Brookhaven National Laboratory

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# Chiral and trace anomalies in DVCS

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based on 2210.13419 and 2305.09431
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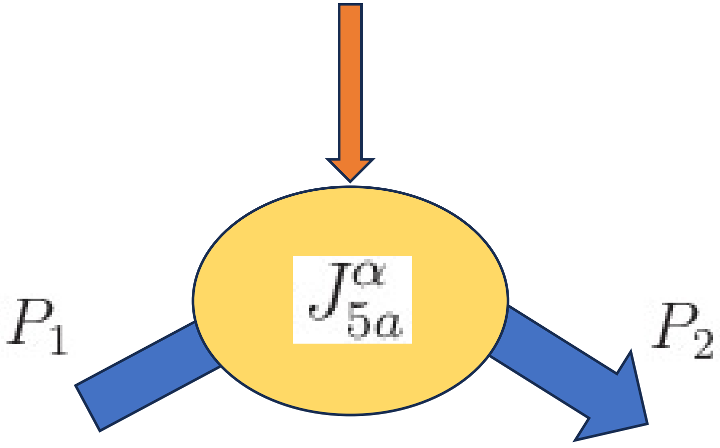
# Circa 1960: Isovector axial form factors

Noether current of SU(2) chiral symmetry $q \to e^{i\alpha^a \tau^a \gamma_5} q$

$$J^\alpha_{5a} = \sum_q \bar{q}\gamma^\alpha\gamma_5 \frac{\tau^a}{2} q$$




$$t = (P_2 - P_1)^2 \equiv l^2$$

Nucleon form factors

$$\langle P_2 | J^\alpha_{5a} | P_1 \rangle = \bar{u}(P_2) \left[ \gamma^\alpha \gamma_5 F_A(t) + \frac{l^\alpha \gamma_5}{2M} F_P(t) \right] \frac{\tau^a}{2} u(P_1)$$

pseudovector pseudoscalar

# Chiral symmetry breaking and pion pole

In massless QCD, the current is conserved $\partial_\alpha J^\alpha_{5a} = 0$

$$2MF_A(t) + \frac{tF_P(t)}{2M} = 0 \quad \Rightarrow \quad F_P(t) \approx \frac{-4M^2 g_A^{(3)}}{t}$$

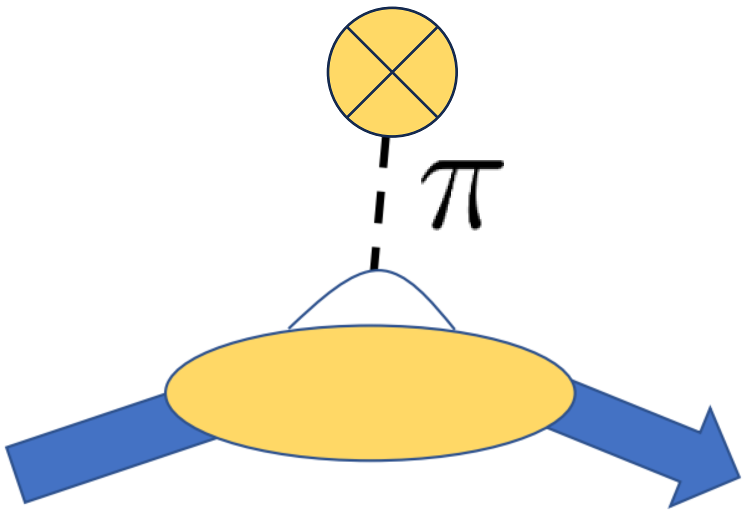
Pole at $t = 0$ from massless particle exchange

In real QCD with finite quark masses ,

$$\frac{1}{t} \to \frac{1}{t - m_\pi^2}$$

Pion nearly massless due to spontaneously broken chiral symmetry Nambu (1960)

# Pion pole in GPD




GPD = x-dependent form factor

$$F_P(t) = \int_{-1}^{1} dx \left( \tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \right) \approx \frac{-4M^2 g_A^{(3)}}{t}$$

Massless pole already in GPD

$$\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \sim \frac{\theta(\xi - |x|)}{t}$$

Penttinen, Polyakov, Goeke (1999)

??

First indication from lattice QCD? (Note that $\xi = 0$ in their paper.)
Bhattacharya et al. (2023)

# Singlet axial form factors

Nucleon form factor of $J_5^\alpha = \sum_q \bar{q}\gamma^\alpha\gamma_5 q$

$$\langle P_2|J_5^\alpha|P_1\rangle = \bar{u}(P_2)\left[\gamma^\alpha\gamma_5 g_A(t) + \frac{l^\alpha\gamma_5}{2M}g_P(t)\right]u(P_1)$$

$g_A(0) = \Delta\Sigma$ quark spin contribution to the nucleon spin

In massless QCD, the current is conserved due to axial U(1) symmetry

$$2Mg_A(t) + \frac{tg_P(t)}{2M} = 0 \quad \Rightarrow \quad \frac{g_P(t)}{2M} \approx -\frac{2M\Delta\Sigma}{t}$$

Pole at $t = 0$ from massless $\eta_0$ meson exchange

# Chiral anomaly

Quantum mechanically, the current is not conserved $\partial_\alpha J_5^\alpha = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu}\tilde{F}_{\mu\nu}$

$$\Rightarrow \quad \frac{g_P(t)}{2M} = \frac{1}{t}\left( i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2Mg_A(t)\right)$$

anomaly pole $\qquad$ $\eta_0$ pole

In real QCD, there is no massless pole in $g_P(t)$ due to pole cancellation

Pole shifted to the physical $\eta'$ meson mass via resummation of $1/N_c$ series Witten (1979), Veneziano (1979)

$$\frac{1}{t} + \frac{m_{\eta'}^2}{t^2} + \frac{m_{\eta'}^4}{t^3} + \cdots = \frac{1}{t - m_{\eta'}^2}$$

$$m_{\eta'}^2 \sim \langle (F\tilde{F})(F\tilde{F})\rangle \sim 1/N_c$$

Any implications for the corresponding GPD?

$$g_P(t) = \sum_q \int_{-1}^{1} dx \tilde{E}_q(x,\xi,t)$$

# Gravitational form factors

QCD energy momentum tensor

$$\Theta^{\mu\nu} = \sum_f \bar{\psi}_f \gamma^{(\mu} i D^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}$$

Nucleon form factors

$$\langle P_2 | \Theta^{\alpha\beta} | P_1 \rangle = \bar{u}(P_2) \left[ A(t) \frac{P^\alpha P^\beta}{M} + (A(t) + B(t)) \frac{P^{(\alpha} i \sigma^{\beta)\lambda} l_\lambda}{2M} + D(t) \frac{l^\alpha l^\beta - g^{\alpha\beta} t}{4M} \right] u(P_1)$$

In massless QCD, $\Theta^{\alpha\beta}$ is traceless due to conformal symmetry

$$A(t) + \frac{B(t)}{4M^2} t - \frac{3D(t)}{4M^2} t = 0$$

$$\frac{3}{4} D(t) \approx \frac{M^2}{t} A(t) \qquad (t \to 0)$$

Pole at $t = 0$ from massless spin-0 glueball exchange

# Trace anomaly

Quantum mechanically, the trace is nonzero $$(\Theta)^{\alpha}_{\alpha} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

$$\Rightarrow \quad \frac{3}{4} D(t) \approx -\frac{M}{t} \left( \frac{\langle P_2 | \frac{\beta(g)}{2g} F^2 | P_1 \rangle}{\bar{u}(P_2) u(P_1)} - M A(t) \right)$$

anomaly pole glueball pole

In real QCD, there is no massless pole in $D(t)$ due to pole cancellation

Poles in $D(t)$ at physical glueball masses.

Mamo, Zahed (2021)
Fujita, YH, Sugimoto, Ueda (2022)

# Take-home message

Anomalies relate form factors

Chiral anomaly
$$2Mg_A(t) + \frac{tg_P(t)}{2M} = i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

Trace anomaly
$$M\left(A(t) + \frac{B(t)}{4M^2}t - \frac{3D(t)}{4M^2}t\right)\bar{u}(P_2)u(P_1) = \langle P_2|\frac{\beta(g)}{2g}F^{\mu\nu}F_{\mu\nu}|P_1\rangle$$

Form factors are moments of GPDs

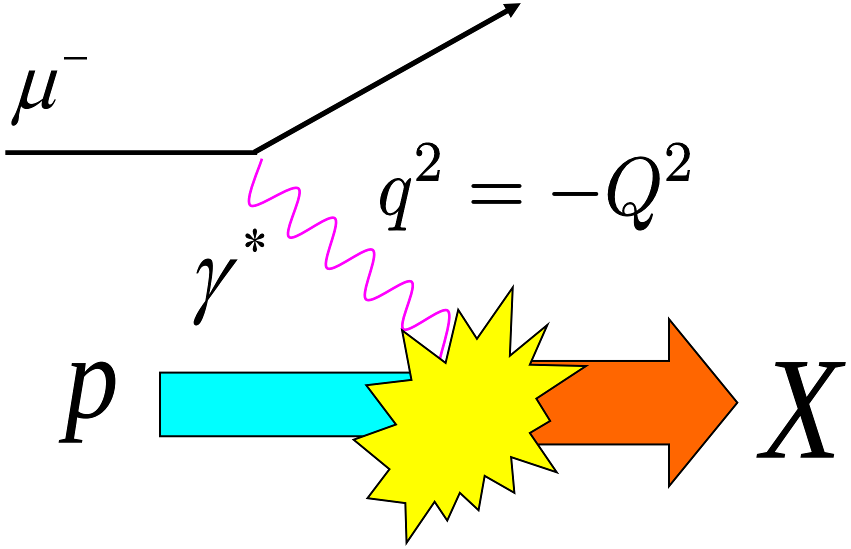
$$g_P(t) = \sum_q \int_{-1}^{1} dx\,\tilde{E}_q(x,\xi,t) \qquad A_q(t) + \xi^2 D_q(t) = \int_{-1}^{1} dx\,x H_q(x,\xi,t)$$

→ Anomalies relate/constrain GPDs!

# Circa 1990: Proton spin crisis

Longitudinal double spin asymmetry in polarized DIS

$$A_{LL} = \frac{\mu^{\uparrow}p^{\downarrow} - \mu^{\uparrow}p^{\uparrow}}{\mu^{\uparrow}p^{\uparrow} + \mu^{\uparrow}p^{\downarrow}} \sim \left(1 + \frac{\sigma_L}{\sigma_T}\right)\frac{2xg_1}{F_2}$$




$\Delta\Sigma$

$$\int_0^1 dx g_1(x) = \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \mathcal{O}(\alpha_s)$$

$\Delta\Sigma \sim 1$ in the quark model

$\Delta\Sigma \approx 0.3 \ll 1$ in QCD

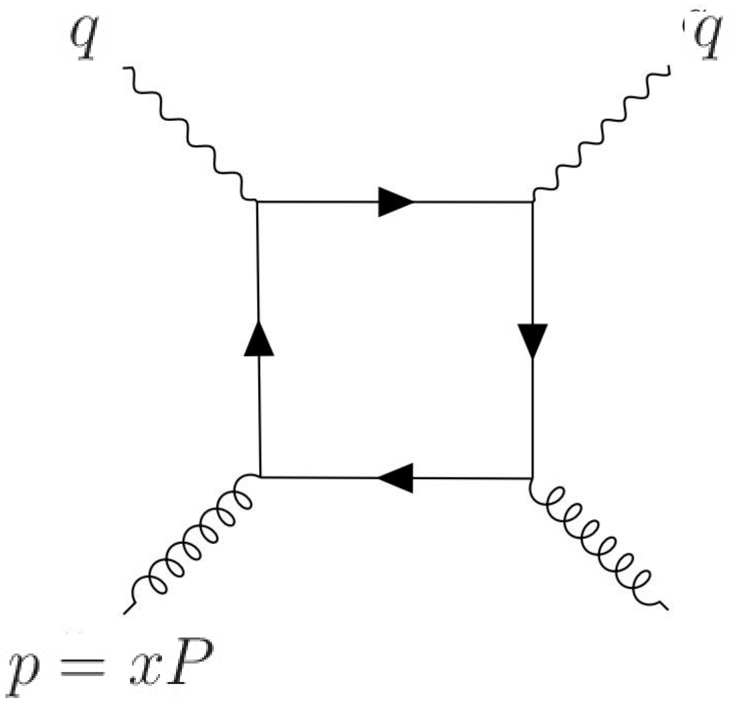
"spin crisis"

# The box diagram (forward kinematics)

One-loop correction to $g_1(x)$, gluon channel

$$g_1(x) \sim \frac{\alpha_s}{2\pi}\left(\ln\frac{Q^2}{m^2}\Delta P_{qg}(x) + \delta C_{qg}(x)\right) \otimes \Delta G(x)$$

polarized gluon PDF



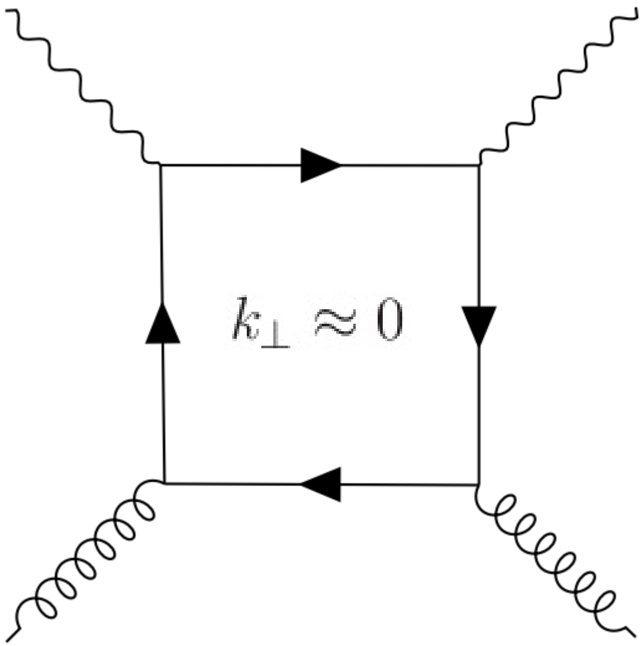

hard coefficient function

$$\delta C_{qg}(x) = (2x-1)\left(\ln\frac{1-x}{x} - 1\right) + 2(1-x)$$

a lot of controversy over this term in the past

# An infrared sensitivity

The $1-x$ term comes from the infrared region of the box diagram




$$(1-x)\int_0^{Q^2} dk_\perp^2 \frac{m_q^2}{(k_\perp^2+m_q^2)^2}$$ = finite!

but the coefficient function should be dominated by UV…

Absorb this `anomalous' gluon contribution into a re-definition of quark spin

Altarelli, Ross (1988)
Carlitz, Collins, Mueller (1988)

$$\Delta\widetilde{\Sigma} = \Delta\Sigma + \frac{n_f \alpha_s}{2\pi}\Delta G$$

Expect $\Delta\tilde{\Sigma} \sim 1$

If $\Delta G$ is large and positive, this can explain the smallness of $\Delta\Sigma$ .

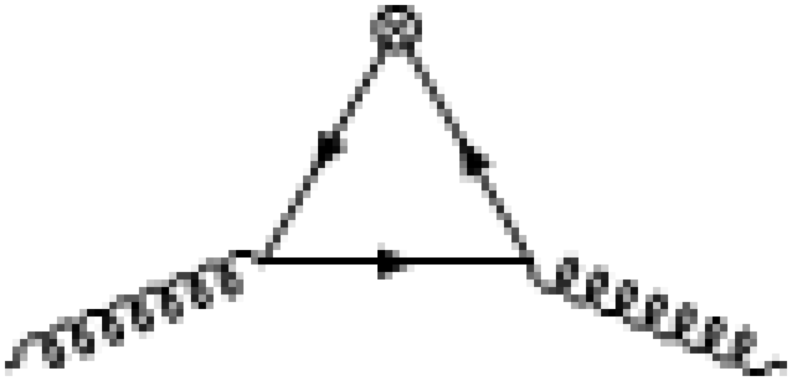
# A critique

Jaffe, Manohar (1989)

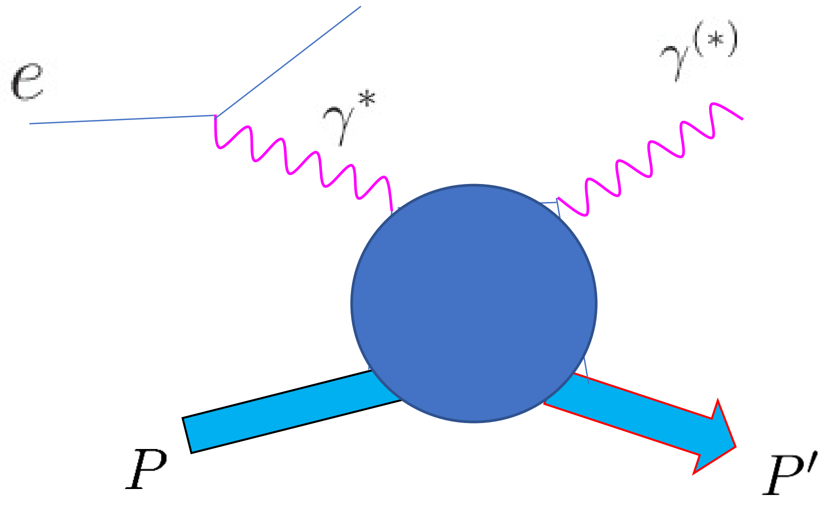
The authors of refs. [12, 13] suggest that the triangle diagram provides a *local* probe of the gluon distribution in the target. If this were true, $\Delta\Gamma$ would be protected from infrared problems and the calculation would be reliable in the usual sense. However, we believe there are strong arguments that the triangle is not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the $\eta'$ a mass★.

direction? The answer lies in the triangle diagram. For massless quarks and on-shell gluons, the off-forward matrix element of the triangle diagram (see fig. 3) coincides with the matrix element of $-i(l^\mu/l^2)(\alpha_s/2\pi)\mathrm{Tr}\, F\tilde{F}$ [54]. This result is regularization-independent. In QCD, the pole at $l^2=0$ is unphysical and is cancelled by non-triangle contributions to the matrix element of $A^0_\mu$. With the aid

IR sensitivity of the box diagram → signal of chiral anomaly
Natural to regularize by off-forward kinematics

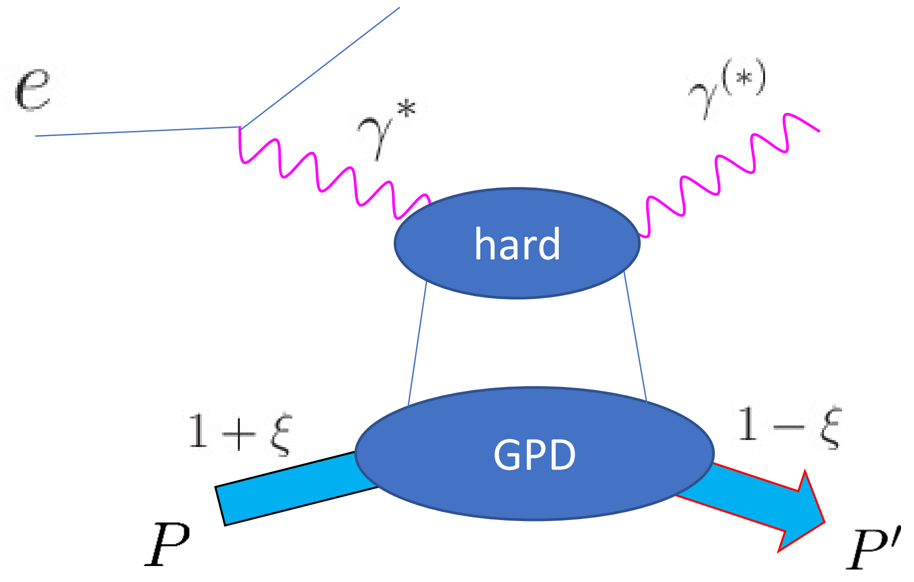


# Deeply Virtual Compton Scattering




Generalized
Bjorken limit

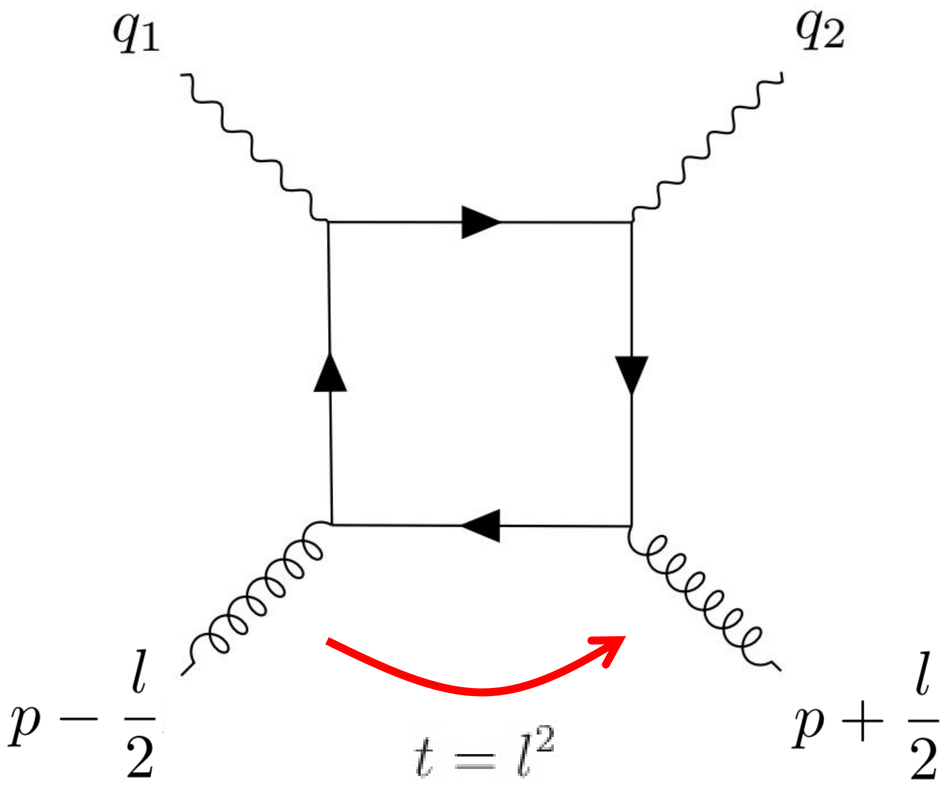
$$Q^2 \gg |t|, \Lambda^2_{\rm QCD}$$




Factorization proof Collins, Freund (1998); Ji, Osborne (1998)

$$T^{\mu\nu}(x_B,\xi,t) = \sum_{a=q,g} \int \frac{dx}{x} C_a^{\mu\nu}\left(\frac{x_B}{x},\frac{\xi}{x}\right) f_a(x,\xi,t) + \mathcal{O}(1/Q^2)$$

# Box diagram (off-forward)




In all previous works on DVCS, the hard part was computed at $\xi \neq 0$ and $t = 0$

Naively, introducing $t \neq 0$ only produces higher twist corrections of order $t/Q^2$

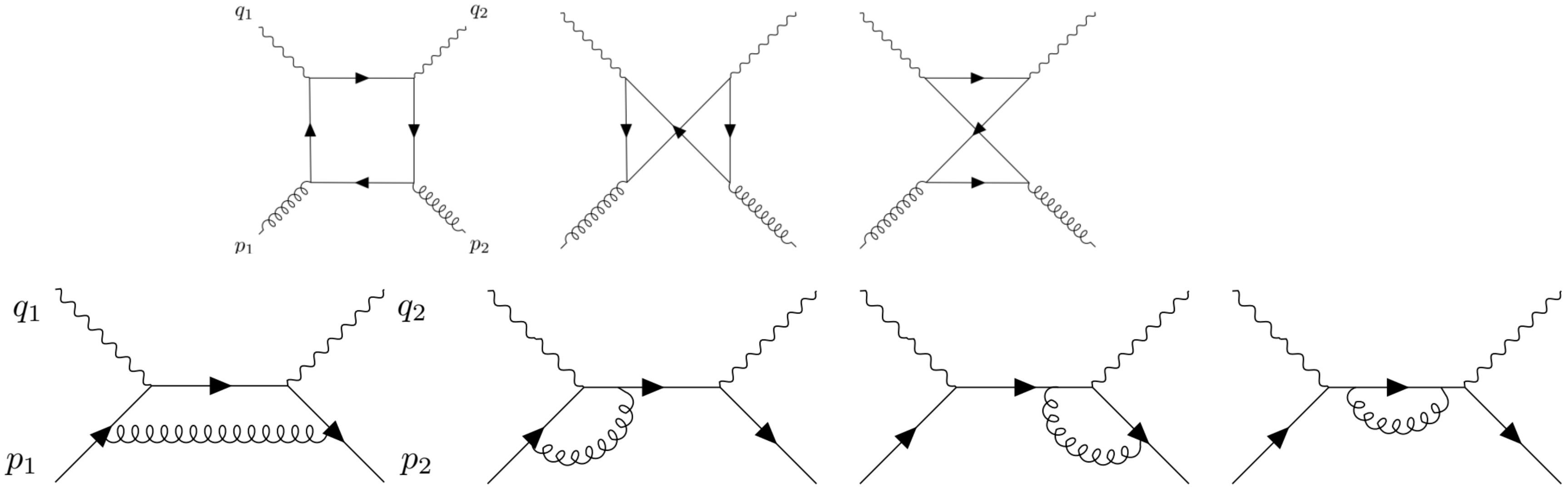
However, calculations with $t \neq 0$ can reveal anomaly poles. Tarasov, Venugopalan (2019,2021)

$t \neq 0$ also naturally cuts off the collinear singularity $\frac{1}{\epsilon} \to \ln \frac{Q^2}{-t}$

Assume $\Lambda_{QCD}^2 \ll |t| \ll Q^2$ for the moment.

# One-loop calculation

$$T^{\mu\nu} = \frac{g_\perp^{\mu\nu}}{2P^+}\bar{u}(P_2)\left[\gamma^+\mathcal{H} + \frac{i\sigma^{+\nu}l_\nu}{2M}\mathcal{E}\right]u(P_2) - i\frac{\epsilon_\perp^{\mu\nu}}{2P^+}\bar{u}(P_2)\left[\gamma^+\gamma_5\tilde{\mathcal{H}} + \frac{\gamma^5 l^+}{2M}\tilde{\mathcal{E}}\right]u(P_2)$$




We find

Complete GPD evolution kernel as the coefficient of $\ln Q^2/t$

a pole $1/t$ in 3 out of the 4 leading twist Compton form factors $\mathcal{H}, \mathcal{E}, \tilde{\mathcal{E}}$

# Chiral anomaly pole in $\tilde{\mathcal{E}}$

$$\tilde{\mathcal{E}} \sim \frac{\alpha_s}{t} \tilde{A} \otimes \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) W \tilde{F}_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

twist-four GPD associated with the operator $F^{\mu\nu}\tilde{F}_{\mu\nu}$

Tarasov, Venugopalan (2019)
YH (2020)
Radyushkin, Zhao (2021)

$$\tilde{A}(x, x_B, \xi) = \frac{8T_R}{x} \frac{(1-\hat{x}) \ln \frac{\hat{x}-1}{x} + (\hat{x} - \hat{\xi}) \ln \frac{\hat{x}-\xi}{\hat{x}} - (\hat{x} \to -\hat{x})}{1 - \hat{\xi}^2}$$

imaginary part of this at $\xi = 0$ agrees with Tarasov, Venugopalan (2019)

# Trace anomaly pole in $\mathcal{H}, \mathcal{E}$

$$\mathcal{H} \sim -\mathcal{E} \sim \frac{\alpha_s}{t} A \otimes \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \frac{\langle P_2 | F^{\mu\nu}(-z^-/2) W F_{\mu\nu}(z^-/2) | P_1 \rangle}{\bar{u}(P_2) u(P_1)}$$

opposite sign!

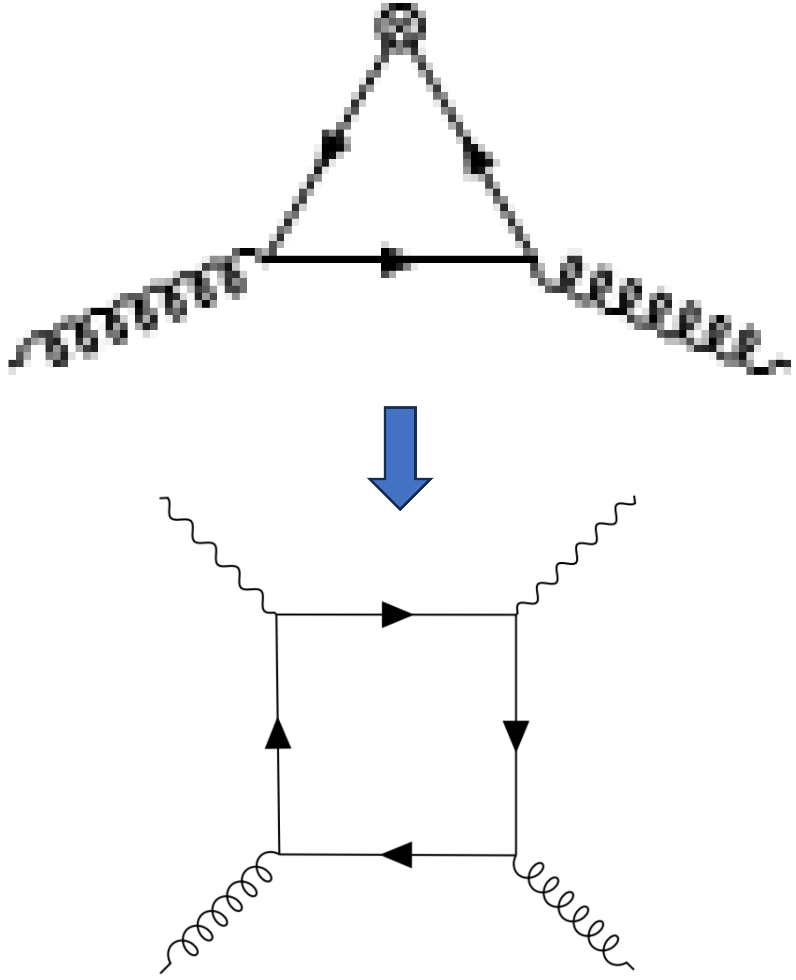
twist-four GPDs associated with the operator $F^{\mu\nu} F_{\mu\nu}$
relevant to nucleon mass decomposition

YH, Zhao (2020); Radyushkin Zhao (2021)

$$A(x, x_B, \xi) = \frac{2T_R}{x} \left( 1 + \frac{\hat{x}(1-\hat{x}) \ln \frac{\hat{x}-1}{\hat{x}} + \hat{x}(\hat{x}-\hat{\xi}) \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} + (\hat{x} \to -\hat{x})}{1-\hat{\xi}^2} \right)$$

# From triangle to box

Poles found in the box diagram are x-dependent generalizations of the triangle anomaly



$$\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu}\tilde{F}_{\mu\nu}$$

$$\Rightarrow \quad \langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi}\frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}^a_{\alpha\beta} | p_1 \rangle$$

Adler, Bell, Jackiw (1969)

$$(\Theta_{\rm QED})^\mu_\mu = \frac{\alpha_{em}}{6\pi} F^2$$

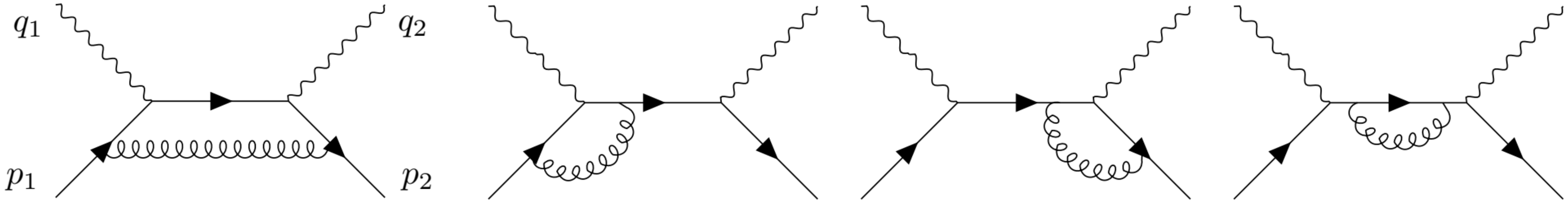
$$\Rightarrow \quad \langle p_2 | \Theta^{\mu\nu}_{\rm QED} | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2}\left( p^\mu p^\nu + \frac{l^\mu l^\nu - l^2 g^{\mu\nu}}{4} \right) \langle p_2 | F^{\alpha\beta} F_{\alpha\beta} | p_1 \rangle + \dots .$$

Giannotti, Mottola (2009)

Armillis, Coriano, Delle Rose (2010)

# Single and double IR poles

Quark channel diagrams contain single and double IR poles




$$\begin{aligned}\delta C_1^q(\hat{x},\hat{\xi}) = &-\frac{\left(\frac{-t}{Q^2}\right)^{-\epsilon_{IR}}}{\epsilon_{\rm IR}^2(1-\hat{x})} - \frac{3\left(\frac{-t}{Q^2}\right)^{-\epsilon_{IR}}}{2\epsilon_{\rm IR}(1-\hat{x})} + \frac{1-2\hat{x}-2\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)}\ln\frac{\hat{x}-1}{\hat{x}} + \frac{(\hat{x}-\hat{\xi})(-1+\hat{x}^2+3\hat{x}\hat{\xi}+3\hat{\xi}^2)}{(1-\hat{x}^2)(1-\hat{\xi}^2)\hat{\xi}}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}} \\ &+ \frac{1+\hat{x}^2-2\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)}\ln^2\frac{\hat{x}-1}{\hat{x}} + \frac{\hat{x}}{2(1-\hat{\xi}^2)\hat{\xi}}\ln^2\frac{\hat{x}-\hat{\xi}}{\hat{x}} + \frac{-1-\hat{x}^2+2\hat{\xi}^2}{2(1-\hat{x}^2)(1-\hat{\xi}^2)}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\ln\frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})} \\ &+ \frac{\hat{x}}{(1-\hat{\xi}^2)\hat{\xi}}\mathrm{Li}_2\frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{1+\hat{x}^2-2\hat{\xi}^2}{(1-\hat{x})(1-\hat{\xi}^2)}\left(\mathrm{Li}_2\frac{1-\hat{\xi}}{1-\hat{x}} + \mathrm{Li}_2\frac{1+\hat{\xi}}{1-\hat{x}}\right) + (\hat{x}\to-\hat{x}),\end{aligned}$$

# Tension with QCD factorization?

$$P_{qg} \ln \frac{Q^2}{-t} \langle F^{+\mu} F^{+}_{\mu} \rangle$$

$$\Delta P_{qg} \ln \frac{Q^2}{-t} \langle F^{+\mu} \tilde{F}^{+}_{\mu} \rangle$$

Expected, should be absorbed into twist-2 GPDs

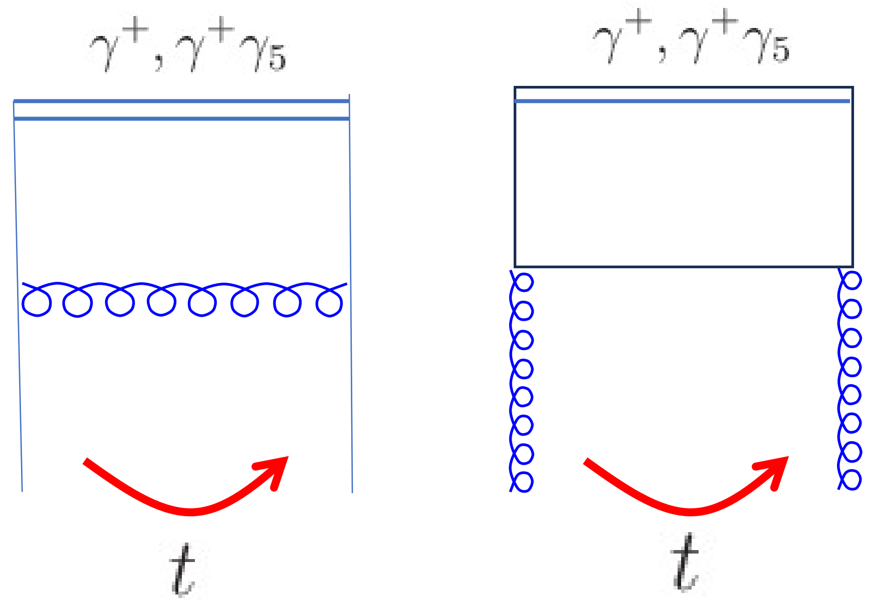
$$\frac{1}{t}\langle FF \rangle, \ \frac{1}{t}\langle F\tilde{F} \rangle$$

Twist-4 GPDs not suppressed by $\frac{1}{Q^2}$

$$\frac{1}{\epsilon_{\rm IR}^2}\left(\frac{-t}{Q^2}\right)^{-\epsilon} \langle \bar{q}\gamma^+ q \rangle$$

Uncancelled double IR pole (no real-virtual cancellation in DVCS)

# GPDs at one-loop




We also computed the quark GPD of a quark and gluon keeping $t \neq 0$ and found the same types of singularities

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2 | \bar{q}(-z/2) W \gamma^+ \gamma_5 q(z^-/2) | p_1 \rangle$$

$$= \begin{cases} \frac{\alpha_s T_R}{2\pi}\left[(1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1}\left(\frac{2x-1-\xi^2}{(1-\xi^2)^2}\left(\frac{\left(\frac{\tilde{\mu}^2}{-l^2}\right)^\epsilon}{\epsilon_{\mathrm{UV}}} - \ln\frac{(1-x)^2}{1-\xi^2}\right) - 2\frac{1-x}{(1-\xi^2)^2}\right) + \frac{2il^+\epsilon^{\epsilon_1\epsilon_2^* lp}}{t}\frac{1-x}{1-\xi^2}\right] & x > \xi \\ \frac{\alpha_s T_R}{2\pi}\Bigg[(1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1}\frac{\left(\frac{\tilde{\mu}^2}{-l^2}\right)^\epsilon}{\epsilon_{\mathrm{UV}}}\frac{-1}{(1+\xi)^2} + \frac{2il^+\epsilon^{\epsilon_1\epsilon_2^* lp}}{t}\frac{1}{1+\xi} & x < \xi \\ \quad + (1-\xi^2) i\epsilon^{+p\epsilon_2^*\epsilon_1}\frac{1}{(1-\xi^2)^2}\left[-2\xi\ln(\xi^2-x^2) + (1+\xi^2)\ln(1-x^2) - 2x\ln\frac{(1-x)(x+\xi)}{(1+x)(\xi-x)} - 2(1+\xi^2)\ln(1+\xi) + 4\xi\ln(2\xi) + 2\xi - 2\right]\Bigg] & \end{cases}$$

# Infrared subtraction

$\langle F^{\mu\nu} ... F_{\mu\nu} \rangle$

$$\int_0^1 dx A(x, x_B, \xi) \mathcal{F}(x, \xi, t)$$

$$= 2T_R \int_0^1 dx C_0(x, x_B) \left[ \int_x^1 \frac{dx'}{x'} K\left(\frac{x}{x'}, \frac{\xi}{x'}\right) \mathcal{F}(x', \xi, t) - \theta(\xi - x) \int_0^1 \frac{dx'}{x'} L\left(\frac{x}{x'}, \frac{\xi}{x'}\right) \mathcal{F}(x', \xi, t) \right]$$

leading order Compton kernel

$$C_0(x, x_B) = \frac{1}{x - x_B + i\epsilon} + \frac{1}{x + x_B - i\epsilon}$$

$$K(x, \xi) = \frac{x(1-x)}{1-\xi^2}, \qquad L(x, \xi) = \frac{x(\xi - x)}{1-\xi^2}$$

Absorb the $1/t$ poles of Compton amplitude into twist-2 GPDs

Similarly, $\frac{1}{\epsilon_{\rm IR}}, \frac{1}{\epsilon_{\rm IR}^2}$ can also be absorbed.

→ Factorization restored

# The fate of anomaly poles

After absorbed into twist-2 GPD, the anomaly pole becomes a part of the GPD

$$\sum_q (\tilde{E}_q(x,\xi,t)+\tilde{E}_q(-x,\xi,t)) = \frac{T_R n_f \alpha_s}{\pi}\frac{M^2}{t}\tilde{C}^{\rm anom}\otimes\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)W\tilde{F}_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}+\cdots$$

integrate over x

$$\frac{g_P(t)}{2M} = \frac{1}{t}\left(i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2Mg_A(t)\right)$$

exactly reproduce the anomaly pole!

Twist-2 and twist-4 GPDs related by the chiral anomaly

Cancel the pole at $t=0$ with the nonperturbative $\eta_0$ meson pole.
Support of the pole not limited to the ERBL region

# D-term and gluon condensate

Trace anomaly pole induces the Polyakov-Weiss D-term of unpol GPDs

$$H_q^{\rm PW}(x,\xi,t) = -E_q^{\rm PW}(x,\xi,t) = \theta(\xi - |x|) D_q(x/\xi, t)$$

$$\sum_q D_q(z,t) \approx -\frac{T_R n_f \alpha_s}{\pi} z(1-|z|) \frac{M}{t} \left( \frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - \frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\bigg|_{t=0} \right) + \cdots$$

anomaly pole glueball pole (added by hand)

integrate

$$\sum_q D_q(t) \approx -\frac{M}{t} \left( \frac{\langle P_2|\frac{T_R n_f \alpha_s}{6\pi} F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - \frac{\langle P_2|\frac{T_R n_f \alpha_s}{6\pi} F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\bigg|_{t=0} \right) + \cdots$$

Compare with the full relation

$$\frac{3}{4} D(t) \approx -\frac{M}{t} \left( \frac{\langle P_2|\frac{\beta(g)}{2g} F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)} - M A(t) \right)$$

# Decomposition of the trace anomaly

$$T^{\mu}_{\mu} = (T_q)^{\mu}_{\mu} + (T_g)^{\mu}_{\mu} = \frac{\beta}{2g}F^2 + m(1+\gamma_m)\bar{\psi}\psi$$

YH, Rajan, Tanaka (2018) — 2-loop
Tanaka (2019, 2022) — 3-loop
Ahmed, Chen, Czakon (2022) — 4-loop

$$\langle P|\eta_{\mu\nu}T^{\mu\nu}_{qR}|P\rangle = \langle P|\Big\{ (m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F\,(m\bar{\psi}\psi)_R + \frac{1}{3}n_f\,(F^2)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\Big[(m\bar{\psi}\psi)_R\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right) + (F^2)_R\left(\frac{17C_An_f}{27} + \frac{49C_Fn_f}{54}\right)\Big]\Big\}|P\rangle\,,$$

The box diagram only reproduces the leading term of the quark part $(T_q)^{\mu}_{\mu}$

$$\frac{\beta(g)}{2g} = \frac{T_R n_f \alpha_s}{6\pi} + \cdots$$

Expect anomaly poles to all orders in perturbation theory, also in gluon GPDs

→ Build up the D-term order by order

# Conclusions

Anomalies relate form factors
Form factors are moments of GPDs

→ Anomalies relate GPDs

GPDs encode profound aspects of QCD such as chiral symmetry breaking and the origin of mass.

$t \neq 0$ regularization (probably) equivalent to $\overline{\mathrm{MS}}$ after the subtraction of finite terms.
More roundabout, but more physical and reveals the connection to QCD anomalies.