QED Radiative Corrections for Exclusive Reactions

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Towards Improved Hadron Femtography with Hard Exclusive Reactions

Jefferson Lab, August 7-11, 2023

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Plan of talk

Radiative corrections for charged lepton scattering

- Soft photon emission, spin independence
- Single-Spin Asymmetries of a Bethe-Heitler process
- Two-Photon exchange for DVMP
- Implications for DVCS
- Outlook

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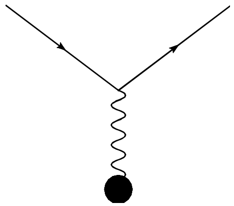
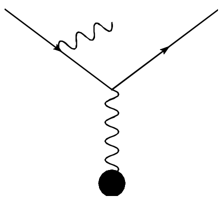
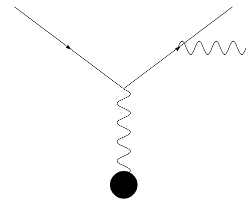
Basics of QED radiative corrections



(First) Born approximation

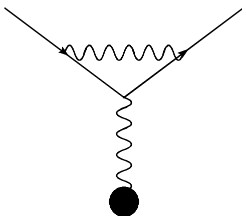


Initial-state radiation



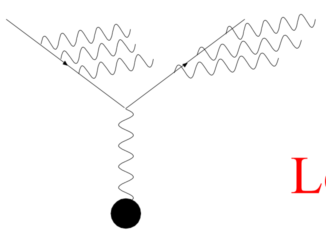
Final-state radiation

Cross section ~ dω/ω => integral diverges logarithmically: **IR catastrophe**



Vertex correction => cancels divergent terms; Schwinger (1949)
Assumed $Q^2/m_e^2 >> 1$

$$\sigma_{\exp} = (1+\delta)\sigma_{Born}, \quad \delta = \frac{-2\alpha}{\pi}\{(\ln\frac{E}{\Delta E} - \frac{13}{12})(\ln\frac{Q^2}{m_e^2} - 1) + \frac{17}{36} + \frac{1}{2}f(\theta)\}$$



Leonard Maximon, GWU, made important contributions to the field

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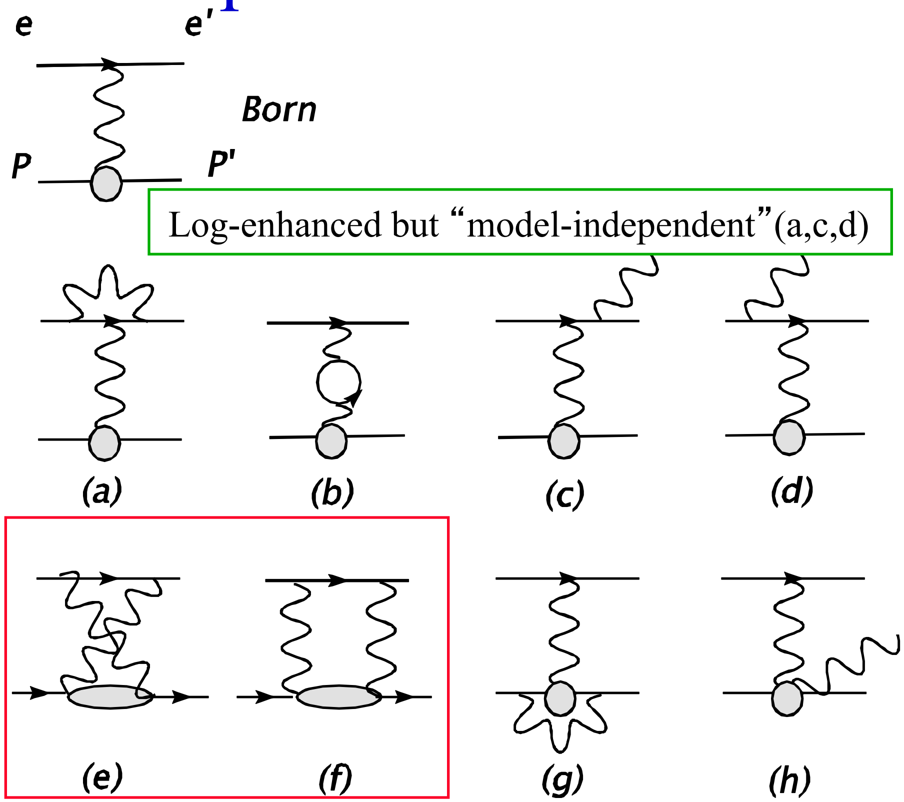
Complete radiative correction in O(α_{QED})



Radiative Corrections to elastic ep:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and Virtual Compton (g,h)
- Corrections (e-h) depend on the nucleon structure

Two-photon corrections: no large logs, but dependent on nucleon structure

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Basic Approaches to QED Corrections

- L.W. Mo, Y.S. Tsai, Rev. Mod. Phys. 41, 205 (1969); Y.S. Tsai, Preprint SLAC-PUB-848 (1971).
  - Considered both elastic and inelastic inclusive cases. No polarization.
- D.Yu. Bardin, N.M. Shumeiko, Nucl. Phys. B127, 242 (1977).
  - Covariant approach to the IR problem. Later extended to inclusive, semi-exclusive and exclusive reactions with polarization.
    - RADGEN: Monte Carlo of p(e,e')X including radiative events
- E.A. Kuraev, V.S. Fadin, Yad.Fiz. 41, 7333 (1985); E.A. Kuraev, N.P.Merenkov, V.S. Fadin, Yad. Fiz. 47, 1593 (1988).
  - Developed a method of electron structure functions based on Drell-Yan representation, used at e^+e^- colliders;
  - Extended for SIDIS in AA et al, JETP 98, 403 (2004).
- Structure-function approach further developed in Liu, Melnitchouk, Qiu, Sato, PRD 104, 094033 (2021)

Two-Photon Exchange Overview

Progress in Particle and Nuclear Physics 95 (2017) 245–278

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Review

Two-photon exchange in elastic electron–proton scattering

CrossMark

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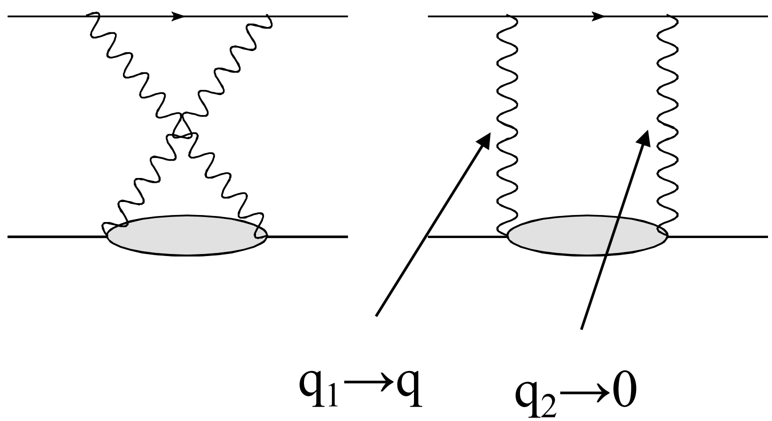
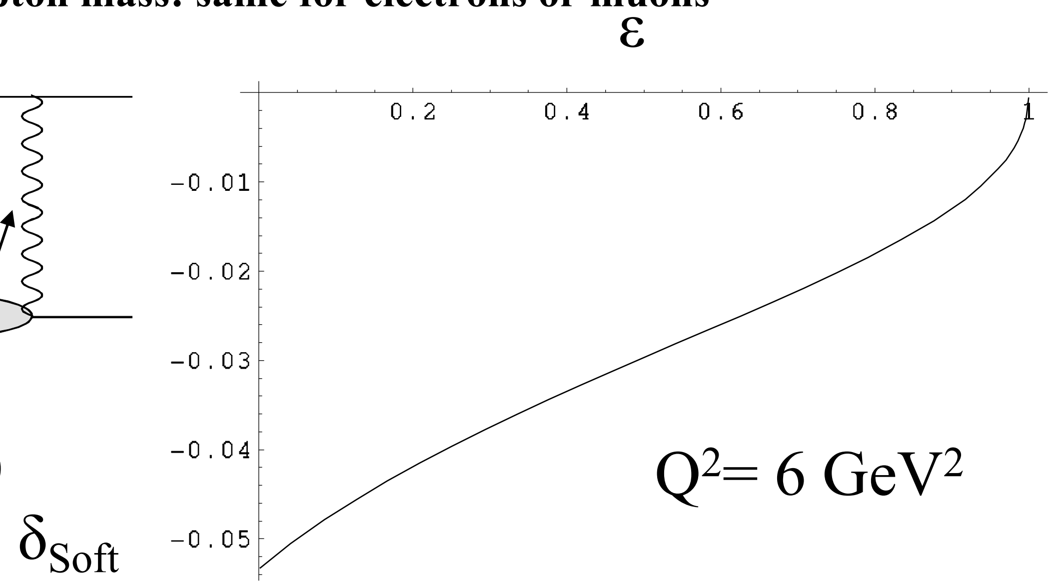
Separating *soft* 2-photon exchange

- Tsai; Maximon & Tjon (k→0); similar to Coulomb corrections at low Q^2
- Grammer &Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to cross section
- **Already included in experimental data analysis**
- NB: Corresponding effect to polarization transfer and/or asymmetry is zero
- **Correction is independent of lepton mass: same for electrons or muons**





A similar approach can be applied for any exclusive reaction or SIDIS. Special care of soft/hard photon separation

Full Calculation of Bethe-Heitler Contribution

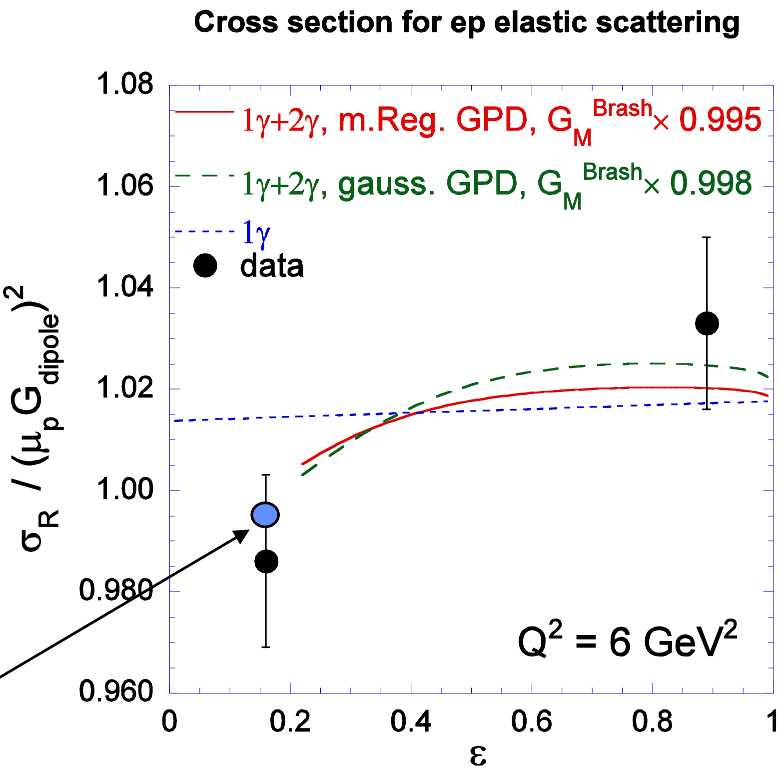
*AA et al., using MASCARAD code (**Phys.Rev.D64:113009,2001**)*
Full calculation including soft and hard bremsstrahlung

Radiative leptonic tensor in full form
AA et al, ***PLB 514, 269 (2001)***

$$L^{r}{}_{\mu\nu} = -\frac{1}{2} Tr(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1+\gamma_5\hat{\xi}_e)(\hat{k}_1 + m)\overline{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\mu - \frac{\gamma_\mu \hat{k}\gamma_\alpha}{2k\cdot k_1} - \frac{\gamma_\alpha \hat{k}\gamma_\mu}{2k\cdot k_2}$$

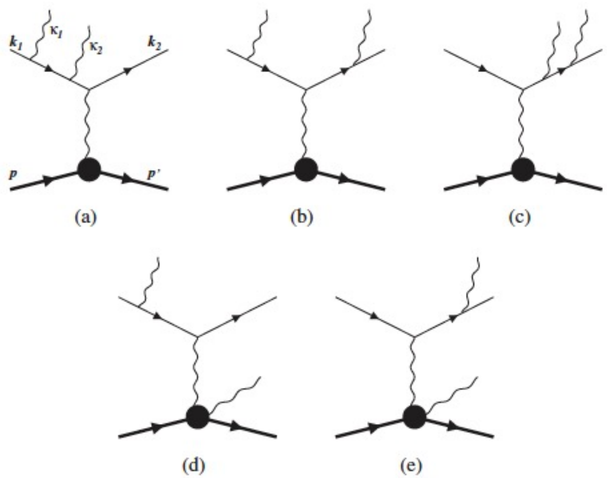
$$\Gamma_{\alpha\nu} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\nu - \frac{\gamma_\alpha \hat{k}\gamma_\nu}{2k\cdot k_1} - \frac{\gamma_\nu \hat{k}\gamma_\alpha}{2k\cdot k_2}$$

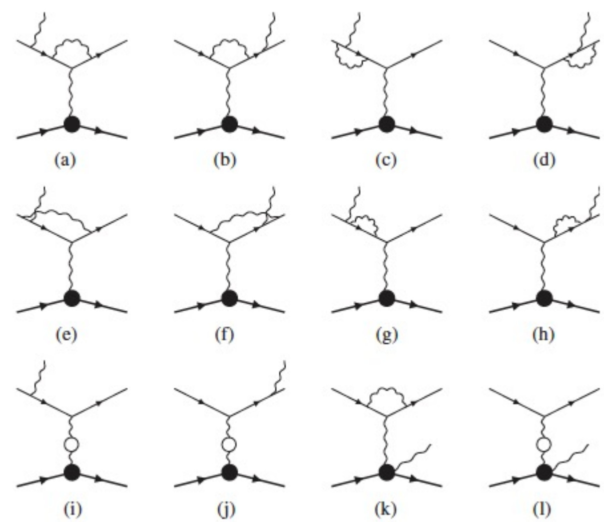


Additional effect of full soft+hard brem → +1.2% correction to ε-slope

Rad.Correction to Single-Spin Asymmetries of VCS

- Evaluation of QED radiative corrections for single-spin asymmetries in Virtual Compton Scattering experiments Vanderhaeghen et al. Phys. Rev. C 62, 025501 (2000) for beam SSA in VCS) – soft photon approximation
- Akushevich, Ilyichev, *Radiative effects in deep virtual Compton scattering,* Phys Rev D 98, 013005 (2018), included hard photon emission





Rad Correction to DVCS beam asymmetry kinematic cuts

- Vmin is a parameter related to the invariant mass of the undetected state (small Vmin removes hard-photon emission)

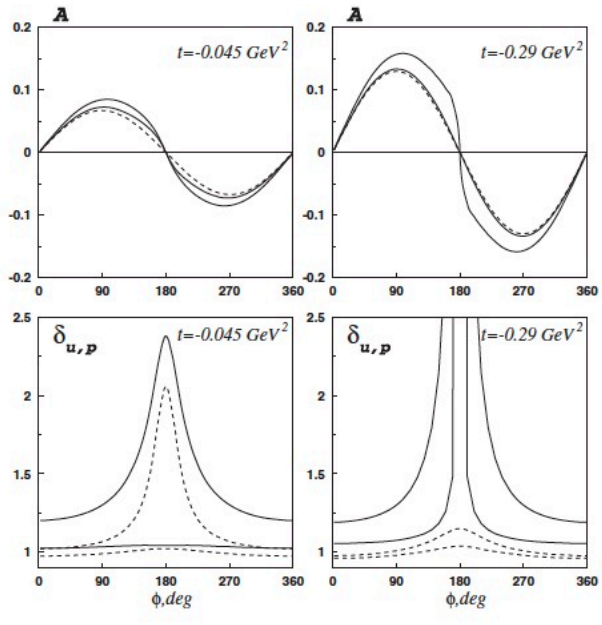


FIG. 5. The ϕ-dependence of the asymmetry (upper) and RC factors (lower plots). The dashed curve at the upper plots gives the $\sigma_{1\gamma}$ and the solid curve shows the observed cross sections with $V^2_{\rm cut} = 0.3$ GeV^2 (the curve closer to dashed curve) and without cuts. Dashed and solid curves at the bottom plots show $\delta_{u,p}$ with and without the cut, respectively. The curves with higher values corresponds to δ_p, i.e., $\delta_p > \delta_u$. Kinematical variables used for this example were $x = 0.1$, $Q^2 = 2$ GeV^2, and $E_{\rm beam} = 11$ GeV.

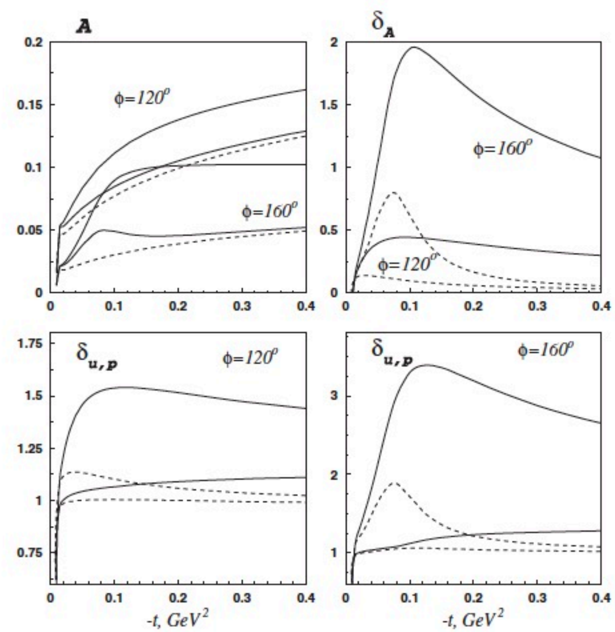


FIG. 6. The t-dependence of the asymmetry (upper left), RC to asymmetry (upper right), and RC factors (lower plots). Dashed curve for A gives the $\sigma_{1\gamma}$ and solid curved show the observed cross sections with $V^2_{\rm cut} = 0.3$ GeV^2 (the curve closer to dashed curve) and without cuts. Dashed and solid curves at the other three plots show $\delta_{A,u,p}$ with and without the cut, respectively. Kinematical variables used for this example were $x = 0.1$, $Q^2 = 2$ GeV^2, and $E_{\rm beam} = 11$ GeV.

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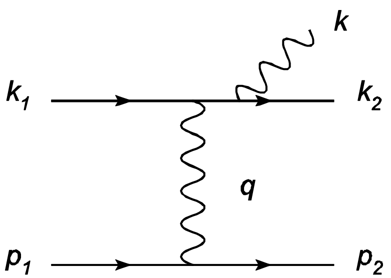
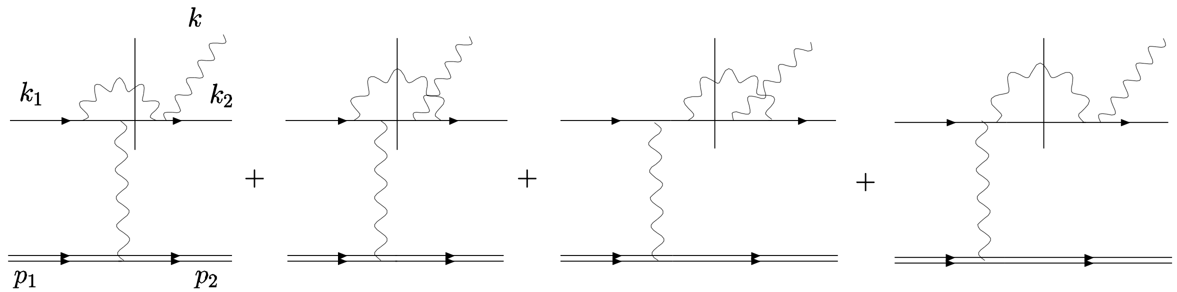
QED Loops also Generate Single-Spin Asymmetries

AA, Konchatnij, Merenkov, *Single-spin asymmetries in the Bethe-Heitler process e- + p ---> e- + gamma + p from QED radiative corrections,* J.Exp.Theor.Phys.102:220-233, 2006; hep-ph/0507059

- SSA in Bethe-Heitler process is due to interference between (real) tree-level amplitude and QED loops = O(α) correction that contain absorptive parts



Tree-level real amplitude



QED loop diagrams

Expression for beam SSA

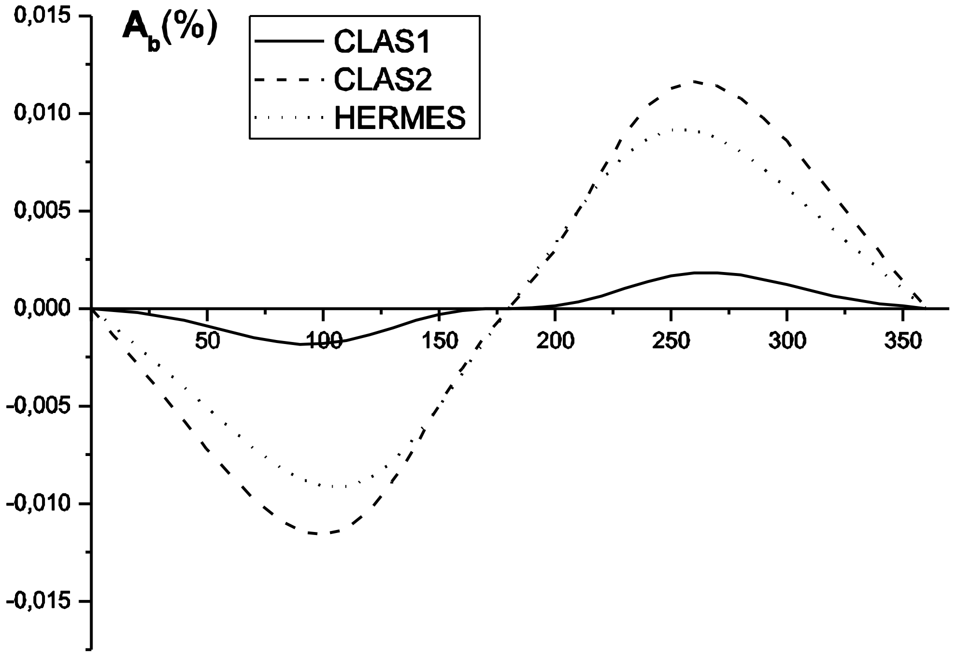
$$P^{(1)}_{\mu\nu} H_{\mu\nu} = \frac{2\pi(k_1 k_2 q p_1)}{st}(F_1^2 - \frac{q^2}{4M^2}F_2^2)[(2V - s + q^2)\overline{B}_1 + (2X - s - u)\overline{B}_2],$$

$$\overline{B}_1 = \frac{2(u^2 - 2s^2 - su)}{uc} + \frac{2bc}{c^2} + \frac{4b^2}{t^2} - \frac{4b}{t}(1 + \frac{b}{t})\log(1 + \frac{t}{u}),$$

$$\overline{B}_2 = \frac{6s}{c} - \frac{2(2b - t)}{t} + 4(-1 + \frac{ub}{t^2} - \frac{s}{t})\log(1 + \frac{t}{u})$$

- Results are expressed in terms of analytic functions of Mandelstam invariants
- Free of infrared and mass singularities
- No large logarithms appear
- In addition to α, proportional to q^2 that is small in DVCS kinematics
- Similar formulas obtained for target SSA; similar suppression takes place

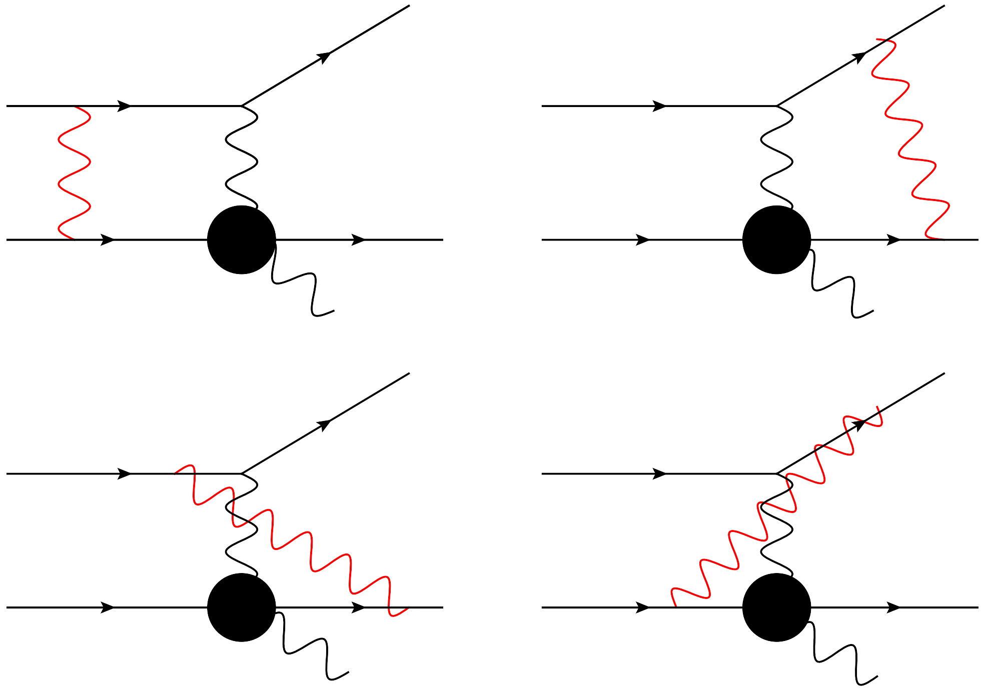
Numerical results



Asymmetry less than 0.015% due to O(α)+additional kinematic suppression

Missing: Soft TPE for VCS

- Photon coupling to external charged lines
- Results are independent of hadronic models
- IR-finite due to cancellation with real-photon emission



VCS

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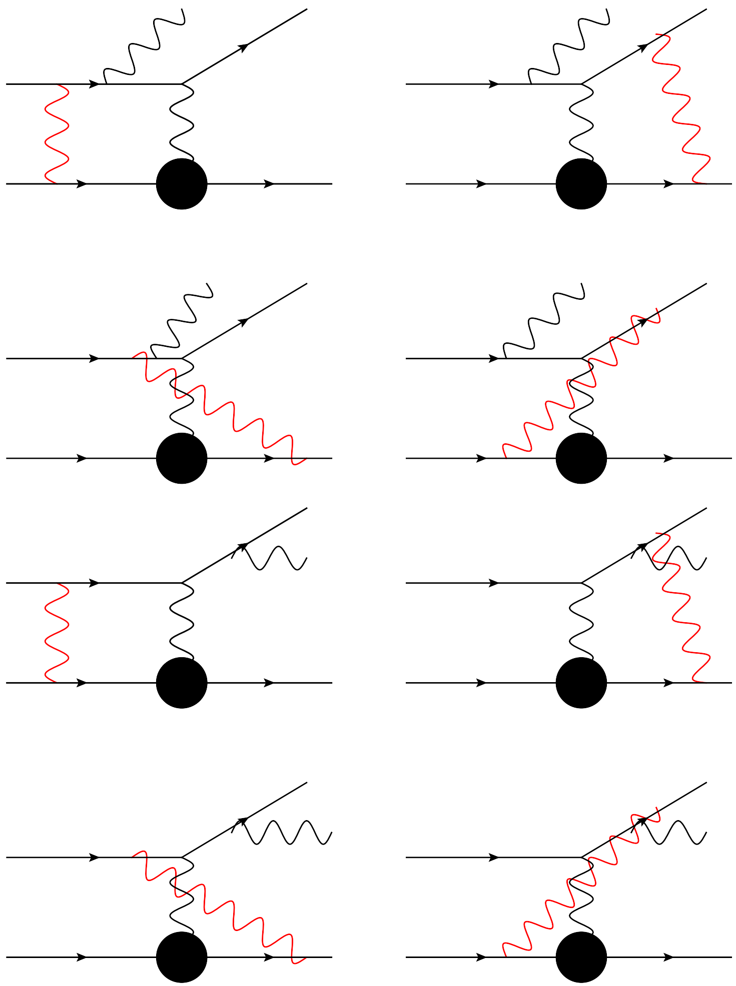
Missing: TPE to Bethe-Heitler

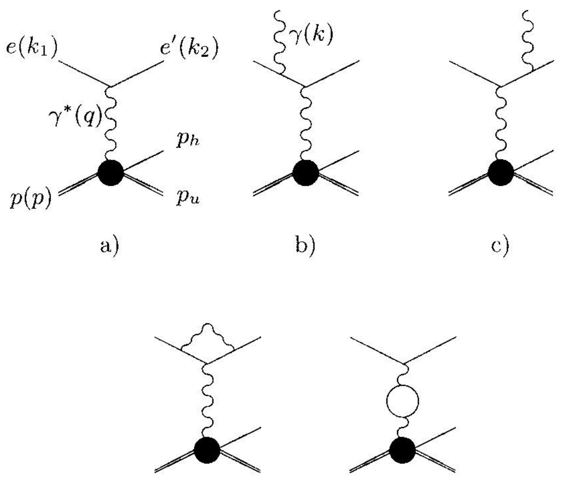


For BH+VCS
give “soft factors”

Important:
TPE corrections to VCS-BH
intereference terms are C-even

RC for Exclusive Electroproduction of Pions

- **AA, Akushevich, Burkert, Joo, Phys.Rev.D66, 074004 (2002)**
  - Conventional RC, precise treatment of phase space, <u>no peaking approximation</u>, no dependence on hard/soft photon separation; Can be used for any exclusive electroproduction of 2 hadrons, e.g., d(e,e' p)n (EXCLURAD code) or any exclusive states



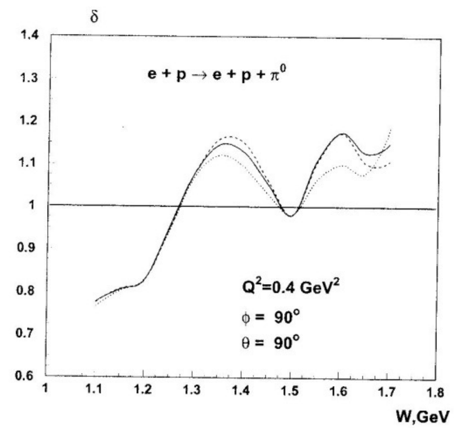
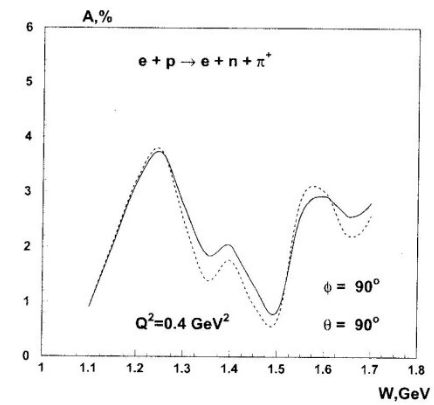


FIG. 3: W-dependence of RC to the cross section of neutral pion production. The models used are MAID2000 (solid curve), MAID'98 [34] (dashed curve) and AO [35] (dotted curve).



- Requires model/phenomenology input for $\gamma^* N \rightarrow \pi N$ amplitudes
- Used in data analysis at Jlab (and MIT, HERMES, MAMI,…)

Radiative Corrections for Exclusive Processes

- Photon emission is a part of any electron scattering process: accelerated charges radiate
- Exclusive electron scattering processes such as $p(e,e' h_1)h_2$ are in fact inclusive $p(e,e' h_1)h_2\ n\gamma$,

  where we can produce an infinite number of low-energy photons
- But low-energy photons do not affect polarization observables, thanks to Low theorem

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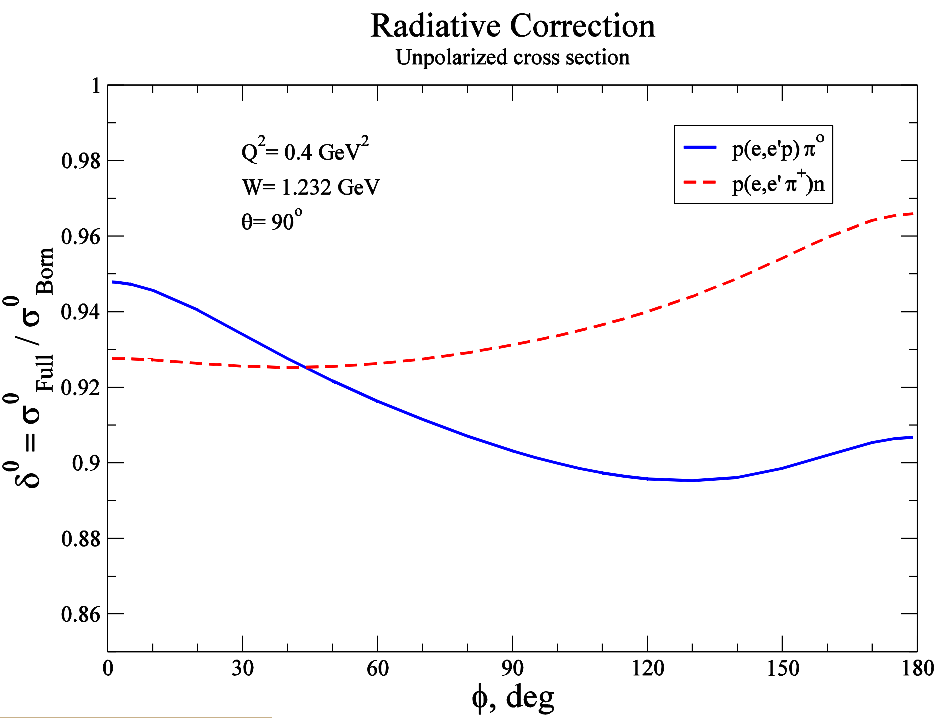
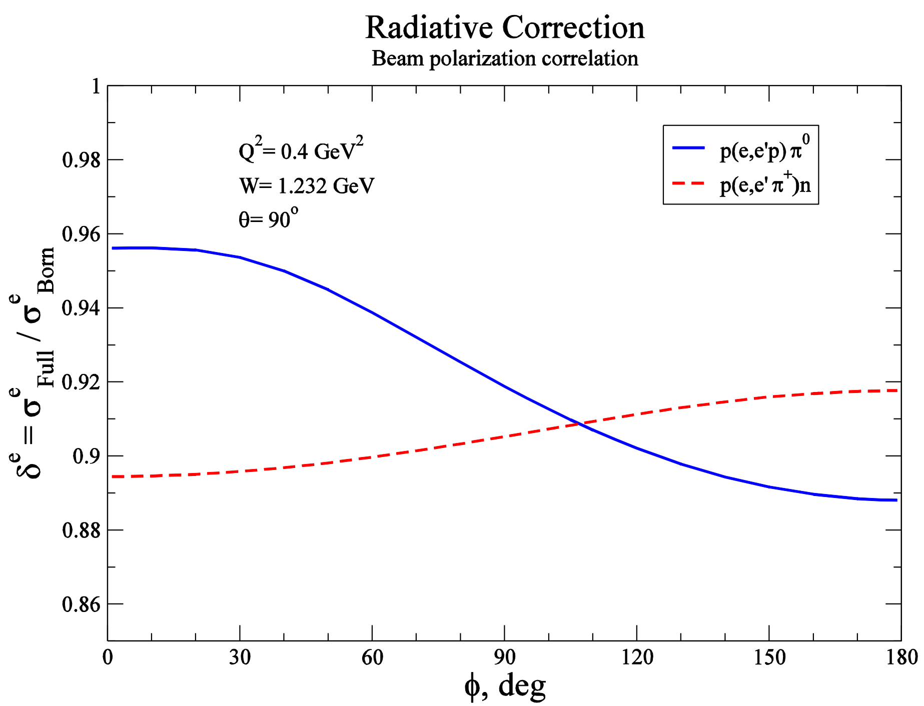
Exclurad updated to include polarization (work with K. Joo)

- Corrections to single-spin beam and target asymmetries and double-spin beam-target asymmetry
  - Target polarized along the beam direction





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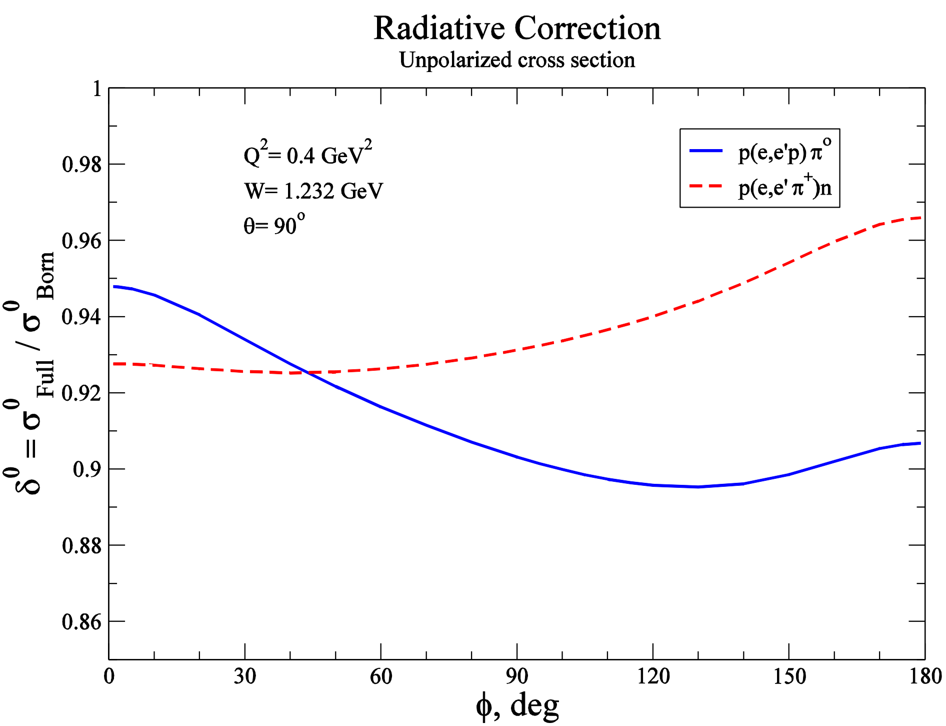
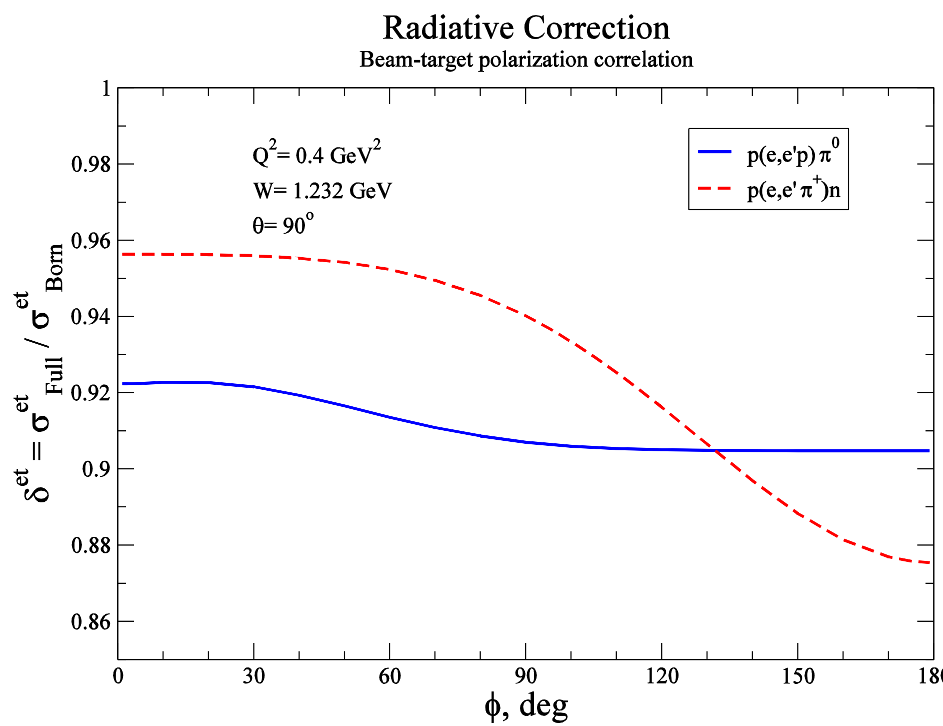
RC for beam-target asymmetry

If kinematic cuts for the radiated photon are tight (below 2nd pion production threshold, correction to polarization asymmetry is under <1%)





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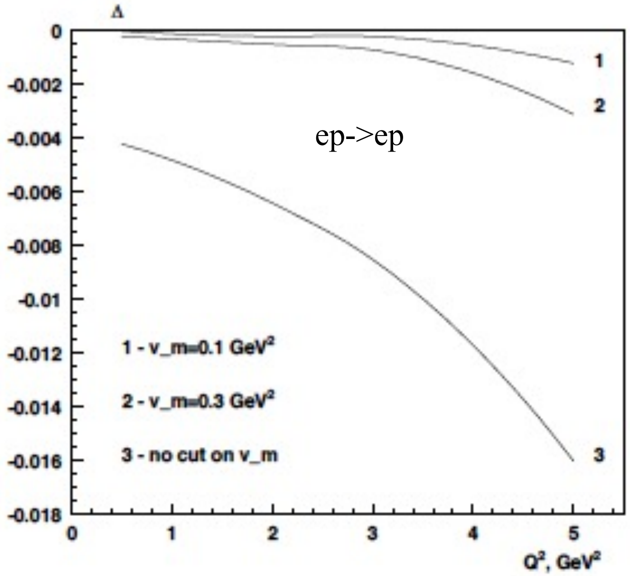
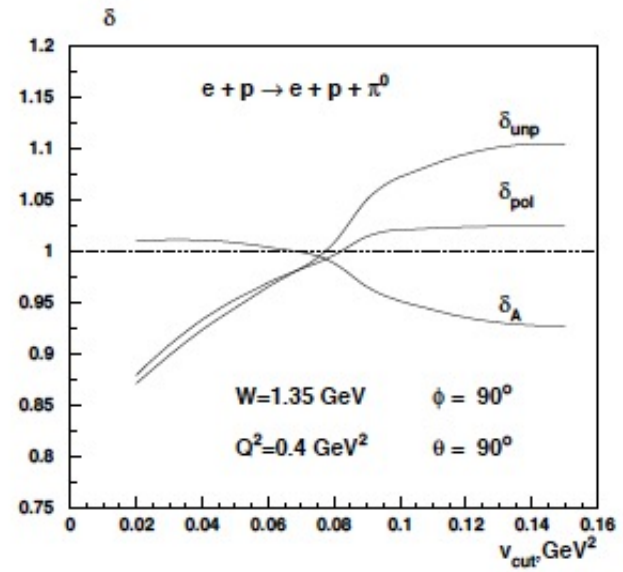
RC for Spin Asymmetries

- RC is zero for soft photons (can be enforced by kinematic cuts for brem photons, but not for TPE)

  =>RC to spin asymmetries strongly depend on kinematic cuts
- Important to use no soft approximation for calculations of spin asymmetries

RC dependence on the cuts





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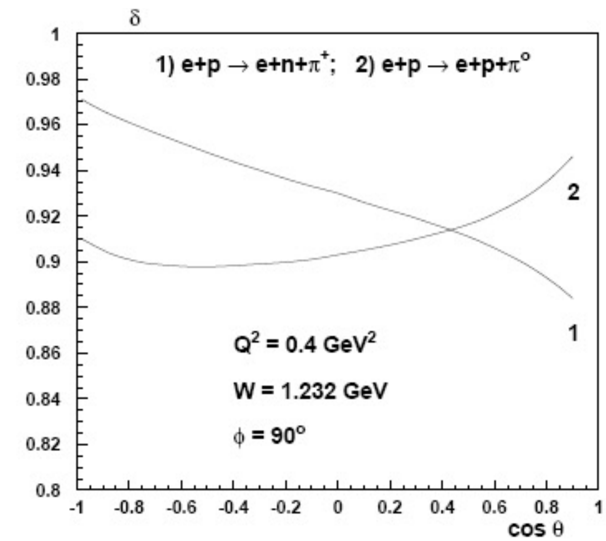
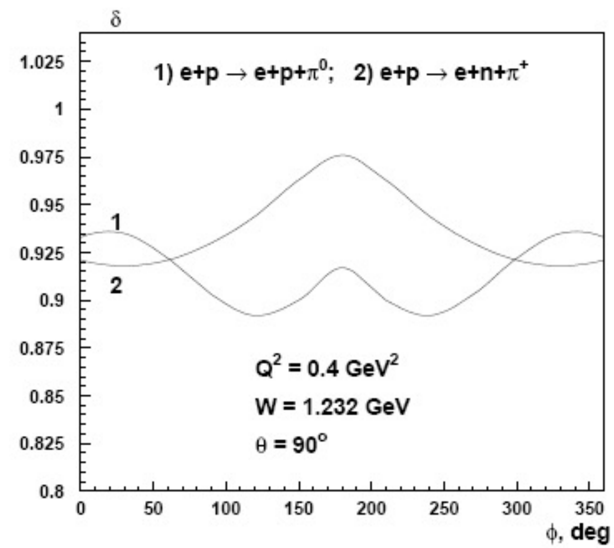
Angular Dependence of Rad.Corrections

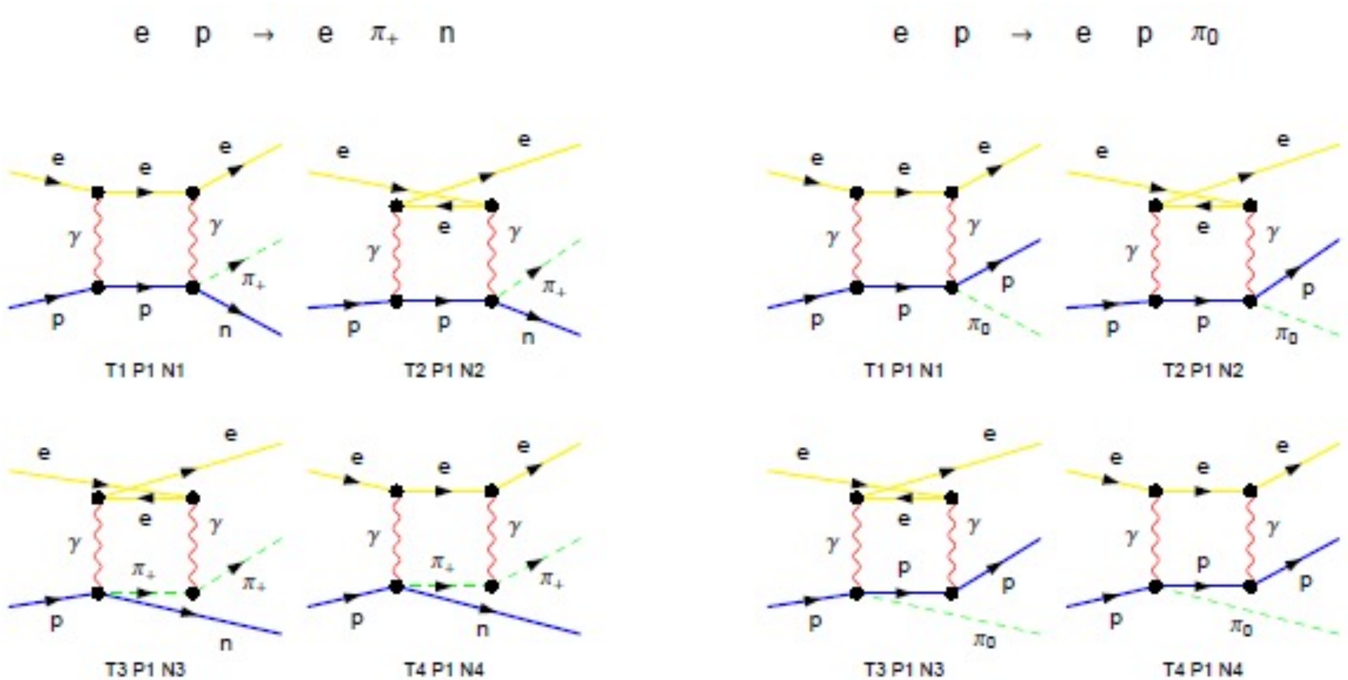
- Rad.Corrections introduce additional angular dependence on the experimentally observed cross section of electroproduction processes, both exclusive and semi-inclusive





Two-Photon Exchange in Exclusive Electroproduction of Pions (same for muons!)

- Standard contributions: EXCLURAD
- *Additional contributions due to two-photon exchange*, calculated by AA, Aleksejevs, Barkanova, arXiv:1207.1767 (Phys.Rev. D88 (2013) 053008) Calculated in soft-photon approximation



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TPE for Pion Production: IR regularization

- AA, Aleksejevs, Barkanova, arXiv:1207.1767 (Phys.Rev. D88 (2013) 053008)
- Need to add real photon emission to cancel IR divergence
- Use a finite photon mass for intermediate steps; photon mass dependence cancels in the end after adding TPE and real-photon emission
- Expressed results in terms of Passarino-Veltman integrals
- Obtained analytic results for the limit

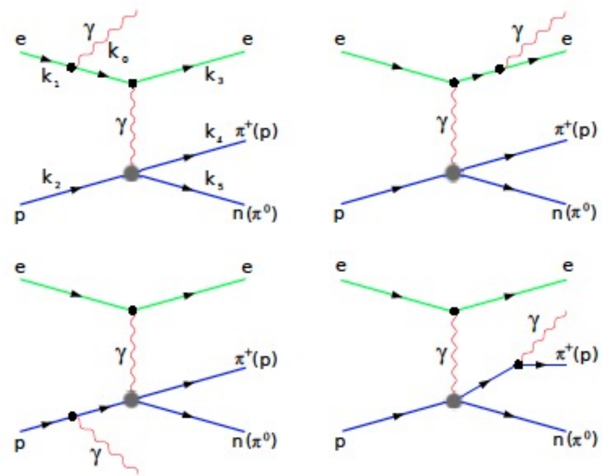
of zero electron mass

“Soft” TPE: a necessary step before includ

“hard” TPE, need to subtract soft terms

at the quark level and add at the hadron

level



TPE: some details of the calculation

- Brem+TPE, neglecting the electron mass
- Soft photons factorize at the amplitude level,

$$M_1^{SPT} = -\frac{\alpha}{2\pi} S \cdot C_0(\{k_1, m_1\}, \{-k_2, m_2\}) \cdot M_0.$$

- Passarino-Veltman 3-point scalar integral

$$C_0(\{k_i, m_i\}, \{k_j, m_j\}) = \frac{1}{i\pi^2} \int d^4q \frac{1}{q^2} \cdot \frac{1}{(k_i - q)^2 - m_i^2} \cdot \frac{1}{(k_j - q)^2 - m_j^2}.$$

$$\delta_{box}^{SPT} = -\frac{\alpha}{\pi} \mathrm{Re}\big[S \cdot C_0(\{k_1, m_1\}, \{-k_2, m_2\}) + X \cdot C_0(\{k_3, m_3\}, \{k_2, m_2\}) + V_3 \cdot C_0(\{k_3, m_3\}, \{-k_4, m_4\}) + V_1 \cdot C_0(\{k_1, m_1\}, \{k_4, m_4\})\big].$$

Simplified in a small-mass limit, final result reads

$$\delta_{tot}^{SPT} = \delta_{tot}^{IR} + \delta_{tot}^{F}$$

$$\delta_{box}^{IR} = -\frac{\alpha}{\pi} \ln\frac{m_2^2}{\lambda^2} \left[\ln\frac{S}{X} - \ln\frac{V_1}{V_3}\right]$$

$$\delta_{\gamma}^{IR} = -\frac{\alpha}{\pi} \ln\frac{4\Delta\varepsilon^2}{\lambda^2} \left[-\ln\frac{S}{X} + \ln\frac{V_1}{V_3}\right].$$

$$\delta_{box}^{F} = -\frac{\alpha}{\pi}\left[\frac{1}{2}\ln\frac{S}{X} \cdot \ln\frac{S \cdot X}{m_2^4} + \frac{1}{2}\ln\frac{V_3}{V_1} \cdot \ln\frac{V_1 \cdot V_3}{m_4^4} - \pi^2 - Li_2\left(\frac{S + m_2^2}{S}\right) + Li_2\left(\frac{X - m_2^2}{X}\right) + Li_2\left(\frac{V_1 - m_4^2}{V_1}\right) - Li_2\left(\frac{V_3 + m_4^2}{V_3}\right)\right]$$

$$\delta_{\gamma}^{F} = -\frac{\alpha}{\pi}\left[Li_2\left(1 - \frac{\beta_2 \cdot (u_1 - V_1)}{S \cdot m_5^2}\right) + Li_2\left(1 - \frac{m_2^2 \cdot (u_1 - V_1)}{S \cdot \beta_2}\right) - Li_2\left(1 - \frac{\beta_4 \cdot (u_1 - V_1)}{V_1 \cdot m_5^2}\right) - Li_2\left(1 - \frac{m_4^2 \cdot (u_1 - V_1)}{V_1 \cdot \beta_4}\right) - Li_2\left(1 - \frac{\beta_2 \cdot (u_3 - V_3)}{X \cdot m_5^2}\right) - Li_2\left(1 - \frac{m_2^2 \cdot (u_3 - V_3)}{X \cdot \beta_2}\right) + Li_2\left(1 - \frac{\beta_4 \cdot (u_3 - V_3)}{V_3 \cdot m_5^2}\right) + Li_2\left(1 - \frac{m_4^2 \cdot (u_3 - V_3)}{V_3 \cdot \beta_4}\right)\right].$$

TPE at higher Q^2

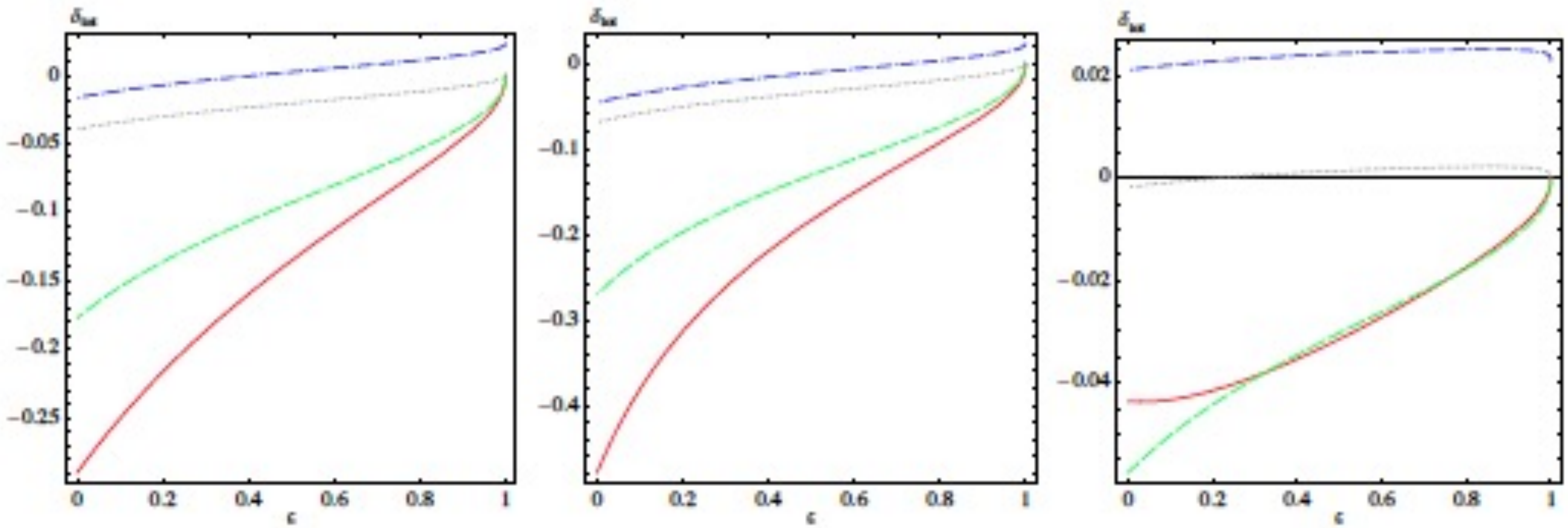


Figure 5: π^0 electroproduction two-photon box correction (for detected proton) dependencies on virtual photon degree of polarization parameter ϵ for momentum transfers $Q^2 = 3.0\,GeV^2$ (left plot), $Q^2 = 7.0\,GeV^2$ (middle plot) and $Q^2 = 0.4\,GeV^2$ (right plot). All plots are given for $\phi_4 = 90°$ and $\theta_4 = 90°$ and $W = 1.232\,GeV$. Dot-dashed curve - SPT, dotted curve - SPT with $\alpha\pi$ subtracted, dashed curve - SPMT, solid curve - FM approach.

TPE effects increase at higher Q^2; SPMT (Maximon-Tjon soft-photon prescription) results in abnormally large corrections

Angular dependence of "soft" corrections

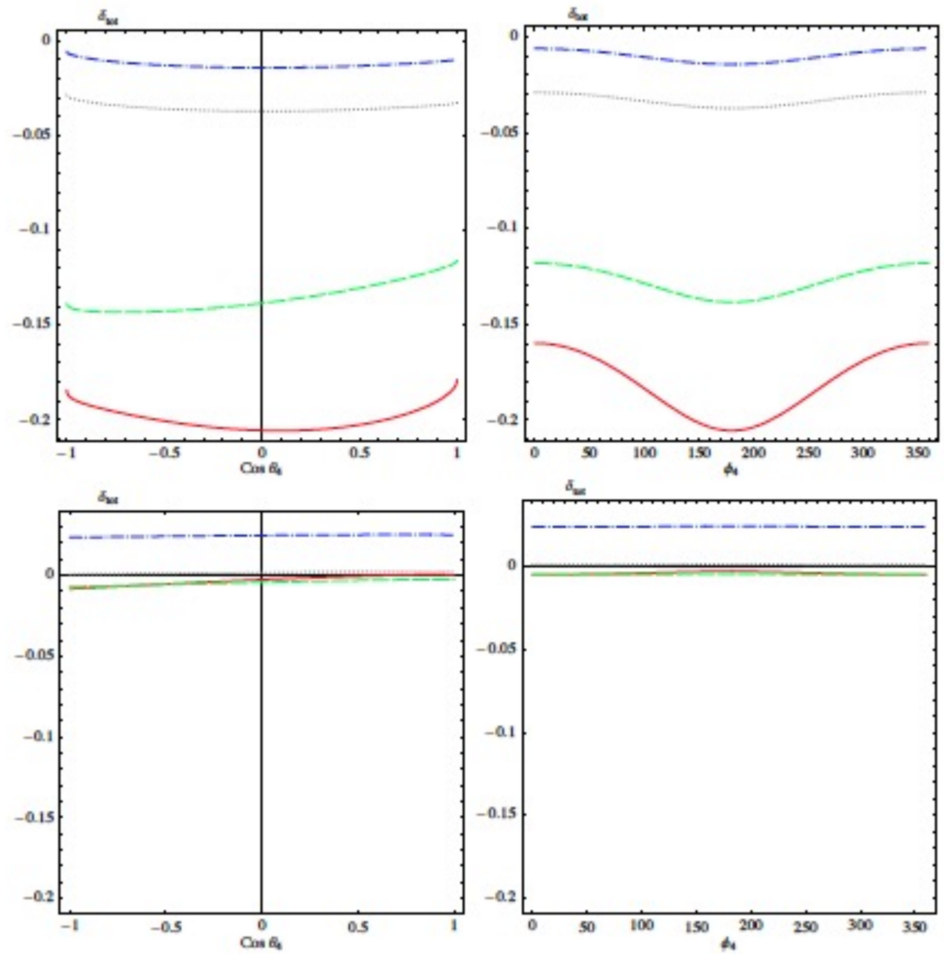
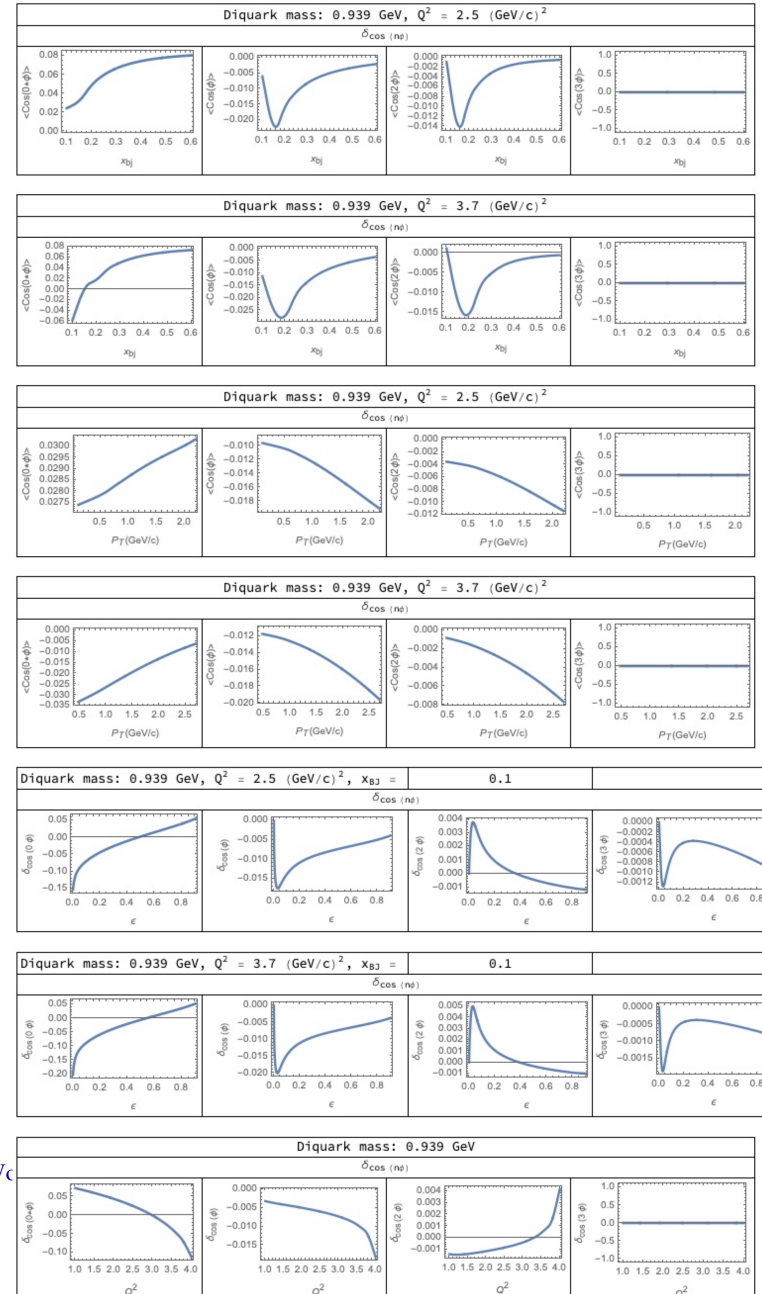


Figure 3: π^0 electroproduction two-photon box correction angular dependencies for the high $Q^2 = 6.36\,GeV^2$ (top row) and low $Q^2 = 0.4\,GeV^2$(bottom row) momentum transfers, $W = 1.232\,GeV$ and $E_{lab} = 5.75\,GeV$. Left column: dependence on $\cos\theta_4$ with $\phi_4 = 180°$. Right column: dependence on ϕ_4 with $\theta_4 = 90°$. Dot-dashed curve - SPT, dotted curve - SPT with $\alpha\pi$ subtracted, dashed curve - SPMT, solid curve - FM approach.

Extension of TPE calculations to SIDIS

- PhD project of Stinson Lee (GWU)
  - Use of a di-quark model
  - Soft photons included
  - Hard TPE in progress



Summary for Exclusive QED corrections

- Rad Corrections for exclusive meson production (EXCURAD code is available with extensions for polarized particles and other final states)

TPE for exclusive reactions

- Two-photon exchange calculated in soft approximation for pion electroproduction
  - Extended to SIDIS (Stinson Lee, GW grad student)
- Can be added to existing codes and/or generators and studied for specific experimental conditions
- Hard TPE is model-dependent and non-negligible
- Positron beams at Jlab may address TPE effects and lepton – hadron emission interferences

Evaluation of Rad Corrections is essential for (1) experiment planning stage to estimate the systematics and (2) data analysis.

- Follow Radiative Corrections Helpdesk www.jlab.org/RC and Rad Cor working group meeting via Center for Nuclear Femtography (planned for Oct-Nov 2023)

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