Threshold quarkonium production and mass structure of the proton

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Outline

1. Trace anomaly, threshold quarkonium production, mass and confinement radii
2. Proton mass sum rule, anomaly contribution, and QCD Higgs mechanism

Trace anomaly and threshold quarkonium production

QCD trace anomaly

- In the massless quark limit, classical chromodynamics (CCD) is scale invariant -> conformal symmetry

Energy-momentum tensor $T^{\mu\nu}$ traceless: $T_{\alpha}^{\alpha} = 0$

- In the quantum version of the theory, QCD, the conformal symmetry is broken by quantum fluctuations -> anomaly.

$$T_{\alpha}^{\alpha} \sim \frac{d\alpha_s(\Lambda)}{d\ln\Lambda} F^2$$

due to UV div., the QCD coupling runs with cut-offs

Anomaly and mass scale

- Anomaly (F^2) is a scalar, providing much needed mass scale of the massless theory.
- The V.E.V of F^2 characterizes the non-perturbative property of the QCD vacuum.

  important for QCD sum rule calculation
- The matrix elements of F^2 in hadrons generate the hadron mass scale.

$$m_H \sim \langle H|F^2|H\rangle$$

Therefore, it is important to measure F^2 experimentally

$\langle p|F^2|p\rangle$: heavy quarkonium probe

D. Kharzeev, Proc. Int. Sch. Phys. Fermi 130, 105 (1996)

- Using a color dipole to measure the scalar field static response. Voloshin (1978), ….

$$E^2 = \frac{1}{2}\left(E^2 + B^2\right) + \frac{1}{2}\left(E^2 - B^2\right)$$

$$= T_g^{00} + F^2$$

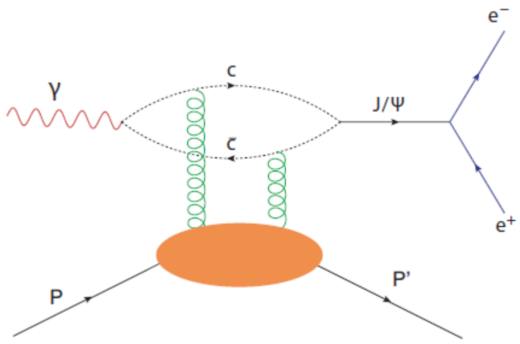
$$= T_g^{00} + \frac{T_\alpha^\alpha}{\alpha_s} \sim \frac{T_\alpha^\alpha}{\alpha_s}$$

- Heavy quarkonium is a natural probe for the scalar matrix element, or mass scale.

photo or electro-production of heavy quarkonium.

D. Kharzeev, Proc. Int. Sch. Phys. Fermi 130, 105 (1996)

- Heavy quarkonium production



only ½ amplitude satisfies the slow moving condition

- Vector Dominance Model (VDM): Questionable but maybe phenomenologically useful .

QCD factorization?

- In photoproduction, because of the heavy quark mass, the difference in rapidity (skewness) between two gluons must be large.
- Factorization formula can be derived in the heavy quark limit, ignoring the intrinsic heavy-quark contribution.
- The cross section depends on gluon GPDs: a new factorization formula.

Guo, Ji, Liu, *Phys. Rev. D* 103 (2021) 9, 096010

Threshold J/psi production: QCD factorization

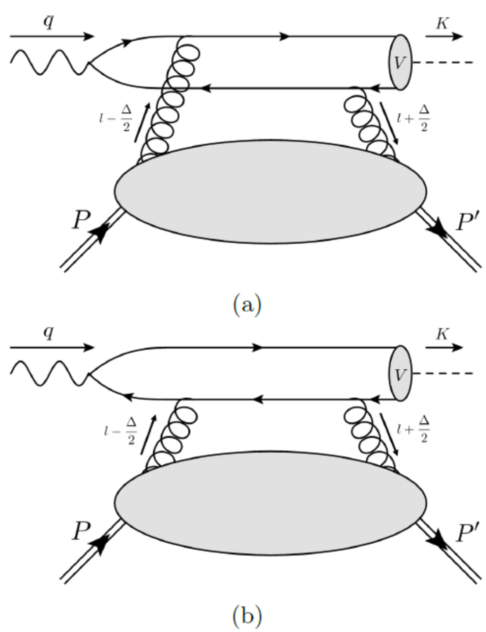


FIG. 4: Examples of leading Feynman diagrams that contribute to heavy vector meson photoproduction.

$$\frac{d\sigma}{dt} = \frac{e^2 e_Q^2}{16\pi\,(W^2 - M_N^2)^2}\frac{1}{2}\sum_{\text{polarization}} |\mathcal{M}(\varepsilon_V,\varepsilon)|^2 \;,$$
$$= \frac{\alpha_{\text{EM}} e_Q^2}{4\,(W^2 - M_N^2)^2}\frac{(16\pi\alpha_S)^2}{3M_V^3}|\psi_{\text{NR}}(0)|^2|G(t,\xi)|^2$$

$$G(t,\xi) = \frac{1}{2\xi}\int_{-1}^{1} \mathrm{d}x \mathcal{A}(x,\xi) F_g(x,\xi,t) \;. \tag{14}$$

where the hard kernel $\mathcal{A}(x,\xi)$ reads

$$\mathcal{A}(x,\xi) \equiv \frac{1}{x+\xi-i0} - \frac{1}{x-\xi+i0} \;. \tag{15}$$

The standard gluon GPD F_g is defined as [40]

$$F_g(x,\xi,t) \equiv$$
$$\frac{1}{(\bar{P}^+)^2}\int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x}\left\langle P' \middle| \mathrm{Tr}\left\{F^{+i}\left(-\frac{\lambda n}{2}\right)F^{+}_{\;i}\left(\frac{\lambda n}{2}\right)\right\}\middle| P\right\rangle \tag{16}$$

Large-ξ expansion

- In the heavy quark limit, the skewness parameter $\xi \to 1$
- One might consider expand the gluon propagators in the large ξ limit (Hatta et al)

$$G(t,\xi) = \sum_{n=0}^{\infty} \frac{1}{\xi^{2n+2}} \int_{-1}^{1} dx x^{2n} F_g(x,\xi,t) \ .$$

$$G(t,\xi) = \frac{1}{\xi^2(\bar{P}+)^2} \langle P' | \frac{1}{2} \sum_{a,i} F^{a,+i}(0) F^{a,+}{}_{i}(0) | P \rangle$$
$$= \frac{1}{2\xi^2(\bar{P}+)^2} \langle P' | T_g^{++} | P \rangle \ , \qquad (2$$

Mass structure: gravitations form factors

- Form factors of EMT for quarks and gluons (Ji,1996)

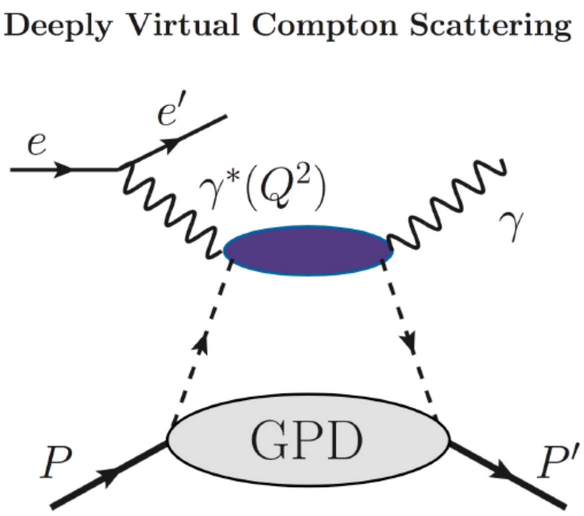
$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M]U(P)\,,$$

- Form factors for the total EMT (Pagels, 1966)

$$\langle P'|T^{\mu\nu}|P\rangle = \bar{u}(P')\Big[A(Q^2)\gamma^{(\mu}\bar{P}^{\nu)} + B(Q^2)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha/2M + C(Q^2)(q^\mu q^\nu - g^{\mu\nu}q^2)/M\Big]u(P)\,,$$

$$A = A_q + A_g,\ B\ \&\ C\ etc.,\ \ \bar{C}_q + \bar{C}_g = 0$$

Deeply virtual Compton scattering



$$\int_{-1}^{1} dx\ xH(x,\xi,t) = A(t) + \xi^2\ C(t)\ ,$$
$$\int_{-1}^{1} dx\ xE(x,\xi,t) = B(t) - \xi^2\ C(t)\ .$$

(Ji, Melnitchouk, Song, 1997)

Mass form factor/distribution

$$\langle P' \left| T^{00} \right| P \rangle = \bar{u} \left(P' \right) u(P) G_m(Q^2) \ .$$

where

$$G_m(Q^2) = \left[M A \left(Q^2 \right) - B(Q^2) \frac{Q^2}{4M} + C(Q^2) \frac{Q^2}{M} \right]$$

Trace anomaly, mass scale, and scalar form factor

- Form factor of the scalar density

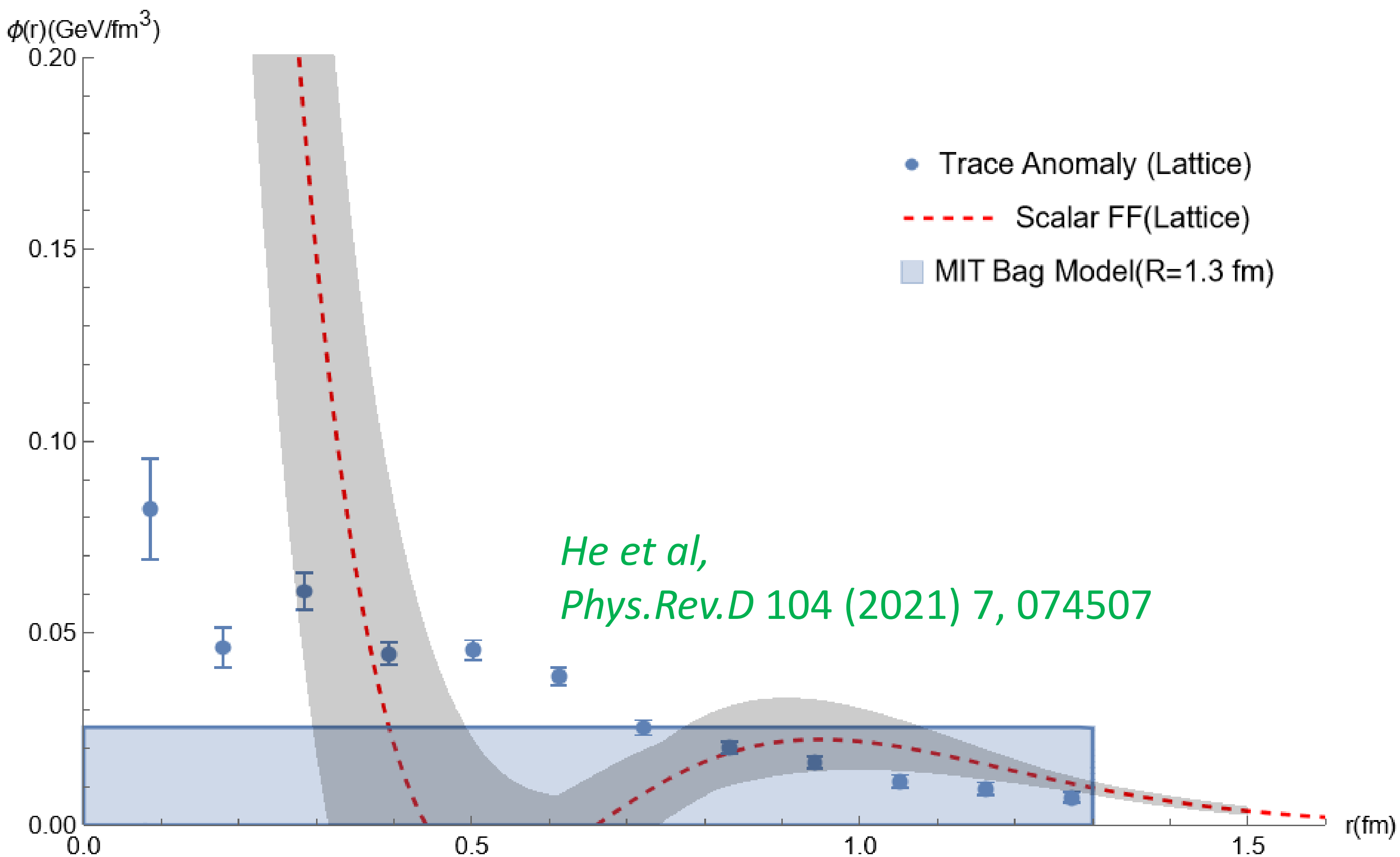
$$\langle P' | T^{\mu}_{\mu} | P \rangle = \bar{u}(P') u(P) G_s(Q^2) \ ,$$

where,

$$G_s(Q^2) = \left[MA(Q^2) - B(Q^2)\frac{Q^2}{4M} + C(Q^2)\frac{3Q^2}{M} \right]$$

- Fourier transformation of G_s gives us the scalar field distribution inside the Nucleon
- Dynamical MIT "bag constant".
- One can determine the mass scale without directly measuring F^2 matrix element! (EMT conservation)

Scalar field (QAE) distribution inside the proton



He et al,
Phys.Rev.D 104 (2021) 7, 074507

Scalar and mass radii

- Definition:

$$\langle r^2\rangle_{s,m} = -6\frac{dG_{s,m}(Q^2)}{dQ^2} ,$$

$$\langle r^2\rangle_s = -6\frac{dA(Q^2)}{dQ^2} - 18\frac{C(0)}{M^2}$$

$$\langle r^2\rangle_m = -6\frac{dA(Q^2)}{dQ^2} - 6\frac{C(0)}{M^2} ,$$

- The difference

$$\langle r^2\rangle_s - \langle r^2\rangle_m = -12\frac{C(0)}{M^2}$$

- Conjecture $\langle r^2\rangle_s > \langle r^2\rangle_m$ or C(0)<0

Scalar radius

- Scalar field radius might be similar to confinement radius
- The radius

$$\langle r^2 \rangle_s = -6\frac{dA(Q^2)}{dQ^2} - 18\frac{C(0)}{M^2}$$

- MIT bag scalar radius

$$r_s^2 = \frac{3}{5}R^2,\ r_s = 1.3fm$$

Proton mass sum rule, anomaly contribution, and QCD Higgs mechanism

The proton mass sum rule

- Physics consideration:
  - Mass is energy, $m = E/c^2$

    What is the individual sources of energy?

    terms in the QCD Hamiltonian

$$\mathrm{E}_0 = \sum_i \quad E_i, \quad \mathrm{E}_0 = \int d^3\vec{r}\, E(\vec{r})$$

E_i shall be calculable in theory, measurable in exp.

  - Symmetry

    Lorentz symmetry is one of the most important physics constraints.

- The sum rule is unique!

X. Ji, PRL *Phys. Rev. Lett.* 74 (1995) 1071

QCD energies in the nucleon

- Four different types

$$H_{\rm QCD} = H_q + H_m + H_g + H_a \, .$$

$$H_q = \int d^3\vec{x}\ \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi,$$ Quark energy

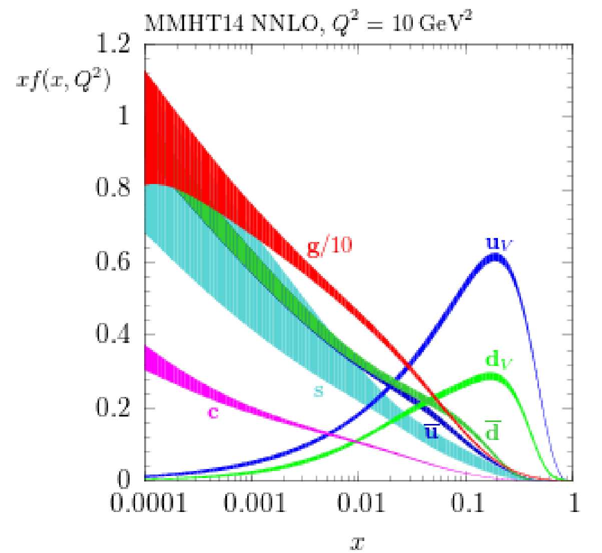
$$H_m = \int d^3\vec{x}\ \bar{\psi}m\psi,$$ Quark mass

$$H_g = \int d^3\vec{x}\ \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2),$$ Gluon energy

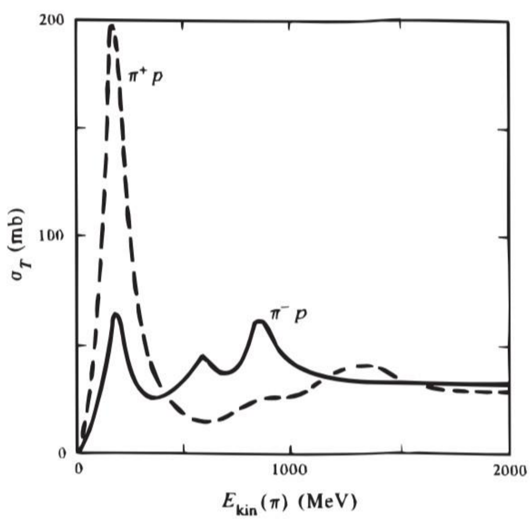
$$H_a = \int d^3\vec{x}\ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2 - \mathbf{B}^2).$$ Quantum Anomalous Energy (QAE)

Proton mass content from data (Ji,1995)

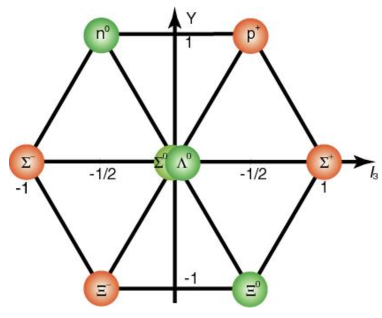
- Quark energy and gluon energy can be measured through quark and gluon parton distributions in high energy scattering



- Quark mass contribution can be measured through π-N σ-term, and through analysis of masses of strange baryon.

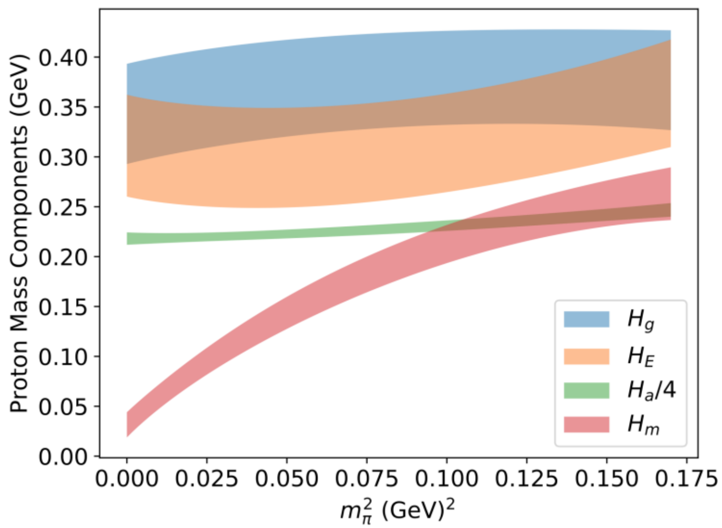


- There is no direct exp. information on QAE

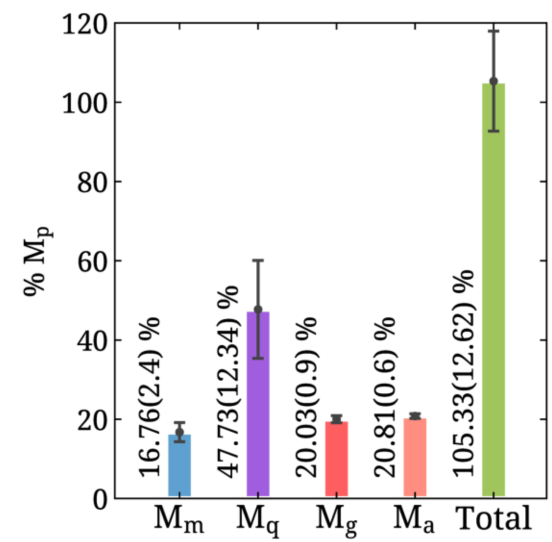


Proton mass on the lattice

To date no direct calculation of the trace anomaly



Y.-B. Yang *et al.*, (χQCD), PRL 121, 212001 (2018)



C. Alexandrou *et al.*, (ETMC), PRL 119, 142002 (2017)
C. Alexandrou *et al.*, (ETMC), PRL 116, 252001 (2016)

Trace anomaly only constrained through sum-rules not calculated directly.

Combining contributions

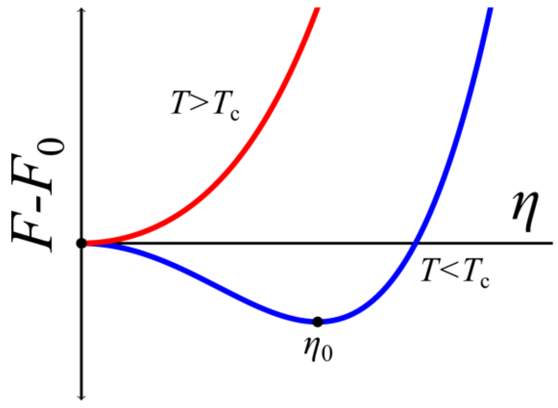
- One can combine the four contributions to form 2 or 3 term decompositions.

  C. Lorce

  Metz et al, see Metz talk
- In doing so, however, physics suffers.
- One of the most important pieces of physics in mass sum rule is the anomaly contribution, which points to a Higgs-like mechanism for the proton mass.

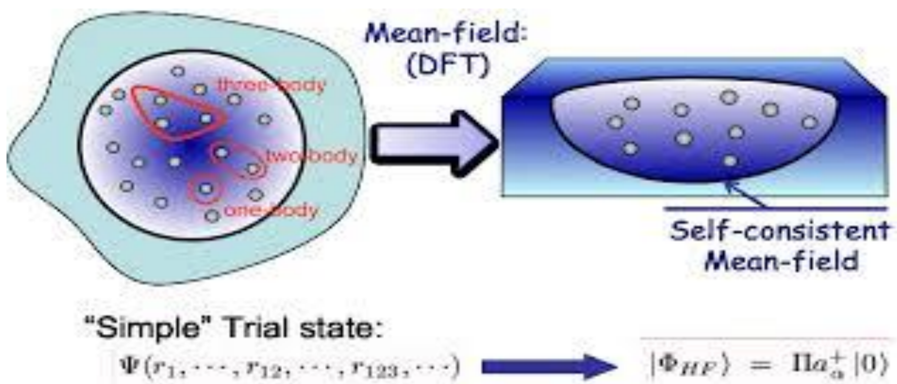
Energy of a scalar field

- Scalar field is special because it can have a vacuum condensate: non-vanishing expectation value in the physical vacuum. $\langle \eta \rangle \neq 0$, which stores the internal energy (latent heat)



Mean field theory for nuclear structure

- Traditional theory for nuclear structure: mean field theory

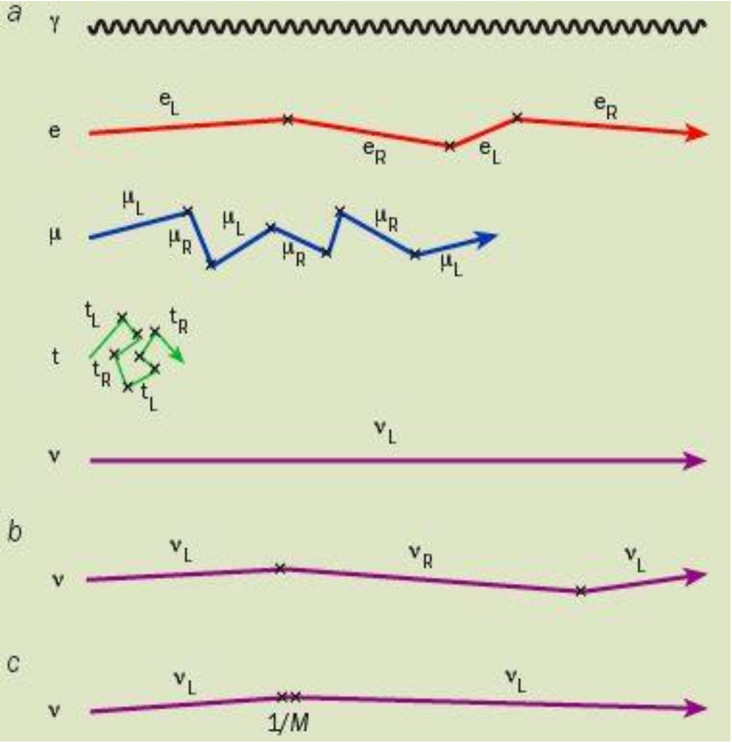


- Two important features:
  a) Mean field depth is about ~40 MeV
  b) Large spin-orbit splitting for nuclear shells.

Masses of electrons and leptons: Higgs mechanism

- There is a scalar field H, which interacts with the fermions $g\bar{\psi}\psi H$. H acquires an expectation value in the vacuum after SSB, $\langle H \rangle = v = 246$ GeV, hence the fermion mass,

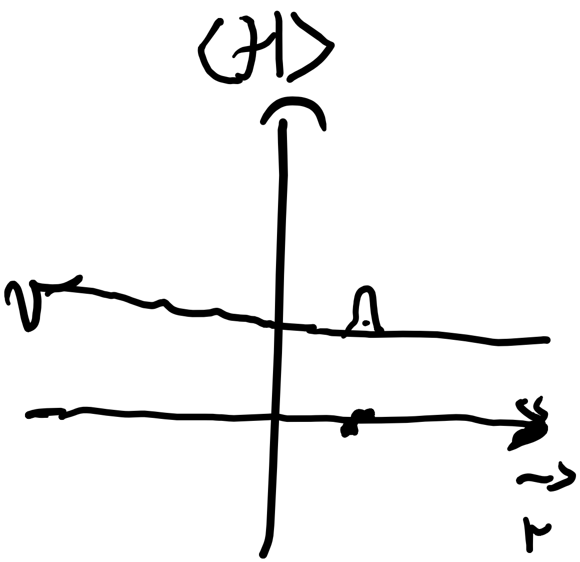
$$m = gv$$

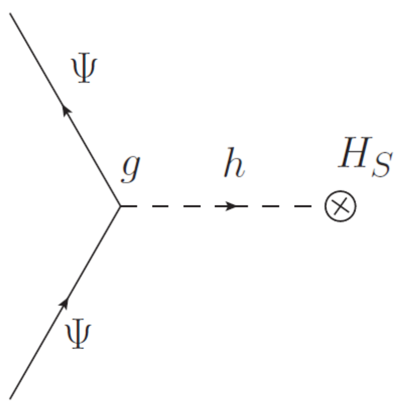


Dynamical picture of the fermion mass in Higgs mechanism

- Part of the fermion mass comes from the dynamical excitation of the higgs field in the presence of fermion

$$m_f \sim \langle f|H_S|f\rangle \sim \langle f|h|f\rangle$$

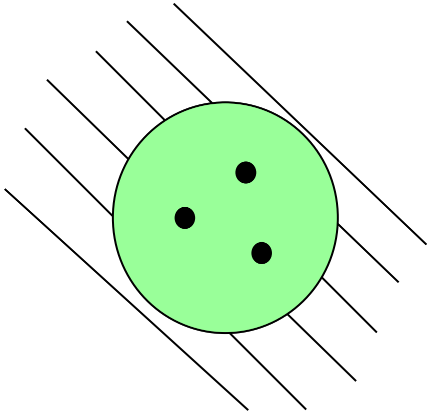




Quantum anomalous energy (QAE) contribution to the proton mass:

- The scalar field has a VEV: $\langle 0|F^2|0\rangle$
- QAE comes from the scalar response to the presence of the quarks.

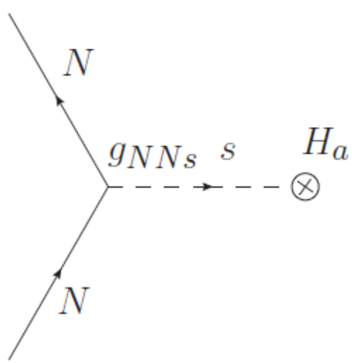
$$\phi = F^2 - \langle 0|F^2|0\rangle$$



- QCD Higgs mechanism, with gluon scalar as a dynamical Higgs field.

QAE as a dynamical response

- $E_a \sim \langle N | F^2 | N \rangle$
- This matrix element can also be calculated through dynamical scalar excitations



$$\langle N|\phi|N\rangle = \sum_s \frac{g_{NNs} f_s}{m_s^2} \; .$$

Test of the QCD "Higgs mechanism"

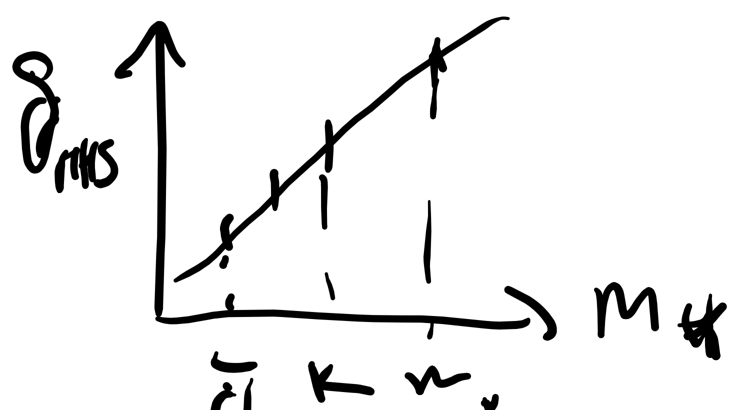
- The couplings of the scalars with the hadrons are proportional to the hadron masses.

$$g_{HHs} \sim m_H$$

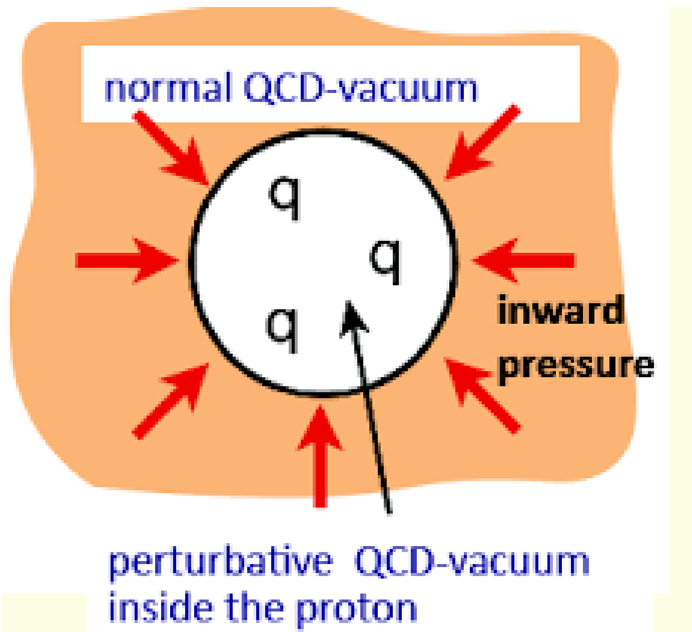
this also works for pion and kaon.

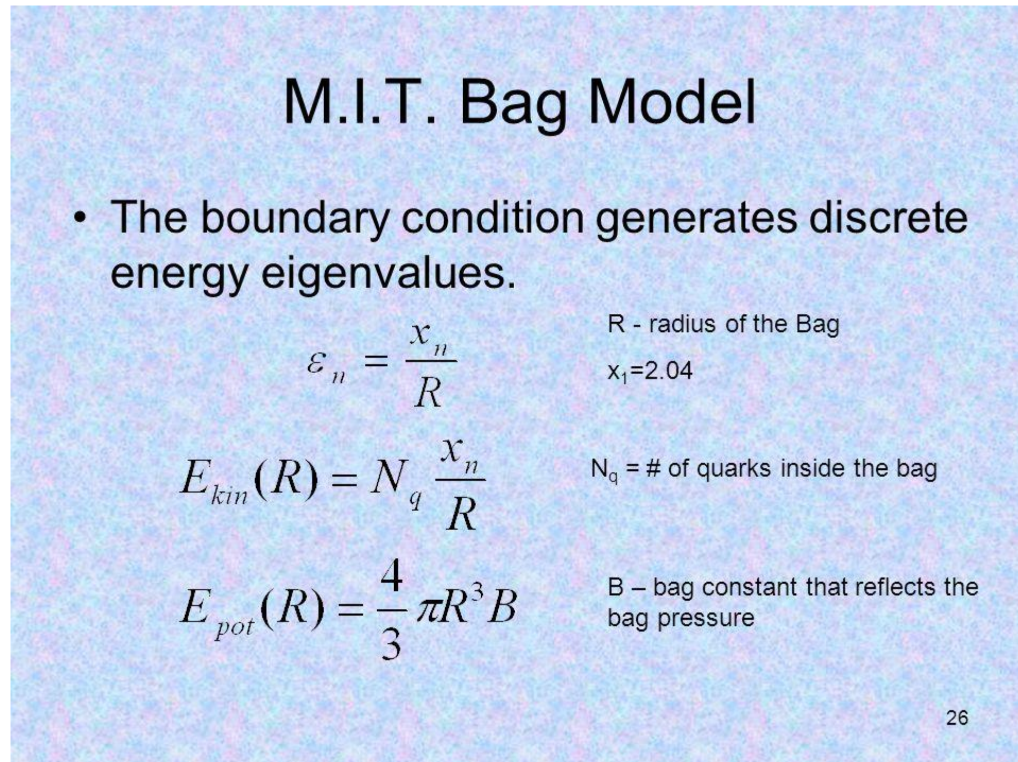
One can do the similar test as one does for Higgs particles at LHC but much more complicated

- Scalar spectrum



Physics of QAE in the MIT bag model





Mass = quark kinetic energy + B(scalar-field condensate)

Conclusions

- Threshold heavy-quarkonium production can probe the gluon contributions to the energy-momentum tensor form factors, which give us important information about the mass and scalar radii.
- The origin of proton mass is closely related to gluon scalar field, similar to EW Higgs mechanism. Anomaly contribution is a critical part of the mass.