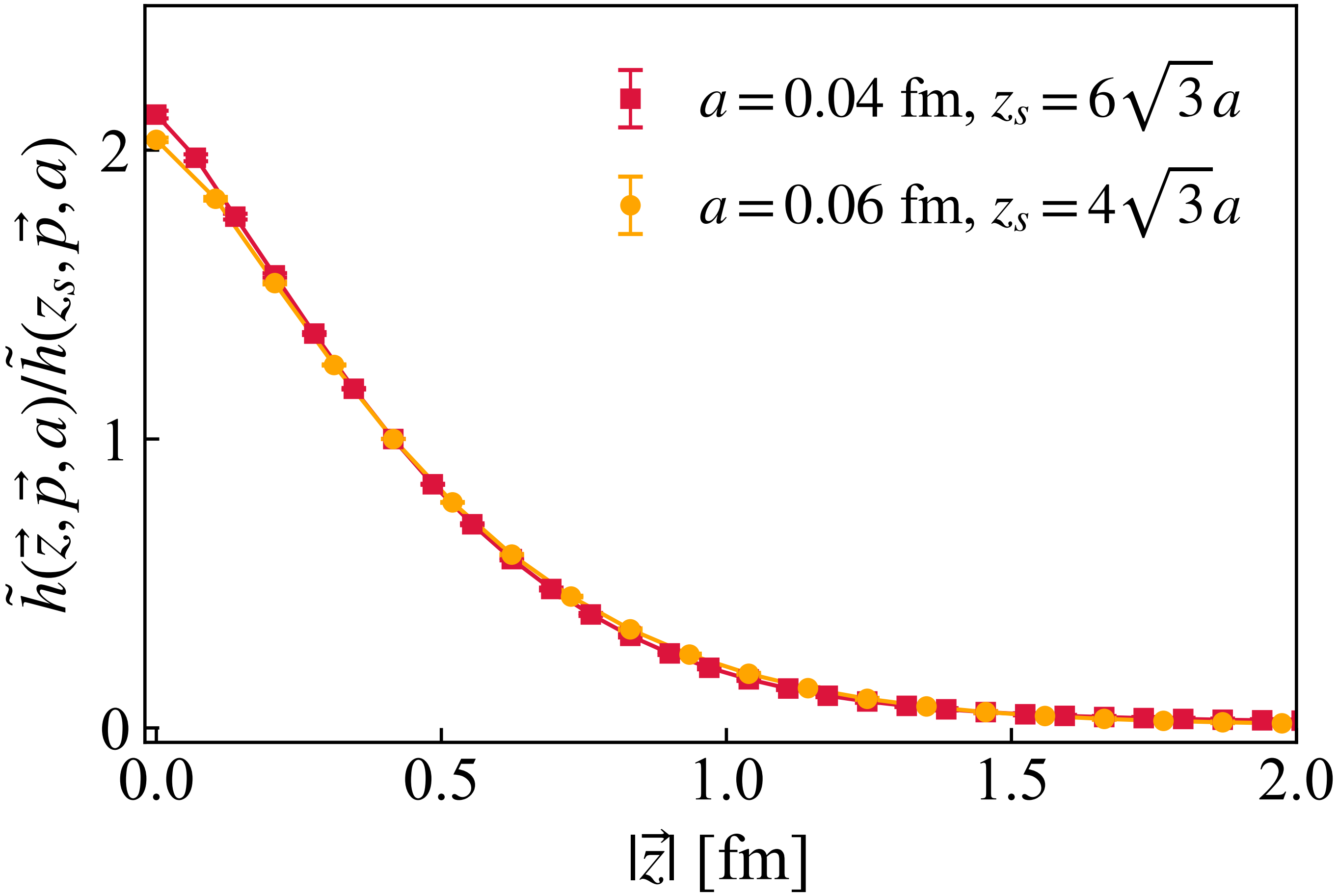
$a = 0.04$ fm

$m_\pi = 300$ MeV

$L_s^3 \times L_t = 64^4$

$N_{\rm cfg} = 12$

$\vec{z} = (1,1,1)z$



Nice continuum limit except for the discretization effects at *z~a* !

Consistency at short distance

Double ratio:

$$\mathcal{M}(z, P^z, a) = \frac{\tilde{h}(z, P^z, a)}{\tilde{h}(z,0,a)} \frac{\tilde{h}(0,0,a)}{\tilde{h}(0,P^z, a)}$$

K. Orginos et al., PRD **96** (2017).

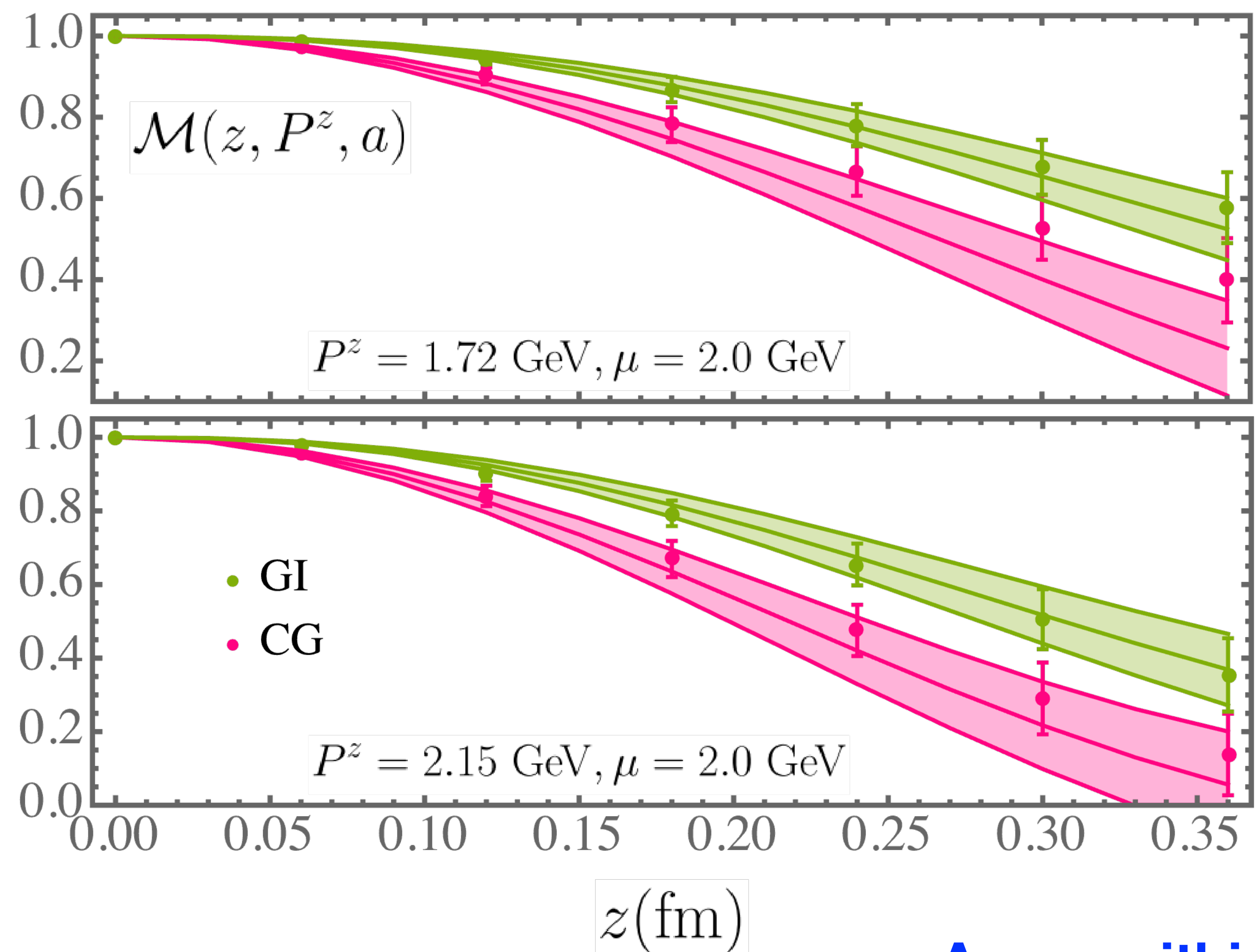
Parameterize PDF $f_v(x) \sim x^\alpha(1-x)^\beta$

↓

Fit α, β from the GI matrix elements

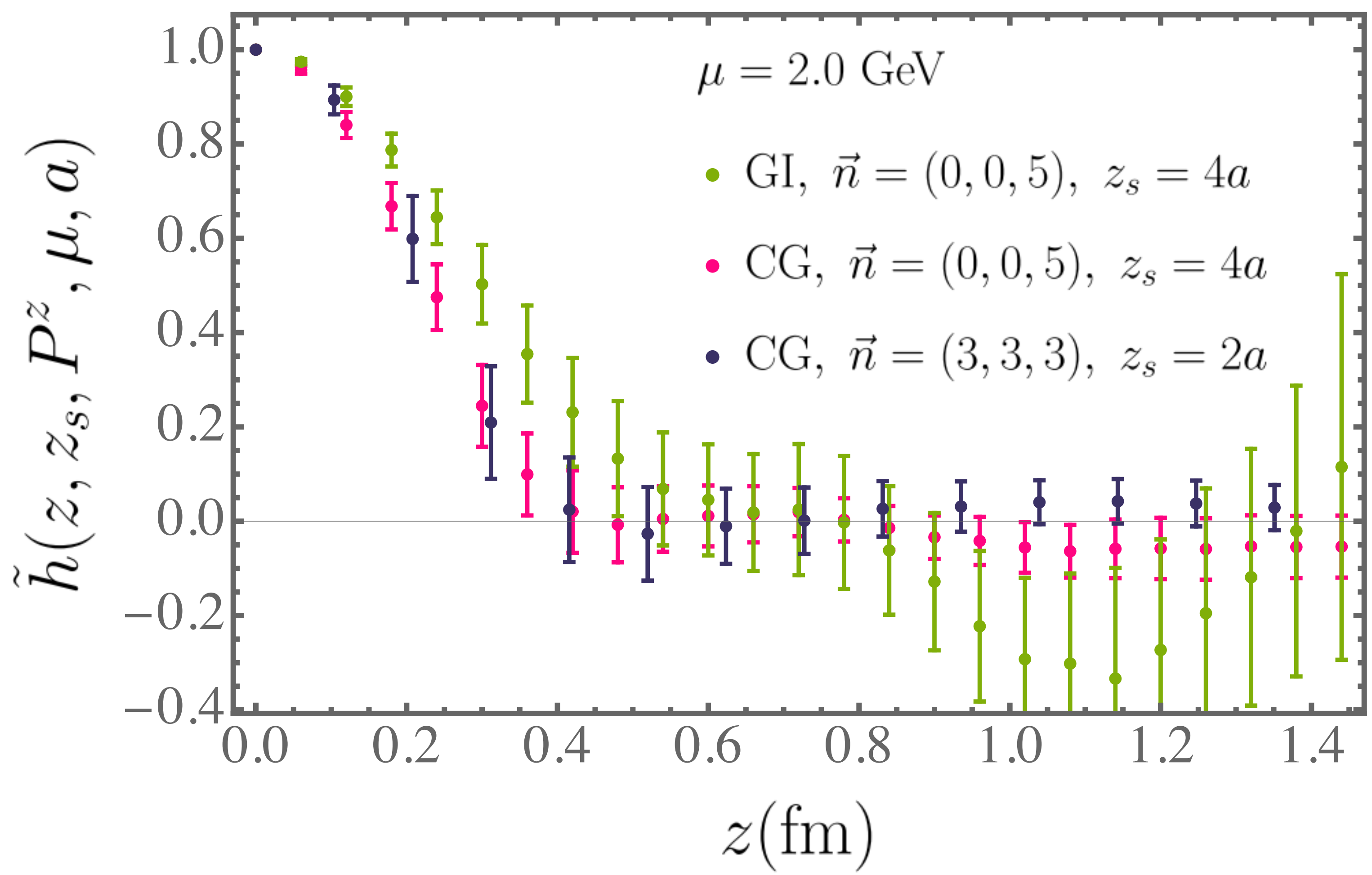
↓

Match fitted PDF to the CG matrix elements



Agree within 1σ !

Hybrid scheme renormalization with LRR



$$|z| \leq z_s\,, \quad \frac{h(z, P^z, a)}{h(z,0,a)}$$

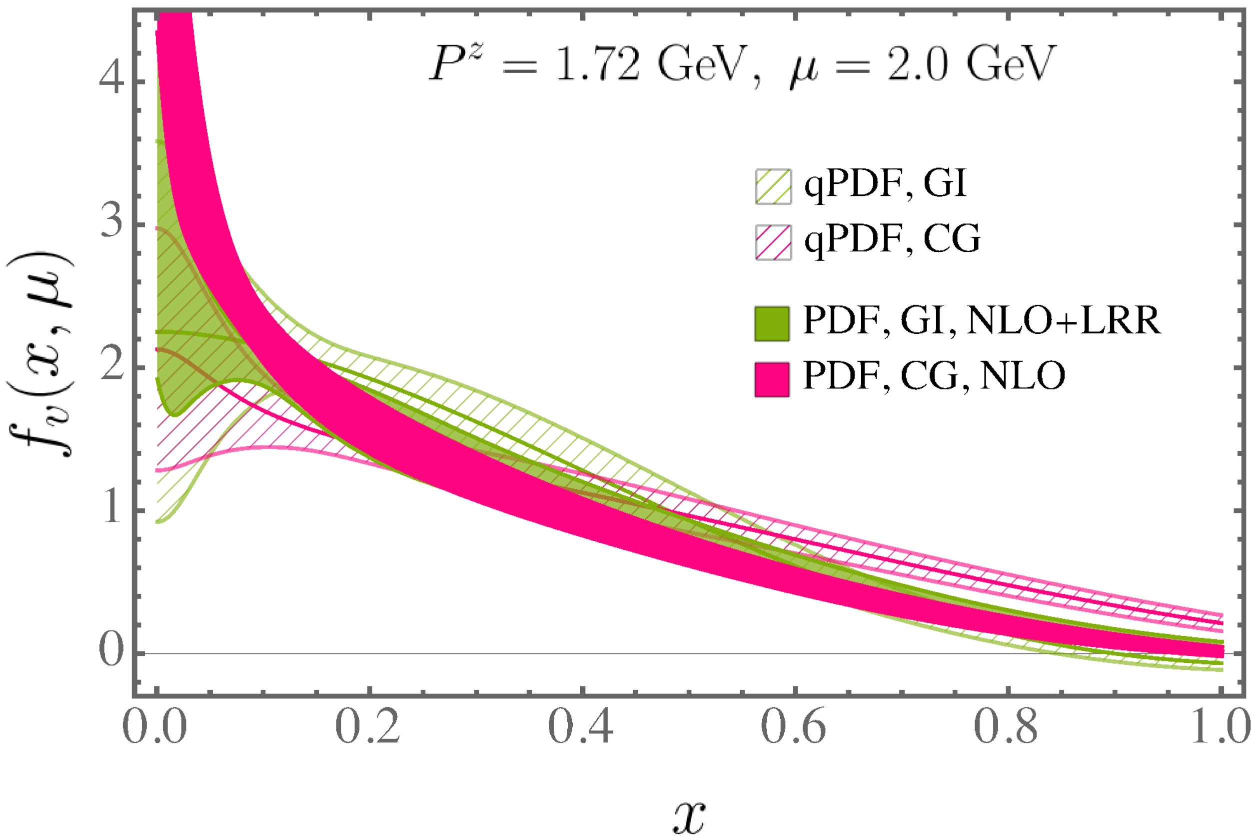
$$|z| > z_s\,, \quad e^{(\delta m(a)+\bar{m}_0)|z|}\frac{h(z, P^z, a)}{h(z_s,0,a)}$$

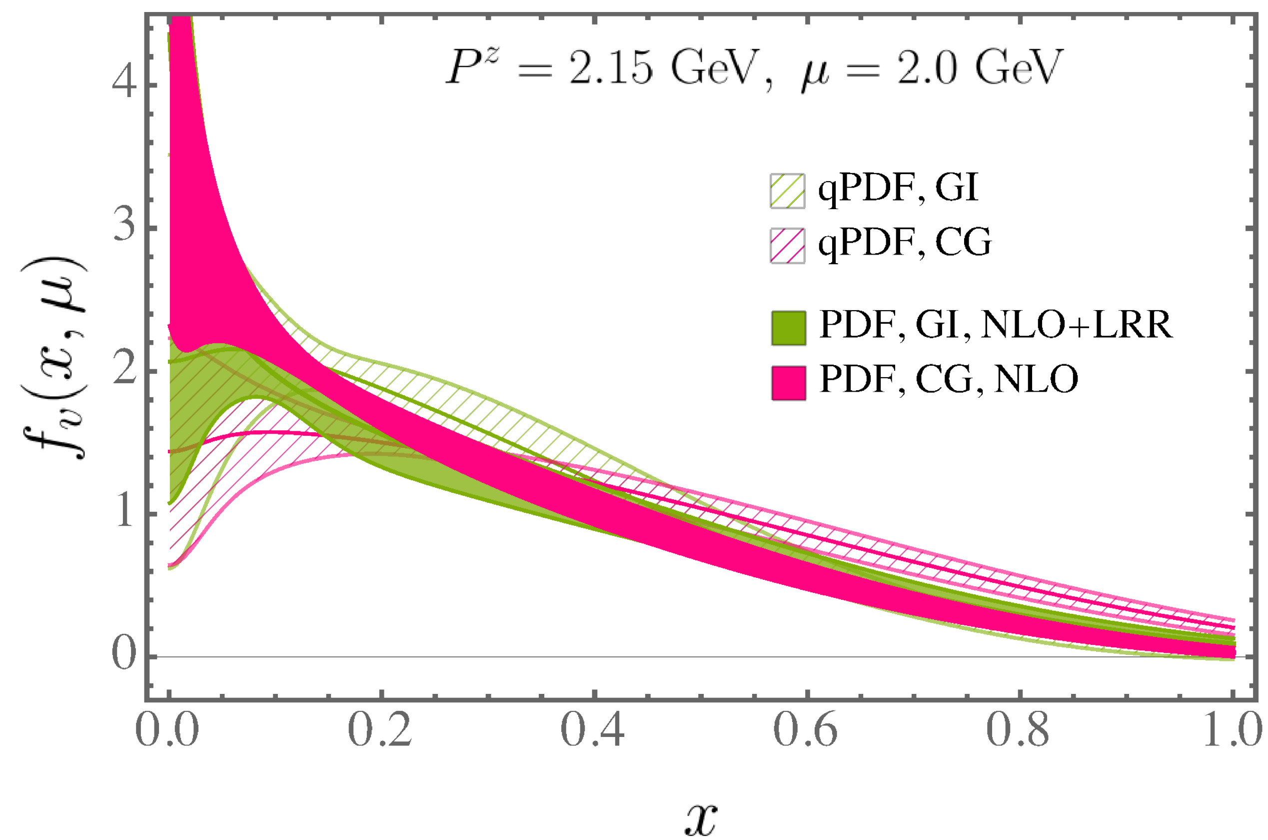
X. Ji, YZ, et al., NPB **964** (2021).

- **Both CG matrix elements and their errors remain small at large |z|, which leads to better controlled Fourier transform;**
- **Off-axis and on-axis momenta matrix elements are at similar precision, despite half the statistics for the former.**

Perturbative matching

Comparison of the GI and CG quasi-PDF methods:

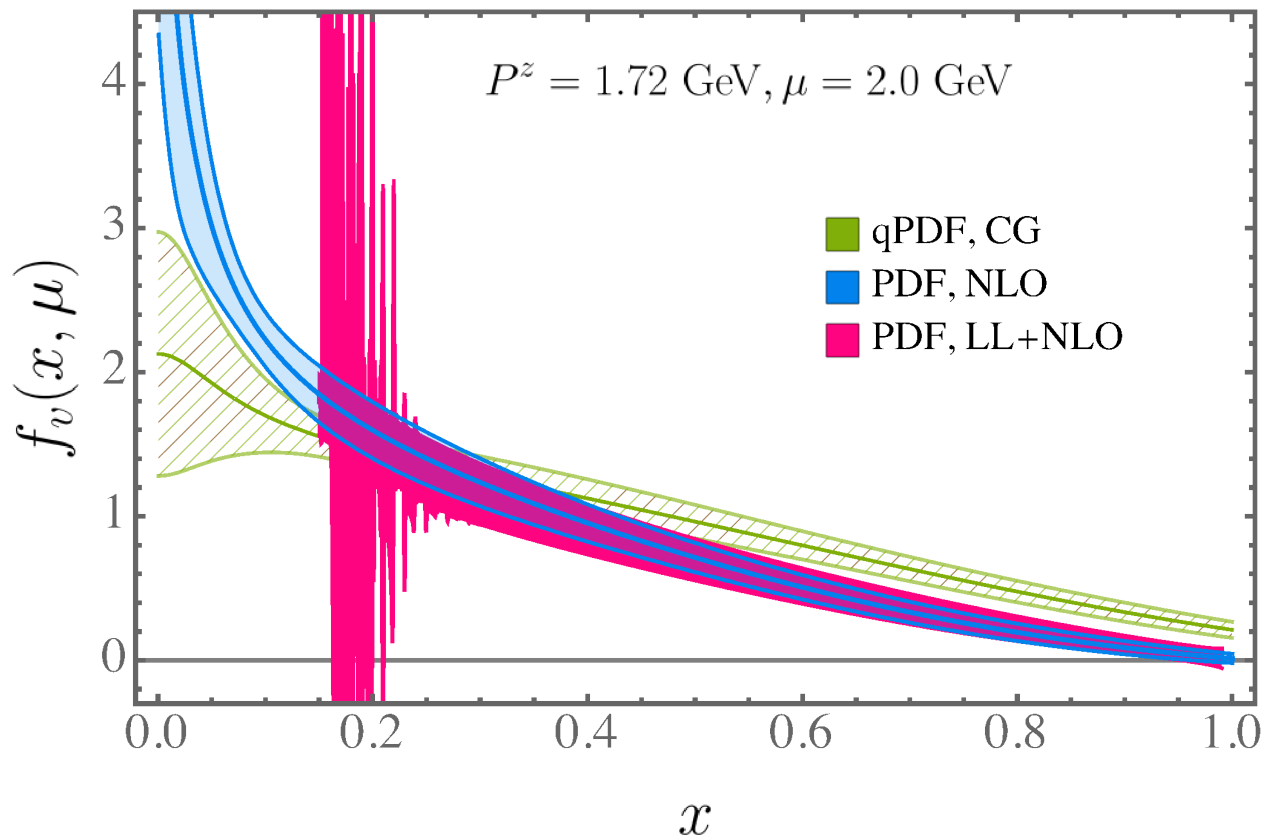


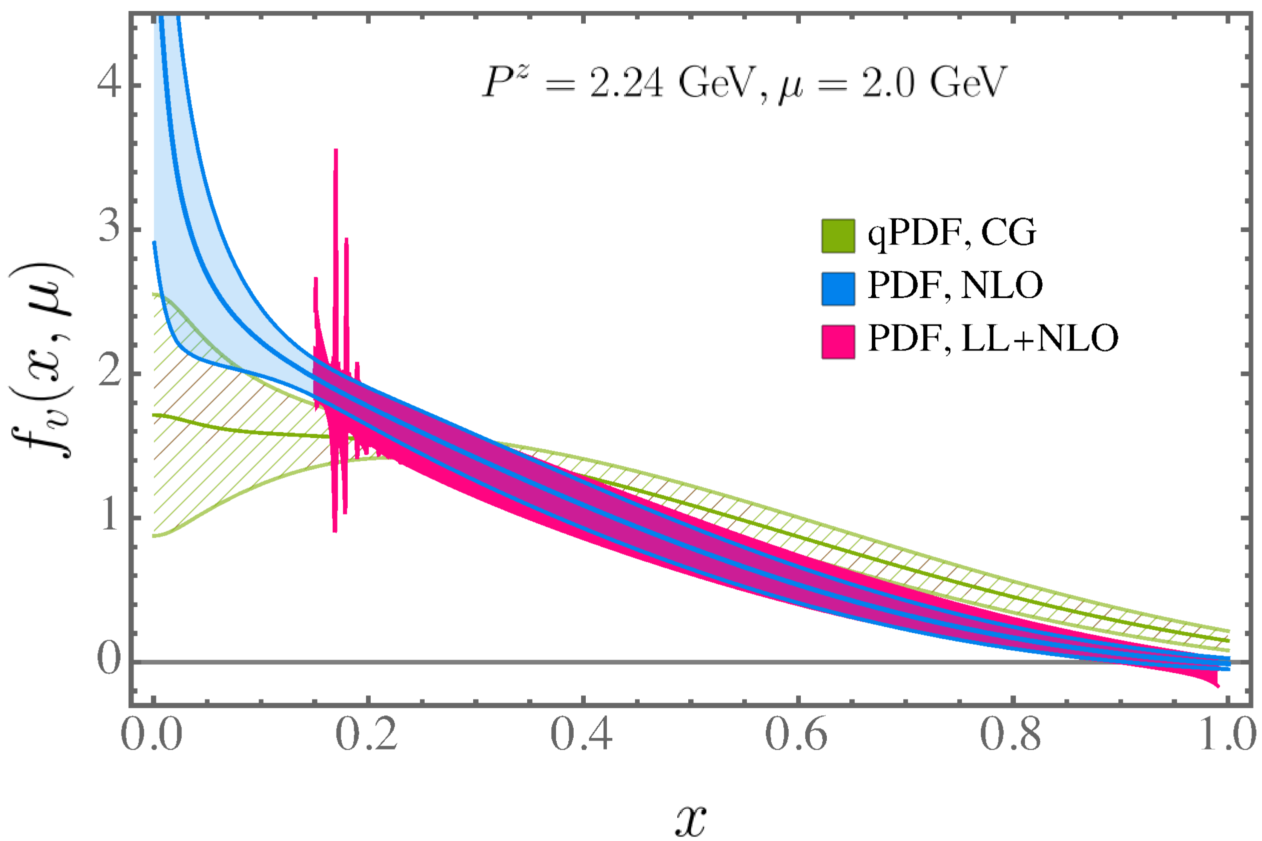


While the quasi-PDFs are different by at least 1σ, the matched results are consistent for $x \gtrsim 0.2$, demonstrating the universality in LaMET !

NLO V.S. Leading-logarithmic (LL) small-*x* resummation

Inverse matching at $\mu = \kappa \cdot 2xP^z$ → DGLAP evolution from $\kappa \cdot 2xP^z$ to $\mu = 2$ GeV → Vary $\kappa \in [1/\sqrt{2}, \sqrt{2}]$ to estimate scale uncertainty

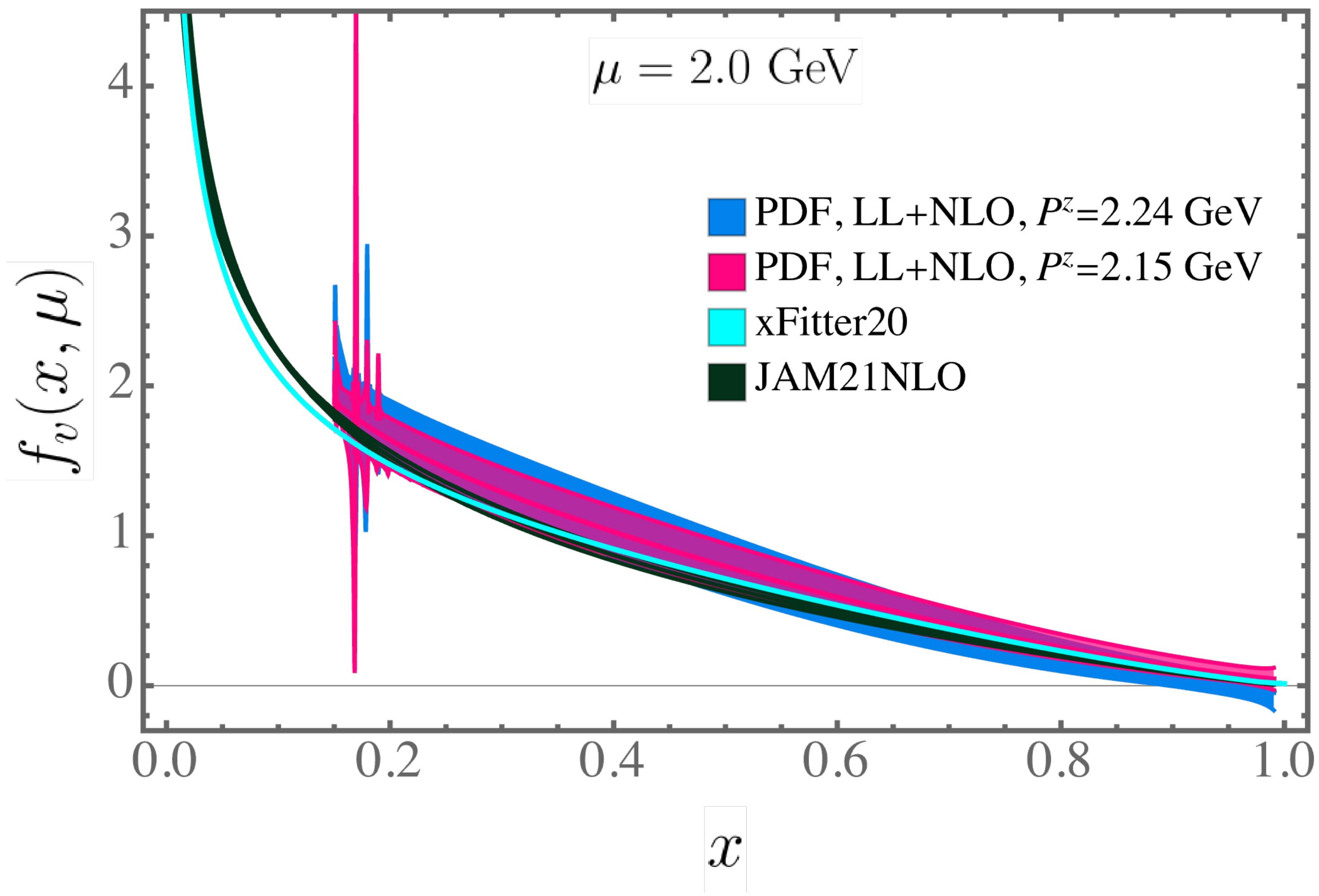




Small-*x* resummation makes almost no difference for $x \gtrsim 0.4$, but becomes important at smaller x and is out of control at $2xP^z \sim 0.8$ GeV where $\alpha_s \sim 1$.

Final result

Comparison with global fits

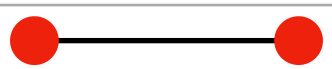


Global fits at NLO

- **JAM21NLO**, PRL 127 (2021);
- **xFitter (2020)**, PRD 102 (2020).

- **Agreement with global fits for $x \gtrsim 0.2$ within the (large) error;**
- **Precision can be considerably improved with larger statistics.**

Comparison between GI and CG quasi-PDFs

	Momentum direction	Renormalization	Gribov copies	Power corrections	Mixing	Higher-order corrections
 Gauge-invariant (GI)	$(0,0,n_z)$ $(n_x,0,0)$ $(0,n_y,0)$	Linear divergence + vertex and wave function renormalization	N/A	$\Lambda_{\rm QCD}^2/P_z^2$ w. renormalon subtraction	Lorentz symmetry	Available at NNLO now
 Coulomb gauge (CG)	(n_x, n_y, n_z) 🙂	Wave function renormalization 🙂	Affecting IR (long range) region 🧐	$\Lambda_{\rm QCD}^2/\vec{p}^2$	3D rotational symmetry 🙁🧐	May be hard to go beyond NLO 🧐

Summary

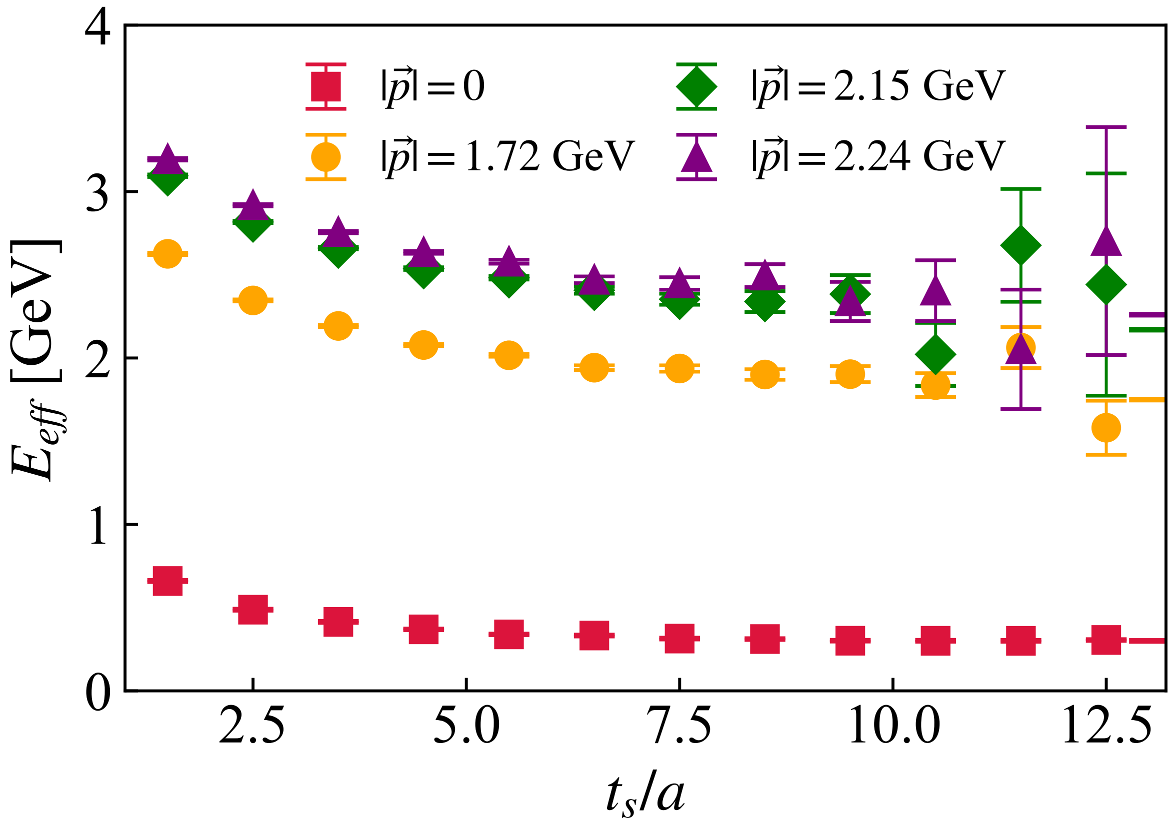
- We verify the factorization of CG quasi-PDF to the PDF at NLO;
- We demonstrate the universality in LaMET through the equivalence of CG and GI quasi-PDF methods;
- The CG correlations have the advantages of access to larger off-axis momenta (at a lower computational cost), absence of linear divergence, and enhanced long-range precision;
- It is almost free to compute the GI and CG matrix elements at the same time.

Outlook

- **Open questions:**
  - Effects of Gribov copies seem negligible, but should be further studied;
  - Threshold resummation is necessary and similar to the GI quasi-PDF;
  - OPE and mixings complicated by breaking of Lorentz symmetry.
- **Broader applications:**
  - GPDs. Straightforward extension from the PDF.
  - TMDs. Staple-shaped Wilson lines with infinite extension.
    - Absence of Wilson line provides much convenience in computation and renormalization;
    - Factorization should be provable as boosted quarks in a physical gauge capture the right collinear degrees of freedom.

Bare matrix elements

Effective mass



3pt/2pt ratio