Generalized Parton Distributions through Universal Moment Parameterization (GUMP): Towards global analysis at non zero skewness

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Outline

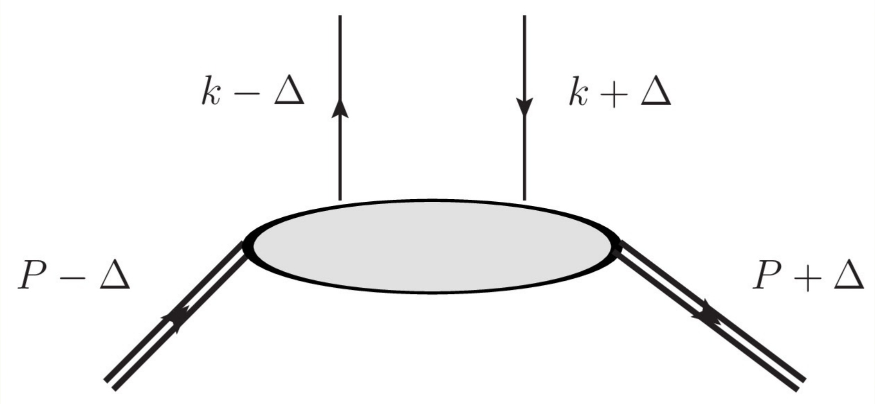
- GPD Review
- GUMP Program
  - Conformal moment parameterization
- First Step Towards Global Analysis: u and d quarks
  - Simplified GPD moment ansatz
  - Experimental and lattice input
- Non-zero Skewness Global Fit
- GPD Extraction
  - Ambiguity in the ERBL/DA-like region
  - D-terms vs DA-like terms
- Moving Forward
- Conclusions

GPDs

- GPDs generalize the well known PDFs to encode full 3 dimensional information on the quarks and gluons within hadrons

$$f(x) \to F(x, \xi, t)$$

$$x \sim \text{parton momentum fraction},\ \xi \sim \text{longitudinal momentum transfer},$$
$$t = \Delta^2 \sim \text{momentum transfer squared}$$



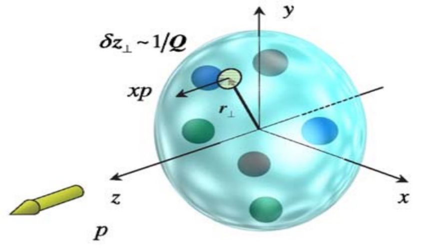
GPDs

- Polarization of the hadron and its parton constituents connects GPDs to the distribution of angular momentum within hadrons (*X. Ji* 1997)
  - Ji sum rule

$$J_i = \frac{1}{2}\int_0^1 \mathrm{d}x\, x\,[H_i(x,\xi) + E_i(x,\xi)]$$

- Related via a Fourier transform to the impact parameter distribution of partons (*M. Burkardt* 2003)

$$\rho(x, r_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot r_\perp} H(x, 0, \Delta_\perp^2)$$



- Related to bulk properties of hadron states encoded in form factors

$$\int \mathrm{d}x\, xH_i(x,\xi,t) = A_i(t) + (2\xi)^2 C_i(t), \quad \int \mathrm{d}x\, xE_i(x,\xi,t) = B_i(t) - (2\xi)^2 C_i(t)$$

GUMP program: Moment Parameterization

- Parameterize GPDs by directly parameterizing their conformal moments

$$F_i(x,\xi,t) = \sum_{j=0}^{\infty} (-1)^j p_{i,j}(x,\xi) \mathcal{F}_{i,j}(\xi,t)$$

(D. Mueller and A. Schafer 2006)

- Expansion based on eigenfunctions of evolution – Gegenbauer polynomials

$$(-1)^j p_j(x,\xi) = \xi^{-j-1} \frac{2^j \Gamma(\frac{5}{2}+j)}{\Gamma(\frac{3}{2})\Gamma(j+3)} \left[1 - \left(\frac{x}{\xi}\right)^2\right] C_j^{3/2}\left(\frac{x}{\xi}\right)$$

conformal wave function

$$\int_{-1}^{1} \frac{\mathrm{d}x'}{|\xi|} \mathcal{K}\left(\frac{x}{\xi}, \frac{x'}{\xi}\right) C_j^{3/2}\left(\frac{x}{\xi}\right) = \gamma_j C_j^{3/2}\left(\frac{x}{\xi}\right)$$

GPD evolution kernel

GUMP program: Moment Parameterization

- Conformal moment parameterization has nice features for fitting GPDs
- Simple evolution implementation – conformal moments are multiplicatively renormalized at LO
  - Follows from using eigenfunctions of evolution kernel
- Polynomiality condition (*X. Ji* 1998) automatically enforced on conformal moments

$$F_{i,n}(\xi,t)=\int_{-1}^{1}\mathrm{d}x\,x^{n-1}F(x,\xi,t)=\sum_{k=0,\ \mathrm{even}}^{n}\xi^k F_{i,n,k}(t)$$

$$\downarrow$$

$$\mathcal{F}_{i,j}(\xi,t)=\sum_{k=0,\ \mathrm{even}}^{j+1}\xi^k \mathcal{F}_{i,j,k}(t)$$

First Step Toward Global GPD Analysis

- Apply in GUMP program for global analysis of u and d quark GPDs at non-zero skewness with LO scale evolution
- Parameterize each GPD moment with five parameters

$$F_{i,j,0} = N_i B(j+1-\alpha_i, 1+\beta_i) \frac{j+1-\alpha_i}{j+1-\alpha_i(t)} \beta(t)$$

Euler Beta Function

Regge trajectory

$$\beta(t) = e^{-b|t|}$$

$$\alpha(t) = \alpha + \alpha' t$$

- Take each moment to be a power series in skewness – polynomiality condition

$$F_{i,j} = F_{i,j,0}(t) + \xi^2 R_{\xi^2} F_{i,j,0}(t) + \xi^4 R_{\xi^4} F_{i,j,0}(t)...$$

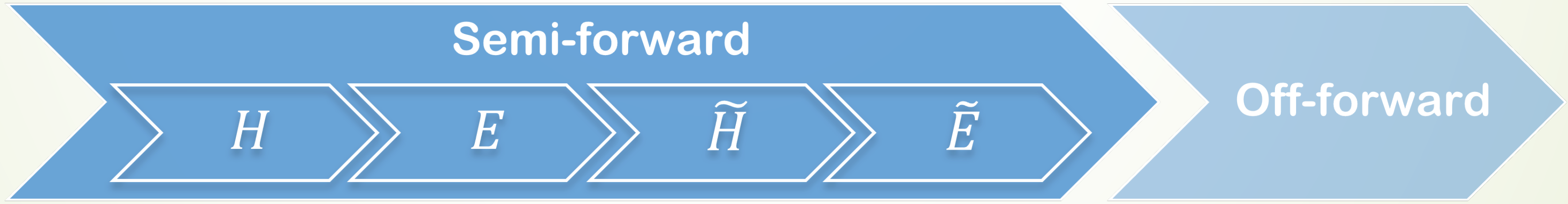
First Step Toward Global GPD Analysis

- The number of parameters needed for modelling all the species of GPD grows very quickly
- We impose extra constraints for simplicity

GPDs species and flavors	Fully parameterized	GPDs linked to	Proportional constants
H_{u_V} and $\widetilde{H}_{u_V}$	✔	-	-
E_{u_V} and $\widetilde{E}_{u_V}$	✔	-	-
H_{d_V} and $\widetilde{H}_{d_V}$	✔	-	-
E_{d_V} and $\widetilde{E}_{d_V}$	✘	E_{u_V} and $\widetilde{E}_{u_V}$	$R_{d_V}^{E/\widetilde{E}}$
$H_{\bar{u}}$ and $\widetilde{H}_{\bar{u}}$	✔	-	-
$E_{\bar{u}}$ and $\widetilde{E}_{\bar{u}}$	✘	$H_{\bar{u}}$ and $\widetilde{H}_{\bar{u}}$	$R_{\mathrm{sea}}^{E/\widetilde{E}}$
$H_{\bar{d}}$ and $\widetilde{H}_{\bar{d}}$	✔	-	-
$E_{\bar{d}}$ and $\widetilde{E}_{\bar{d}}$	✘	$H_{\bar{d}}$ and $\widetilde{H}_{\bar{d}}$	$R_{\mathrm{sea}}^{E/\widetilde{E}}$
H_g and $\widetilde{H}_g$	✔	-	-
E_g and $\widetilde{E}_g$	✘	H_g and $\widetilde{H}_g$	$R_{\mathrm{sea}}^{E/\widetilde{E}}$

Non-zero Skewness Global Fit

- Even with constraints, lots of parameters!
  - Very high dimensional space to navigate for best fit
  - Very computationally demanding to do error propagation
- We employ a sequential fit, starting with forward (PDF, t-dependent PDF) constraints for each GPD species then apply the off-forward constraints from DVCS data



Semi-Forward Inputs

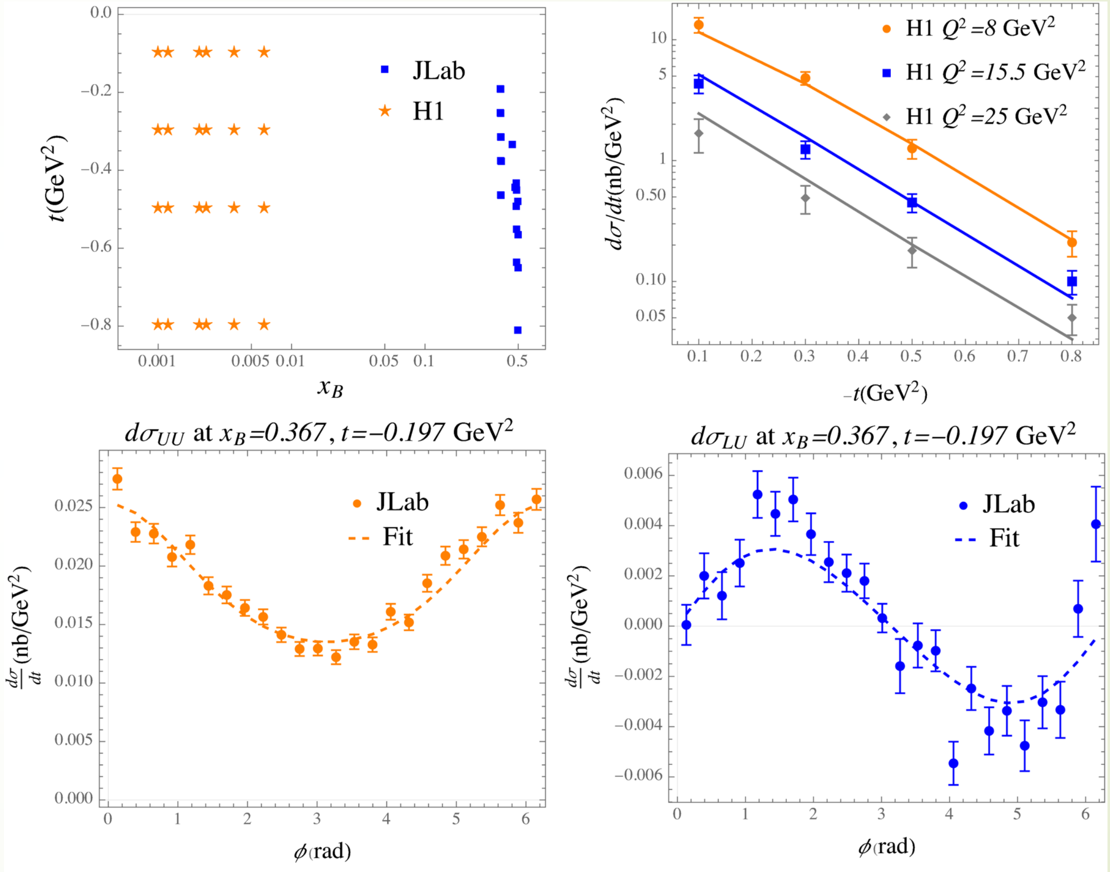
- JAM (2022) PDF global analysis results
  - Full global analysis should in principle fit to PDF sensitive data directly, but here we fit to JAM results
  - Limited number of points taken to avoid need for more sophisticated forward limit
- Globally extracted electromagnetic form factors (*Z. Ye et al* 2018)
- Lattice GPDs (*Alexandrou et al* 2020) and form factors (*Alexandrou et al* 2022)
  - x, t -dependent GPDs (semi-forward limit)

Off-Forward Inputs

- DVCS measurements from JLab (*CLAS* 2019 & 2021, *Hall A* 2018 & 2022) and HERA (*H1* 2010)
- Only using t-dependent cross sections due to practical limitations
- Far more points from JLab data than from HERA from φ-dependence and both UU and LU polarization channels
- Off-forward lattice GPDs not used in fitting, but can supply crucial constraints for future work!

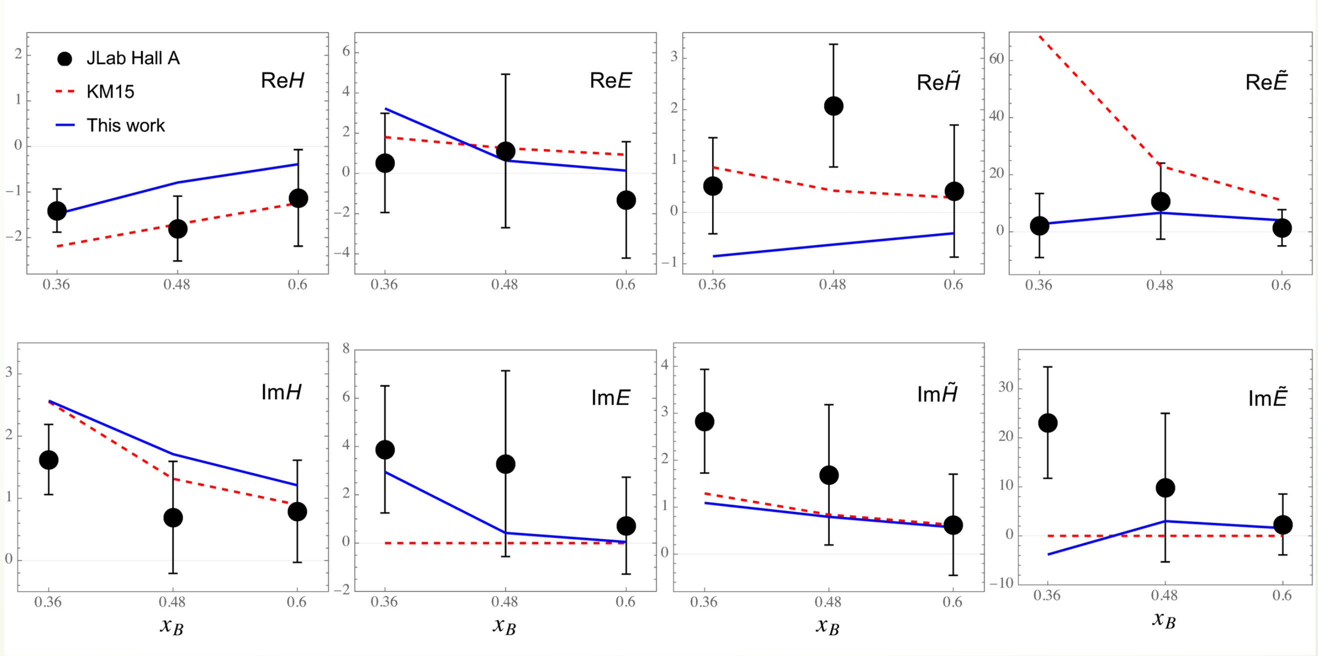
Non-zero Skewness Global Fit

- Total χ^2/dof is approximately 1.4
- Some agreement with both JLAB and H1 data
- Gluon GPDs not well constrained at non-zero skewness
  - Only contribute to DVCS through evolution at LO
- Error propagation is not yet implemented
  - Very computationally expensive with so many parameters!



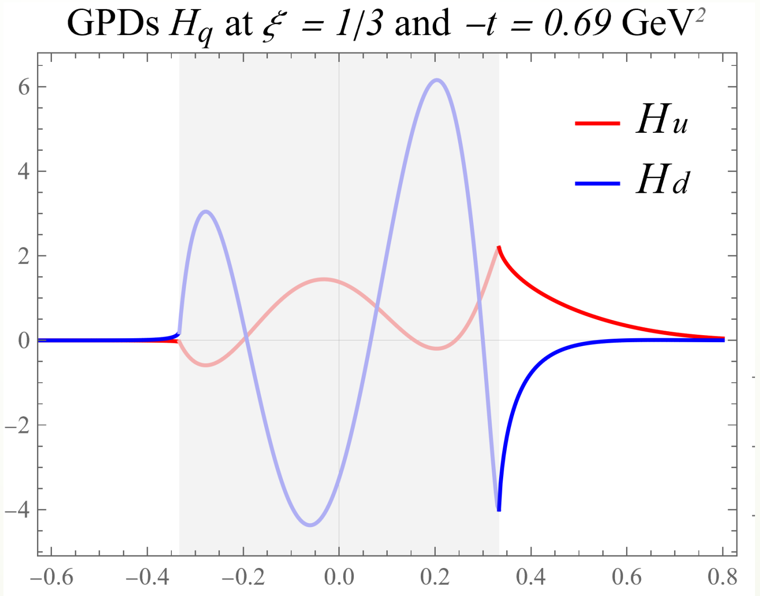
Non-zero Skewness Global Fit

- CFFs from fit are mostly consistent with local extraction from JLAB Hall A data as well as KM15 extractions
- Some inconsistencies can be expected from degeneracies in CFF contribution to cross sections – need more polarization configurations!



Extracted GPDs

- GPDs are mostly constrained on the $\xi = x$ line and in the DGLAP region $|\xi| < |x|$
- ERBL region shows large oscillations which are characteristic of the Gegenbauer polynomials used in the moment expansion



Ambiguity in ERBL Region

- We can add terms in the moment expansion which only contribute to the ERBL region

$$(-1)^j p_j(x,\xi) = \xi^{-j-1} \frac{2^j \Gamma(\frac{5}{2}+j)}{\Gamma(\frac{3}{2})\Gamma(j+3)} \left[1 - \left(\frac{x}{\xi}\right)^2\right] C_j^{3/2}\left(\frac{x}{\xi}\right), \quad |x| < |\xi|$$

- This suggests an interpretation of the GPDs in terms of quark and antiquark pieces as well as a ERBL region distribution amplitude (DA) piece

$$F_q(x,\xi,t) = F_{\hat{q}}(x,\xi,t) \mp F_{\bar{q}}(-x,\xi,t) + F_{q\bar{q}}$$

quark $x > -\xi$

antiquark $-x > -\xi$

DA $\xi > x > -\xi$

Connection to D-term

- These DA terms don't have a large affect on CFFs, but they do contain information related to the various D-terms in QCD, ex.
  - Gravitational form factor C/D

$$\int_{-1}^{1} \mathrm{d}x\, x H_q(x,\xi,t) = A_q(t) + (2\xi)^2 C_q(t)$$

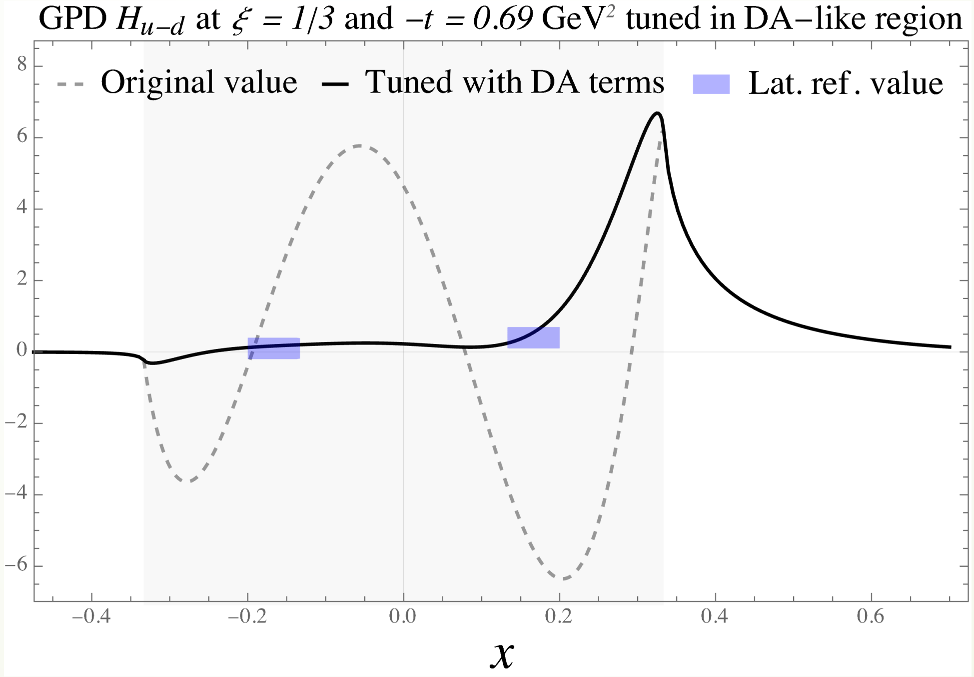
  - Dispersion relation subtraction term

$$F(\xi,t,Q^2) = \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \left(\frac{1}{\xi-\xi'} \mp \frac{1}{\xi+\xi'}\right) \mathrm{Im}\left[F(\xi'-i0,t,Q^2)\right] + \mathcal{C}(t,Q^2)$$

- By constraining the DA terms with further experimental data and lattice calculations, we can access the mechanical properties of hadrons contained in these D-terms!

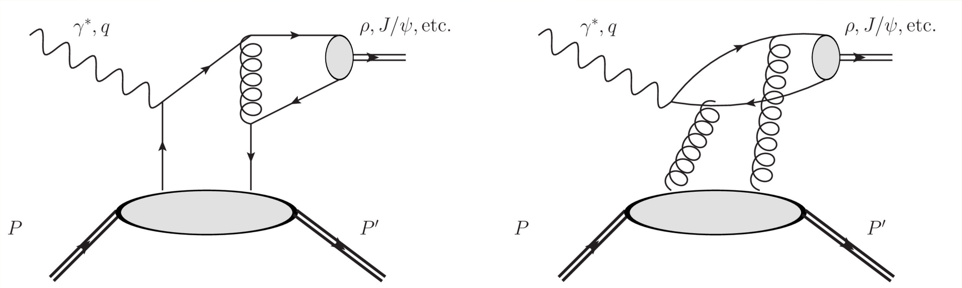
Constraining DA Terms

- Adding in lattice GPD calculations can give us constrains directly in the ERBL region
- Adding just a few terms to the moment expansion can remove the unphysical oscillations



Moving Forward: Adding in Gluons!

- DVCS at LO is only sensitive to gluon GPDs through scale evolution
- Using Deeply Virtual Meson Production (DVMP) gives a direct probe of gluons at LO



- Light vector mesons have similar sensitivity to quarks and gluons
  - KM framework applied to produce simultaneous fits of DVCS and DVMP for ρ^0 meson production with data from HERA *(see Marija's talk on Wednesday)*
- Add heavy vector meson to obtain better constraints on gluon GPDs – use J/ψ production!

Deeply Virtual J/ψ Production

- Charm quark contribution for nucleon target is negligible – direct probe of gluons
- Complementary with GUMP work on quark GPDs, but mostly sensitive to small-x_B region whereas JLab data combined with HERA gives better constraint at moderate x_B
- Caveat: mass of the J/ψ is too large for usual collinear factorization

$$M_{J/\psi}^2/Q_{\text{max bin}}^2 \approx 9/20 \rightarrow \text{corrections of order } 1/2$$

- Need to take heavy mass corrections into account – non-relativistic (NR) QCD!

Non-relativistic model approach

- Encoding the J/ψ formation into NR matrix elements

$$\Gamma[J/\psi \to l^+l^-] \approx \frac{2e_c^2\pi\alpha_{EM}^2}{3m_c^2}\langle\mathcal{O}_1\rangle_{J/\psi}\left(1-\frac{8\alpha_s}{3\pi}\right)^2$$

- Maintain the form of the factorization theorem for the process – still sensitive to leading twist GPDs (*D. Y. Ivanov et al* 2004)

$$\mathcal{M} = \left(\frac{\langle\mathcal{O}_1\rangle_V}{m_c}\right)^{1/2}\sum_i F_i(x,\xi,t)\otimes_x H_i(x,\xi)$$

- Systematically improvable with relativistic, higher twist, and NLO QCD corrections
- Bridge between electroproduction and photoproduction regimes

Implementing NR J/ψ Production in GUMP

- LO framework used for previous global fit does not match data in HERA kinematics
- NLO evolution is known in moment space (*Mueller et al* 2013)
- NLO NR matrix element for J/ψ formation is known (*D. Y. Ivanov et al* 2004)
- Finite mass corrections for hard scattering are only known in momentum fraction space
  - Mass corrections make the convolutions for converting to conformal moment space much more complicated
- Adaptive numerical complex integral to construct GPD from moments is computationally expensive

Implementing NR J/ψ Production in GUMP

- Implemented NLO evolution
  - Corrections are large enough to fix scaling problems from LO!
- Switching over to a fixed order integral evaluation allows significant reduction in computation time with < 1% extra numerical error
  - Terms in the integral can now be memorized and only calculated a few times (as in Gepard)
- Conversion of NLO finite mass hard scattering terms to moment space is on going

Future Improvements/Additions

- Implement J/ψ electroproduction fits with NLO
- Add threshold J/ψ production – potentially constrain D-term/DA-terms
- Add quark flavors and implement ρ^0 and ϕ electroproduction
- Full simultaneous global analysis with DVCS and DVMP contributions
- Implement t-integrated cross sections – speed up for NLO could make t-integrated cross sections practical

Conclusions

- Global fit combining experimental data and lattice calculations to constrain GPDs at non-zero skewness
- Developing the GUMP program to include gluon GPDs in global analysis through J/ψ production data
- Implementing NLO corrections
- Several directions for future improvements available

Backup Slides

Best Fit χ^2 Breakdown

Sub-fits	χ^2	N_{data}	$\chi^2_\nu \equiv \chi^2/\nu$
Semi-forward			
tPDF H	281.7	217	1.41
tPDF E	59.7	50	1.36
tPDF $\widetilde{H}$	159.3	206	0.84
tPDF $\widetilde{E}$	63.8	58	1.23
Off-forward			
JLab DVCS	1413.7	926	~ 1.53
H1 DVCS	19.7	24	~ 0.82
Off-forward total	1433	950	**1.53**
Total	2042	1481	**1.40**

Best Fit Parameters

Vector GPDs H and E		Axial-vector GPDs $\widetilde{H}$ and $\widetilde{E}$	
Parameter	Value (uncertainty)	Parameter	Value (uncertainty)
$N^H_{u_V}$	4.923 (89)	$N^{\widetilde{H}}_{u_V}$	4.833 (429)
$\alpha^H_{u_V}$	0.216 (7)	$\alpha^{\widetilde{H}}_{u_V}$	-0.264 (34)
$\beta^H_{u_V}$	3.229 (23)	$\beta^{\widetilde{H}}_{u_V}$	3.186 (122)
$\alpha'^H_{u_V}$	2.347 (51)	$\alpha'^{\widetilde{H}}_{u_V}$	2.182 (175)
$N^H_{\bar{u}}$	0.163 (8)	$N^{\widetilde{H}}_{\bar{u}}$	0.070 (33)
$\alpha^H_{\bar{u}}$	1.136 (10)	$\alpha^{\widetilde{H}}_{\bar{u}}$	0.538 (112)
$\beta^H_{\bar{u}}$	6.894 (207)	$\beta^{\widetilde{H}}_{\bar{u}}$	4.229 (1320)
$N^H_{d_V}$	3.359 (170)	$N^{\widetilde{H}}_{d_V}$	-0.664 (170)
$\alpha^H_{d_V}$	0.184 (18)	$\alpha^{\widetilde{H}}_{d_V}$	0.248 (76)
$\beta^H_{d_V}$	4.418 (77)	$\beta^{\widetilde{H}}_{d_V}$	3.572 (477)
$\alpha'^H_{d_V}$	3.482 (171)	$\alpha'^{\widetilde{H}}_{d_V}$	0.542 (103)
$N^H_{\bar{d}}$	0.249 (12)	$N^{\widetilde{H}}_{\bar{d}}$	-0.086 (42)
$\alpha^H_{\bar{d}}$	1.052 (10)	$\alpha^{\widetilde{H}}_{\bar{d}}$	0.495 (137)
$\beta^H_{\bar{d}}$	6.554 (216)	$\beta^{\widetilde{H}}_{\bar{d}}$	2.554 (897)
N^H_g	2.864 (108)	$N^{\widetilde{H}}_g$	0.243 (304)
α^H_g	1.052 (8)	$\alpha^{\widetilde{H}}_g$	0.631 (330)
β^H_g	7.413 (165)	$\beta^{\widetilde{H}}_g$	2.717 (2865)
$N^E_{u_V}$	0.181 (38)	$N^{\widetilde{E}}_{u_V}$	7.993 (3480)
$\alpha^E_{u_V}$	0.907 (17)	$\alpha^{\widetilde{E}}_{u_V}$	0.800 (116)
$\beta^E_{u_V}$	1.102 (245)	$\beta^{\widetilde{E}}_{u_V}$	6.415 (1577)
$\alpha'^E_{u_V}$	0.461 (86)	$\alpha'^{\widetilde{E}}_{u_V}$	2.076 (933)
$N^E_{d_V}$	-0.223 (47)	$N^{\widetilde{E}}_{d_V}$	-2.407 (1239)
R^E_{sea}	0.768 (169)	$R^{\widetilde{E}}_{\mathrm{sea}}$	38 (8)
$R^H_{u,2}$	0.229 (0.032)	$R^{\widetilde{H}}_{u,2}$	0.246 (81)
$R^H_{d,2}$	-2.639 (202)	$R^{\widetilde{H}}_{d,2}$	1.656 (375)
$R^E_{u,2}$	0.799 (285)	$R^{\widetilde{E}}_{u,2}$	2.684 (171)
$R^E_{d,2}$	3.404 (1157)	$R^{\widetilde{E}}_{d,2}$	38 (2)
b^H_{sea}	3.448 (133)	$b^{\widetilde{H}}_{\mathrm{sea}}$	9.852 (1330)