# Quark GPD Phenomenology

## From twist-2 to twist-3

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**Aug 8 2023**

Nuclear Femtography Workshop
JLAB Aug 2023

# Outline

- Review of DVCS Theory and Phenomenological Approaches
- GPDs from Universal Moment Parameterization (GUMP): results so far
- Twist-3 GPDs and observables in DVCS

# Timeline of DVCS Cross Section Calculations

X. Ji, PRD 55 (1997) 71114
**(Ji)**

- First attempt, twist-2

Belitsky, Mueller, Kirchner, Nuc Phys B 629 (2002) 323
**(BMK)**

- Full twist-2 + WW twist-3, certain light cone choice made, kinematical approximations made, all polarization channels covered

Belitsky, Mueller, Kirchner, Phys Rev D 82 (2010) 074010
**(BMK)**

- Kinematic improvements made to 2001 work, but doesn't cover all polarization channels

Braun, Manashov, Muller, Pirnay, PRD89, (2019) 074022
**(BMMP)**

- Extension of BMK's work, incorporating higher order target and mass corrections

B. Kriesten et al., Phys Rev D 101 (2020) 054021
**(UVa)**

- Genuine twist-3 CFFs used, physics connections to other processes made, all polarizations covered

Y. Guo, X. Ji, K. Shiells, JHEP 12 (2021) 103
**(GSJ)**

- Full twist-2 + WW twist-3, optimal light cone choice found, no kinematical approximations used, all polarization channels covered

Y. Guo, X. Ji, K. Shiells, B. Kriesten (2022) JHEP 06 (2022) 096
**(GSJ)**

- Extension of 2021 work with genuine twist-3 CFFs

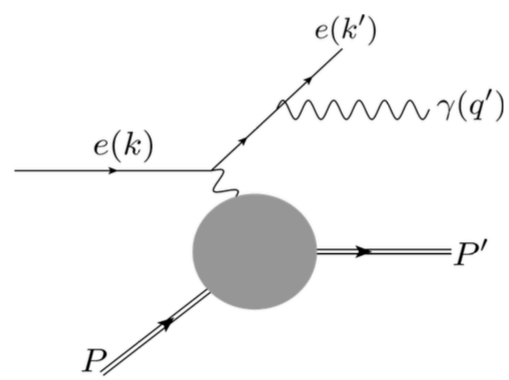
# GSJ Formalism (Guo, Shiells, Ji)

- Considers the 5-fold differential DVCS Cross section

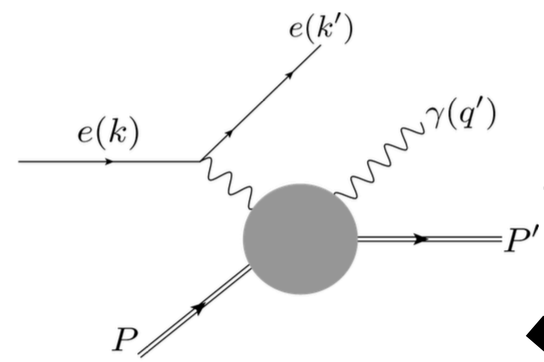
$$\frac{d^5\sigma}{dx_B dQ^2 d|t| d\phi d\phi_S} = \frac{\alpha_{EM}^3 x_B y^2}{16\pi^2 Q^4 \sqrt{1+\gamma^2}} |\mathcal{T}|^2$$

- Comes from 2 amplitudes:

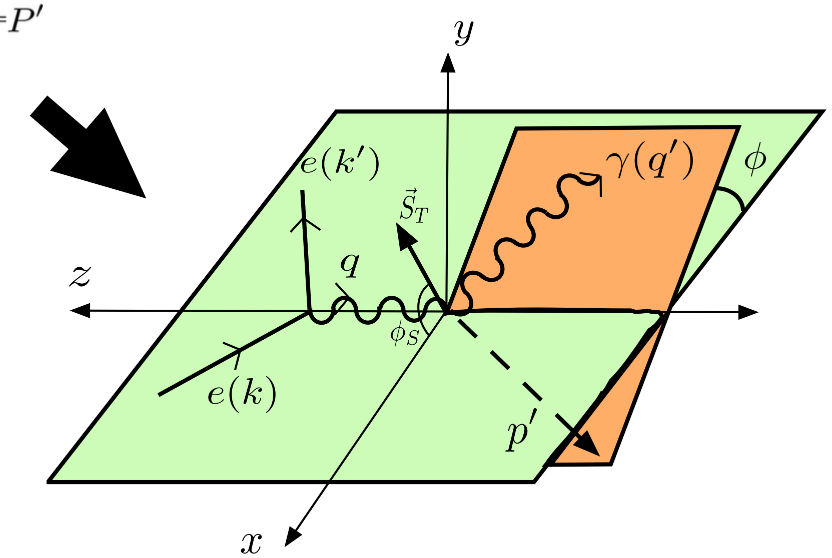
$$\mathcal{T} = \mathcal{T}_{BH} + \mathcal{T}_{DVCS}$$




Bethe-Heitler




DVCS




- Expands the Compton amplitude with respect to a general light-cone direction, expressing in terms of universally-defined twist-2 quark-quark GPDs

- Allows for a polarized beam and target

# Harmonic Structure

- The GSJ formalism expresses both pure DVCS and interference cross sections into products between $\phi$-dependent scalar coefficients $\times$ $\phi$-independent irreducible CFF expressions

$$\sigma = \text{(scalar coefficient)} \times \text{(CFF expression)}$$

- All the scalar coefficients can be expressed in terms of **harmonic series**

e.g. unpolarized coefficients:

$$h^U = \sum_{n=0}^{3} h_n^U \cos(n\phi) \qquad A^{I,U} = \frac{Q^4}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=0}^{3} a_n^{I,U} \cos(n\phi)$$

- **Leading twist dominates the lower-order harmonic coefficients**, while the higher-order harmonics involve higher twist contributions and are kinematically suppressed

- General idea: we can **fit harmonic coefficients to the data**, acquiring equations which constrains the CFFs — this works for both cross sections and asymmetries

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- All 3 parts of the cross section ( | DVCS |$^2$ ,BH, DVCS-BH Interference) are sensitive to the helicities of the beam and target

$$\frac{d^5\sigma}{dx_B dQ^2 d|t| d\phi d\phi_S} = \frac{d^5\sigma^{(2)}}{dx_B dQ^2 d|t| d\phi d\phi_S}(x_B, t, Q, E_b, \phi, \mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}})$$

+ "twist-3"

- This covers 6 distinct channels of beam/target polarizations: UU, LU, UL, LL, UT, LT. **Each channel has a different dependence of the CFFs**

- Phenomenological rule-of-thumb:

$$\sigma^{UU}_{\text{DVCS}} = \frac{2\pi\Gamma}{Q^4}\sum_n h^U_n(E_b; x_B, t, Q^2)\mathcal{D}(\mathcal{F}^2)\cos(n\phi)$$

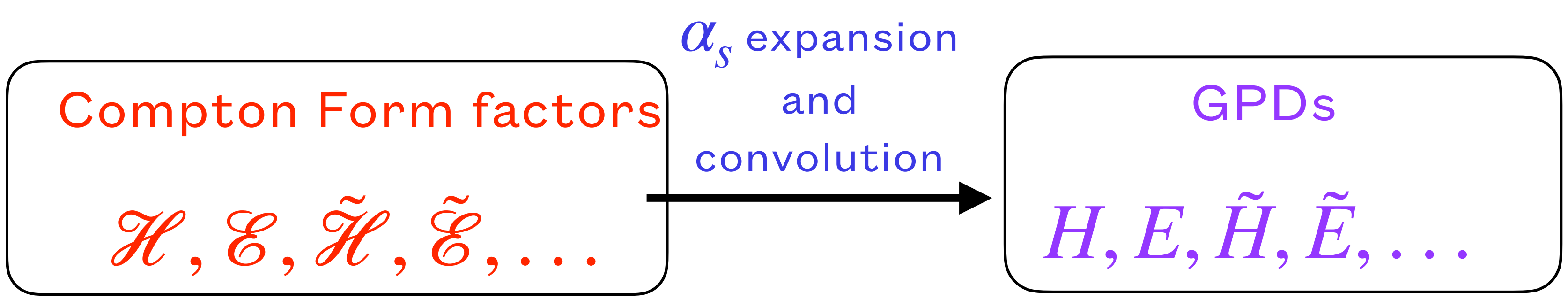
$$\#constr \approx \textstyle\sum_{\text{pol.}}(\#E_b) \times (\#harm/pol.)$$

$$\mathcal{F} = \mathcal{H}, \mathcal{E}, ...(x_B, t, Q^2)$$

For stable CFF extraction (unique solution): $\#constr \geq \#param$

twist 2 CFFs $\Rightarrow$ 8 param

# Extraction of GPDs




$$\mathcal{H}(\xi,t) \equiv \int_{-1}^{1} \mathrm{d}x C_{(0)}^{q[-]}(x,\xi) H(x,\xi,t) \ , \quad \mathcal{E}(\xi,t) \equiv \int_{-1}^{1} \mathrm{d}x C_{(0)}^{q[-]}(x,\xi) E(x,\xi,t) \ ,$$

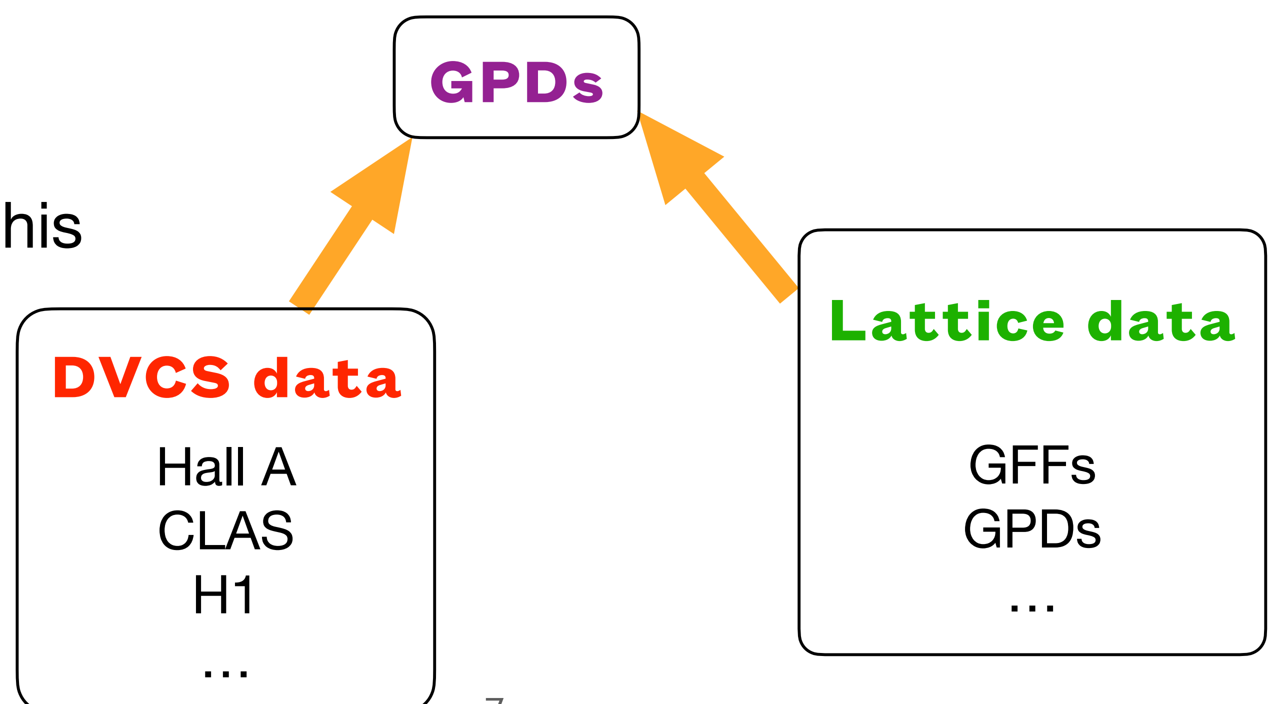
$$\tilde{\mathcal{H}}(\xi,t) \equiv \int_{-1}^{1} \mathrm{d}x C_{(0)}^{q[+]}(x,\xi) \tilde{H}(x,\xi,t) \ , \quad \tilde{\mathcal{E}}(\xi,t) \equiv \int_{-1}^{1} \mathrm{d}x C_{(0)}^{q[+]}(x,\xi) \tilde{E}(x,\xi,t) \ ,$$

$\alpha_s = 0$:

$$C_{(0)}^{q[\pm]} = -Q_q^2 \left( \frac{1}{x - \xi + i0} \mp \frac{1}{x + \xi - i0} \right) ,$$

- We can fit a **GPD model** simultaneously to DVCS data points AND lattice QCD data

We have developed this architecture!



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# GUMP: GPDs from Universal Moment Parameterization

- Based on the conformal moment expansion of GPDs

Mueller & Schaefer NPB 739 (2006) 1

- This leads to *dual parameterization* or *Mellin Barnes* frameworks

- Mathematically, one is essentially expanding the GPD into a basis of orthogonal functions

| | PDFs | | GPDs |
|---|---|---|---|
| Formal summation | $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \delta^{(n)}(x) f_{n+1}$ | Generalization → / ← Forward Limit | $F(x,\xi,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\xi) \mathcal{F}_j(\xi,t)$ — conformal moments |
| | ↕ Equivalent | | ↕ Sommerfeld-Watson transformation |
| Contour integral | $f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-s} f_s \mathrm{d}s$ | | $F(x,\xi,t) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \frac{p_j(x,\xi)}{\sin(\pi[j+1])} \mathcal{F}_j(\xi,t)$ |

Mellin-Barnes Respresentation

- Polynomiality condition:

$$\mathcal{F}_j(\xi,t) = \boxed{\mathcal{F}_{j,0}(t)} + \boxed{\xi^2 \mathcal{F}_{j,2}(t)} + \mathcal{O}(\xi^4)$$

Nonzero-skewness case

**Zero-skewness case :**

*t*-dependent PDF

MODEL: $$\mathcal{F}_{j,0}(t) = N_0 B(j+1-\alpha_0, 1+\beta_0) \frac{j+1-\alpha_0}{j+1-\alpha_0+\alpha_0' t}$$

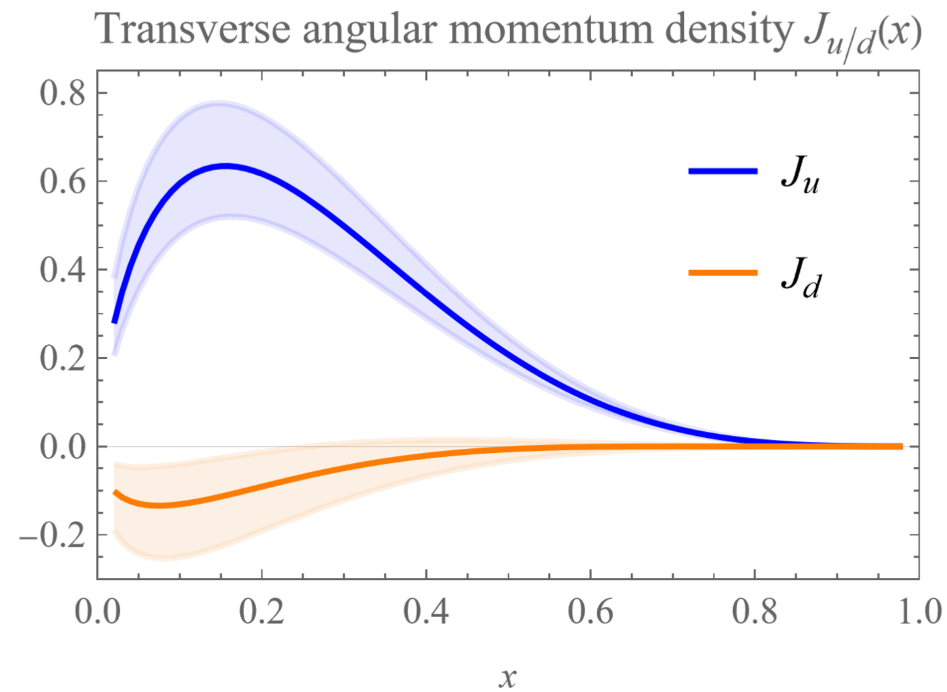
- Direct constraints from $t$-dependent lattice GFFs
- Additional constraints from globally-determined PDFs
- Also data points on lattice-computed $t$-dependent isovector GPDs
- Only valence distributions are considered here, as sea contributions highly suppressed

# Zero-skewness Results

*JHEP* 09 (2022) 215

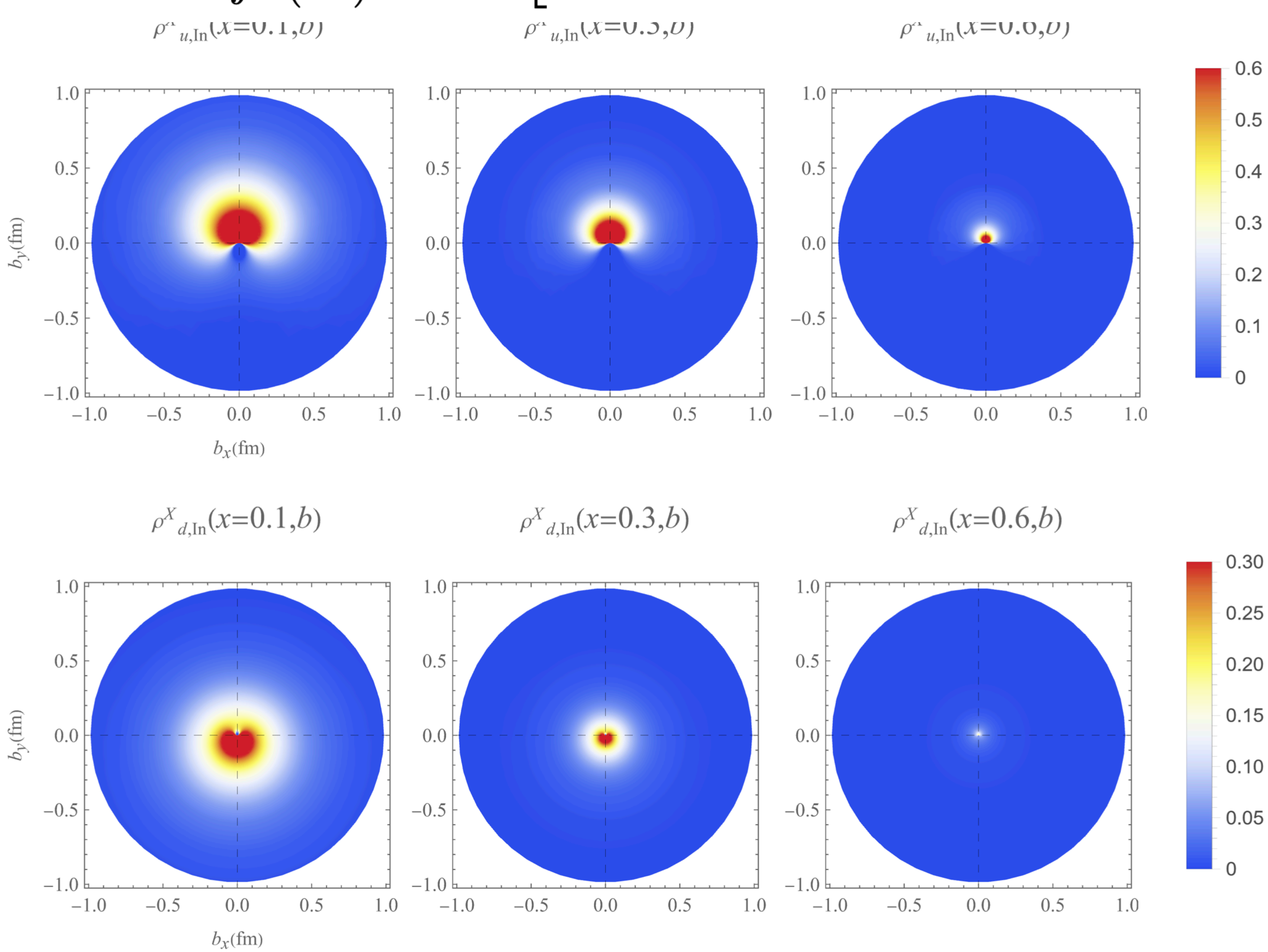
Quark intrinsic AM density:

$$J_{u/d}(x,t) = \frac{1}{2}x\left(H_{u/d}(x,t) + E_{u/d}(x,t)\right)$$




For a proton transversely polarized in the x-direction:

$$\rho^X_{q,\mathrm{In}}(x,\boldsymbol{b}) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\left[H_q(x,-\boldsymbol{\Delta}^2) + \frac{i\Delta_y}{2M}\left(H_q(x,-\boldsymbol{\Delta}^2) + E_q(x,-\boldsymbol{\Delta}^2)\right)\right]$$




Irony: most sought femtography quantities are at $\xi = 0$

# Nonzero-skewness Study

*JHEP* 05 (2023) 150

$$\mathcal{F}_{j,k}(t) = \sum_{i=1}^{i_{\max}} N_{i,k} B(j+1-\alpha_{i,k}, 1+\beta_{i,k}) \frac{j+1-k-\alpha_{i,k}}{j+1-k-\alpha_{i,k}(t)} \beta(t)$$

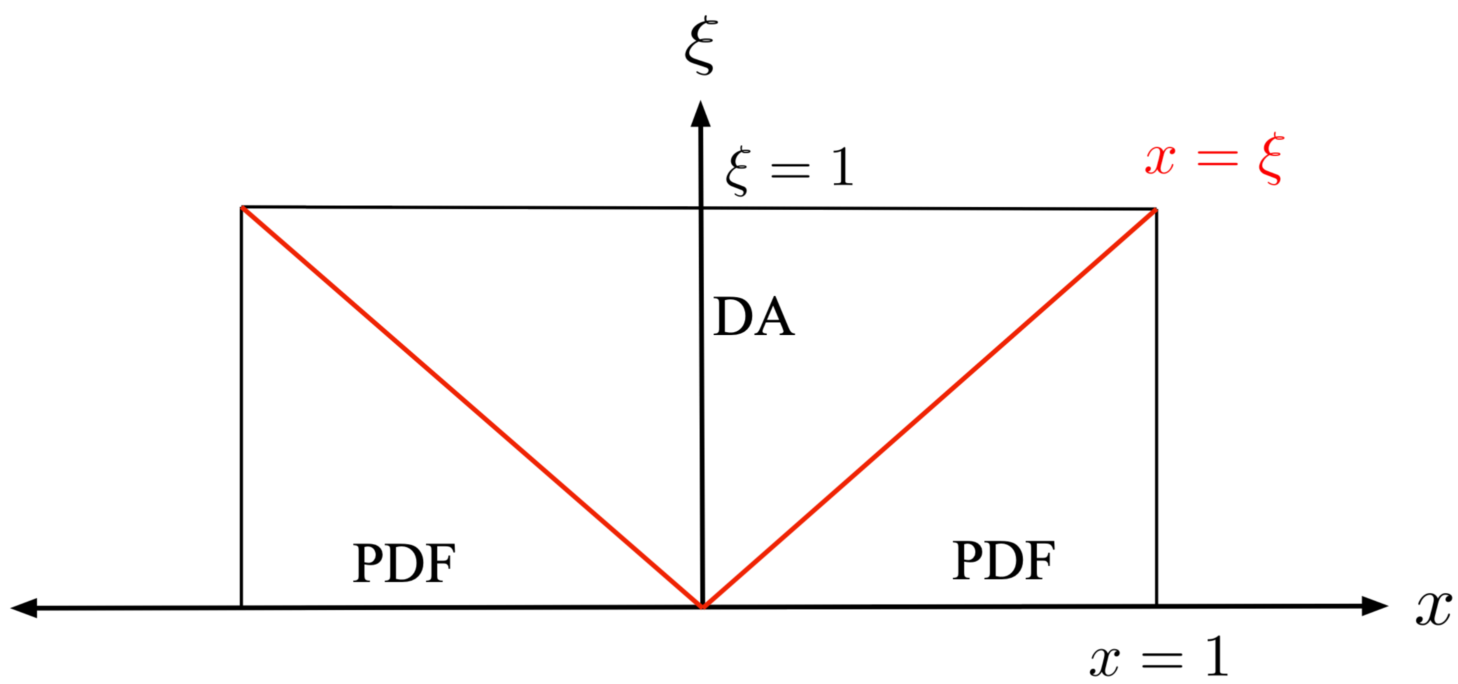
$$\beta(t) = \exp(bt)$$

- Deconvolution problem means there aren't enough constraints from off-forward measurements to fully constrain all the GPDs

- To reduce the number of parameters, we assume a proportionality to the zero-skewness moments:

$$\mathcal{F}_{j,k}(t) = R_k \mathcal{F}_{j-k,0}(t)$$


$$\mathcal{F}_j(\xi, t) = \mathcal{F}_{j,0}(t) + \xi^2 \mathcal{F}_{j,2}(t) + \mathcal{O}(\xi^4)$$




- GPDs do not naturally distinguish their quark and antiquark components, especially in the DA region

$$F_q(x,\xi,t) \equiv F_{\hat{q}}(x,\xi,t) \mp F_{\bar{q}}(-x,\xi,t) + F_{q\bar{q}}(x,\xi,t)$$

$-$ vector
$+$ axial vector

| flavour label | quark only | antiquark only | quark-antiquark |
|---|---|---|---|
| | $x > -\xi$ | $x > -\xi$ | $\xi > x > -\xi$ |

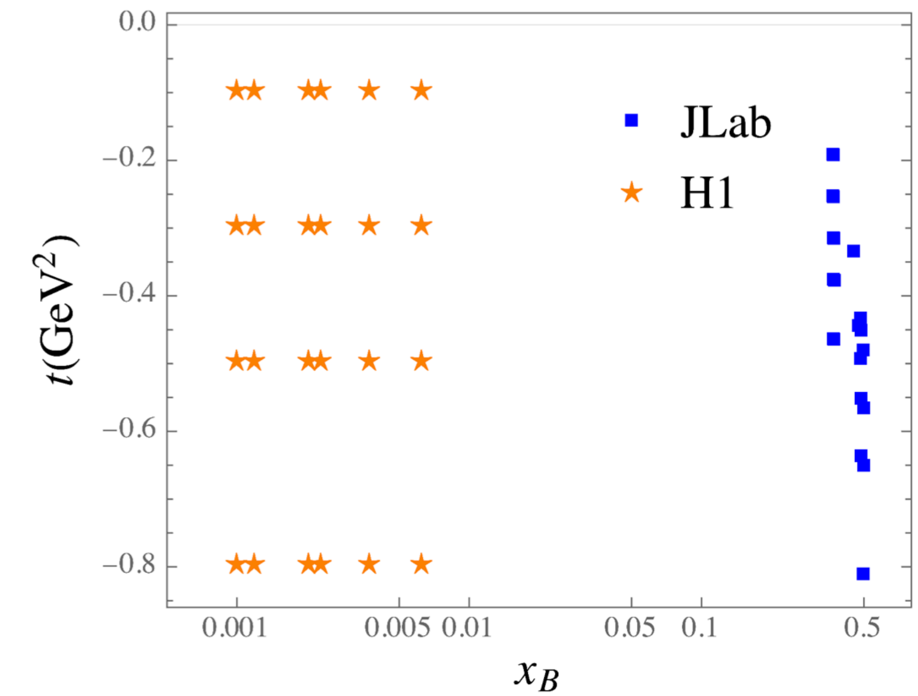
- We choose basis:
$\{\hat{q}, \bar{q}, g\} \to \{q_v, \bar{q}, g\} \otimes \{q = u, d\} \otimes \{F = H, E, \tilde{H}, \tilde{E}\}$

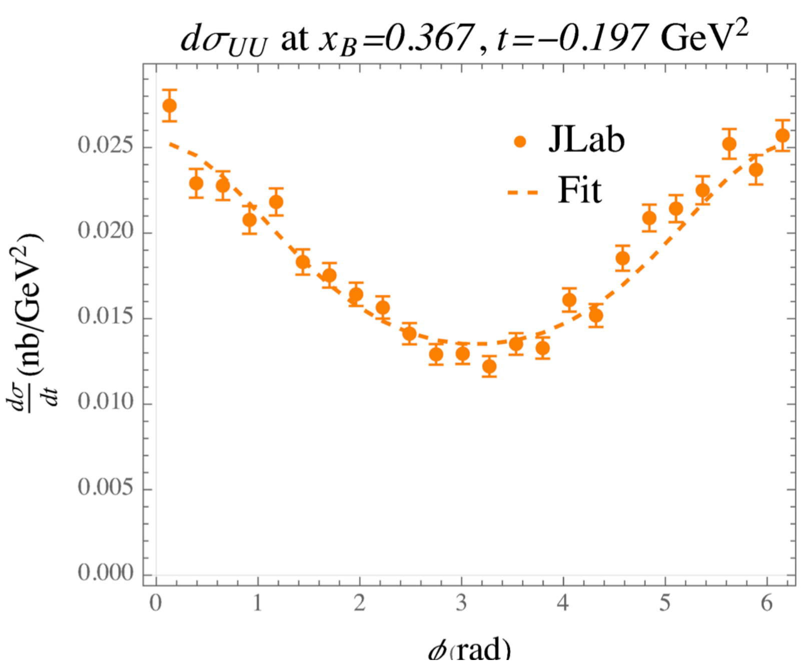
$$\Rightarrow 20 \quad \text{GPDs}$$

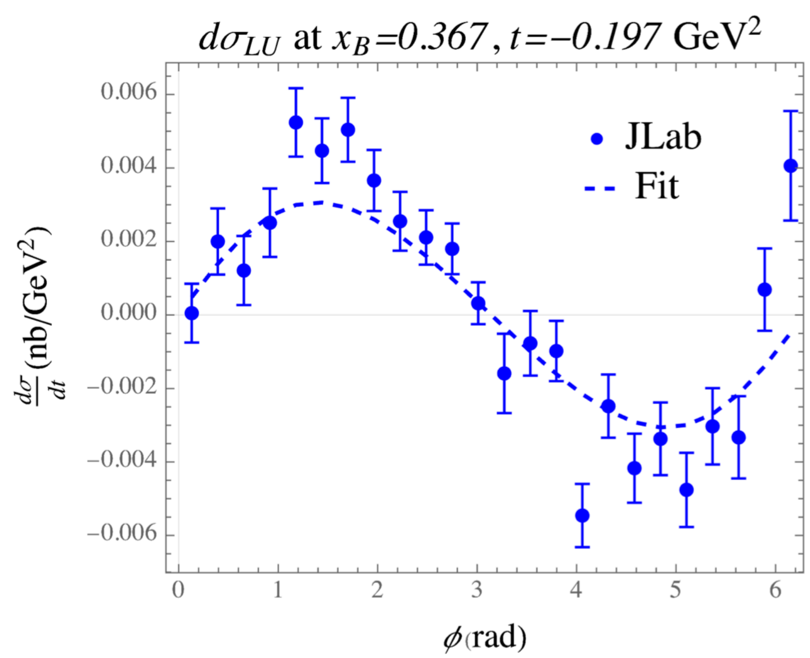
That's 20 functions of 3 variables….

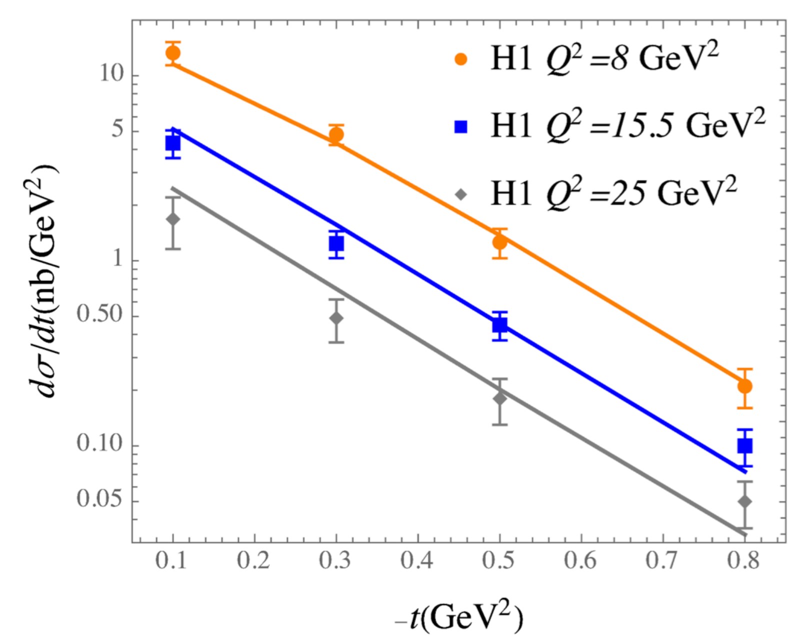
$$\chi^2_{\text{tot}} = \chi^2_{\text{fwd}} + \chi^2_{\text{off-fwd}} = \chi^2_{H} + \chi^2_{E} + \chi^2_{\widetilde{H}} + \chi^2_{\widetilde{E}} + \chi^2_{\text{off-fwd}}$$

| Sub-fits | $\chi^2$ | $N_{\text{data}}$ | $\chi^2_\nu \equiv \chi^2/\nu$ |
|---|---|---|---|
| **Semi-forward** | | | |
| $t$PDF $H$ | 281.7 | 217 | 1.41 |
| $t$PDF $E$ | 59.7 | 50 | 1.36 |
| $t$PDF $\widetilde{H}$ | 159.3 | 206 | 0.84 |
| $t$PDF $\widetilde{E}$ | 63.8 | 58 | 1.23 |
| **Off-forward** | | | |
| JLab DVCS | 1413.7 | 926 | $\sim$ 1.53 |
| H1 DVCS | 19.7 | 24 | $\sim$ 0.82 |
| Off-forward total | 1433 | 950 | **1.53** |
| **Total** | 2042 | 1481 | **1.40** |

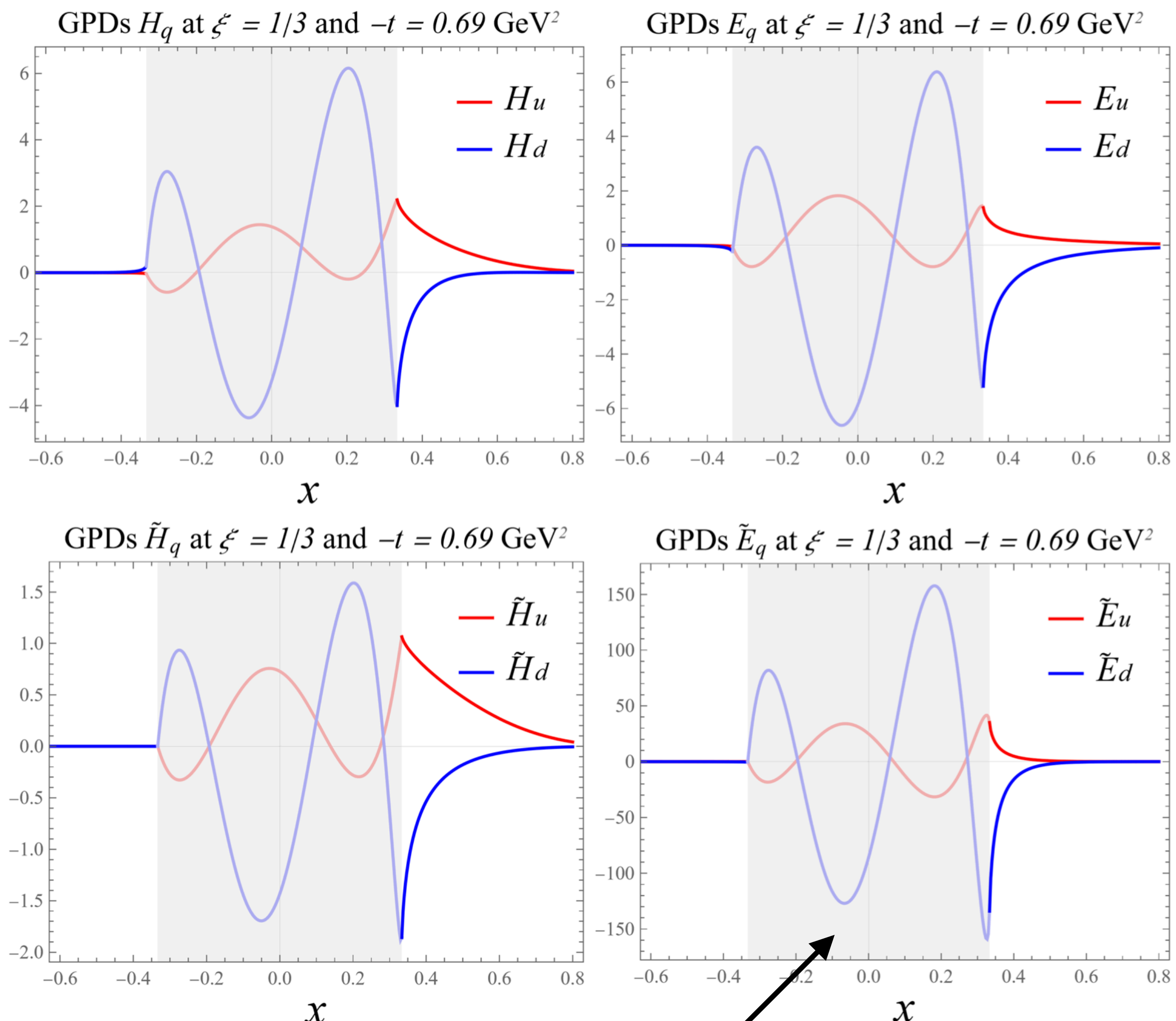


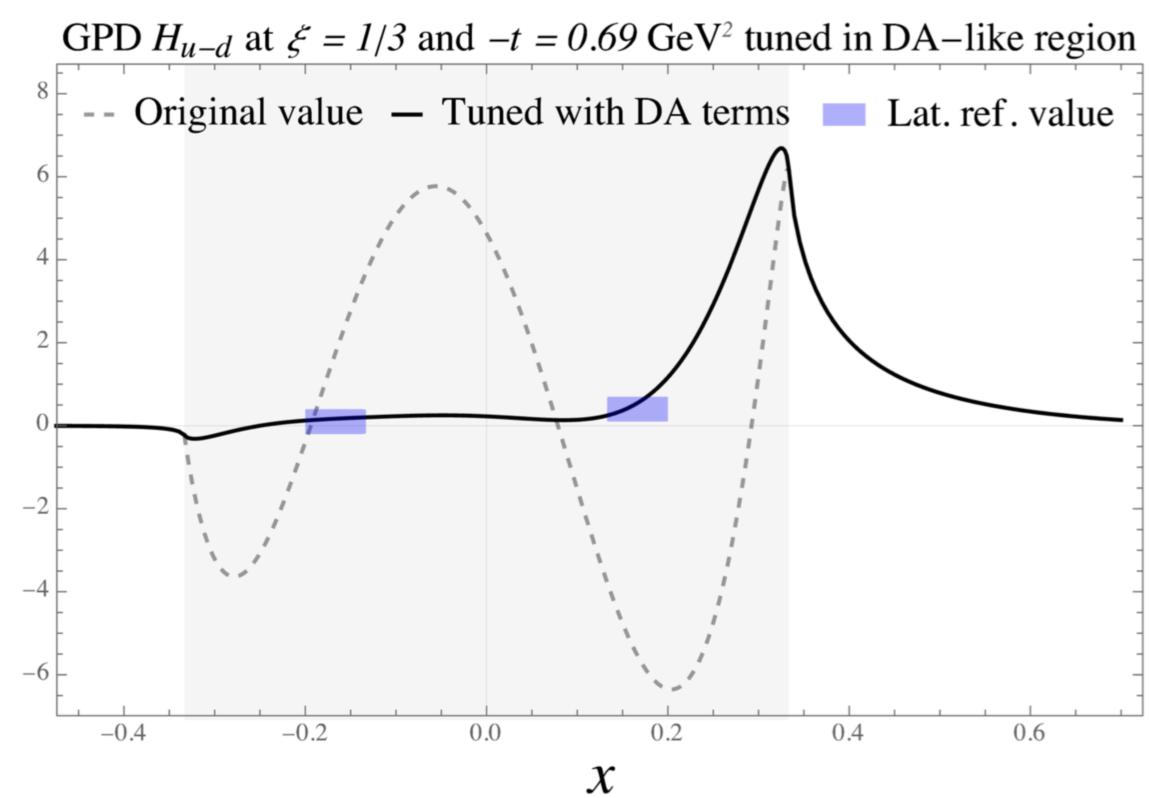



$$F_q(x,\xi,t) \equiv F_{\hat{q}}(x,\xi,t) \mp F_{\bar{q}}(-x,\xi,t) + F_{q\bar{q}}(x,\xi,t)$$




- We can include the DA GPD term, which is important for finite $\xi$. Here we've tweaked it to show one can achieve good agreement with the isovector lattice predictions there
- However, insufficient constraints exist to include $F_{q\bar{q}}$ in a global analysis




- GPDs acquire an oscillatory behaviour in the DA region due to the conformal wave functions (~ Gegenbauer polynomials)

# Twist 3 Phenomenology in DVCS

- Start by expanding the Compton tensor:

$$T^{\mu\nu} \equiv i \int \mathrm{d}^4 x e^{i(q+q')z/2} \left\langle P', S' \middle| \mathrm{T}\left\{ J^\mu\left(\frac{z}{2}\right) J^\nu\left(-\frac{z}{2}\right)\right\} \middle| P, S \right\rangle$$
$$= \quad T^{\mu\nu}_{(2)} + T^{\mu\nu}_{(3)} + \cdots$$

- $T^{\mu\nu}_{(2)}$ involves 4 twist-2 GPDs: $H, E, \widetilde{H}, \widetilde{E}$

- $T^{\mu\nu}_{(3)}$ involves 8 twist-3 GPDs: $H_{2T}, H'_{2T}, E_{2T}, E'_{2T}, \widetilde{H}_{2T}, \widetilde{H}'_{2T}, \widetilde{E}_{2T}, \widetilde{E}'_{2T}$

defined in: Meissner, Metz, Schlegel, JHEP 08 (2009) 056

- But they arise in degenerate pairs:

Belitsky, Radyushkin Phys Rept 418 (2005) 1

$$\bar{H}_{2T} = H_{2T} - H'_{2T}$$
$$\bar{E}_{2T} = E_{2T} - E'_{2T}$$
$$\vdots$$

- So effectively, there are only 4 new tw-3 GPDs introduced: $\bar{H}_{2T}, \bar{H}'_{2T}, \bar{E}_{2T}, \bar{E}'_{2T}$

# A look at Twist 3 Scalar Coefficients

JHEP 06 (2022) 096

$$d\sigma_{\text{Total}} = d\sigma_{\text{BH}} + d\sigma_{\text{DVCS}} + d\sigma_{\text{INT}}$$

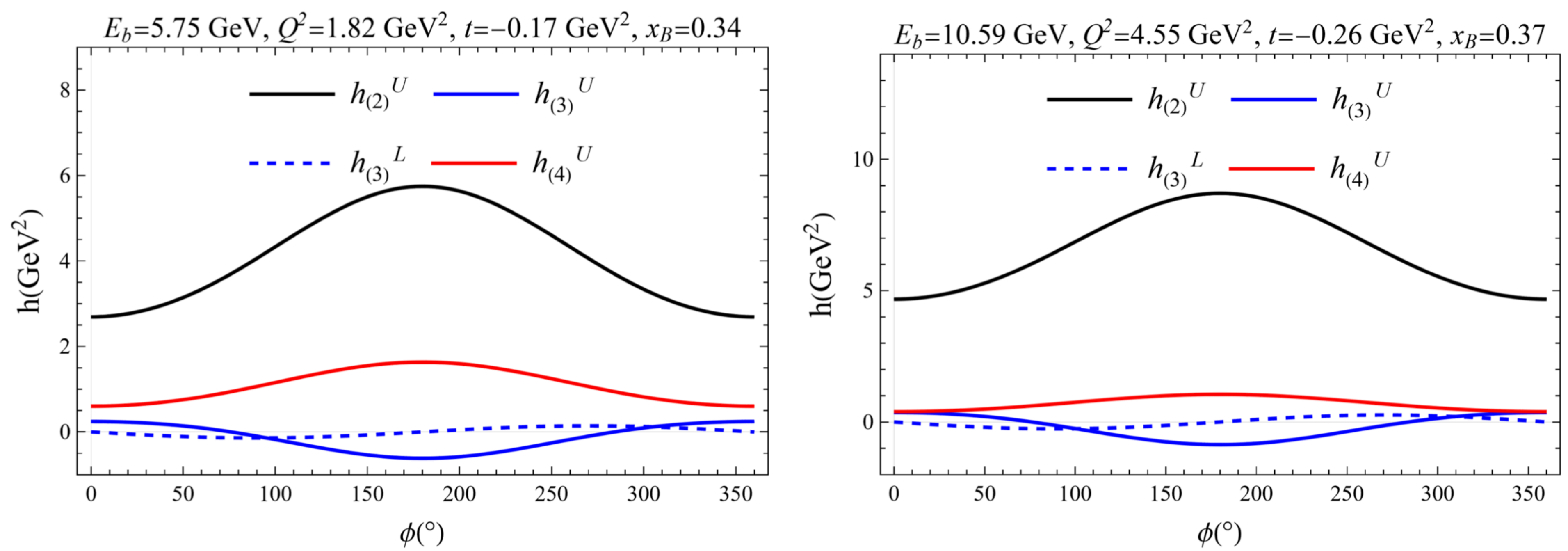
(5-fold cross section $ep \to ep\gamma$)

$$d\sigma_{\text{DVCS}} = \frac{\alpha_{\text{EM}}^3 x_B y^2}{16\pi^2 Q^4 \sqrt{1+\gamma^2}} \frac{1}{Q^4} \Big\{ F_{\text{UU}} + (2\Lambda_L) F_{\text{UL}} + (2\Lambda_T)\left(\cos(\phi_S - \phi) F_{\text{UT,in}} + \sin(\phi_S - \phi) F_{\text{UT,out}}\right)$$
$$+ (2h)\left[ F_{\text{LU}} + (2\Lambda_L) F_{\text{LL}} + (2\Lambda_T)\left(\cos(\phi_S - \phi) F_{\text{LT,in}} + \sin(\phi_S - \phi) F_{\text{LT,out}}\right)\right] \Big\}$$

$$F_{UU} = F_{UU}^{(2)} + F_{UU}^{(3)} + F_{UU}^{(4)}$$

$$\mathcal{F} = \text{CFF}$$

$$\sim h^U_{(2)}(\mathcal{F}^{(2)} \times \mathcal{F}^{(2)}) \qquad \sim h^U_{(3)}(\mathcal{F}^{(2)} \times \mathcal{F}^{(3)}) \qquad \sim h^U_{(4)}(\mathcal{F}^{(3)} \times \mathcal{F}^{(3)})$$

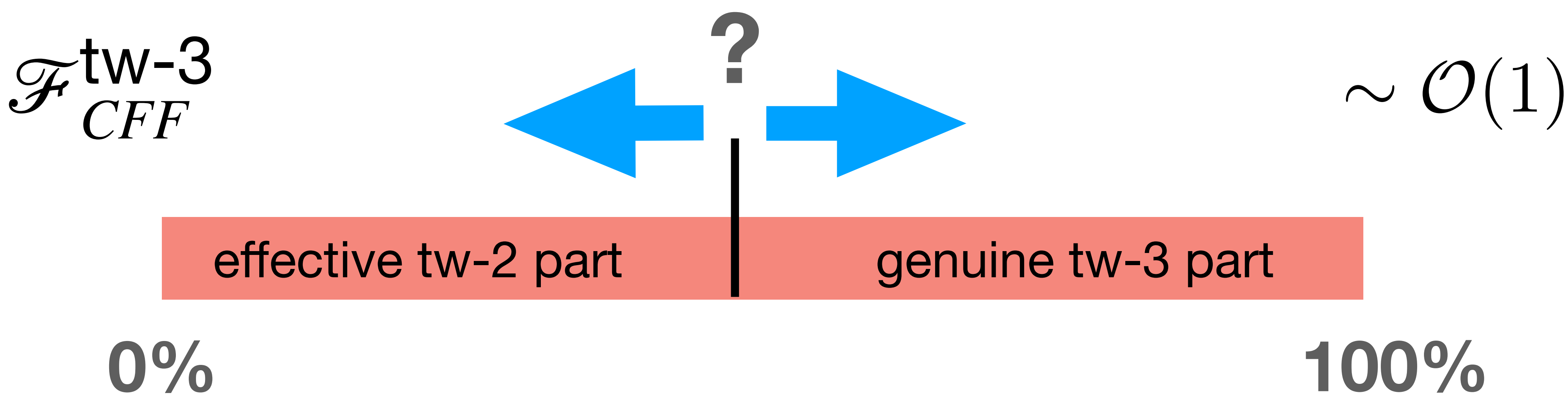



- Even greater suppression is seen for twist-3 Interference ($d\sigma_{\text{INT}}$) coefficients and for EIC kinematics

- Due to Lorenz invariance and QCD eom, tw2 GPDs are related to tw3 ones: Wandura-Wilczek

$$W^{[\gamma^\mu]} \approx \frac{\Delta^\mu}{n\cdot\Delta} n_\nu W^{[\gamma^\nu]} + \int_{-1}^{1} \mathrm{d}u W_+(x,u,\xi) G^\mu(u,\xi) + i\widetilde{\epsilon}^{\mu\nu} \int_{-1}^{1} \mathrm{d}u W_-(x,u,\xi) \widetilde{G}_\nu(u,\xi)$$

- The true qGq twist-3 contributions are difficult to estimate, however the overall twist-3 CFF is unitless and is expected to be $\sim \mathcal{O}(1)$



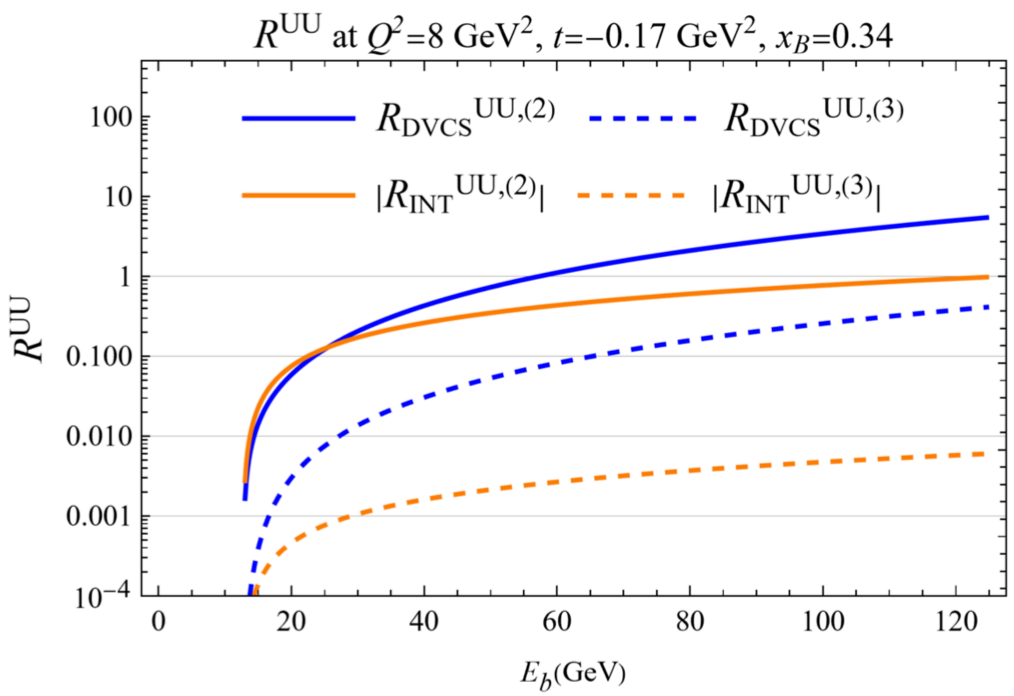

- Given the typical high kinematical suppression attached to the tw-3 CFFs, If one is plotting the total cross section, estimating the CFF with only its twist-2 part is still somewhat reasonable

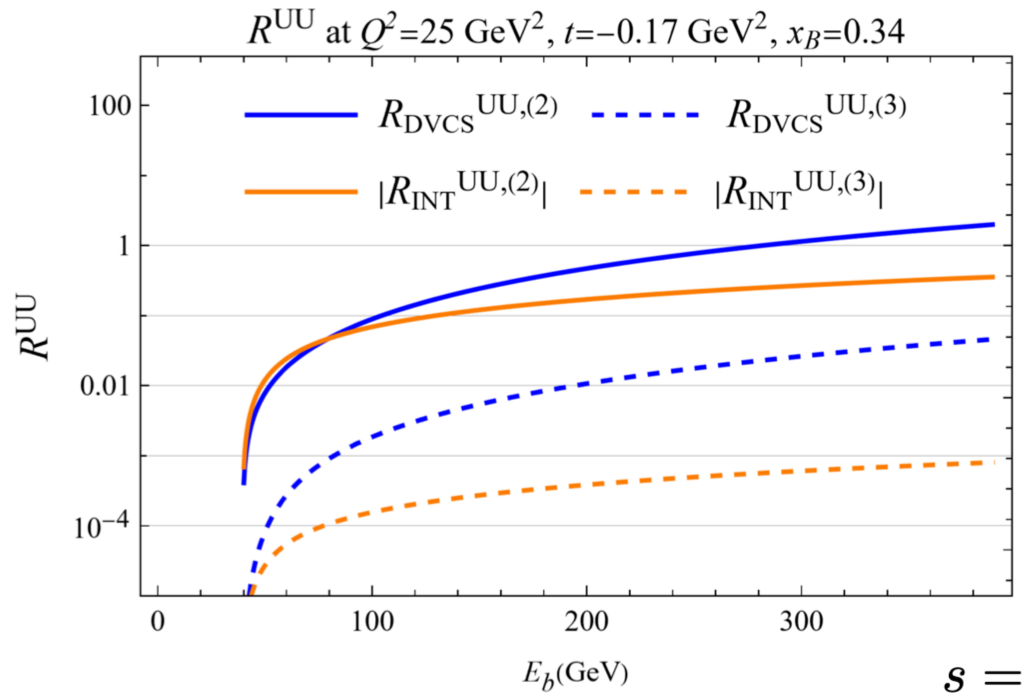
- But if estimating/plotting a unique observable which is of subleasing twist, a better job is needed!

# $Q^2$ and $E_b$ Dependence of twist-3 Cross Section

$$R_i^{\mathrm{UU}}(x_B, Q^2, t) = \frac{\mathrm{d}^3\sigma_i^{\mathrm{UU}}}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}|t|} \left( \frac{\mathrm{d}^3\sigma_{\mathrm{BH}}^{\mathrm{UU}}}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}|t|} \right)^{-1} = R_i^{\mathrm{UU},(2)} + R_i^{\mathrm{UU},(3)}$$

$$i \in \{\mathrm{DVCS}, \mathrm{INT}\}$$




$$s = M^2 + 2ME_b$$

- At higher beam energies the DVCS and INT cross sections gain ground against the BH background ("small-$y$ enhancement"), however the tw-3 part slowly gains ground on the tw-2 part at really large beam energies!

- N.B. it's not just about the $Q^2$ value, kinematically we always get contributions in the form: $\frac{-t}{Q^2}$ , $\frac{M^2 x_B^2}{Q^2}$

# Tw 3 GPDs and OAM of quarks

NPB 969 (2021) 115440

Jaffe Manohar Sum Rule: $$\frac{1}{2}\Delta q + \Delta G + l_q^z + l_g^z = \frac{1}{2}$$

canonical OAM $$l_q^z(x) = \int dy G_{q,D,3}(x,y) + \int dy \mathcal{P}\frac{1}{y-x} G_{q,F,3}(x,y)$$ partonic density is complicated!

$$J_q^z = \int dx \left( x G_{q,3}(x) - \frac{1}{2} g_1(x) \right)$$

Kinetic OAM: $$G_{q,3}(x,\xi,t) = \widetilde{E}_{2T}(x,\xi,t) - \xi E_{2T}(x,\xi,t)$$

much simpler looking

- In the sum rule we set $\xi = 0$, so we actually don't need $E_{2T}$, but we still can't get $E'_{2T}$ from DVCS alone

# Tw 3 DVCS Structure Functions

- It would appear *impossible* to completely isolate twist-3 GPDs (ahem….CFFs) in any DVCS observable. But there are 2 possible approaches to help with that task:

### 1. Charge-even DVCS Observables:

- Of the 8 possible polarization channels: UU, UL, LU, LL, UT in, UT out, LT in, LT out there are 4 in which pure DVCS component starts at sub-leading twist:

$$F_{LU}$$
$$F_{UL}$$
$$F_{UT,\mathrm{in}}$$
$$F_{LT,\mathrm{out}}$$

- Therefore, charge odd cross section sums: $d\sigma(e^-) + d\sigma(e^+)$ remove the interference component, leaving "twist-3" DVCS contributions isolated

- Here are all their structure functions:

$$F_{\rm LU}^{(3)} = -4h_{(3)}^{\rm L}{\rm Im}\Bigg[-\mathcal{E}^*\bar{\mathcal{H}}_{2T}+\mathcal{H}^*\bar{\mathcal{E}}_{2T}+\left(\mathcal{H}+\frac{t}{4M^2}\mathcal{E}\right)^* 2\bar{\tilde{\mathcal{H}}}_{2T}-\xi(\mathcal{H}+\mathcal{E})^*\bar{\tilde{\mathcal{E}}}_{2T}$$
$$+\xi\tilde{\mathcal{E}}^*\bar{\mathcal{H}}_{2T}-\xi\tilde{\mathcal{H}}^*\bar{\mathcal{E}}_{2T}+\tilde{\mathcal{H}}^*\bar{\tilde{\mathcal{E}}}_{2T}\Bigg],$$

$$F_{\rm UL}^{(3)} = 4h_{(3)}^{\prime\rm U}{\rm Im}\Bigg[-\xi\tilde{\mathcal{E}}^*\bar{\mathcal{H}}_{2T}+\left(\tilde{\mathcal{H}}-\frac{\xi^2}{1+\xi}\tilde{\mathcal{E}}\right)^*\bar{\mathcal{E}}_{2T}+2\left(\tilde{\mathcal{H}}+\xi\left(\frac{t}{4M^2}-\frac{\xi}{1+\xi}\right)\tilde{\mathcal{E}}\right)^*\bar{\tilde{\mathcal{H}}}_{2T}$$
$$-\xi\left(\tilde{\mathcal{H}}+\frac{\xi}{1+\xi}\tilde{\mathcal{E}}\right)^*\bar{\tilde{\mathcal{E}}}_{2T}+\mathcal{E}^*\bar{\mathcal{H}}_{2T}-\xi\left(\mathcal{H}+\frac{\xi}{1+\xi}\mathcal{E}\right)^*\bar{\mathcal{E}}_{2T}+\left(\mathcal{H}+\frac{\xi}{1+\xi}\mathcal{E}\right)^*\bar{\tilde{\mathcal{E}}}_{2T}\Bigg]$$

$$F_{\rm UT,in}^{(3)} = \frac{4}{N}h_{(3)}^{\prime\rm U}{\rm Im}\Bigg[2\left((\xi^2-1)\tilde{\mathcal{H}}+\xi^2\tilde{\mathcal{E}}\right)^*\bar{\mathcal{H}}_{2T}+2\xi\left(\xi\tilde{\mathcal{H}}+\left(\frac{\xi^2}{1+\xi}+\frac{t}{4M^2}\right)\tilde{\mathcal{E}}\right)^*\bar{\mathcal{E}}_{2T}$$
$$-N^2\left(\tilde{\mathcal{H}}+\frac{\xi}{1+\xi}\tilde{\mathcal{E}}\right)^*\bar{\tilde{\mathcal{H}}}_{2T}-2\xi\left(\tilde{\mathcal{H}}-\xi\left(\frac{\xi}{1+\xi}-\frac{t}{4M^2}\right)\tilde{\mathcal{E}}\right)^*\bar{\tilde{\mathcal{E}}}_{2T}$$
$$+2\left((\xi^2-1)\mathcal{H}+\xi^2\mathcal{E}\right)^*\bar{\mathcal{H}}_{2T}+2\xi\left(\xi\mathcal{H}+\left(\frac{\xi^2}{1+\xi}+\frac{t}{4M^2}\right)\mathcal{E}\right)^*\bar{\mathcal{E}}_{2T}$$
$$-2\left(\xi\mathcal{H}+\left(\frac{\xi^2}{1+\xi}+\frac{t}{4M^2}\right)\mathcal{E}\right)^*\bar{\tilde{\mathcal{E}}}_{2T}\Bigg],$$

$$F_{\rm LT,out}^{(3)} = \frac{4}{N}h_{(3)}^{\rm L}{\rm Re}\Bigg[2\left((\xi^2-1)\mathcal{H}+\xi^2\mathcal{E}\right)^*\bar{\mathcal{H}}_{2T}+2\left(\xi^2\mathcal{H}+\left(\xi^2+\frac{t}{4M^2}\right)\mathcal{E}\right)^*\bar{\mathcal{E}}_{2T}$$
$$-N^2(\mathcal{H}+\mathcal{E})^*\bar{\tilde{\mathcal{H}}}_{2T}-2\xi\left(\mathcal{H}+\frac{t}{4M^2}\mathcal{E}\right)^*\bar{\tilde{\mathcal{E}}}_{2T}$$
$$+2\left((\xi^2-1)\tilde{\mathcal{H}}+\xi^2\tilde{\mathcal{E}}\right)^*\bar{\mathcal{H}}_{2T}+2\xi^2\left(\tilde{\mathcal{H}}+\frac{t}{4M^2}\tilde{\mathcal{E}}\right)^*\bar{\mathcal{E}}_{2T}$$
$$-2\xi\left(\tilde{\mathcal{H}}+\frac{t}{4M^2}\tilde{\mathcal{E}}\right)^*\bar{\tilde{\mathcal{E}}}_{2T}\Bigg],$$

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- As you can see, **you still need to input the twist-2 CFFs** first, and there is no way around that…

- But at least your measurements aren't lost in the noise of a dominant twist-2 signal — your signal is essentially coming from these twist-3 quantities

- Measuring the 4 observables places enough constraints to get all 4 tw-3 quark CFFs $\bar{H}_{2T}, \bar{H}'_{2T}, \bar{E}_{2T}, \bar{E}'_{2T}$, at some kinematical point

## 2. Charge-odd (Interference) Observables

- Opposite to charge-even, these remove the pure DVCS (and BH) and isolate the Interference structure functions

- The twist-3 contributions are then proportional to elastic **FFs $\times$ tw-3 CFFs**

$$F^{\mathrm{I}}_{\mathrm{UU},(3)} = -\mathrm{Re}\Bigg\{ A^{\mathrm{I,U}}_{(3)} \left[ F_1 (\bar{\mathcal{E}}_{2T} + 2\bar{\tilde{\mathcal{H}}}_{2T})^* - F_2 \left( \bar{\mathcal{H}}_{2T} - \frac{t}{4M^2} 2\bar{\tilde{\mathcal{H}}}_{2T} \right)^* \right]$$
$$+ B^{\mathrm{I,U}}_{(3)} (F_1 + F_2) \bar{\tilde{\mathcal{E}}}^*_{2T} + C^{\mathrm{I,U}}_{(3)} (F_1 + F_2) \left[ \xi \bar{\mathcal{H}}_{2T} + \frac{t}{4M^2} \left( \xi \bar{\mathcal{E}}_{2T} - \bar{\tilde{\mathcal{E}}}_{2T} \right) \right]^* \Bigg\},$$

$$F^{\mathrm{I}}_{\mathrm{LU},(3)} = \mathrm{Im}\Bigg\{ A^{\mathrm{I,L}}_{(3)} \left[ F_1 (\bar{\mathcal{E}}_{2T} + 2\bar{\tilde{\mathcal{H}}}_{2T})^* - F_2 \left( \bar{\mathcal{H}}_{2T} - \frac{t}{4M^2} 2\bar{\tilde{\mathcal{H}}}_{2T} \right)^* \right]$$
$$+ B^{\mathrm{I,L}}_{(3)} (F_1 + F_2) \bar{\tilde{\mathcal{E}}}^*_{2T} + C^{\mathrm{I,L}}_{(3)} (F_1 + F_2) \left[ \xi \bar{\mathcal{H}}_{2T} + \frac{t}{4M^2} \left( \xi \bar{\mathcal{E}}_{2T} - \bar{\tilde{\mathcal{E}}}_{2T} \right) \right]^* \Bigg\},$$

$$\vdots$$

- But now you have dominant twist-2 signal added to these, and it needs to be subtracted out, but any subset of the 4 polarization channels can be applied

# Closing Remarks

- For now the focus needs to be on better-constraining the twist-2 GPDs
- This is best achieved with more lattice GFFs, especially isoscalar $u + d$
- As well as more DVCS polarization observables with controlled tw-3 contamination (ideal kinematics)
- Once we get a better handle on the twist-2 GPDs, we can better study twist-3 quantities
- It is IMPOSSIBLE to isolate quark OAM from DVCS alone: need other observables (DDVCS perhaps)
- It is also seemingly impossible to extract twist-3 GPDs from DVCS without having a knowledge of twist-2 CFFs

# Back Up Material

## Quark GPDs

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x} \left\langle P', S' \left| \bar{\psi}\left(-\frac{\lambda n}{2}\right) W_{-\frac{\lambda}{2},\frac{\lambda}{2}} \Gamma \psi\left(\frac{\lambda n}{2}\right) \right| P, S \right\rangle = \bar{u}(P', S') \mathcal{F}_{q,\Gamma}(x, \bar{P}, \Delta, n) u(P, S)$$

$$\mathcal{F}_{q,\gamma^+} = H_q(x,t,\xi)\gamma^+ + E_q(x,t,\xi)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}$$

$$\mathcal{F}_{q,\gamma^\perp} = \frac{\Delta^\perp}{M} G_{q,1}(x,t,\xi) + \Delta^\perp \not{n} G_{q,2}(x,t,\xi) + \frac{i\sigma^{\perp\rho}\Delta_\rho}{2M} G_{q,3}(x,t,\xi) + M i\sigma^{\perp\rho} n_\rho G_{q,4}(x,t,\xi) \ ,$$

# Other kinematical effects