Recent advances in GPD and CFF extraction from hard exclusive processes

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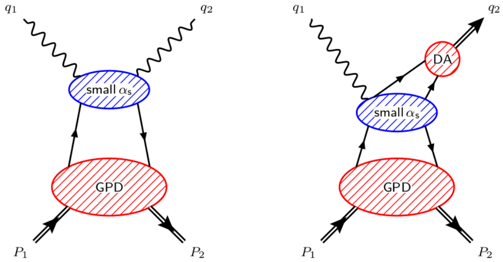
Towards Improved Hadron Femtography with Hard Exclusive Reactions 2023

Outline

Accessing GPDs

- exclusive processes such as DVCS and DVMP

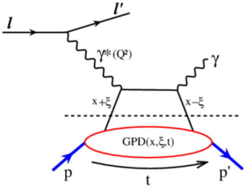


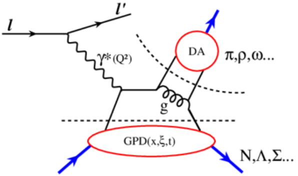
(a) DVCS (b) DVMP

- at leading twist four complex Compton/transition form factors $\mathcal{H}\left(\xi,t,Q^2\right)$, $\mathcal{E}\left(\xi,t,Q^2\right)$, $\widetilde{\mathcal{H}}\left(\xi,t,Q^2\right)$, $\widetilde{\mathcal{E}}\left(\xi,t,Q^2\right)$
- also double DVCS, timelike Compton scattering, etc.

Factorization and GPDs

- factorization theorem [Collins et al. '97, '98]
- DVCS: transversal photon, DVMP: longitudinal photon and meson at twist-2





H. Moutarde et al.

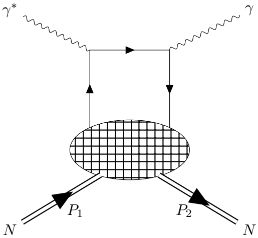
- GPDs enter a convolution as the soft unperturbative part

CFFs: $$\mathcal{F}^A\left(\xi,\Delta^2,Q^2\right)=\int_{-1}^{1}\frac{dx}{2\xi}\,\underbrace{{}^{A}T\left(x,\xi\middle|\alpha_s\left(\mu_R\right),\frac{Q^2}{\mu_F^2}\right)}_{\text{hard scale}}\underbrace{F^A\left(x,\eta,\Delta^2,\mu_F^2\right)}_{\text{soft scale}},\quad A\in\{q,g\}$$

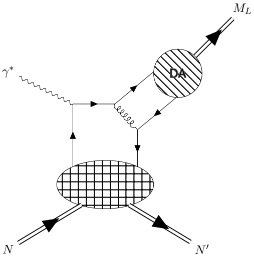
TFFs: $$\mathcal{F}^A\left(\xi,\Delta^2,Q^2\right)=\frac{fC_F}{QN_c}\int_{-1}^{1}\frac{\mathrm{d}x}{2\xi}\int_0^1\mathrm{d}v\,\underbrace{\varphi(v)}_{\text{soft scale}}\,\underbrace{{}^{A}T\left(x,v,\xi\middle|\alpha_s\left(\mu_R\right),\frac{Q^2}{\mu_F^2},\frac{Q^2}{\mu_\varphi^2},\frac{Q^2}{\mu_R^2}\right)}_{\text{hard scale}}\underbrace{F^A\left(x,\eta,\Delta^2\mu_F^2\right)}_{\text{soft scale}}$$

Handbag diagrams

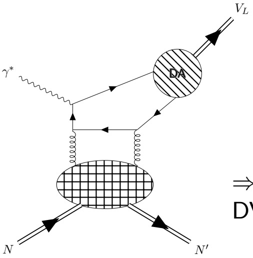
DVCS at LO



DVMP at LO



a) quark subprocess



b) gluon subprocess

$\Rightarrow$ gluons enter DVMP at LO!

Modelling GPDs

GPD evolution

- evolution in x space complicated, mixing of components

$$\frac{1}{x^{p_a}}\frac{\partial H^a\left(x,\eta,t,\mu^2\right)}{\partial\log\left(\mu^2\right)}=\alpha_s\left(\mu^2\right)\sum_{b\in\{q,g\}}\int_x^1\frac{\mathrm{d}z}{\eta}K^{ab,(0)}\left(\frac{z}{\eta},\frac{\eta}{x}\right)\frac{H^b\left(z,\eta,t,\mu^2\right)}{z^{p_b}}$$

- we use conformal moments

$$F_n(\eta,t)=\int_{-1}^{1}dx c_n(x,\eta)F(x,\eta,t)$$

$$c_n(x,\eta)=\eta^n\frac{\Gamma\left(\frac{3}{2}\right)\Gamma(1+n)}{2^n\Gamma\left(\frac{3}{2}+n\right)}C_n^{\frac{3}{2}}\left(\frac{x}{\eta}\right)$$

- $C_n^{3/2}$ Gegenbauer polynomials for quarks (5/2 for gluons)
- analytic continuation $n\to j\in\mathbb{C}$
- evolution diagonal in j space at LO

$$\mu\frac{d}{d\mu}F_j^q\left(\eta,t,\mu^2\right)=-\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)}F_j^q\left(\eta,t^2,\mu^2\right)$$

Valence quark GPDs

- valence quarks modeled in x space ($q = u, d$) at crossover line $x = \eta$ (no Q^2 evolution)

$$\Im\mathfrak{m}\mathcal{H}(\xi,t) \stackrel{LO}{=} \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t)+\frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t)+\frac{2}{9}H^{\textsf{sea}}(\xi,\xi,t)\right]$$

$$H_q^{\textsf{val}}(x,x,t)=\frac{n_q r_q}{1+x}\left(\frac{2x}{1+x}\right)^{-\alpha_v(t)}\left(\frac{1-x}{1+x}\right)^{b_q}\frac{1}{1-\frac{1-x}{1+x}\frac{t}{M_q^2}},\quad q=u,d$$

$$\alpha_v(t)=0.43+0.85t/\mathrm{GeV}^2$$

- fixed parameters: n PDF normalization, $\alpha(t)$ Regge trajectory

Sea quark and gluon GPDs

- sea quarks modelled in j space
- $SO(3)$ partial waves expansion

$$F_j^a(\eta,t) = \sum_{\substack{J=J_{\min} \\ \text{even}}}^{j+1} F_j^{J,a}(t)\eta^{j+1-J}\hat{d}^J_{\alpha,\beta}(\eta), \quad J = j+1, j-1, j-3, \ldots$$

- leading contribution

$$F_j^{J,a}(\eta=0,t) = N^a \frac{\mathrm{B}\left(1-\alpha^a+j, \beta^a+1\right)}{\mathrm{B}\left(2-\alpha^a, \beta^a+1\right)} \frac{\beta(t)}{1-\frac{t}{\left(m_j^a\right)^2}},$$

$$\left(m_j^a\right)^2 = \frac{1+j-\alpha^a}{\alpha'^a}, \quad \beta(t) = \left(1-\frac{t}{M^2}\right)^{-p}, \quad a = \{s,g\}$$

- full NLO QCD Q^2 evolution

- partial wave expansion implemented simply in Mellin-Barnes integral

$$\boldsymbol{\mathcal{H}} = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\xi^{-j-1}\left[i+\tan\left(\frac{\pi j}{2}\right)\right]\times$$
$$\times\left[[\mathbb{C}\otimes\mathbb{E}]_j + [\mathbb{C}\otimes\mathbb{E}]_{j+2}\boldsymbol{S} + [\mathbb{C}\otimes\mathbb{E}]_{j+4}\boldsymbol{T}\right]\boldsymbol{H}_j^{(l)}$$

- 10-15 parameters

Dispersion relations

- CFFs constrained by dispersion relations

$$\mathfrak{Re}\,\mathcal{H}(\xi,t) \stackrel{LO}{=} \Delta(t) + \frac{1}{\pi}\mathrm{P.V.}\int_0^1 \mathrm{d}x \left(\frac{1}{\xi - x} - \frac{1}{\xi + x}\right) \mathfrak{Im}\,\mathcal{H}(x,t)$$

- subtraction constant model

$$\Delta(t) = \frac{C}{\left(1 - \frac{t}{M_C^2}\right)^2}$$

- $\Delta_{\mathcal{H}}(t) = -\Delta_{\mathcal{E}}(t)$, $\Delta_{\tilde{\mathcal{H}}}(t) = \Delta_{\tilde{\mathcal{E}}}(t) = 0$
- only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are modelled

Results

NLO DIS+DVCS+DVMP small-x global fit

- First global fits to DIS+DVCS+DVMP HERA collider data [Lautenschlager, Müller, Schäfer, '13, unpublished!]
- hard scattering amplitude corrected in the meantime [Duplančić, Müller, Passek-Kumerički '17]
- [M. Č. et al., '23] preliminary results for NLO DIS+DVCS+DVMP small-x global fit
- we also studied LO fits, fits to DIS+DVCS and fits to DIS+DVMP
- what are the effects of NLO corrections?
- can we get universal GPDs regardless of DVCS and DVMP data?

Cross sections

- DVCS

$$\frac{d\sigma^{\gamma^* N \to \gamma N}}{d\Delta^2} \approx \frac{\pi \alpha_{em}^2}{(W^2+Q^2)^2}\left[|\mathcal{H}|^2 + |\widetilde{\mathcal{H}}|^2 - \frac{\Delta^2}{4M^2}|\mathcal{E}|^2\right]$$

- DVMP

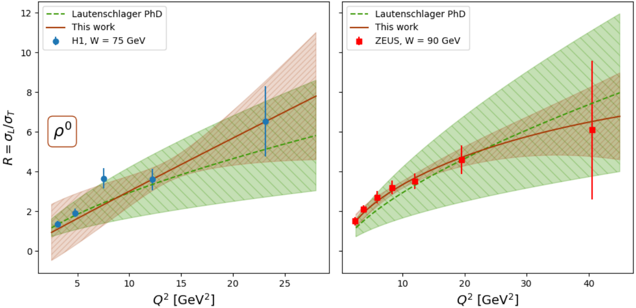
$$\frac{d\sigma^{\gamma^* N \to V N}}{d\Delta^2} \approx \frac{4\pi^2 \alpha_{em} x_B^2}{Q^4}\left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M^2}|\mathcal{E}|^2\right]$$

- for $|\Delta^2| < 1$ GeV^2 CFF $\mathcal{E}$ suppressed by $-\frac{\langle \Delta^2 \rangle}{4M^2} \approx 5 \times 10^{-2}$
- for $\widetilde{\mathcal{H}}$ Regge intercept $\alpha(0) \approx 1/2$, for $\mathcal{H}$ $\alpha(0) \approx 1$, $\widetilde{\mathcal{H}}$ also suppressed
- we ignore valence contributions, only singlet $\mathcal{H}$
- asymptotic distribution amplitude, dominant term in conformal space $\varphi_0 \approx 1$

Changes to original analysis

- for DVMP cut-off at $Q^2 \geq 10$ GeV^2 instead of 4
- no ϕ production
- different parameter constraints
- Q^2 and W dependence in $R = \sigma_L/\sigma_T$

$$R\left(Q^2\right) = \frac{Q^2}{m_{\mathrm{V}}^2}\left(1 + a\frac{Q^2}{m_{\mathrm{V}}^2}\right)^{-p} \rightarrow R(W, Q^2) = \frac{Q^2}{m_\rho^2}\left(1 + a\frac{Q^2}{m_\rho^2}\right)^{-p}\left(1 - b\frac{Q^2}{W}\right)$$



Data & χ^2/N_{pts}

- DIS data: H1 F_2
- DVCS: H1 and ZEUS data, $Q^2 \geq 5.0$ GeV^2
- DVMP: H1 nd ZEUS ρ^0 production, $Q^2 \geq 10.0$ GeV^2
- no t dependence

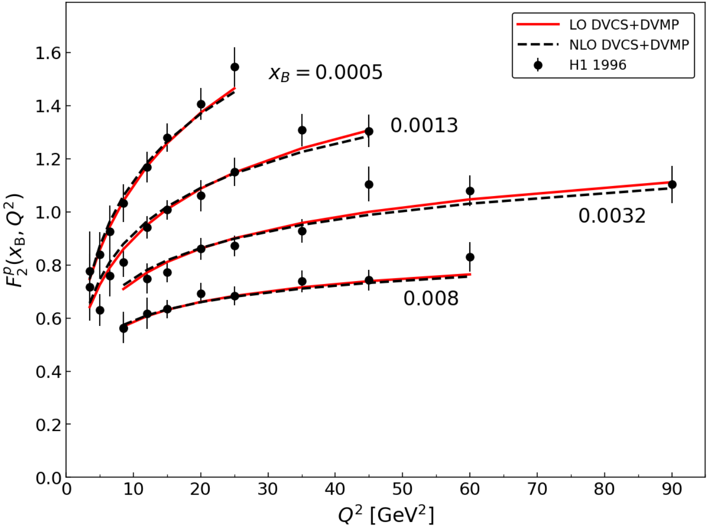
Dataset	N_{pts}	LO DVCS	LO DVMP	LO ALL	NLO DVCS	NLO DVMP	NLO ALL
DIS	85	0.6	0.6	0.6	0.8	0.8	0.8
DVCS	27	0.4	12252.2	0.6	0.6	27269.6	0.8
DVMP	45	49.7	3.1	3.3	11.7	1.5	1.7
Total	157	14.6	2108.3	1.4	3.9	4690.6	1.1

Parameters

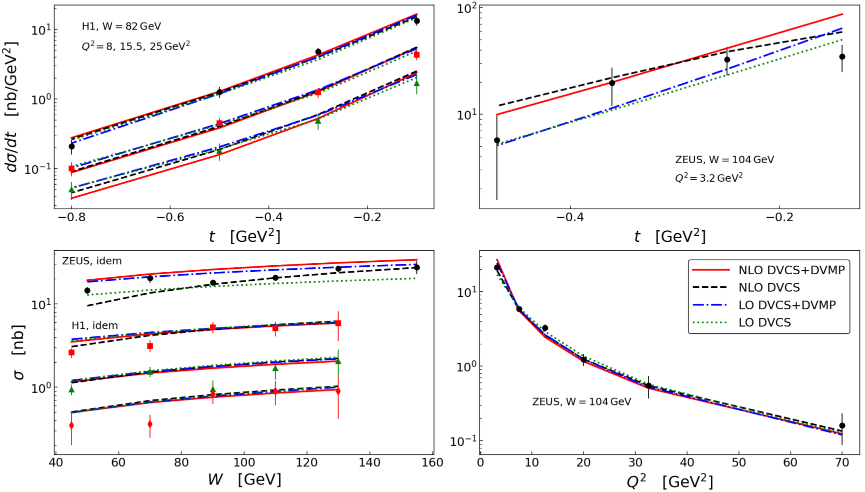
- pure DVMP fits prefer large quark skewness, rely on cancelation from gluons
- subleading waves were larger than leading ones, unstable
- DVMP more sensitive to gluons, we leave more freedom to gluon subleading PWs
- we want a decrease in contribution for PW expansion

	n^{sea}	α_0^{sea}	α'_{sea}	m^2_{sea}	s_2^{sea}	s_4^{sea}	α_0^G	α'_G	m^2_G	s_2^G	s_4^G
unit		1	GeV^{-2}	GeV^2	1	1	1	GeV^{-2}	GeV^2	1	1
initial	0.15	1.00	0.15	0.70	-0.20	0.00	1.00	0.15	0.70	0.00	0.00
limits			(0.0,1.0)	(0,3)	(-0.3,0.3)	(-0.1,0.1)		(0.0,1.0)	(0,3)	(-3.0,3.0)	(-1.0,1.0)
final	0.168	1.128	0.125	0.412	0.280	-0.044	1.099	0.000	0.145	2.958	-0.951
uncert.	0.002	0.011	0.040	0.050	0.032	0.010	0.011	0.010	0.008	0.039	0.025

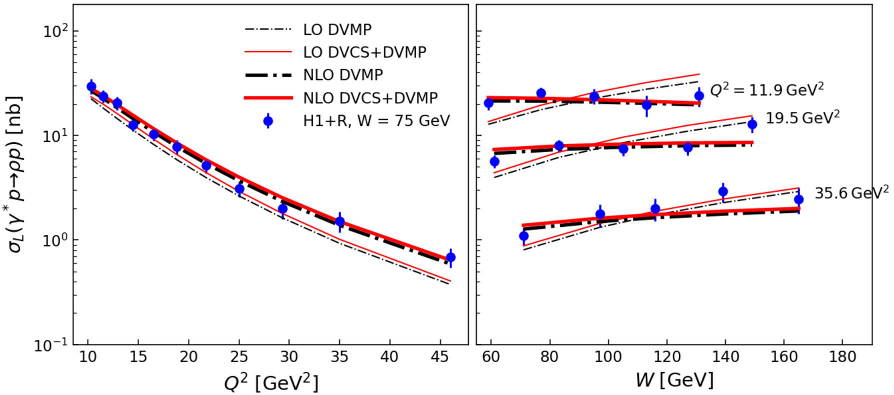
DIS F_2 data description

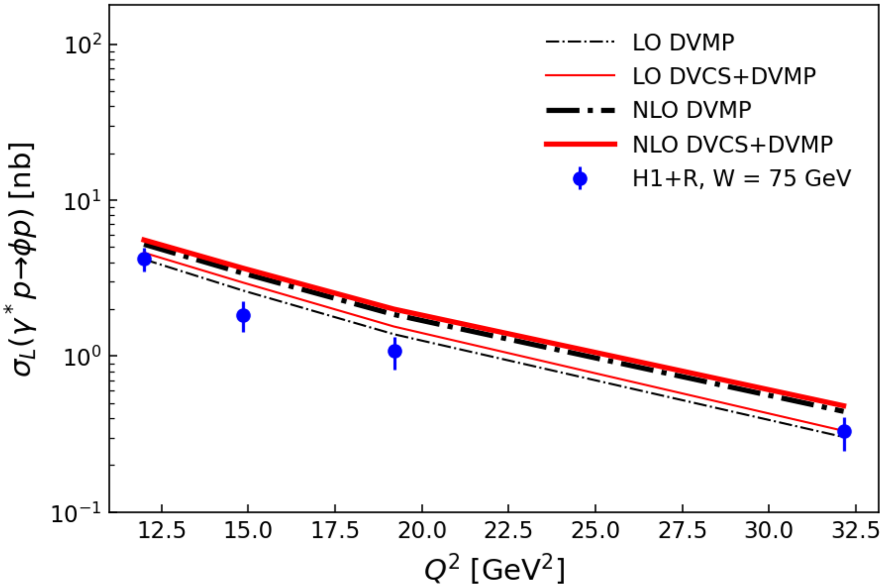


DVCS data description

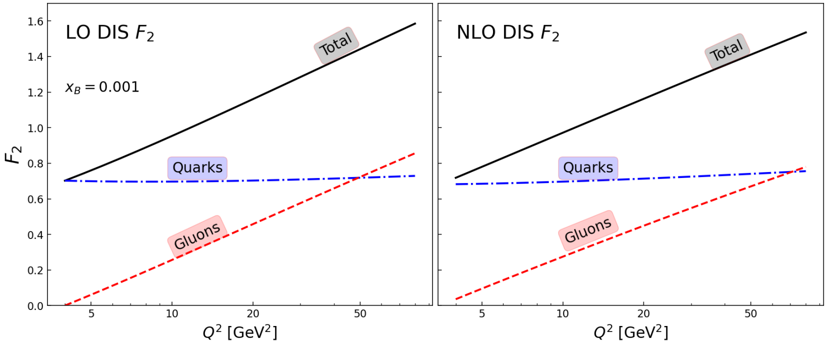


DVMP data description



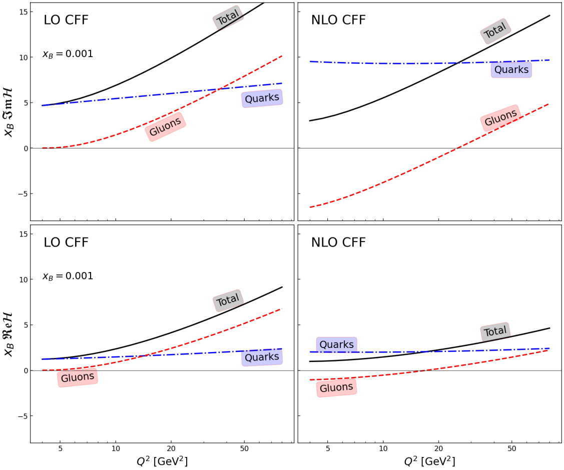

LO DVMP
LO DVCS+DVMP
NLO DVMP
NLO DVCS+DVMP
H1+R, W = 75 GeV
$\sigma_L(\gamma^* p \to \phi p)$ [nb]
Q^2 [GeV^2]
10^2
10^1
10^0
10^{-1}
12.5
15.0
17.5
20.0
22.5
25.0
27.5
30.0
32.5

Quark and gluon contributions: DIS



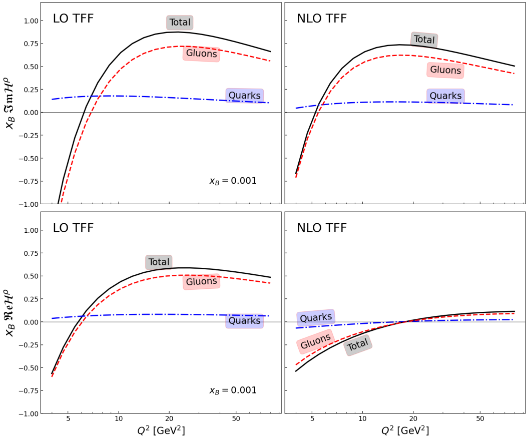
- at LO gluons do not contribute at low Q^2
- not much changes at NLO

Quark and gluon contributions: DVCS



- at LO gluons do not contribute at low Q^2
- at NLO gluons negative at low Q^2

Quark and gluon contributions: DVMP

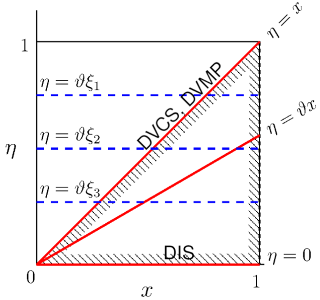


- at LO gluons dominate at low Q^2
- at NLO a much different story, gluons negative at low Q^2, dominate at large Q^2

Skewness ratio

- skewness ratio: ratio of GPD to corresponding PDF

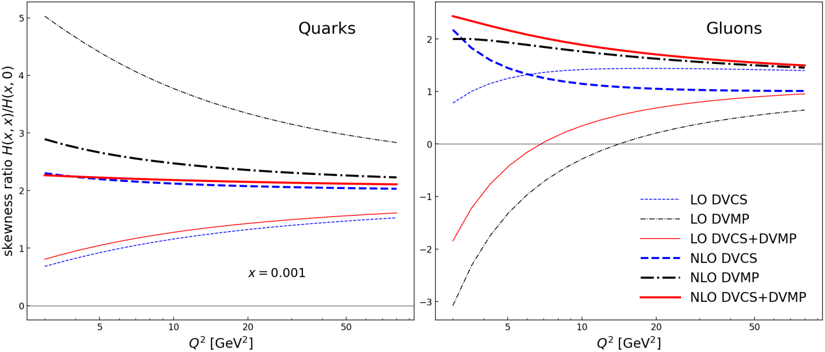
$$r = \frac{H(x, \eta = x)}{q(x)}$$



- conformal (Shuvaev) values, PDFs completely specify GPDs:

$$r^q \approx 1.65, \quad r^G \approx 1$$

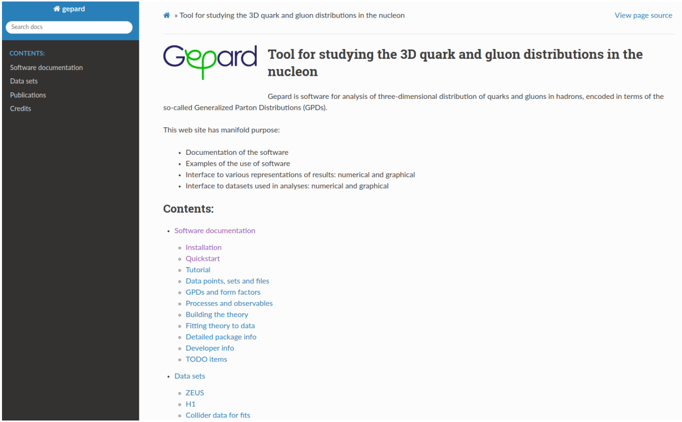
Skewness at LO and NLO



Universal GPD structure emerges at NLO!

Gepard

`https://gepard.phy.hr`


gepard

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» Tool for studying the 3D quark and gluon distributions in the nucleon

View page source

Gepard

Tool for studying the 3D quark and gluon distributions in the nucleon

Gepard is software for analysis of three-dimensional distribution of quarks and gluons in hadrons, encoded in terms of the so-called Generalized Parton Distributions (GPDs).

This web site has manifold purpose:

- Documentation of the software
- Examples of the use of software
- Interface to various representations of results: numerical and graphical
- Interface to datasets used in analyses: numerical and graphical

Contents:

- Software documentation
  - Installation
  - Quickstart
  - Tutorial
  - Data points, sets and files
  - GPDs and form factors
  - Processes and observables
  - Building the theory
  - Fitting theory to data
  - Detailed package info
  - Developer info
  - TODO items
- Data sets
  - ZEUS
  - H1
  - Collider data for fits

Conclusion

- stable fits for higher Q^2 and careful L-T separation
- Q^5 scaling after $Q^2 \approx 10$ GeV^2
- universal GPD structure emerges at NLO