Aspects of the modeling of GPDs at small skewness

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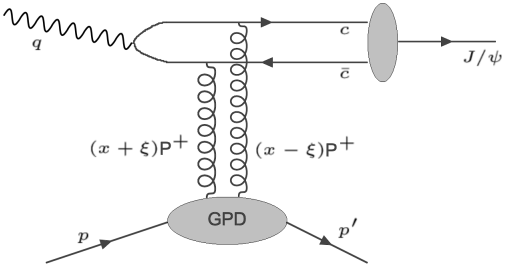
Vector meson production



LO depiction of J/ψ photoproduction. ξ is the **skewness** measuring the transfer of plus-momentum to the hadron. x is the average plus-momentum of the active parton.

Transfer of four-momentum to the hadron → description in the framework of collinear factorization by **generalized parton distributions (GPDs)** and **non-relativistic QCD matrix element** for moderate or small photon virtuality $Q^2 = -q^2$. Hard scale provided by $m_V/2$ **[Jones et al, 2015]**.

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} \approx \frac{x_B}{2}, \quad t = (p' - p)^2$$

Vector meson production

- Vector meson production amplitude up to NLO [Ivanov et al, 2004]:

$$\mathcal{F}(\xi,t) \propto \left(\frac{\langle O_1\rangle_V}{m_V^3}\right)^{1/2} \sum_{a=q,g} \int_{-1}^{1} \mathrm{d}x \, T^a(x,\xi) \, F^a(x,\xi,t) \tag{1}$$

  where $\langle O_1\rangle_V^{1/2}$ is the NR QCD matrix element, T a hard-scattering kernel and $F(x,\xi,t)$ is the GPD.

- The dominant region controlling the imaginary part of the amplitude is:

$$x \approx \xi \approx \frac{x_B}{2} \approx e^{-y} \frac{m_V}{2\sqrt{s}} \tag{2}$$

- At LHCb kinematics *e.g.*, typical values of x_B as low as $\sim 10^{-5}$.

Motivations

- Use this data to probe the gluon PDF at very small x, poorly constrained region with prospects of gluon saturation physics.
- The limit $\xi \to 0$ of GPDs is crucial for **hadron tomography**:

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2) \quad (3)$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron

- What role does this data play with respect to the **deconvolution problem** of reconstructing the full (x, ξ) dependence of GPDs?

GPDs at small skewness

Properties of GPDs **[Müller et al, 1994]**, **[Radyushkin, 1996]** and **[Ji, 1997]**

- The **forward limit** $t \to 0$ – and consequently $\xi \to 0$ – gives back the usual PDFs:

$$H^q(x, \xi = 0, t = 0) = f^q(x) \tag{4}$$
$$H^g(x, \xi = 0, t = 0) = x f^g(x) \tag{5}$$

- Assuming the t-dependence can be parametrized and evacuated from the picture (not obvious!), when $x \gg \xi$, negligible asymmetry between incoming $(x - \xi)$ and outgoing $(x + \xi)$ parton longitudinal momentum fraction → **smooth limit of GPDs**:

$$H(x, \xi, t) \approx H(x, 0, t) \quad \text{for } x \gg \xi \,. \tag{6}$$

Since $\xi \sim 10^{-5}$ at LHCb, one could be tempted to write **GPD = PDF** at small ξ, such as

$$H^g(\xi, \xi = 10^{-5}) \approx H^g(\xi, 0) = \xi f^g(\xi) \tag{7}$$

But there is a problem...

Evolution of GPDs

GPD's dependence on scale is given by **renormalization group equations**.

- In the limit $\xi = 0$, usual DGLAP equation:

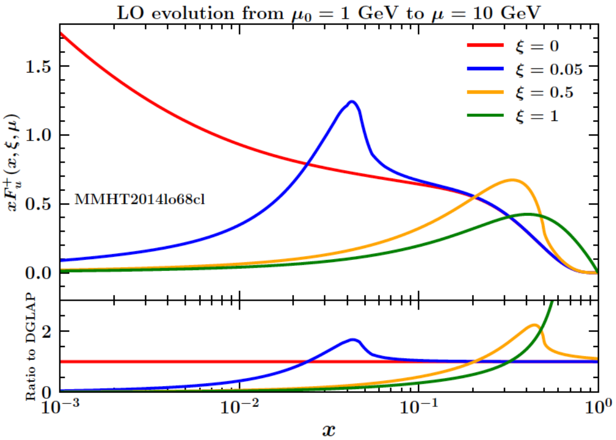
$$\frac{\mathrm{d} f^{q+}}{\mathrm{d}\log\mu}(x,\mu) = \frac{C_F \alpha_s(\mu)}{\pi} \left\{ \int_x^1 \mathrm{d}y \, \frac{f^{q+}(y,\mu) - f^{q+}(x,\mu)}{y-x} \left[1 + \frac{x^2}{y^2}\right] + f^{q+}(x,\mu) \left[\frac{1}{2} + x + \log\left(\frac{(1-x)^2}{x} \right) \right] \right\} \quad (8)$$

- But in the limit $x = \xi$:

$$\frac{\mathrm{d} H^{q+}}{\mathrm{d}\log\mu}(x,x,\mu) = \frac{C_F \alpha_s(\mu)}{\pi} \left\{ \int_x^1 \mathrm{d}y \, \frac{H^{q+}(y,x,\mu) - H^{q+}(x,x,\mu)}{y-x} + H^{q+}(x,x,\mu) \left[\frac{3}{2} + \log\left(\frac{1-x}{2x} \right) \right] \right\} \quad (9)$$

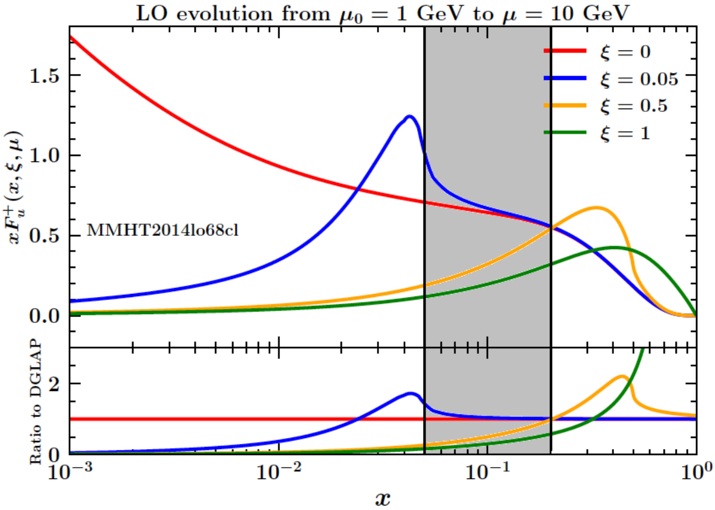
Assuming that GPD = PDF at small ξ and $x \approx \xi$ is incompatible with evolution, which generates an intrinsic ξ dependence!

Evolution of GPDs



Evolution of a PDF with GPD LO evolution for various values of ξ. From **[Bertone et al, 2022]**

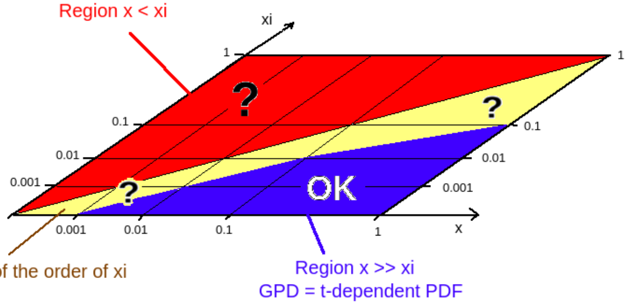
Evolution of GPDs



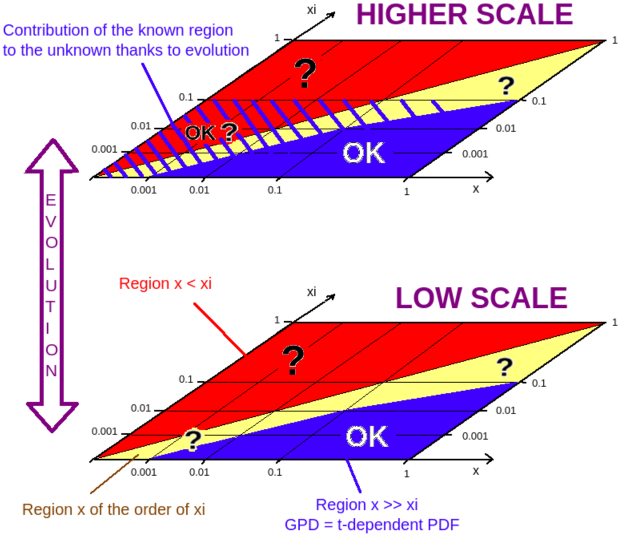
GPDs at small skewness

- Significant asymmetry between incoming and outgoing ($x + \xi \gg x - \xi$) parton momentum means very different dynamics, materialized *e.g.* by a very different behavior under evolution.

No reason for the ξ dependence to be negligible even at very small ξ. Skewness ratios $\frac{H(x,x)}{H(x,0)}$ as large as 1.6 have been advocated at small x. **[Frankfurt et al, 1998] [Shuvaev et al, 1999]**



GPDs at small skewness



- Evolution displaces the GPD from the large x to the small x region
- Significant ξ dependence arises perturbatively in the small x and ξ region
- But how does it compare to the unknown ξ dependence at initial scale?

Obviously depends on the range of evolution, value of x and ξ, and profile of the known t-dependent PDF.

GPDs at small skewness

- The generic idea that the perturbative ξ dependence at higher scale exceeds the initial unknown ξ dependence is the basis of the "Shuvaev transform" modeling of GPDs **[Shuvaev et al, 1999]** applied to HERA and LHC data (see *e.g.* **[Jones et al, 2013]**, **[Flett et al, 2020]**)
- **Shuveav model** conformal moments of GPDs:

$$O_n^q(\xi,\mu) = \xi^n \int_{-1}^{1} \mathrm{d}x \, C_n^{3/2}\left(\frac{x}{\xi}\right) H^q(x,\xi,\mu) \tag{10}$$

The GPD is reconstructed from its conformal moments by the Shuveav operator:

$$H^a(x,\xi,\mu) = S^a(x,\xi,n) \star O_n^a(\xi,\mu) \tag{11}$$

The model approximates the conformal moments by their limit when $\xi = 0$:

$$H^a_{Shuvaev}(x,\xi,\mu) = S^a(x,\xi,n) \star O_n^a(\xi = 0,\mu) \tag{12}$$

GPDs at small skewness

- Shuvaev model:
$$H^a_{Shuvaev}(x, \xi, \mu) = S^a(x, \xi, n) \star O^a_n(\xi = 0, \mu) \tag{13}$$
- Because the LO anomalous dimensions of GPDs are the same as those of PDFs (independent of ξ), this modelling of the ξ dependence is compatible with LO evolution!
- The entire ξ dependence arises from the Shuvaev operator, and the "intrinsic" ξ dependence of the conformal moments is forgotten $\rightarrow$ **assumption of complete dominance of the LO perturbative ξ dependence over any unknown initial ξ dependence**.
- But how good is that assumption? **The hard scale μ must be large enough** for perturbation effects to bring significant contribution, and ξ **must be small enough** for a region $x \gg \xi$ to exist which will control the evolution.

Evolution operators

- LO GPD RGE:

$$\frac{1}{|x^{p_a}|}\frac{dH^q}{d\log\mu}(x,\xi,\mu) = \alpha_s(\mu)\int_0^1 \frac{\mathrm{d}z}{z}\, g^{ab}\left(\frac{z}{x},\frac{\xi}{x}\right)\frac{H^b(z,\xi,t,\mu)}{|z^{p_a}|} \tag{14}$$

  where $p_a = 1$ for gluons $(a = g)$ and 0 for quarks $(a = q)$, and g are the LO GPD splitting functions.

- Solving the LO RGE yields the leading log (LL) evolution operator:

$$\frac{1}{|x^{p_a}|}H^q(x,\xi,\mu) = \int_0^1 \frac{\mathrm{d}z}{z}\,\Gamma^{ab}\left(\frac{z}{x},\frac{\xi}{x};\mu_0,\mu\right)\frac{H^b(z,\xi,t,\mu_0)}{|z^{p_a}|} \tag{15}$$

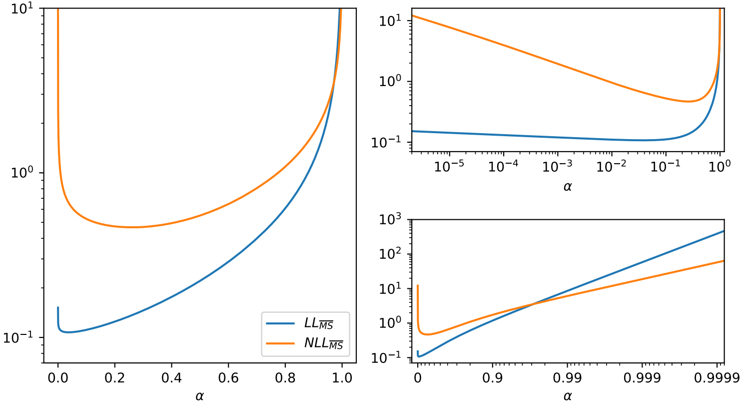
  The LL operator admits a Taylor expansion:

$$\Gamma^{ab}(\alpha,\beta;\mu_0,\mu) = \delta(1-\alpha)+\beta_0\alpha_s(\mu_0)\log\left(\frac{\mu}{\mu_0}\right)g^{ab}(\alpha,\beta)+\beta_0^2\alpha_s^2(\mu_0)\log^2\left(\frac{\mu}{\mu_0}\right)\ldots \tag{16}$$

- Infinite sum of convolutions of the splitting functions g^{ab}, which are distributions (plus-prescriptions), **yet the LL operator is an ordinary function!**

Evolution operators

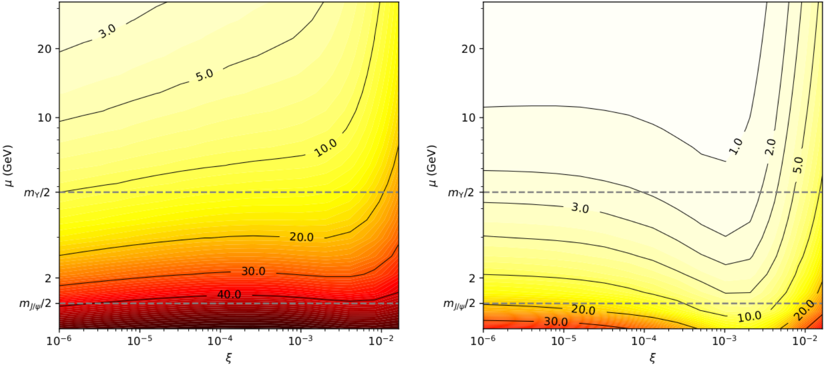
- Example with the limit $\xi \to 0$ (DGLAP LL operator). α is the ratio of final to initial momentum fraction. **small α = small momentum fraction parton radiated by the active parton under evolution**.



Evolution operators

- Since the evolution operator is a simple ordinary positive function, one can the **final scale GPD as a true reweighting of the initial GPD** $\rightarrow$ gives a formal sense to the idea of "displacing" the distribution.
- **Evaluation of the dominance of the perturbative ξ dependence over the initial unknown ξ dependence**:
  - Start from a PDF at a low initial scale $\mu_0 = 1$ GeV (need to be able to apply perturbation theory, so cannot go much below)
  - Produce an arbitrary ξ dependence at initial scale: pessimistic estimate – 60% of uncertainty on the diagonal $x = \xi$ vs. PDF
  - Evolve to higher scale and observe how the LO evolution of the true GPD and the LO evolution of the model GPD = PDF differ
  - Both converge at very large scale as the ξ dependence of evolution overwhelms the initial unknown ξ dependence

Example: working at $t = 0$, with the MMHT2014 PDF [Harland-Lang et al, 2015] at 1 GeV (**prior knowledge of t-dependent PDF**)



Uncertainty on the diagonal of the light sea quarks (left) and gluons (right) depending on $x = \xi$ and μ.
Stronger μ effect for gluons, divergence of PDFs at small x visible.

[HD, Winn, Bertone, 2023]

Dominance of the perturbative ξ dependence

- Strong effect of the hard scale: reduction of uncertainty on the ξ dependence from 60% at 1 GeV down to $10 \sim 20\%$ at $m_{J/\psi}/2$ and $\sim 3\%$ at $m_{\Upsilon}/2$ for the gluons.
- Larger uncertainty for singlet quarks: less radiation under evolution, so less effect of perturbative ξ dependence!
- No clear effect of the ξ dependence below 10^{-3}: as $x = \xi$ becomes smaller, the PDF becomes steeper, so the dominance of the region $x \gg \xi$ at initial scale becomes harder to establish. Trade-off between the fact that x becomes larger compared to ξ and the fact that the PDF becomes larger.
- Foreseeable improvements: use of higher order perturbative evolution. Resummation of $\log(x)$ powers in the anomalous dimensions.

Perspectives

- **Generating perturbatively the ξ dependence offers a well defined functional space for GPDs at small ξ which verifies the main theoretical constraints (polynomiality of Mellin moments, positivity, limits, ...)**
- **By subtracting the degree of freedom of the ξ dependence, we have regularized the deconvolution problem, and we have an evaluation of the uncertainty associated to this regularization.**
- Practical use of the uncertainty exposed before
  - **simple solution**: use directly as an additional systematic uncertainty on the relation of the GPD to the PDF at (μ, x_B)
  - **more sophisticated solution**: use your best flexible model of the t-dependent PDF at the low scale $\mu_0 = 1$ GeV. Evolve it to the hard scale with the full GPD evolution kernel $\rightarrow$ gives GPD-like objects with pure perturbative ξ dependence.
  - Then use Bayesian inference to confront this prior with the actual data, taking into account the previous systematic uncertainty. Either there is good compatibility $\rightarrow$ improves the knowledge of the t-dependent PDF at small x. Or there is incompatibility $\rightarrow$ points out shortcomings in either the current estimate of the PDF at small x, or in the perturbative evolution framework.

Conclusions

- We propose a procedure to evaluate the systematic uncertainty in relating GPDs to PDFs at small x.
- This procedure provides a regularization of the deconvolution problem at small ξ, along with an estimate of the systematic uncertainty associated to the regularization.
- We point out that the uncertainty exhibits only a weak dependence on ξ due to the steep increase of PDFs at small x, but a major dependence on the hard scale of the process. Υ production provides a much safer channel to extract PDFs at small x with limited systematic uncertainty!

Thank you for your attention!

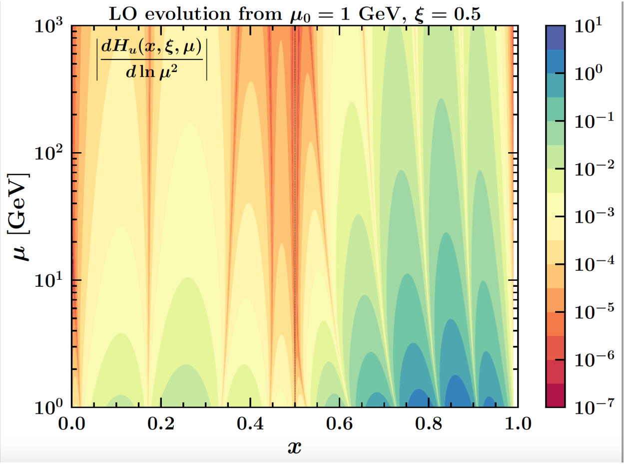
Perspectives

- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) [Berger et al, 2002]. Although its measurement will reduce the uncertainty, especially on $\mathrm{Re}\,\mathcal{H}$ [Jlab proposal PR12-12-001], and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see [Müller et al, 2012]) makes it subject to the same shadow GPDs.
- **Deeply virtual meson production** (DVMP) [Collins et al, 1997] is also an important source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process involves form factors of the general form

$$\mathcal{F}(\xi, t) = \int_0^1 \mathrm{d}u \int_{-1}^1 \frac{\mathrm{d}x}{\xi} \phi(u) T\left(\frac{x}{\xi}, u\right) F(x, \xi, t) \tag{17}$$

  where $\phi(u)$ is the leading-twist meson distribution amplitude (DA).
- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

Shadow GPDs at next-to-leading order



Color plot of an NLO shadow GPD at initial scale 1 GeV^2, and its evolution for $\xi = 0.5$ up to 10^6 GeV^2 via APFEL++ and PARTONS **[Bertone]**.