ELEMENTARY APPROACHES TO RLC CIRCUITS

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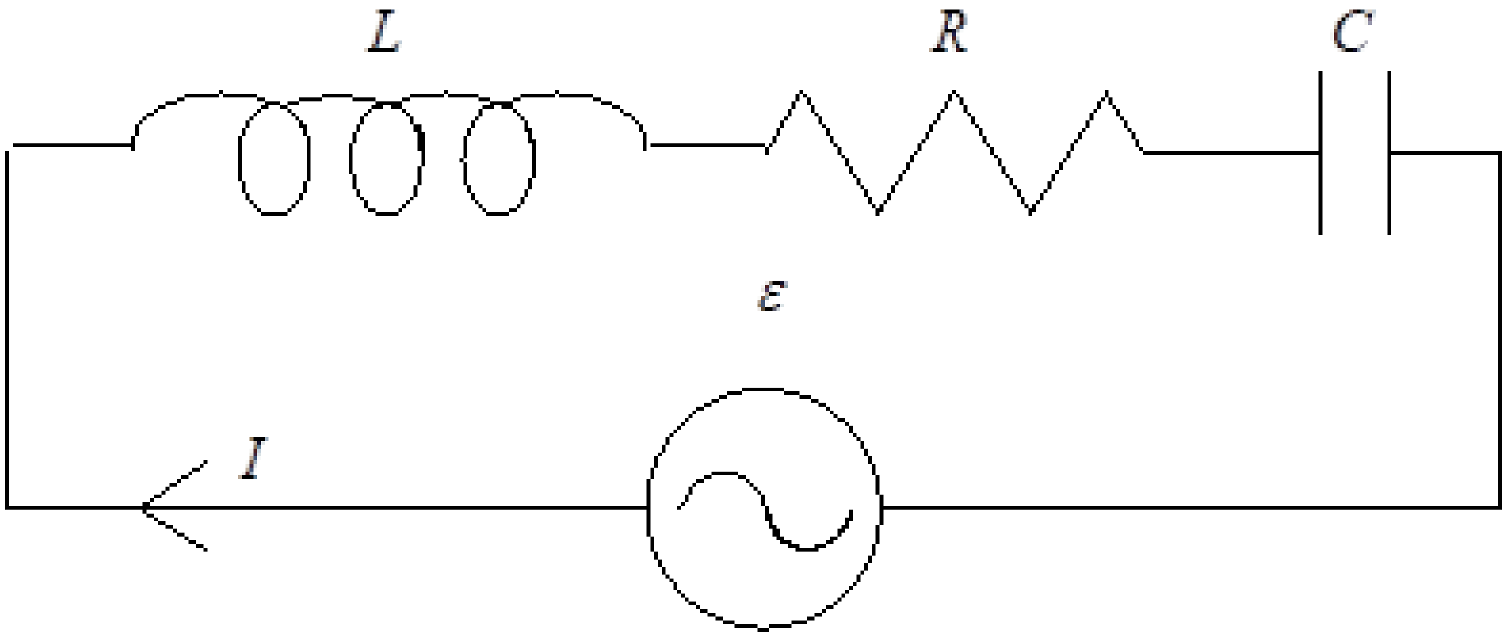
Part 1: THEORY



driver: $V_\varepsilon = \varepsilon_0 \cos \omega t$ (by choice of zero of time)

response: $I = I_0 \cos(\omega t - \phi)$

responds at driving frequency

response lags behind driver

capacitor: $V_C = \dfrac{Q}{C}$

but the current is $\dfrac{dQ}{dt} = I_0 \cos(\omega t - \phi)$

so that $\int dQ = \int I_0 \cos(\omega t - \phi) dt$

which integrates to $Q = \dfrac{I_0}{\omega} \sin(\omega t - \phi)$

and thus $V_C = \dfrac{I_0}{\omega C} \sin(\omega t - \phi)$

Introduce reactance $X_C = \dfrac{1}{\omega C}$ to get $\boxed{V_C = I_0 X_C \sin(\omega t - \phi)}$

inductor: $$V_L = L\frac{dI}{dt} = -L\omega I_0 \sin(\omega t - \phi)$$

introduce reactance $X_L = \omega L$ to get $\boxed{V_L = -I_0 X_L \sin(\omega t - \phi)}$

resistor: $$V_R = IR$$

so that $\boxed{V_R = I_0 R \cos(\omega t - \phi)}$

emf: introduce impedance $Z = \dfrac{\varepsilon_0}{I_0}$

so that $V_\varepsilon = I_0 Z \cos\omega t$

$$V_C = I_0 X_C \sin(\omega t - \phi)$$

$$V_L = -I_0 X_L \sin(\omega t - \phi)$$

$$V_R = I_0 R \cos(\omega t - \phi)$$

Now use the voltage loop rule: $V_\varepsilon = V_R + V_L + V_C$

and divide by I_0 to get

$$Z\cos\omega t = R\cos(\omega t - \phi) - (X_L - X_C)\sin(\omega t - \phi)$$

The two unknowns Z and ϕ are to be expressed in terms of the givens L, R, C, ε_0, and ω.

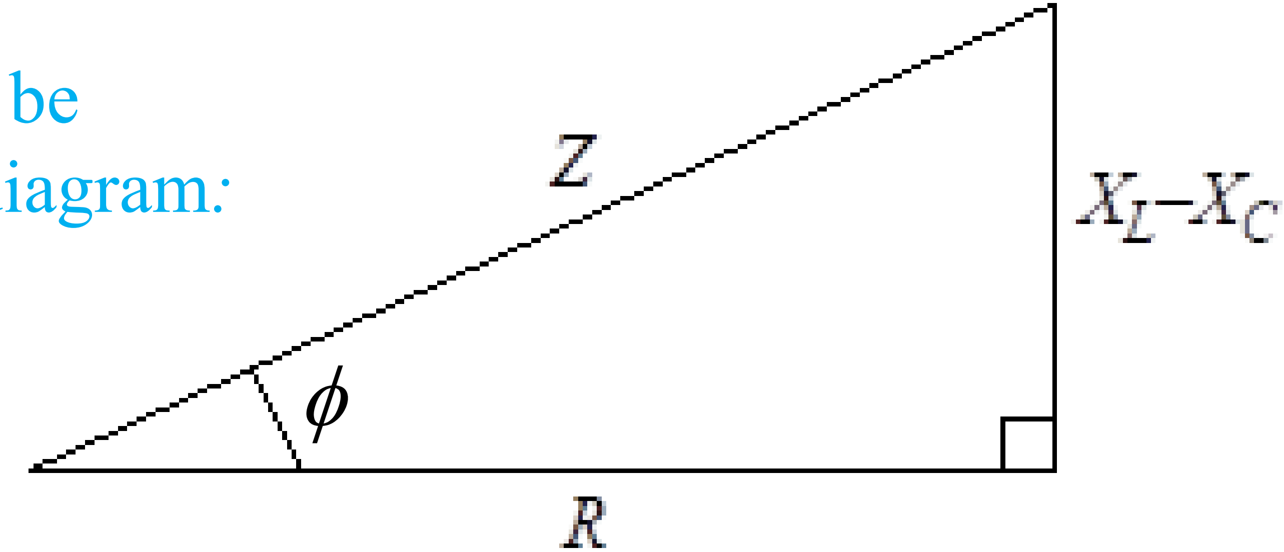
$$Z\cos\omega t = R\cos(\omega t-\phi)-(X_L - X_C)\sin(\omega t-\phi)$$

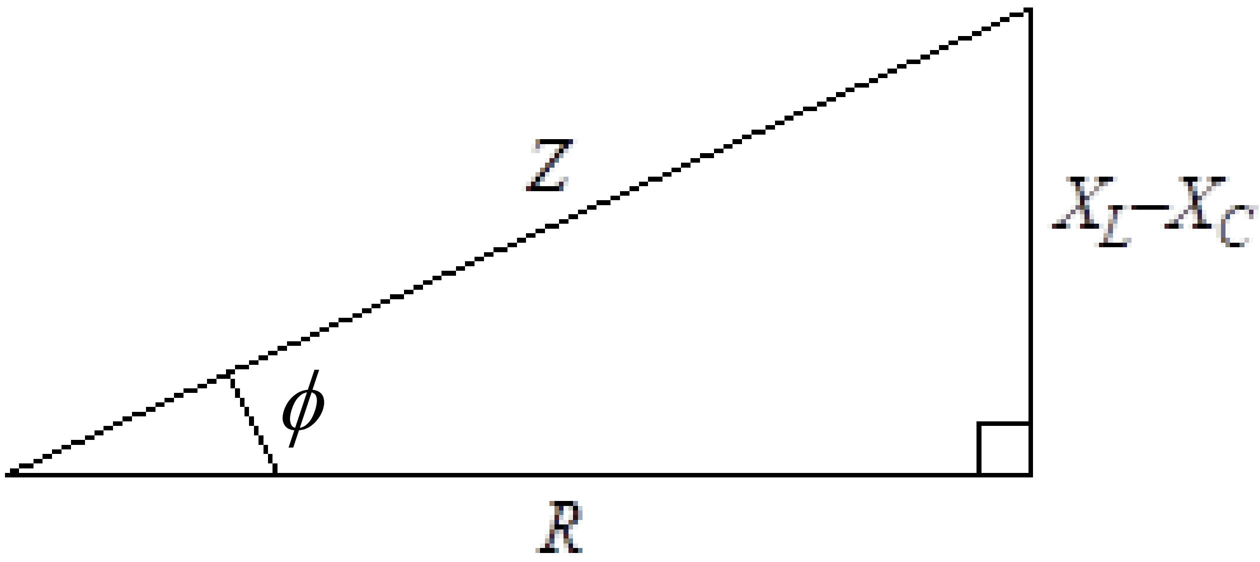
Solve by substituting two orthogonal values of *t*:

$$\omega t = \phi \quad \Rightarrow \quad Z\cos\phi = R$$

$$\omega t = \phi - \tfrac{\pi}{2} \quad \Rightarrow \quad Z\sin\phi = X_L - X_C$$

These two results can be summarized on a triangle diagram:





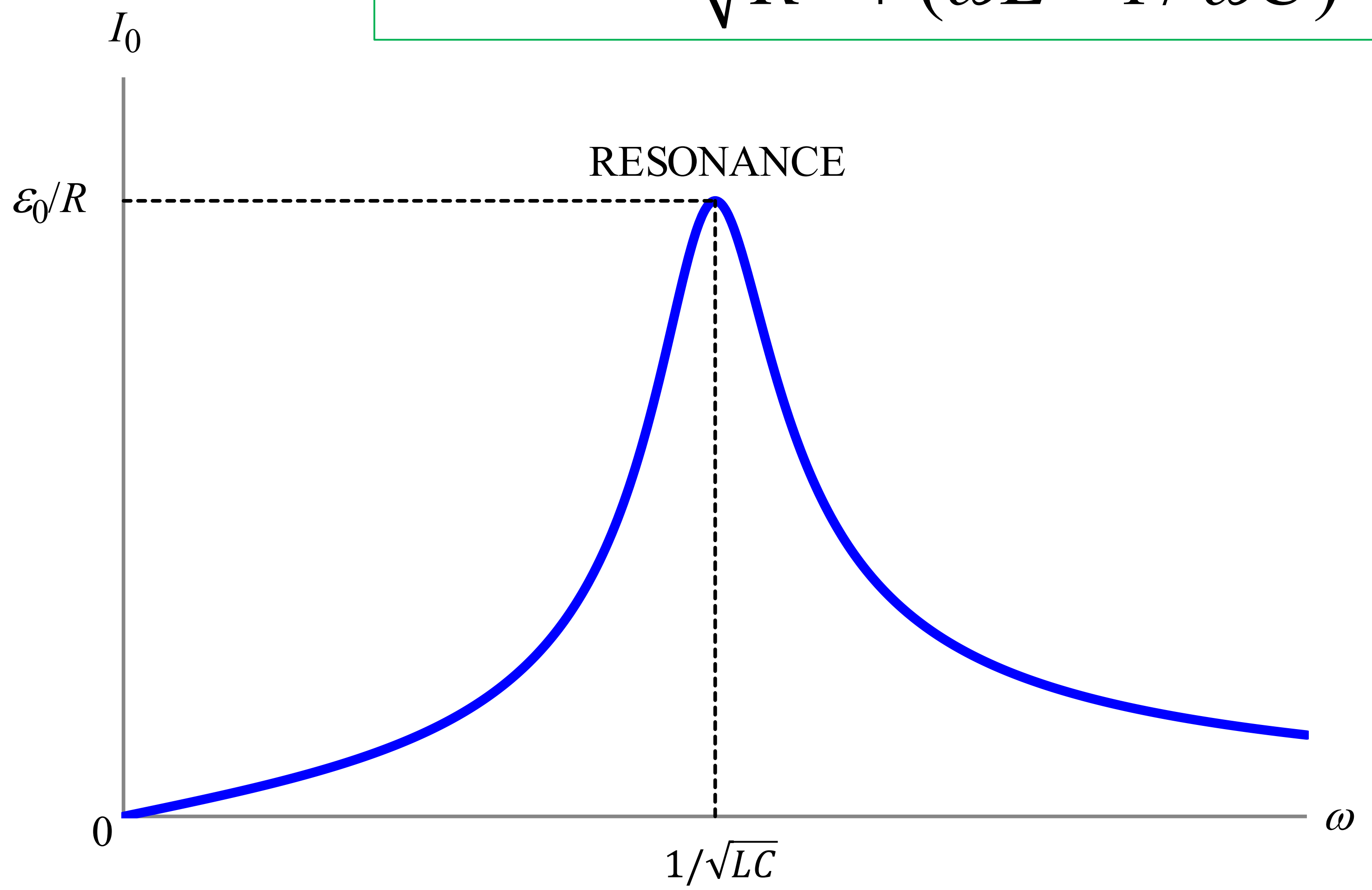
This triangle implies:

the impedance is $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

and the phase shift is $$\phi = \tan^{-1}\frac{X_L - X_C}{R}$$

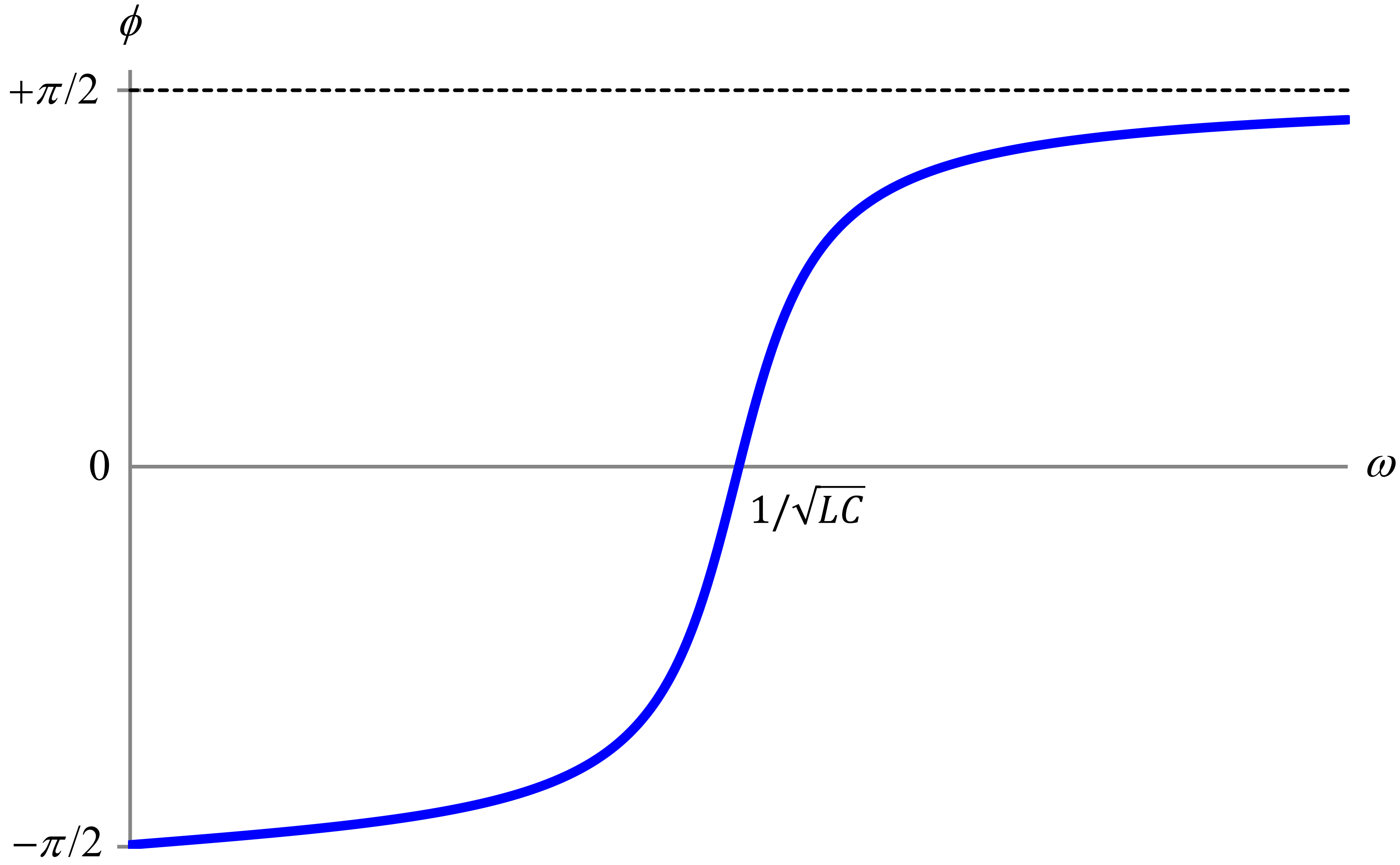
So we can graph the amplitude and phase shift of the response:

current amplitude $$I_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

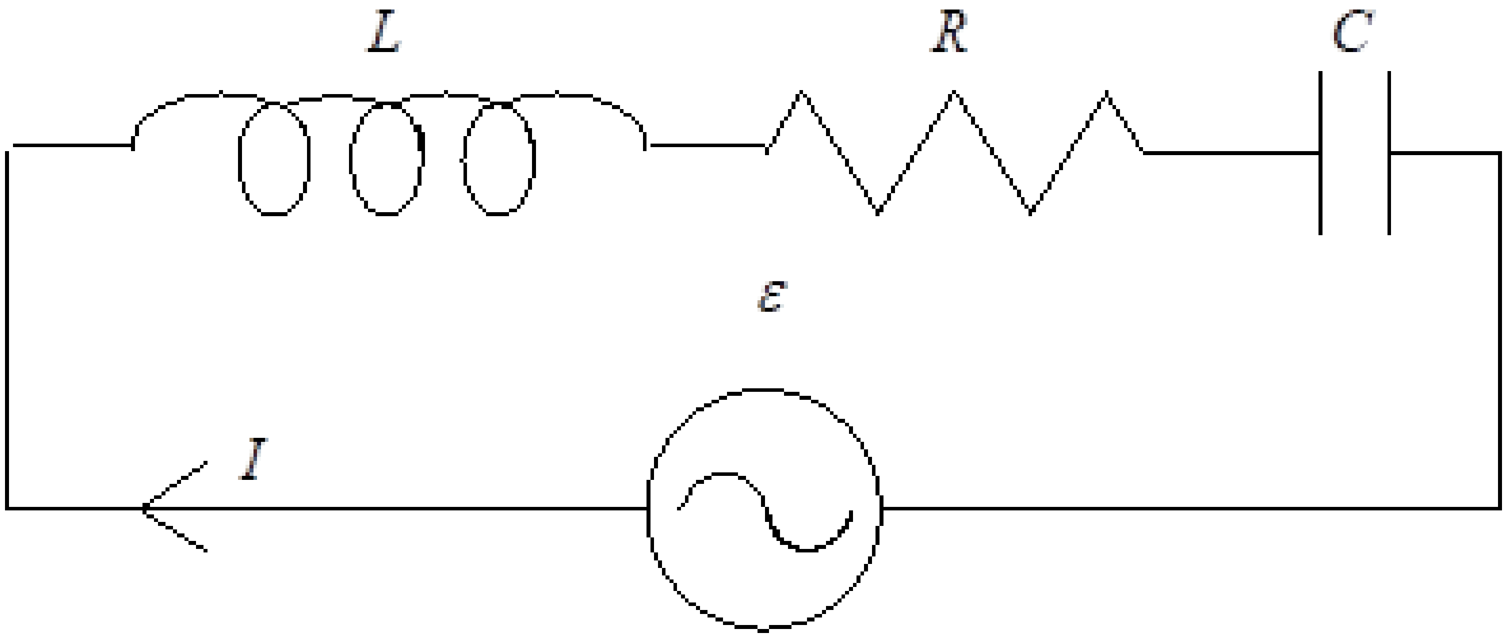


phase lag of the current relative to the emf

$$\phi = \tan^{-1} \frac{\omega L - 1/\omega C}{R}$$

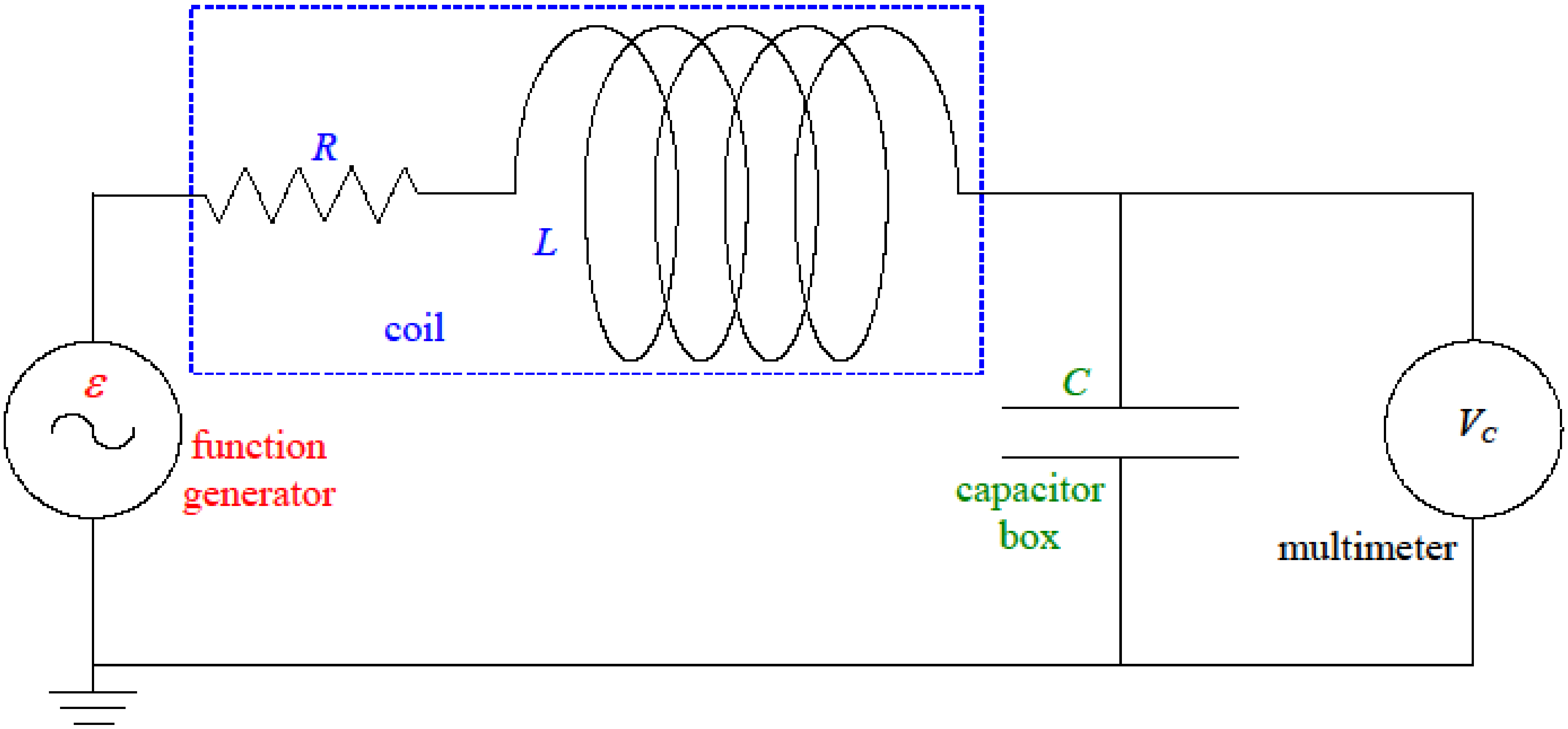


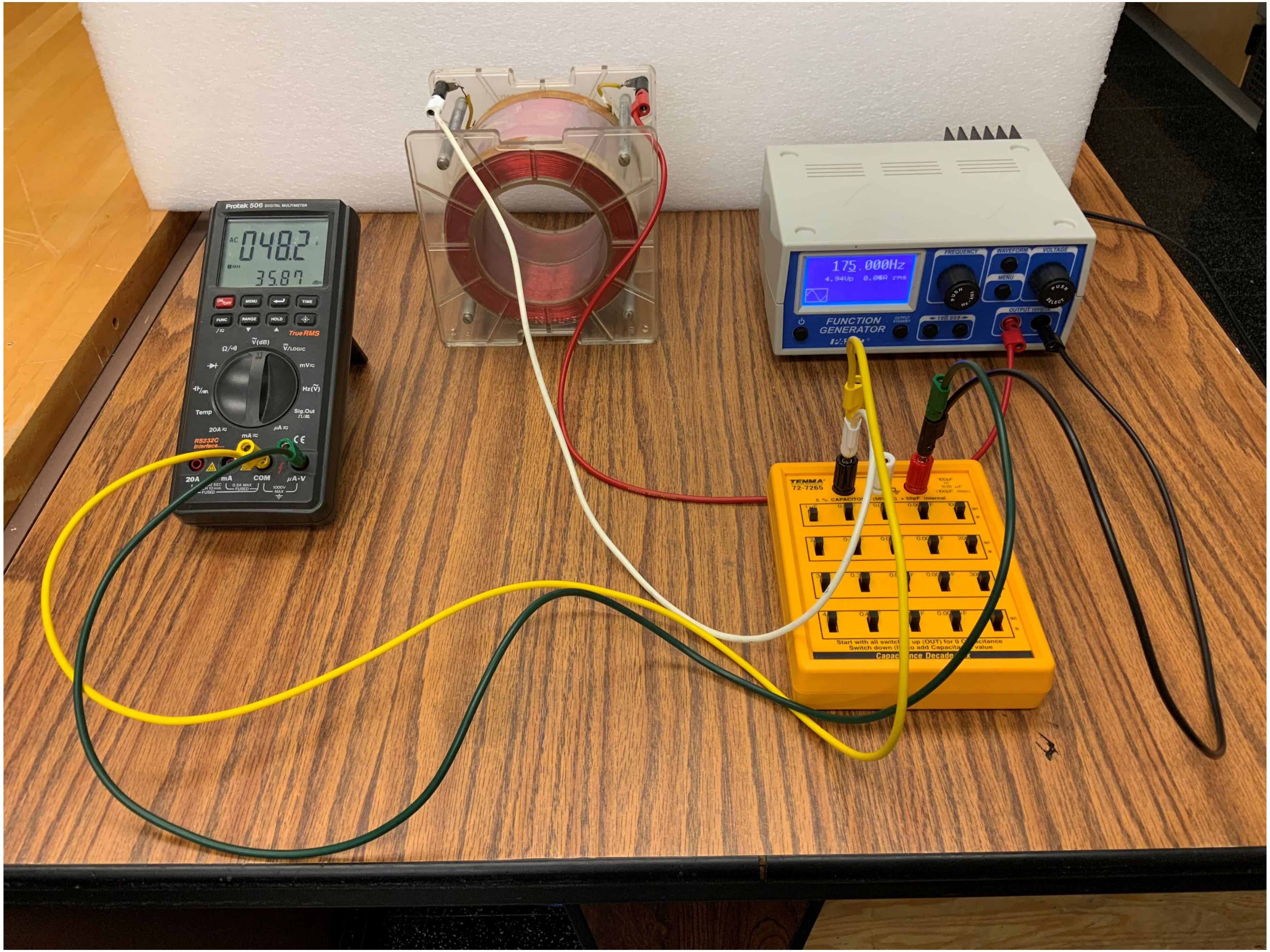
Part 2: EXPERIMENT



- use large coil for L and R
- use sinusoidal function generator for ε
- use variable capacitor box for C
- use multimeter for measurements

Actual circuit configured to measure rms capacitor voltage:




Protek 506
048.2
35.87
True RMS
FUNCTION GENERATOR
175.000Hz
TENMA
72-7265

During setup, use the Protek multimeter to measure:

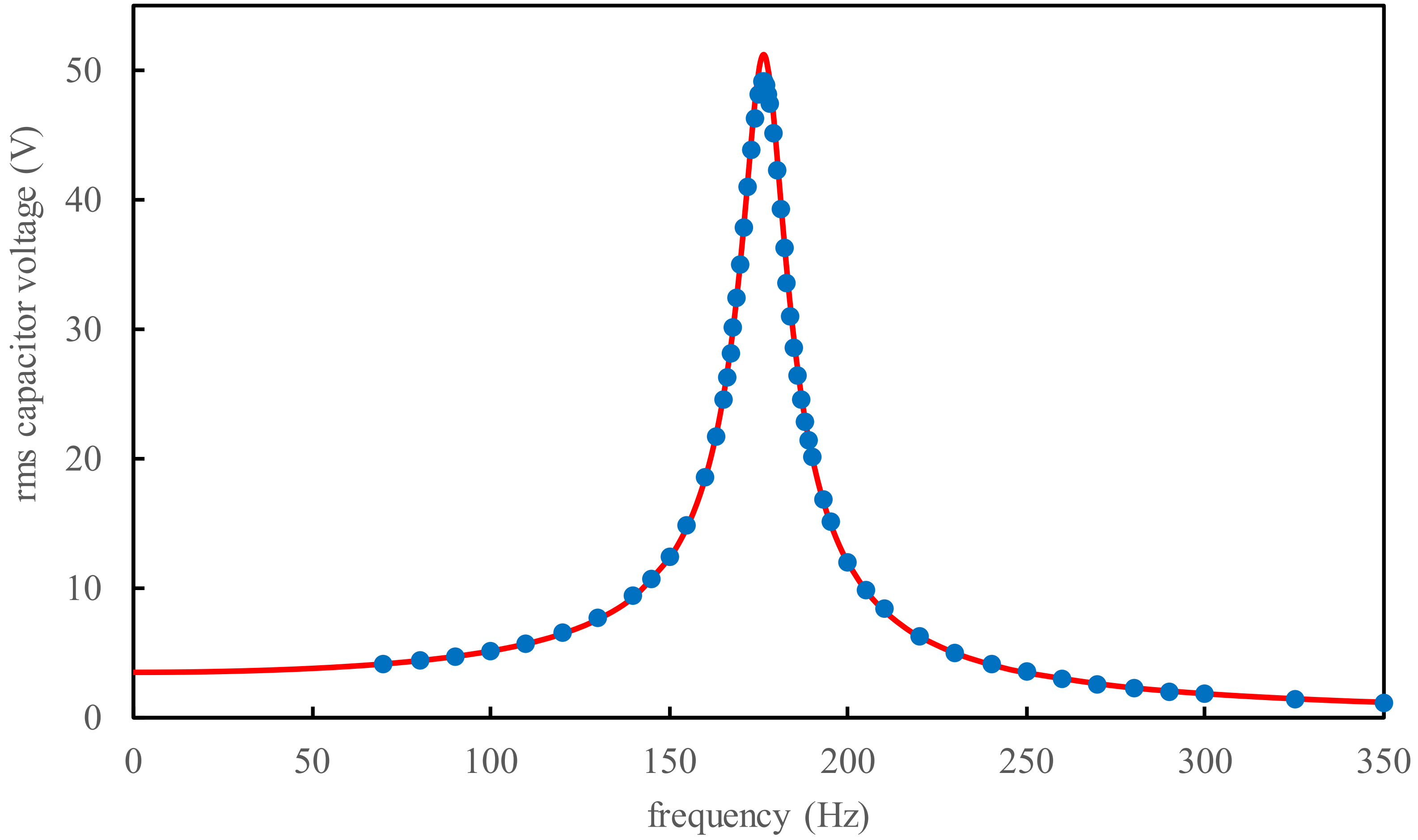
- $L = 0.83$ H
- $C = 0.98$ μF

so the resonance frequency is 176 Hz

- $R = 63\ \Omega$
- $V_{\varepsilon} = 3.5$ V rms

and turn the frequency knob on the Pasco function generator in roughly 10 Hz steps.

$$\frac{V_\varepsilon}{Z} = I = \frac{V_C}{X_C} \quad \Rightarrow \quad V_C = \frac{V_\varepsilon}{\sqrt{(2\pi f RC)^2 + (4\pi^2 f^2 LC - 1)^2}}$$



Blue data points compared to red theory curve with NO free parameters.

Comments are welcome!

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