TWIST-3 GPDs in MODELS

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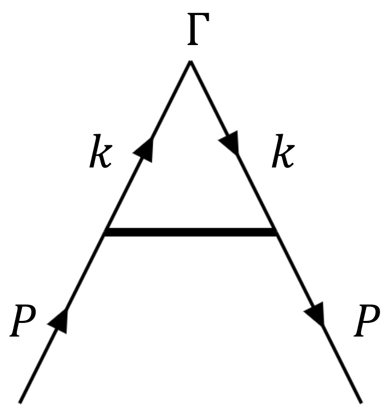
Towards improved hadron femtography with hard exclusive reactions

July 21, 2022

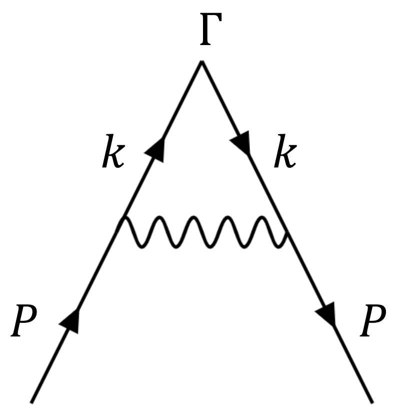
Models used to calculate twist-3 distributions:

- Quark target model (QTM)
- Scalar diquark model (SDM)

SDM



QTM



Outline

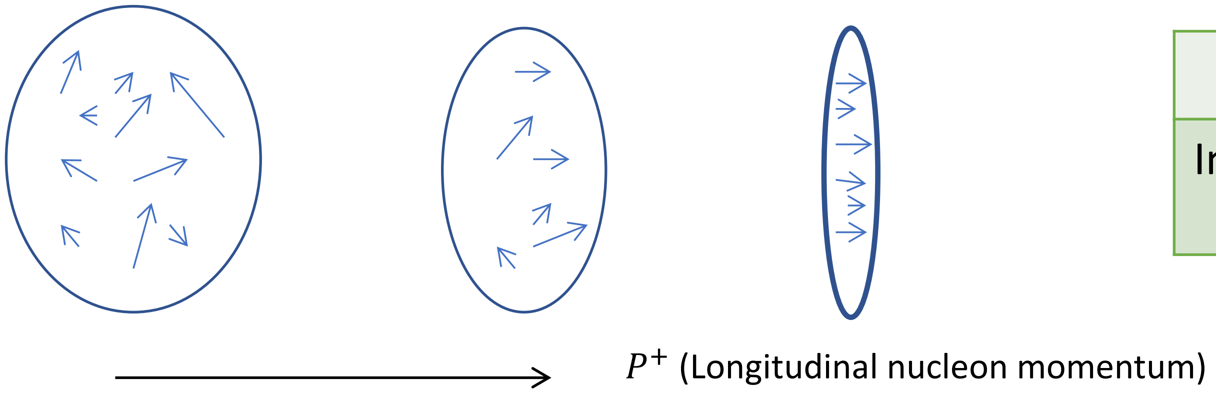
- Discontinuities in twist-3 GPDs
- Singularities in twist-3 PDFs

GPDs

$$F^{[\Gamma]}_{\lambda\lambda'}(P,x,\Delta,N) = \int dk^-\, d^2\vec{k}_T\, W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta)$$

$$= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ik\cdot z}\,\langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\Gamma\,\mathcal{W}(-\tfrac{1}{2}z,\tfrac{1}{2}z\,|\,n)\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=\vec{z}_T=0}\,.$$

Identifying Twist → Behavior under longitudinal momentum boost in the IMF



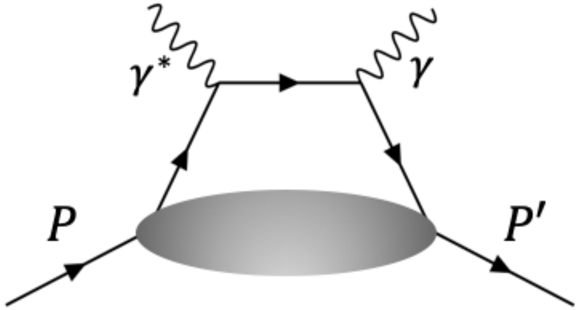
Twist-2	Twist-3	Twist-4
Independent of P^+	$\frac{1}{P^+}$	$\frac{1}{(P^+)^2}$

There are 8 twist-2, 16 twist-3 and 8 twist-4 GPDs

GPDs in the DVCS amplitude

$$T^{\mu\nu} = -i \int d^4x e^{-iq\cdot x} \langle p' | T[J^{\mu}_{\text{e.m.}}(x) J^{\nu}_{\text{e.m.}}(0)] | p \rangle,$$

$$\gamma^*(q) + N(p) \to \gamma(q') + N(p'),$$



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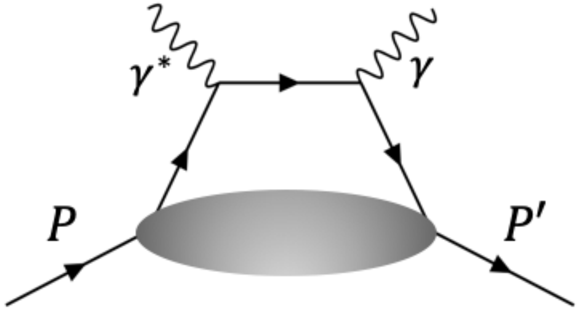
$$T^{\mu\nu} = \frac{1}{2}\int_{-1}^{1} dx \left[\left(-g_{\perp}^{\mu\nu} - \frac{P^{\nu}\Delta_{\perp}^{\mu}}{P\cdot q'} \right) n^{\beta} F_{\beta}(x,\xi,\Delta) C^{+}(x,\xi) + \left(-g_{\perp}^{\nu\alpha} - \frac{P^{\nu}\Delta_{\perp}^{\alpha}}{P\cdot q'} \right) i\epsilon^{\mu}_{\perp\alpha} n^{\beta} \tilde{F}_{\beta}(x,\xi,\Delta) C^{-}(x,\xi) \right.$$
$$\left. - \frac{(q+4\xi P)^{\mu}}{P\cdot q} \left(-g_{\perp}^{\nu\alpha} - \frac{P^{\nu}\Delta_{\perp}^{\alpha}}{P\cdot q'} \right) \left(F_{\alpha}(x,\xi,\Delta) C^{+}(x,\xi) - i\epsilon_{\perp\alpha\beta} \tilde{F}^{\beta}(x,\xi,\Delta) C^{-}(x,\xi) \right) \right],$$



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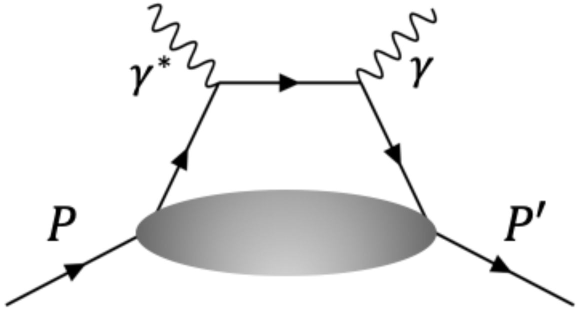

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$$F^\mu = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle p' | \bar{q}(\frac{\lambda}{2}n)\gamma^\mu \mathcal{W}(\frac{\lambda}{2}n, -\frac{\lambda}{2}n) q(-\frac{\lambda}{2}n) | p \rangle$$
$$= \frac{P^\mu}{P^+}\bar{u}(p')\left[\gamma^+ H + \frac{i}{2m}\sigma^{+\nu}\Delta_\nu E\right]u(p) + \bar{u}(p')\left[\frac{\Delta_\perp^\mu}{2m}G_1 + \gamma_\perp^\mu(H+E+G_2) + \Delta_\perp^\mu \frac{\gamma^+}{P^+}G_3 + \tilde{\Delta}_\perp^\mu \frac{\gamma^+\gamma_5}{P^+}G_4\right]u(p)$$

$$\tilde{F}^\mu = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle p' | \bar{q}(\frac{\lambda}{2}n)\gamma^\mu\gamma_5 \mathcal{W}(\frac{\lambda}{2}n, -\frac{\lambda}{2}n) q(-\frac{\lambda}{2}n) | p \rangle \qquad (5)$$
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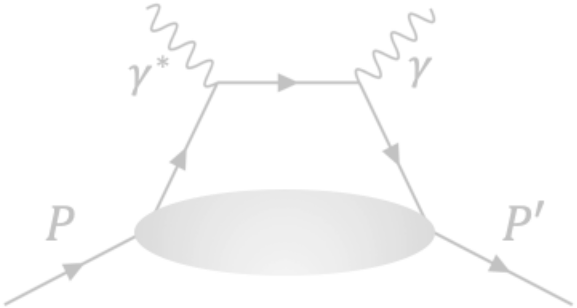

$$T^{\mu\nu} = \frac{1}{2}\int_{-1}^{1} dx\Big[\Big(-g_\perp^{\mu\nu} - \frac{P^\nu\Delta_\perp^\mu}{P\cdot q'}\Big)n^\beta F_\beta(x,\xi,\Delta)C^+(x,\xi) + \Big(-g_\perp^{\nu\alpha} - \frac{P^\nu\Delta_\perp^\alpha}{P\cdot q'}\Big)i\varepsilon^\mu_{\perp\alpha}n^\beta\tilde{F}_\beta(x,\xi,\Delta)C^-(x,\xi)$$
$$-\frac{(q+4\xi P)^\mu}{P\cdot q}\Big(-g_\perp^{\nu\alpha} - \frac{P^\nu\Delta_\perp^\alpha}{P\cdot q'}\Big)(F_\alpha(x,\xi,\Delta)C^+(x,\xi) - i\varepsilon_{\perp\alpha\beta}\tilde{F}^\beta(x,\xi,\Delta)C^-(x,\xi))\Big],$$

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$$= \frac{P^\mu}{P^+}\bar{u}(p')\Big[\gamma^+H + \frac{i}{2m}\sigma^{+\nu}\Delta_\nu E\Big]u(p) + \bar{u}(p')\Big[\frac{\Delta_\perp^\mu}{2m}\boldsymbol{G_1} + \gamma_\perp^\mu(H+E+\boldsymbol{G_2}) + \Delta_\perp^\mu\frac{\gamma^+}{P^+}\boldsymbol{G_3} + \tilde{\Delta}_\perp^\mu\frac{\gamma^+\gamma_5}{P^+}\boldsymbol{G_4}\Big]u(p)$$

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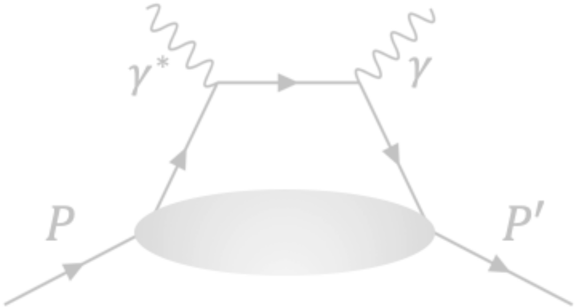

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$$- \frac{(q+4\xi P)^\mu}{P\cdot q'}\Big(-g_\perp^{\nu\alpha} - \frac{P^\nu\Delta_\perp^\alpha}{P\cdot q'}\Big)(F_\alpha(x,\xi,\Delta)C^+(x,\xi) - i\varepsilon_{\perp\alpha\beta}\tilde F^\beta(x,\xi,\Delta)C^-(x,\xi))\Big],$$

Amongst G_1, G_2, G_3, G_4 and $\tilde G_1, \tilde G_2, \tilde G_3, \tilde G_4$

only $\tilde G_2$ survives the forward limit

$$F^\mu = \int_{-\infty}^{\infty} \ldots$$

$$= \frac{P^\mu}{P^+}\bar u(p')\Big[\gamma^+ H + \frac{i}{2m}\sigma^{+\nu}\Delta_\nu E\Big]u(p) + \bar u(p')\Big[\frac{\Delta_\perp^\mu}{2m}G_1 + \gamma_\perp^\mu(H + E + G_2) + \Delta_\perp^\mu\frac{\gamma^+}{P^+}G_3 + \tilde\Delta_\perp^\mu\frac{\gamma^+\gamma_5}{P^+}G_4\Big]u(p)$$

G_1: $\Delta = 0$ → x; G_2: $\Delta = 0$ → […]; G_3: $\Delta = 0$ → x; G_4: $\Delta = 0$ → x

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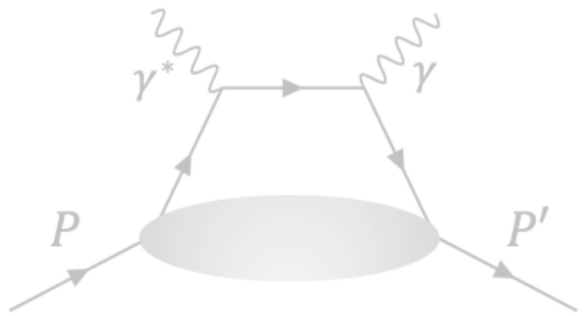

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$$L^{q}_{kin} = -\int dx\, x G_2^{q}(x,\xi=0,t=0)$$

Penttinen, Polyakov, Shuvaev, Strikman, 2000 / Hatta, Yoshida, 2012

$$\tilde{F}^{\mu} = \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\langle p'|\bar{q}(\frac{\lambda}{2}n)\gamma^{\mu}\gamma_5\mathcal{W}(\frac{\lambda}{2}n,-\frac{\lambda}{2}n)q(-\frac{\lambda}{2}n)|p\rangle$$
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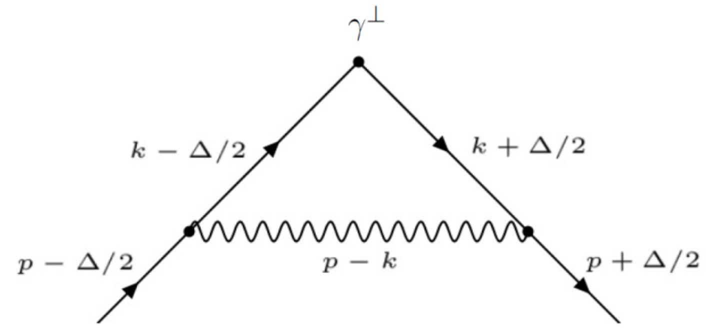
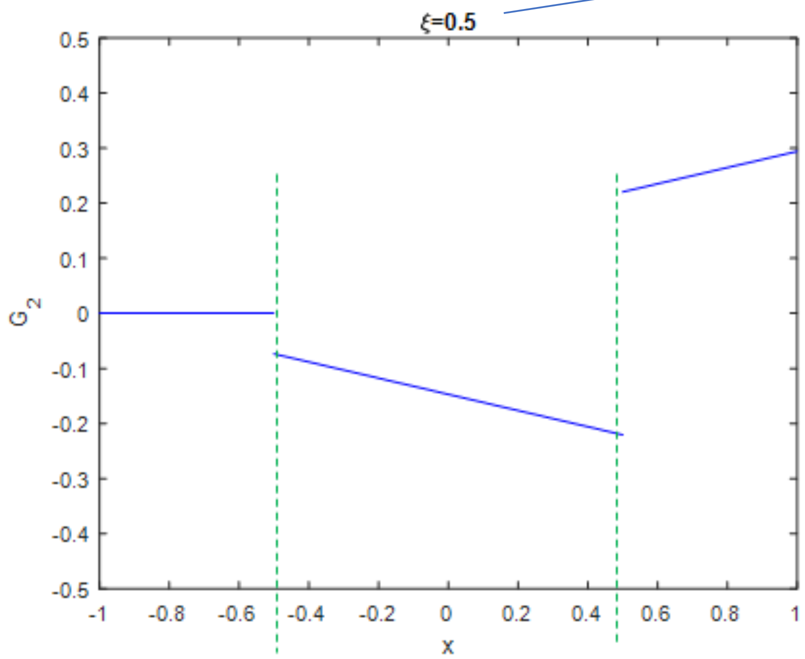
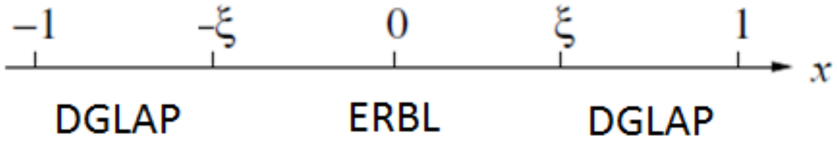
G_2 in Quark Target Model



Quark target model in a symmetric frame

$\xi = \dfrac{\Delta^+}{2p^+}$





$x = \dfrac{k^+}{p^+}$

Δ: The four-momentum transfer,

$P = p - \dfrac{\Delta}{2}\left(P' = p + \dfrac{\Delta}{2}\right)$: The incoming (outgoing) four-momentum,

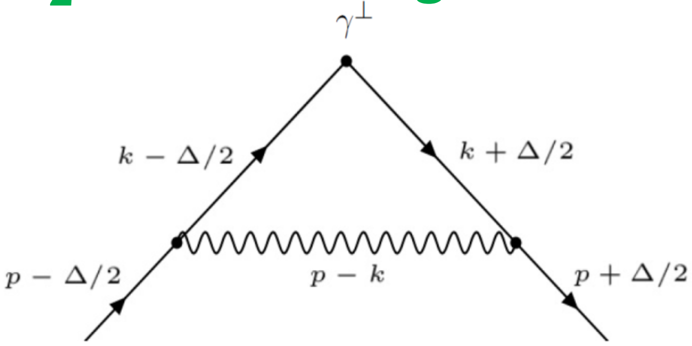
p: The average momentum (with $p_\perp = 0$),

$k - \dfrac{\Delta}{2}\left(k + \dfrac{\Delta}{2}\right)$: The four-momentum before (after) the interaction.

$$G_2 = \begin{cases} \dfrac{g^2}{2\pi^2}\dfrac{(1+x)}{(1-\xi^2)}\ln\Lambda_\perp & \text{for} \quad \xi < x < 1, \\ -\dfrac{g^2}{4\pi^2}\dfrac{(1+x)}{\xi(1+\xi)}\ln\Lambda_\perp & \text{for} \quad -\xi \le x \le \xi, \\ 0 & \text{for} \quad -1 < x < \xi, \end{cases}$$

G_2 has discontinuities

G_2 in Quark Target Model- How does the discontinuity arise?



Quark target model in a symmetric frame

The parameterization

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle P', S'|\,\overline{q}(-\frac{z^-}{2})\gamma^j q(\frac{z^-}{2})\,|P,S\rangle$$
$$= \frac{1}{2p^+}\overline{u}(P',S')\Big[\frac{\Delta_\perp^j}{2M}G_1 + \gamma^j\underbrace{(H+E+G_2)} + \frac{\Delta_\perp^j}{p^+}\gamma^+ G_3 + \frac{i\epsilon_T^{jk}\Delta_\perp^k}{p^+}\gamma^+\gamma_5 G_4\Big]\overline{u}(P,S)$$

Kiptily, Polyakov , Genuine twist-3 contributions to the generalized parton distributions from instantons (2003)

The model

$$-\frac{ig^2}{2}\int\frac{d^4k}{(2\pi)^4}\delta(k^+ - xp^+)\overline{u}(P',S')\gamma^\mu \frac{(\not k + \frac{\not\Delta}{2} + m)}{[(k+\frac{\Delta}{2})^2 - m^2 + i\epsilon]}\gamma^\perp \frac{(\not k - \frac{\not\Delta}{2} + m)}{[(k-\frac{\Delta}{2})^2 - m^2 + i\epsilon]}\gamma^\nu \times\Big[g_{\mu\nu} - \frac{n_\nu(p_\mu - k_\mu)}{p^+ - k^+} - \frac{n_\mu(p_\nu - k_\nu)}{p^+ - k^+}\Big]\frac{1}{[(p-k)^2 - \lambda^2 + i\epsilon]}u(P,S)$$

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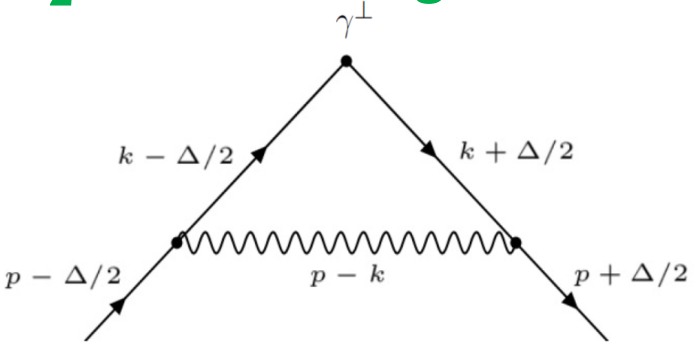

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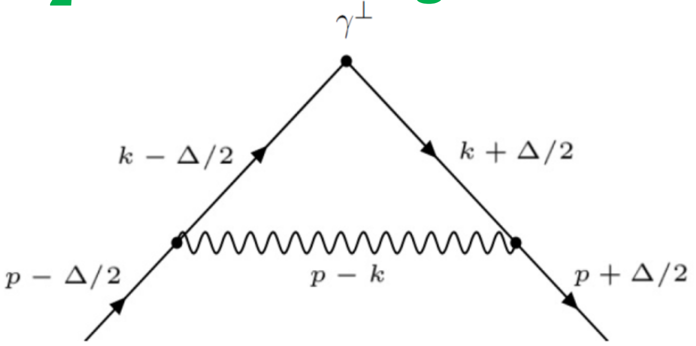
The divergent part of G_2 is calculated as, $-ig^2\int \frac{d^2k_\perp dk^-}{(2\pi)^4}\frac{k^- 8(p^+)^2(1+x)}{[(k + \frac{\Delta}{2})^2 - m^2 + i\epsilon]\,[(k - \frac{\Delta}{2})^2 - m^2 + i\epsilon]\,[(p-k)^2 - \lambda^2 + i\epsilon]}.$

G_2 in Quark Target Model- How does the discontinuity arise?



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Using $(P-k)^2-\lambda^2=2(P^+-k^+)(P^--k^-)-k_\perp^2-\lambda^2$, k^- in the numerator can be replaced by the following expression

$$k^-=\frac{M^2}{2p^+}-\frac{[(p-k)^2-\lambda^2]}{2(p^+-k^+)}-\frac{(k_\perp^2+\lambda^2)}{2(p^+-k^+)}$$

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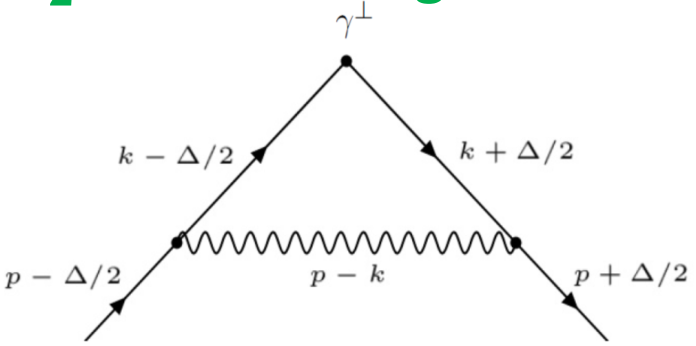

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Kiptily, Polyakov , Genuine twist-3 contributions to the generalized parton distributions from instantons (2003)

The model

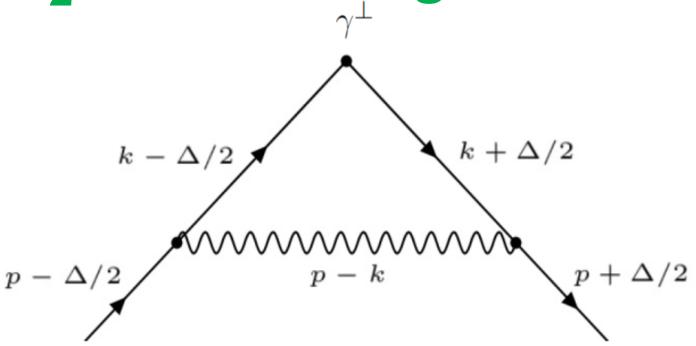
$$-\frac{ig^2}{2}\int\frac{d^4k}{(2\pi)^4}\delta(k^+-xp^+)\overline{u}(P',S')\gamma^\mu\frac{(\not k+\frac{\not\Delta}{2}+m)}{[(k+\frac{\Delta}{2})^2-m^2+i\epsilon]}\gamma^\perp\frac{(\not k-\frac{\not\Delta}{2}+m)}{[(k-\frac{\Delta}{2})^2-m^2+i\epsilon]}\gamma^\nu\times\Big[g_{\mu\nu}-\frac{n_\nu(p_\mu-k_\mu)}{p^+-k^+}-\frac{n_\mu(p_\nu-k_\nu)}{p^+-k^+}\Big]\frac{1}{[(p-k)^2-\lambda^2+i\epsilon]}u(P,S)$$

The divergent part of G_2 is calculated as,
$$-ig^2\int\frac{d^2k_\perp dk^-}{(2\pi)^4}\frac{k^-8(p^+)^2(1+x)}{[(k+\frac{\Delta}{2})^2-m^2+i\epsilon]\,[(k-\frac{\Delta}{2})^2-m^2+i\epsilon]\,[(p-k)^2-\lambda^2+i\epsilon]}.$$

Using $(P-k)^2-\lambda^2=2(P^+-k^+)(P^--k^-)-k_\perp^2-\lambda^2$, k^- in the numerator can be replaced by the following expression

$$k^-=\frac{M^2}{2p^+}-\frac{[(p-k)^2-\lambda^2]}{2(p^+-k^+)}-\frac{(k_\perp^2+\lambda^2)}{2(p^+-k^+)}$$

G_2 in Quark Target Model- How does the discontinuity arise?



Quark target model in a symmetric frame

The parameterization

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle P',S'|\,\overline{q}(-\frac{z^-}{2})\gamma^j q(\frac{z^-}{2})\,|P,S\rangle$$
$$=\frac{1}{2p^+}\overline{u}(P',S')\Big[\frac{\Delta_\perp^j}{2M}G_1+\gamma^j\underbrace{(H+E+G_2)}+\frac{\Delta_\perp^j}{p^+}\gamma^+G_3+\frac{i\epsilon_T^{jk}\Delta_\perp^k}{p^+}\gamma^+\gamma_5G_4\Big]\overline{u}(P,S)$$

Kiptily, Polyakov , Genuine twist-3 contributions to the generalized parton distributions from instantons (2003)

The model

$$-\frac{ig^2}{2}\int\frac{d^4k}{(2\pi)^4}\delta(k^+-xp^+)\overline{u}(P',S')\gamma^\mu\frac{(\not k+\frac{\not\Delta}{2}+m)}{[(k+\frac{\Delta}{2})^2-m^2+i\epsilon]}\gamma^\perp\frac{(\not k-\frac{\not\Delta}{2}+m)}{[(k-\frac{\Delta}{2})^2-m^2+i\epsilon]}\gamma^\nu\times\Big[g_{\mu\nu}-\frac{n_\nu(p_\mu-k_\mu)}{p^+-k^+}-\frac{n_\mu(p_\nu-k_\nu)}{p^+-k^+}\Big]\frac{1}{[(p-k)^2-\lambda^2+i\epsilon]}u(P,S)$$

The divergent part of G_2 is calculated as, $-ig^2\int\frac{d^2k_\perp dk^-}{(2\pi)^4}\frac{k^-8(p^+)^2(1+x)}{[(k+\frac{\Delta}{2})^2-m^2+i\epsilon]\,[(k-\frac{\Delta}{2})^2-m^2+i\epsilon]\,[(p-k)^2-\lambda^2+i\epsilon]}.$

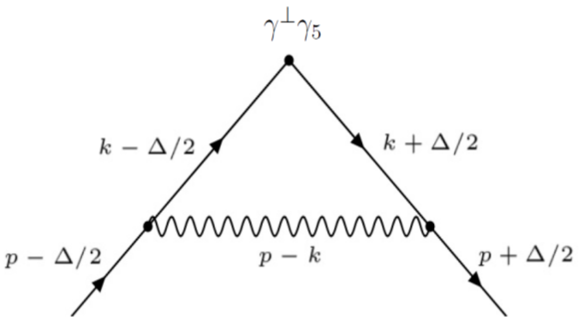
Using $(P-k)^2-\lambda^2=2(P^+-k^+)(P^--k^-)-k_\perp^2-\lambda^2$, k^- in the numerator can be replaced by the following expression

$$k^-=\frac{M^2}{2p^+}-\frac{[(p-k)^2-\lambda^2]}{2(p^+-k^+)}-\frac{(k_\perp^2+\lambda^2)}{2(p^+-k^+)}$$

The second term cancels the propagator in the denominator leading to the following contribution which is nonzero only in the ERBL region, $-\xi<x<\xi$.

$$ig^24p^+\frac{(1+x)}{(1-x)}\int\frac{d^2k_\perp dk^-}{(2\pi)^4}\frac{1}{[(k+\frac{\Delta}{2})^2-m^2+i\epsilon]\,[(k-\frac{\Delta}{2})^2-m^2+i\epsilon]}.$$

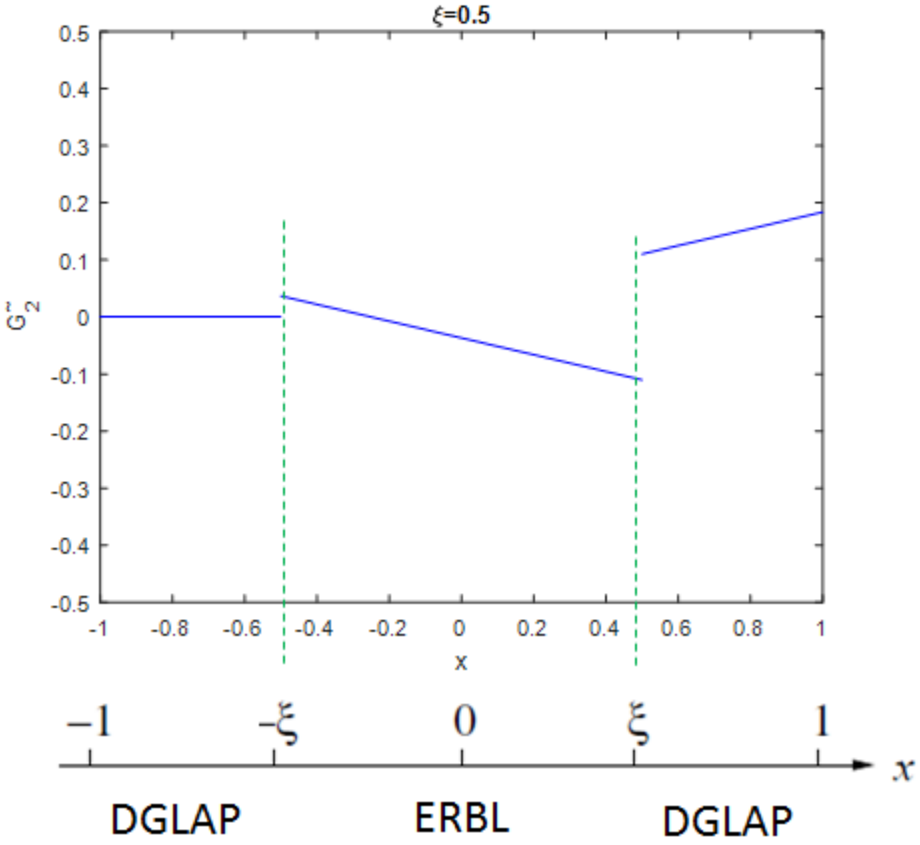
$\widetilde{G}_2$ in Quark Target Model



Quark target model in a symmetric frame

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle P',S'|\,\overline{q}(-\frac{z^-}{2})\gamma^j\gamma_5 q(\frac{z^-}{2})\,|P,S\rangle$$

$$=\frac{1}{2p^+}\overline{u}(P',S')\Big[\frac{\Delta_\perp^j}{2M}\gamma_5(\widetilde{E}+\widetilde{G}_1)+\gamma^j\gamma_5(\widetilde{H}+\widetilde{G}_2)+\frac{\Delta_\perp^j}{p^+}\gamma^+\gamma_5\widetilde{G}_3+\frac{i\epsilon_T^{jk}\Delta_\perp^k}{p^+}\gamma^+\widetilde{G}_4\Big]\overline{u}(P,S).$$



$$\widetilde{G}_2=\begin{cases}\dfrac{g^2}{2\pi^2}\dfrac{(x+\xi^2)}{(1-\xi^2)}\ln\Lambda_\perp & \text{for} \quad \xi<x<1,\\ -\dfrac{g^2}{4\pi^2}\dfrac{(x+\xi^2)}{\xi(1+\xi)}\ln\Lambda_\perp & \text{for} \quad -\xi\le x\le\xi,\\ 0 & \text{for} \quad -1<x<\xi,\end{cases}$$

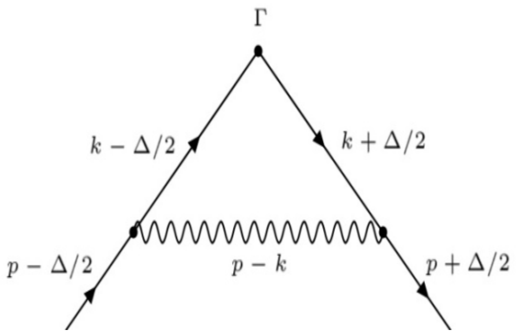
$\widetilde{G}_2$ too has discontinuities

Discontinuities and Factorization

✓: Continuous
✗: Discontinuous

Twist-3 GPD (Vector)	Quark Target Model
G_1	✓
G_2	x
G_3	x
G_4	x

Twist-3 GPD (Axial V.)	Quark Target Model
$\tilde{G}_1$	✓
$\tilde{G}_2$	x
$\tilde{G}_3$	x
$\tilde{G}_4$	x



Quark Target Model

Twist-3 GPDs have discontinuities !

$$\int_{-1}^{1} dx \, \frac{GPD}{x \pm \xi + i\varepsilon}$$

Discontinuities → Divergent scattering amplitudes → Factorization ?

DVCS Amplitude of the nucleon at twist -3 accuracy

Twist	DVCS amplitude involves
Twist-2	$\int_{-1}^{1} dx \; \frac{\text{(Twist-2 GPDs)}}{x \pm \xi + i\epsilon}$
Twist-3	$\int_{-1}^{1} dx \; \frac{\text{(Linear Combinations of Twist-3 GPDs)}}{x \pm \xi + i\epsilon}$

$$\int_{-1}^{1} dx \left[\underbrace{\left(F_\perp^\nu - i\varepsilon^\nu_{\perp\alpha}\widetilde{F}_\perp^\alpha\right)} \frac{1}{x-\xi+i\varepsilon} + \underbrace{\left(F_\perp^\nu + i\varepsilon^\nu_{\perp\alpha}\widetilde{F}_\perp^\alpha\right)} \frac{1}{x+\xi-i\varepsilon} \right]$$

This linear combination of twist-3 GPDs

This linear combination of twist-3 GPDs

Twist-3 generalized parton distributions in deeply-virtual Compton scattering - Fatma Aslan, Matthias Burkardt, Cedric Lorce, Andreas Metz, Barbara Pasquini (2018)

DVCS Amplitude of the nucleon at twist -3 accuracy

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Twist-3	$\int_{-1}^{1} dx \; \frac{\text{(Linear Combinations of Twist-3 GPDs)}}{x \pm \xi + i\epsilon}$

$$\int_{-1}^{1} dx \left[\left(F_\perp^\nu - i\varepsilon^\nu_{\perp\alpha} \widetilde{F}_\perp^\alpha \right) \frac{1}{x - \xi + i\varepsilon} + \left(F_\perp^\nu + i\varepsilon^\nu_{\perp\alpha} \widetilde{F}_\perp^\alpha \right) \frac{1}{x + \xi - i\varepsilon} \right]$$

This linear combination of twist-3 GPDs **CONTINUOUS** at x=ξ

This linear combination of twist-3 GPDs **CONTINUOUS** at x=-ξ

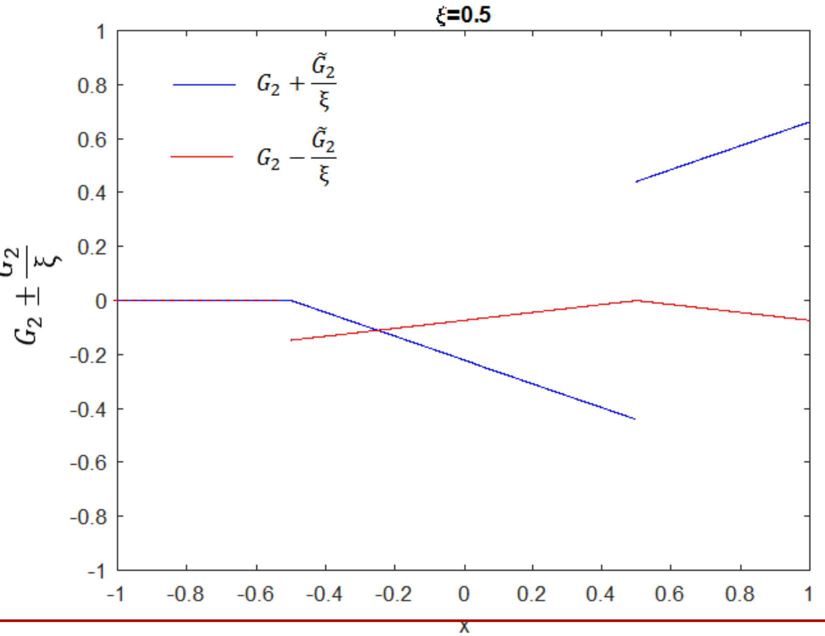
Twist-3 generalized parton distributions in deeply-virtual Compton scattering - Fatma Aslan, Matthias Burkardt, Cedric Lorce, Andreas Metz, Barbara Pasquini (2018)

Example:

The relevant DVCS amplitude involves $G_2 \pm \frac{\tilde{G}_2}{\xi}$

$$\int_{-1}^{1} dx \, \frac{G_2 + \frac{1}{\xi}\tilde{G}_2}{x + \xi + i\varepsilon} \qquad \int_{-1}^{1} dx \, \frac{G_2 - \frac{1}{\xi}\tilde{G}_2}{x - \xi + i\varepsilon}$$

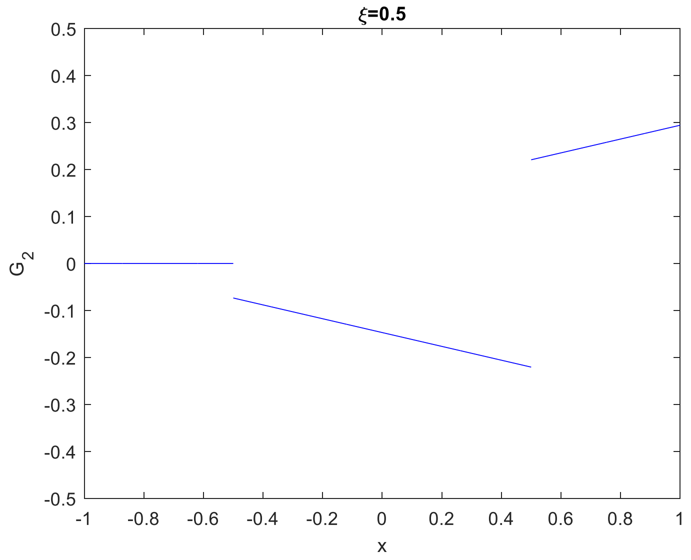
- $G_2 + \frac{1}{\xi}\tilde{G}_2$ continuous at $x = -\xi$
- $G_2 - \frac{1}{\xi}\tilde{G}_2$ continuous at $x = \xi$

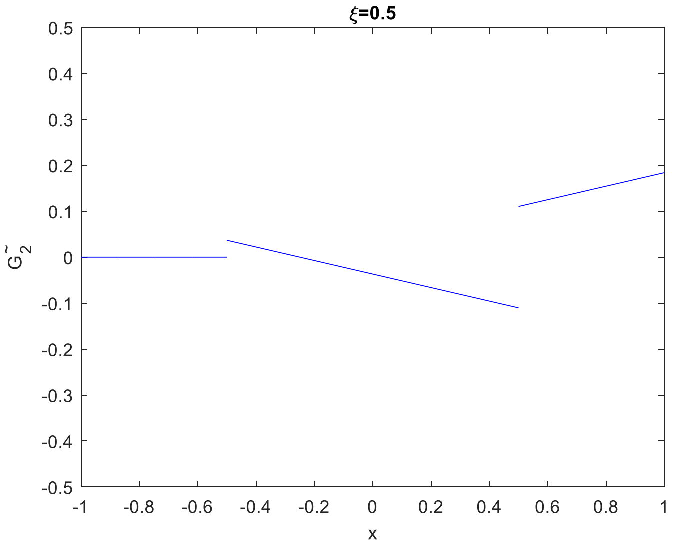


Twist-3 GPDs are discontinuous → Linear combinations of twist-3 GPDs that enter the DVCS amplitude are well-behaved → Twist -3 DVCS factorization is safe

Twist-3 generalized parton distributions in deeply-virtual Compton scattering - Fatma Aslan, Matthias Burkardt, Cedric Lorce, Andreas Metz, Barbara Pasquini (2018)

Factorization is safe, but what about the discontinuities?



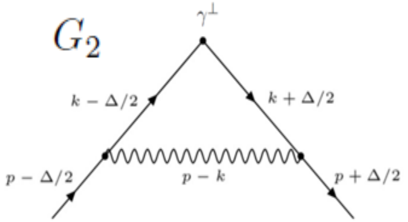


How do they behave?
What do they represent?
What happens in different models?
What about the forward limit?
...

How do the discontinuities behave as ξ→0 ?

G_2 and $\widetilde{G}_2$ in Quark Target Model

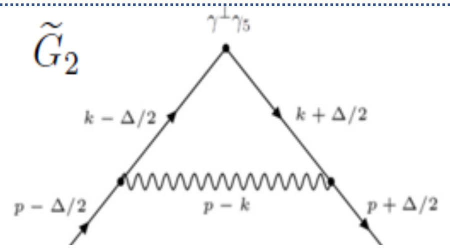
G_2



$$ig^2 4p^+ \frac{(1+x)}{(1-x)} \int \frac{d^2k_\perp dk^-}{(2\pi)^4} \frac{1}{[(k+\frac{\Delta}{2})^2 - m^2 + i\epsilon][(k-\frac{\Delta}{2})^2 - m^2 + i\epsilon]}.$$

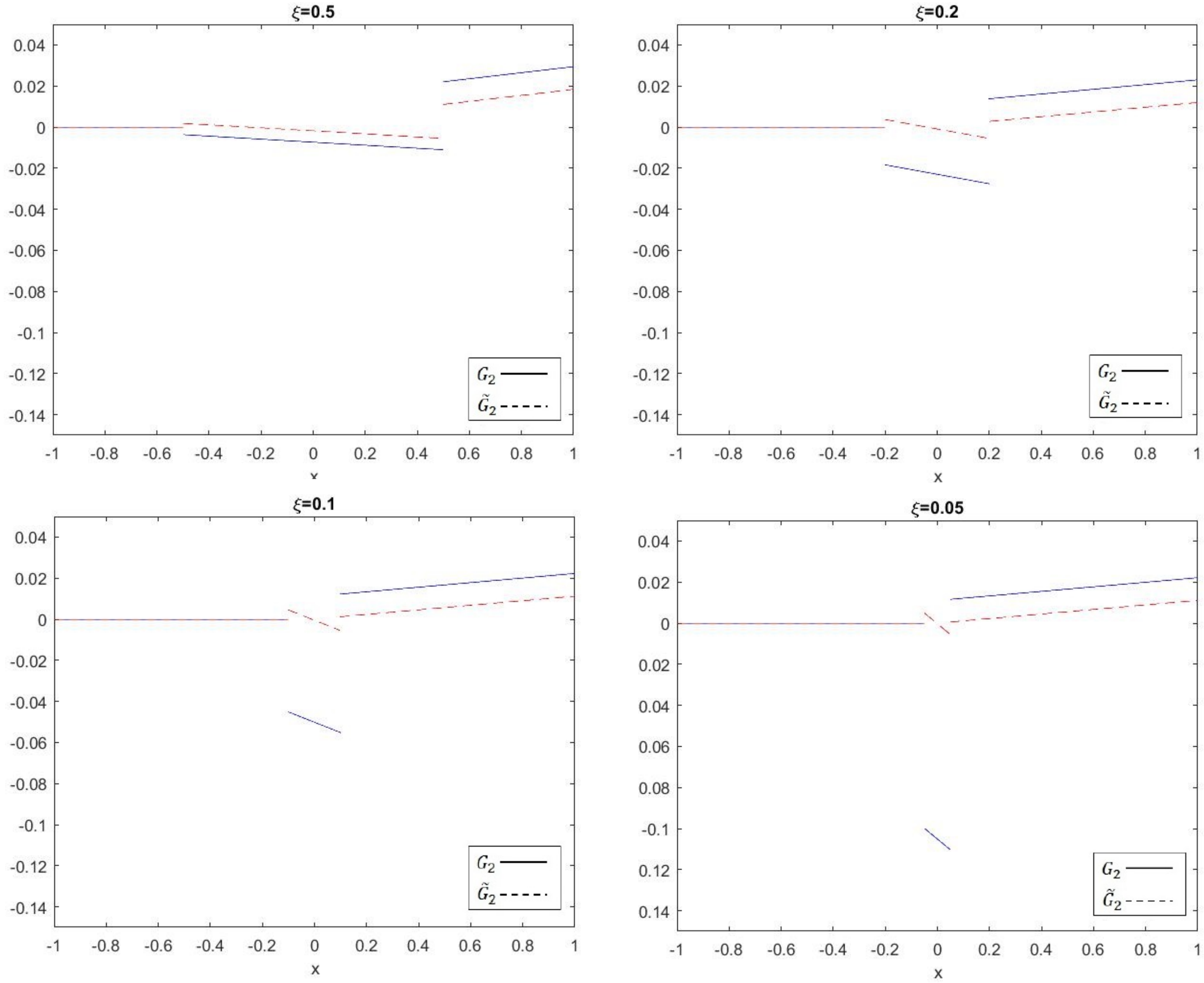
$$\lim_{\xi\to 0} \frac{-g^2}{(2\pi)^2} \frac{(1+x)}{\xi(1-x)} \ln \Lambda_\perp \to \delta(x)$$

$\widetilde{G}_2$



$$ig^2 4p^+ \frac{(x+\xi^2)}{(1-x)} \int \frac{d^2k_\perp dk^-}{(2\pi)^4} \frac{1}{[(k+\frac{\Delta}{2})^2 - m^2 + i\epsilon][(k-\frac{\Delta}{2})^2 - m^2 + i\epsilon]}$$

$$-\frac{g^2}{(2\pi)^2} \frac{(x+\xi^2)}{\xi(1-x)} \ln \Lambda_\perp.$$



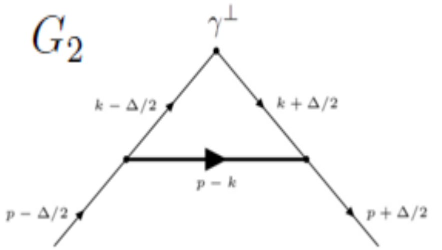
Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Finite

The ERBL region becomes a representation of δ(x)

What happens in different models ?

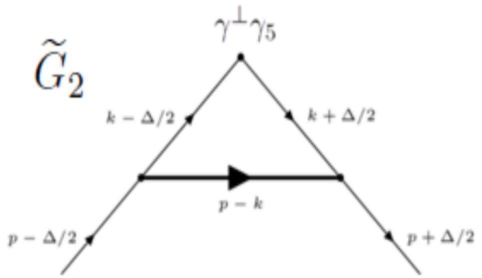
G_2 and $\widetilde{G}_2$ in Scalar Diquark Model

G_2

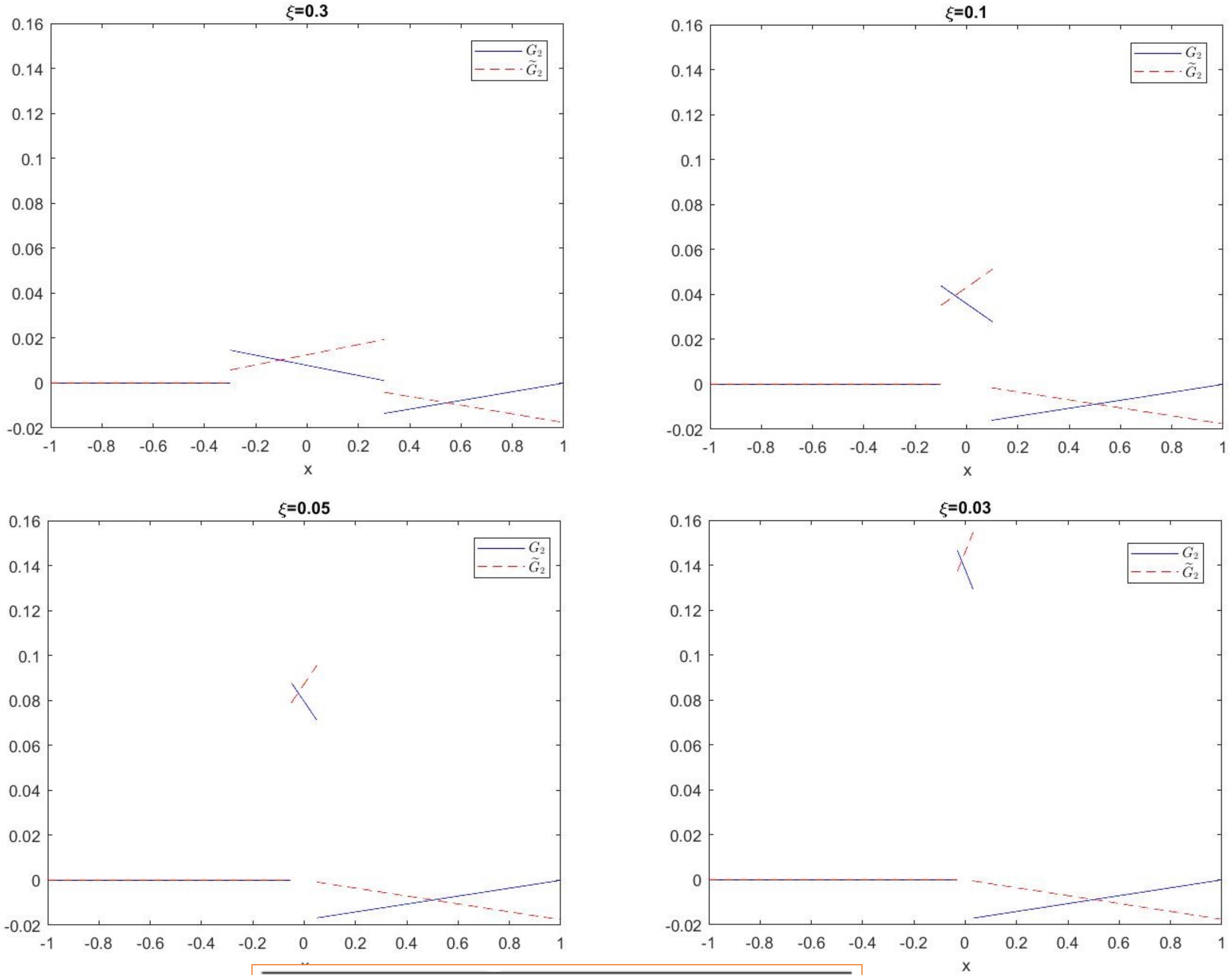


$$G_2 = \begin{cases} -\frac{g^2}{4\pi^2}\frac{(1-x)}{(1-\xi^2)}\ln\Lambda_\perp & \text{for} \quad \xi < x < 1, \\ -\frac{g^2}{16\pi^2}\frac{(2x+\xi-1)}{\xi(1+\xi)}\ln\Lambda_\perp & \text{for} \quad -\xi \le x \le \xi, \\ 0 & \text{for} \quad -1 < x < \xi. \end{cases}$$

$\widetilde{G}_2$



$$\widetilde{G}_2 = \begin{cases} \frac{g^2}{4\pi^2}\frac{(\ldots)}{(1-\xi^2)}\ln\Lambda_\perp & \text{for} \quad \xi < x < 1, \\ \frac{g^2}{16\pi^2}\frac{(2x-2\xi^2+\xi+1)}{\xi(1+\xi)}\ln\Lambda_\perp & \text{for} \quad -\xi \le x \le \xi, \\ 0 & \text{for} \quad -1 < x < \xi. \end{cases}$$



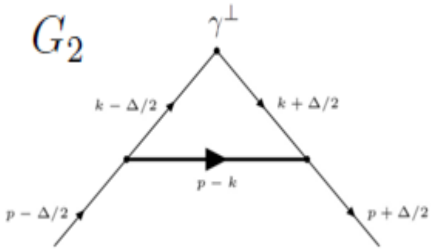
Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Divergent

The ERBL region becomes a representation of δ(x)

What happens in different models ?

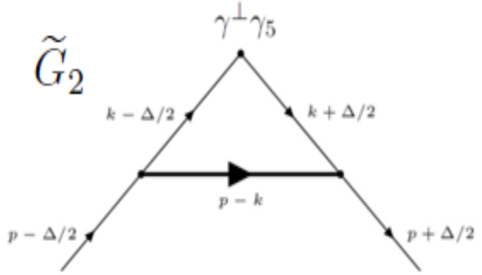
G_2 and $\widetilde{G}_2$ in Scalar Diquark Model

G_2

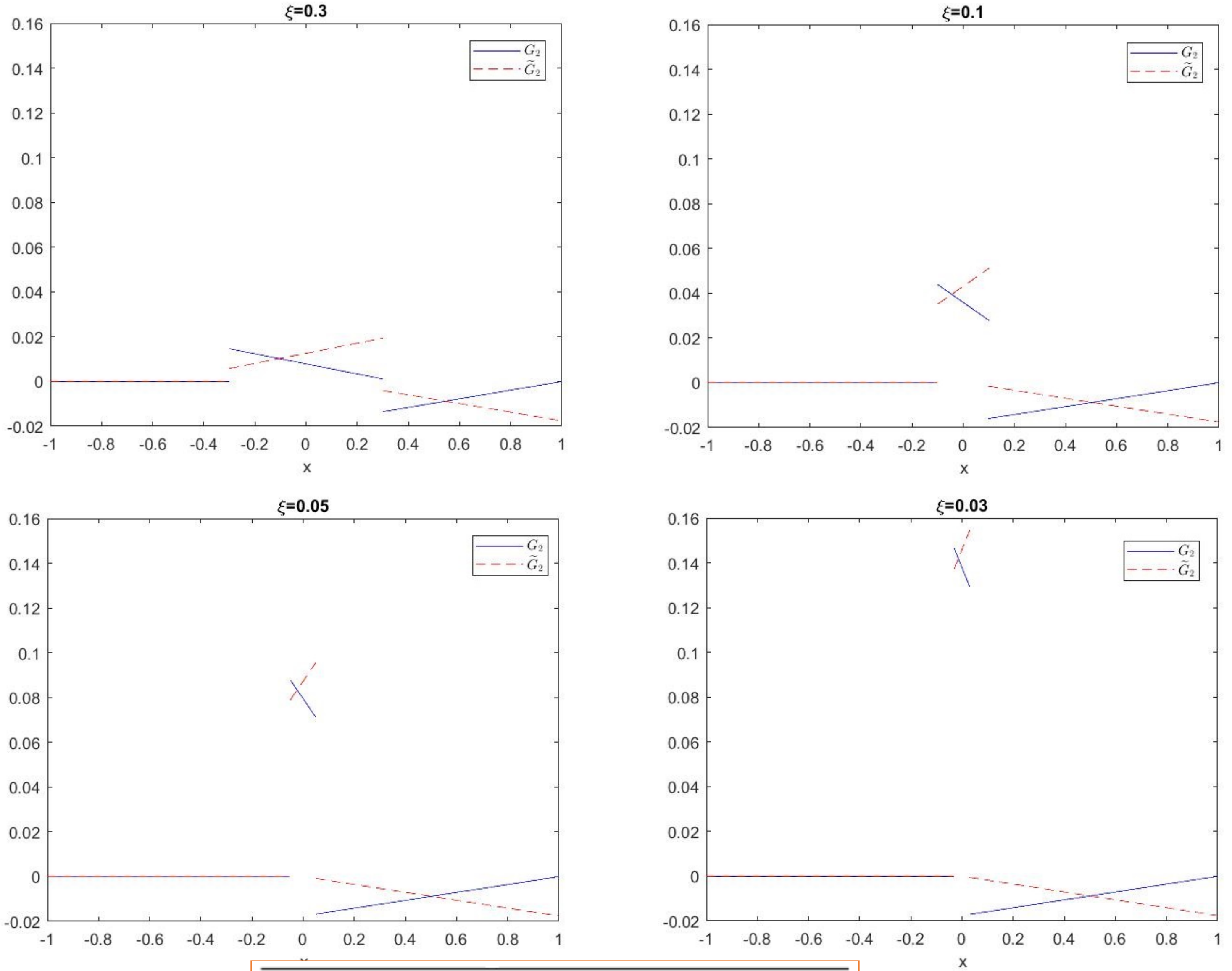


$$G_2 = \begin{cases} -\frac{g^2}{4\pi^2}\frac{(1-x)}{(1-\xi^2)}\ln\Lambda_\perp & \text{for} \quad \xi < x < 1, \\ -\frac{g^2}{16\pi^2}\frac{(2x+\xi-1)}{\xi(1+\xi)}\ln\Lambda_\perp & \text{for} \quad -\xi \le x \le \xi, \\ 0 & \text{for} \quad -1 < x < \xi. \end{cases}$$

$\widetilde{G}_2$



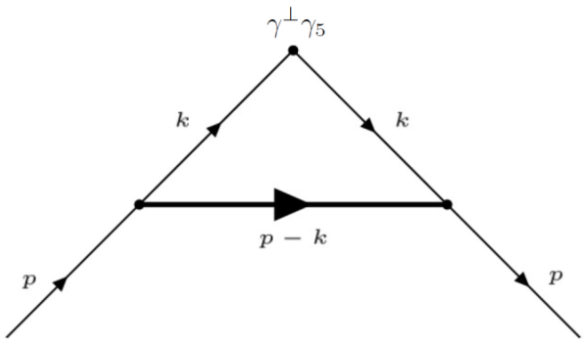
$$\widetilde{G}_2 = \begin{cases} \frac{\text{[illegible]}}{4\pi^2}\frac{\text{[illegible]}}{(1-\xi^2)}\ln\Lambda_\perp & \text{for} \quad \xi < x < 1, \\ \frac{g^2}{16\pi^2}\frac{(2x-2\xi^2+\xi+1)}{\xi(1+\xi)}\ln\Lambda_\perp & \text{for} \quad -\xi \le x \le \xi, \\ 0 & \text{for} \quad -1 < x < \xi. \end{cases}$$



$\widetilde{G}_2$ → ($\Delta = 0$) → g_2

Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Divergent

The forward limit: g_2 and g_2^{Quasi} in SDM



The parameterization

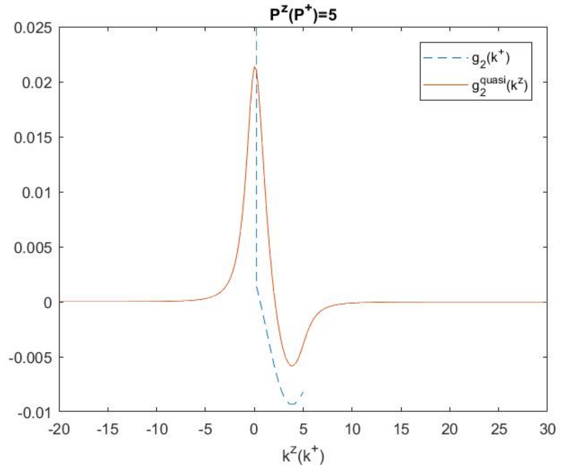
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S|\bar{q}(0)\gamma^\mu \gamma_5 q(\lambda n)|P, S\rangle = 2\{g_1(x)\hat{p}^\mu (S\cdot\hat{n}) + g_T(x) S_\perp^\mu + M^2 g_3(x)\hat{n}^\mu (S\cdot\hat{n})\}$$

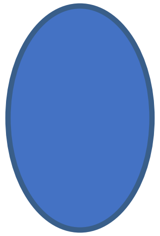
$g_T(x) \rightarrow g_1(x) + g_2(x)$

$$g_2^{quasi} \xrightarrow{P^z\to\infty} g_2$$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

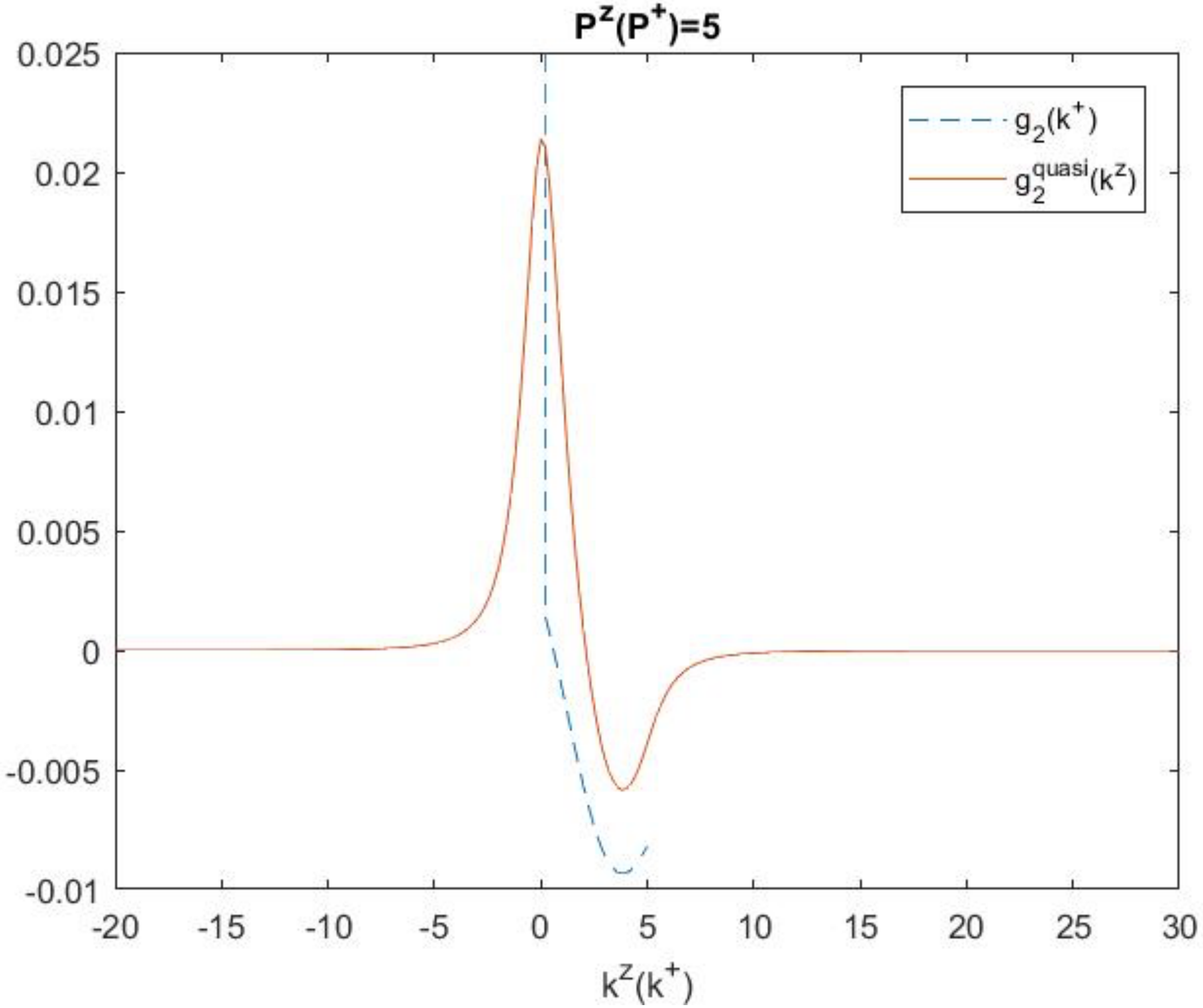
Twist-2	Twist-3
Independent of P^+	$1/P^+$

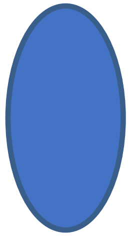




$P^z = P^+ = 5$

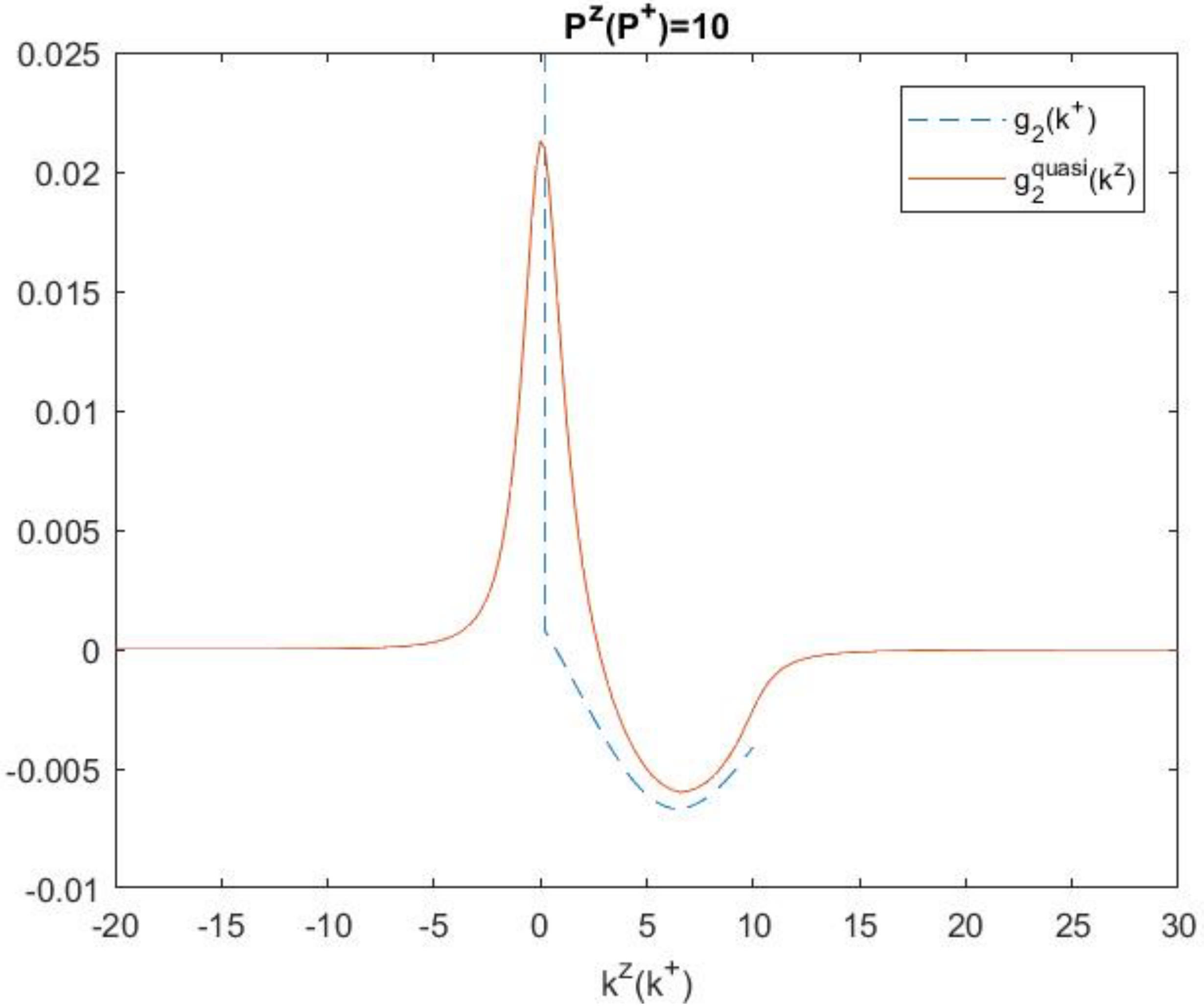
$g_2(\boldsymbol{k^+}),\ \ g_2{}^{quasi}(\boldsymbol{k^z})$

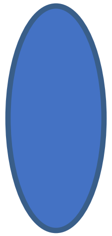




$P^z = P^+ = 10$

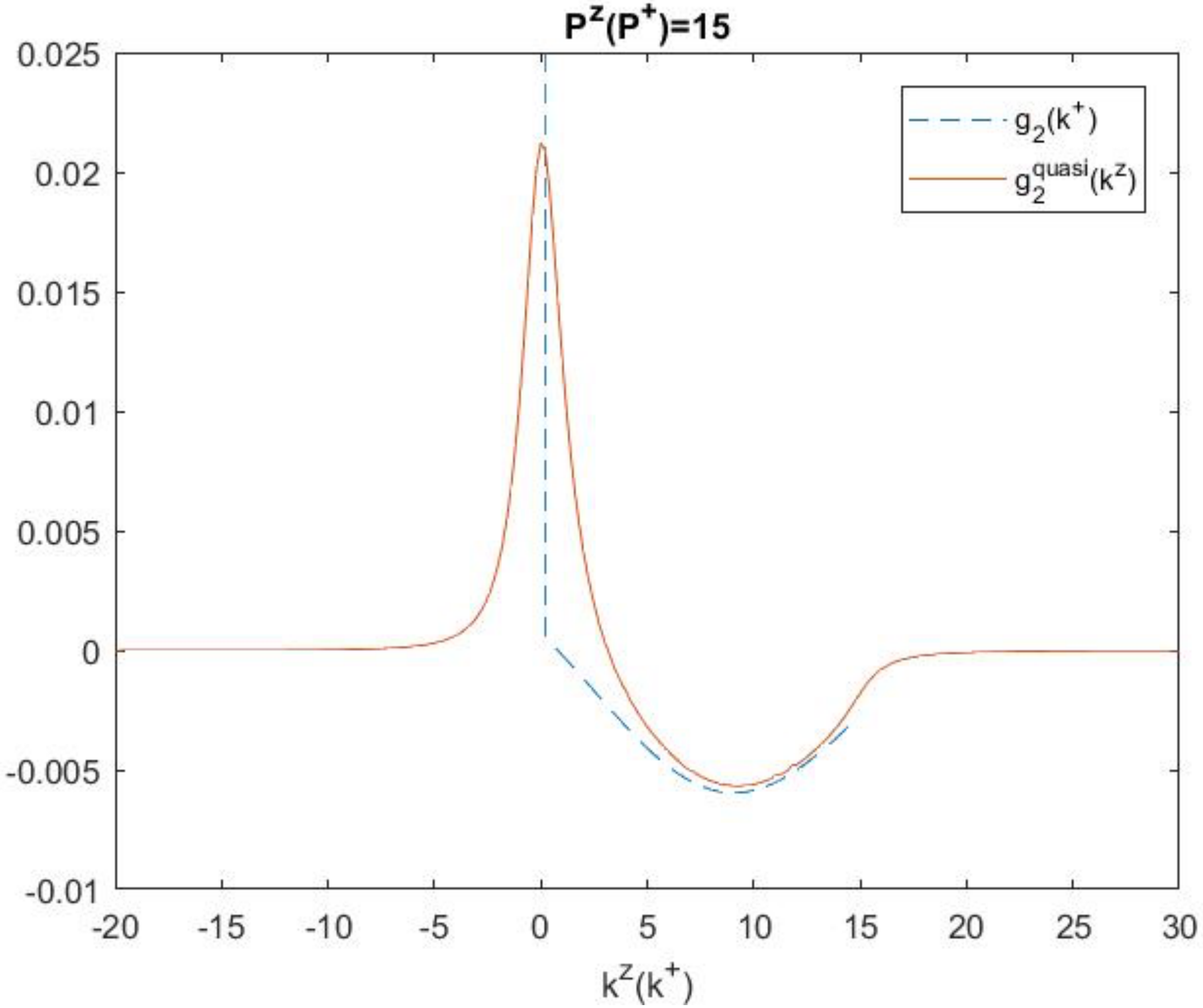
$$g_2(\boldsymbol{k^+}),\ \ g_2{}^{quasi}(\boldsymbol{k^z})$$





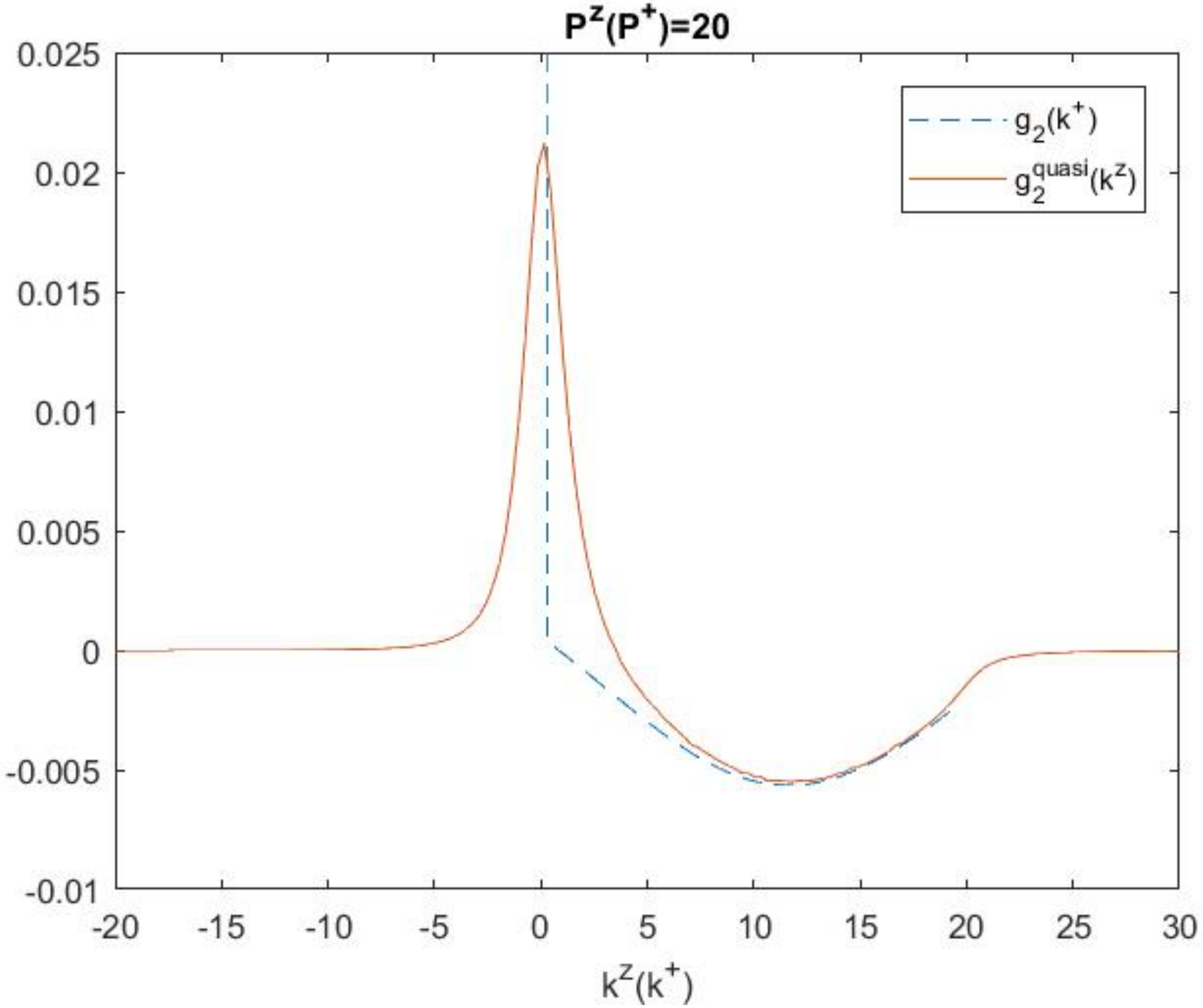
$P^z = P^+ = 15$

$g_2(\boldsymbol{k^+}),\ \ g_2{}^{quasi}(\boldsymbol{k^z})$



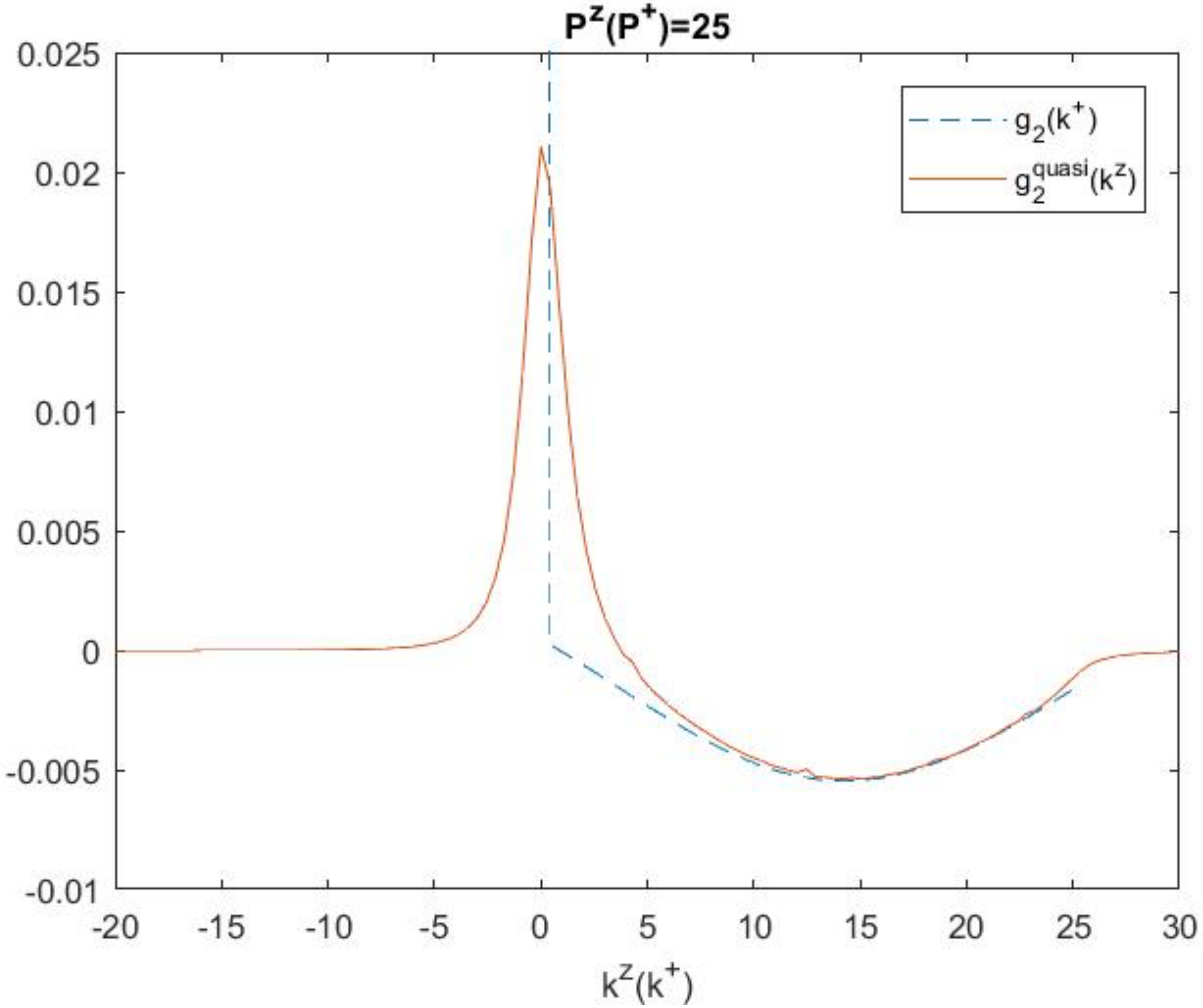
$P^z = P^+ = 20$

$g_2(\boldsymbol{k^+}),\ \ g_2{}^{quasi}(\boldsymbol{k^z})$

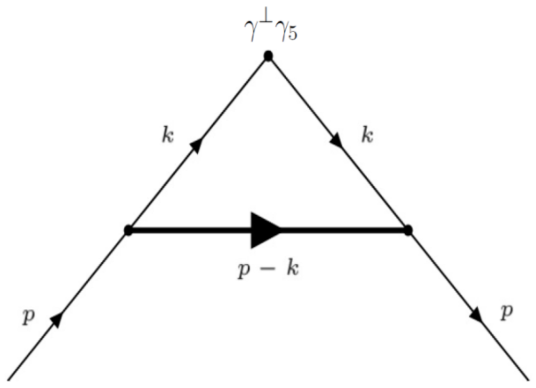


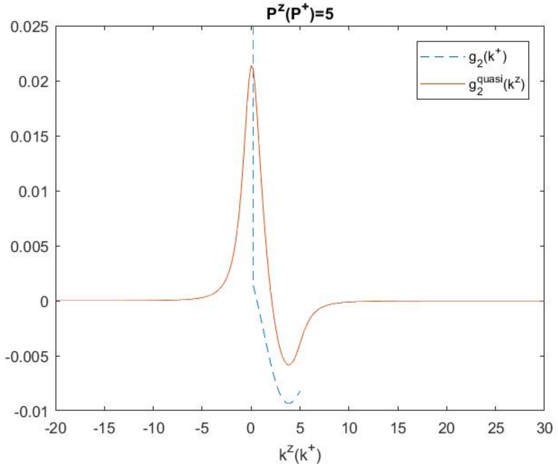
$P^z = P^+ = 25$

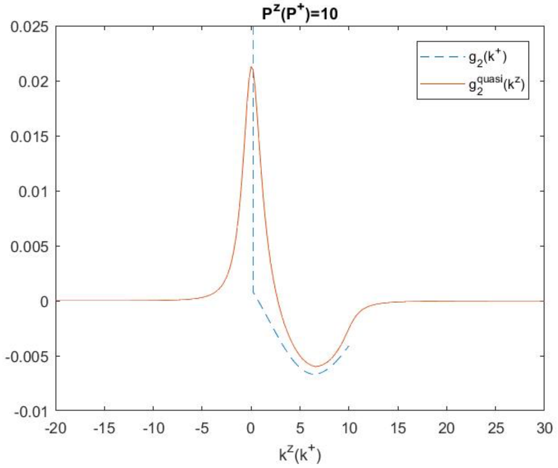
$g_2(\boldsymbol{k^+}),\ \ g_2{}^{quasi}(\boldsymbol{k^z})$

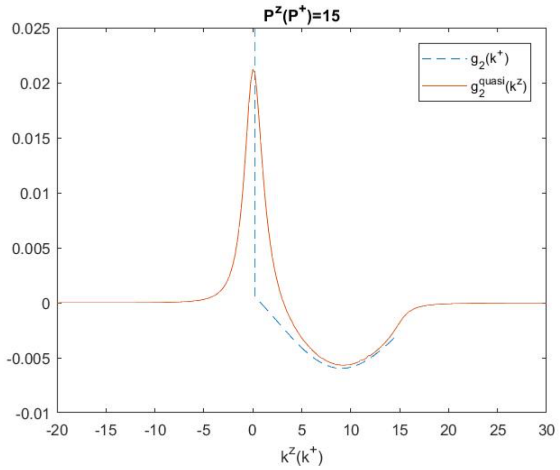


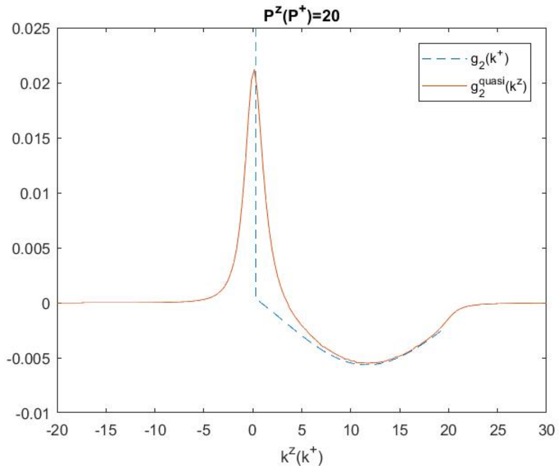
g_2 (k^+) and g_2^{Quasi} (k^z) in SDM

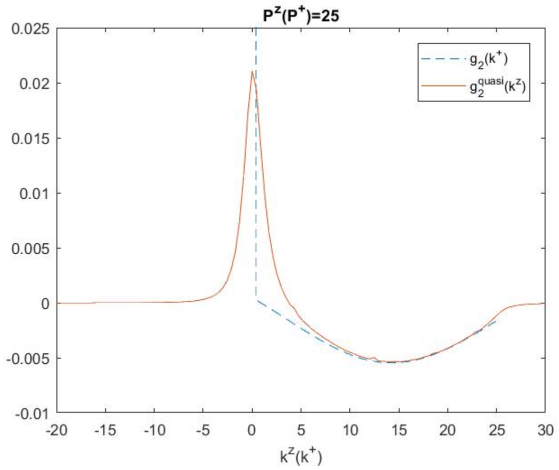








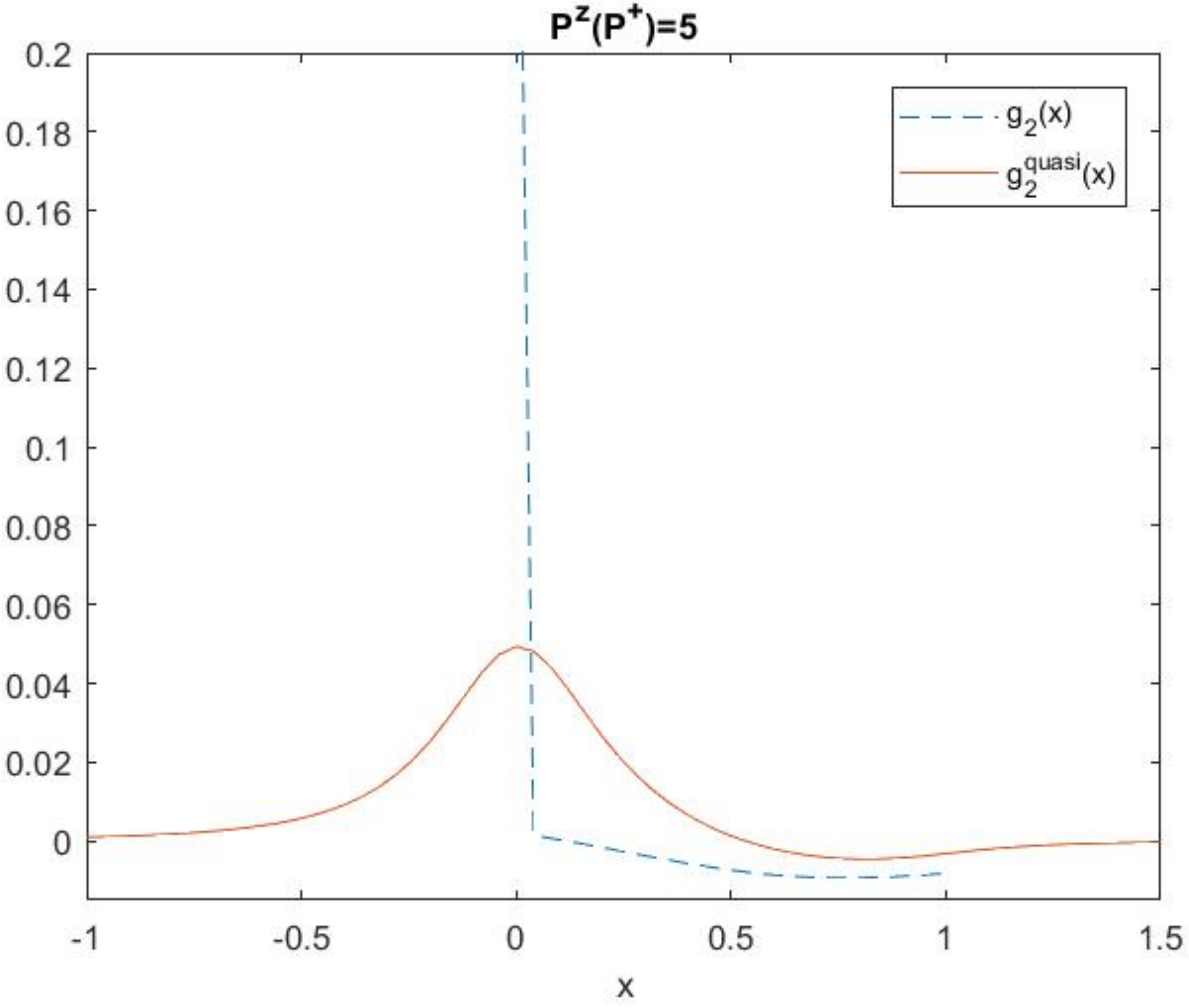




There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.

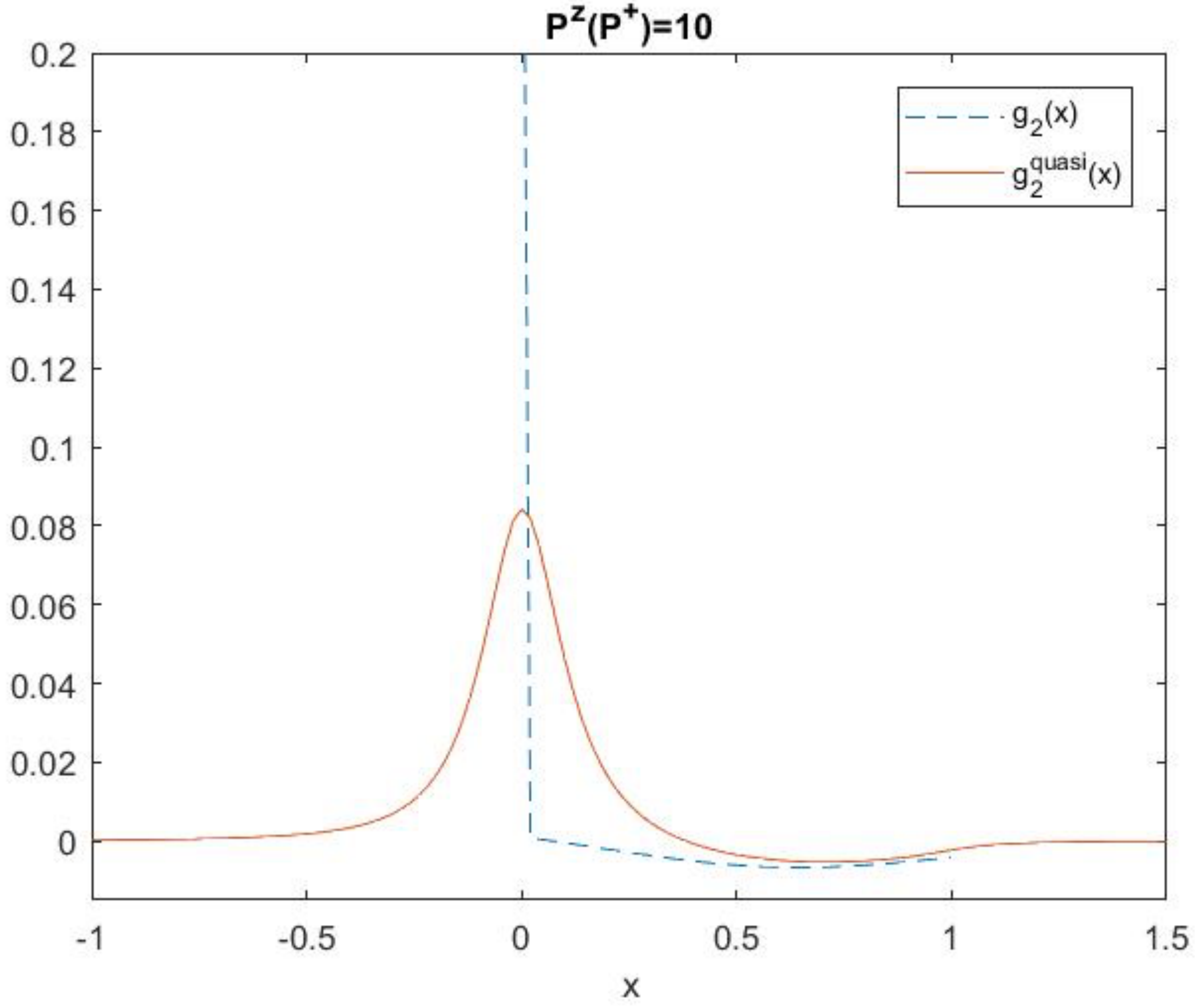
$P^z = P^+ = 5$

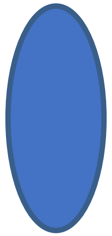
$g_2(x)$, $g_2{}^{quasi}(x)$



$P^z = P^+ = 10$

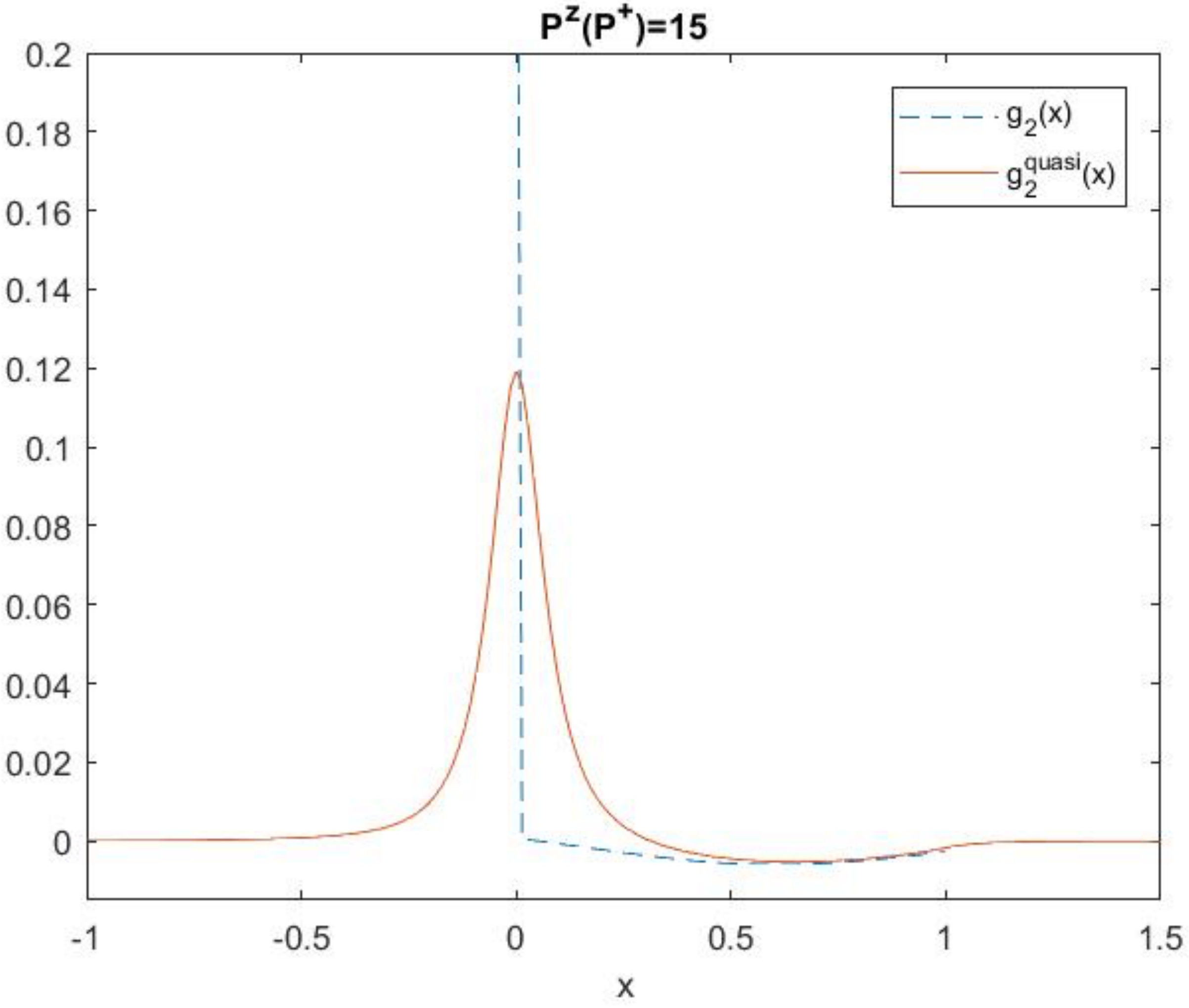
$g_2(\boldsymbol{x}),\ \ g_2{}^{quasi}(\boldsymbol{x})$





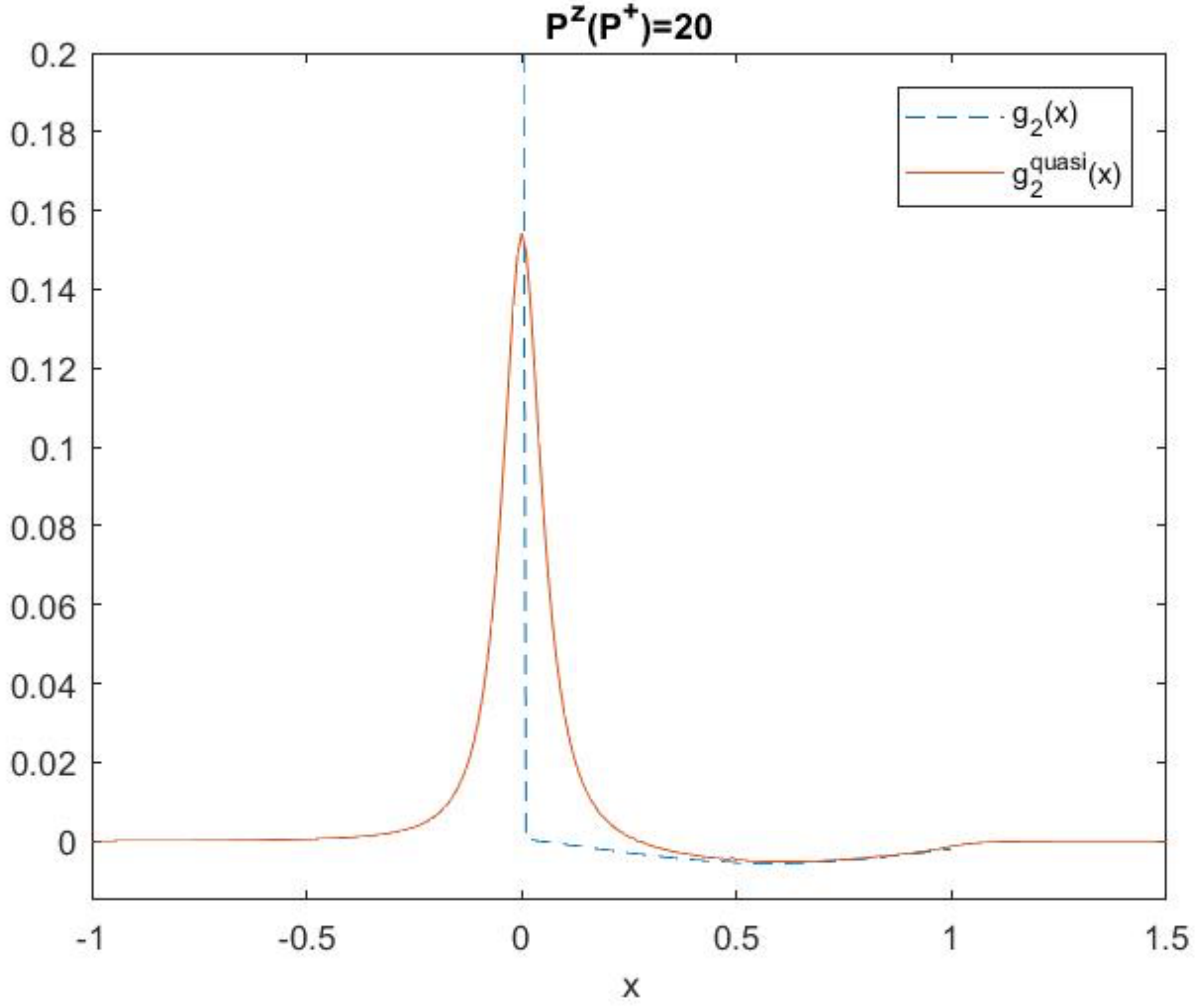
$P^z = P^+ = 15$

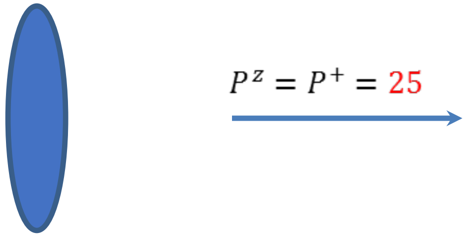
$g_2(x)$, $g_2{}^{quasi}(x)$



$P^z = P^+ = 20$

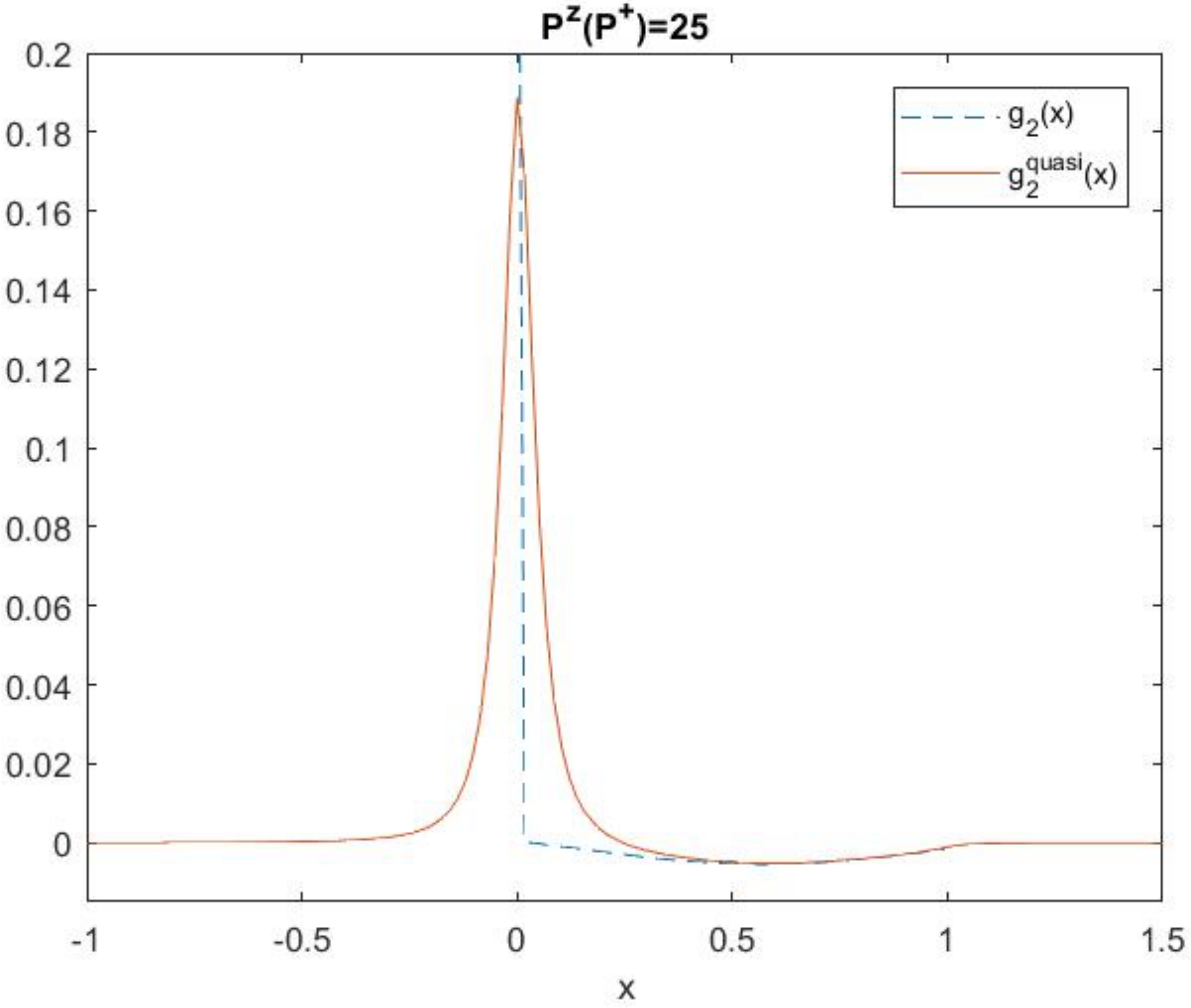
$g_2(x)$, $g_2{}^{quasi}(x)$



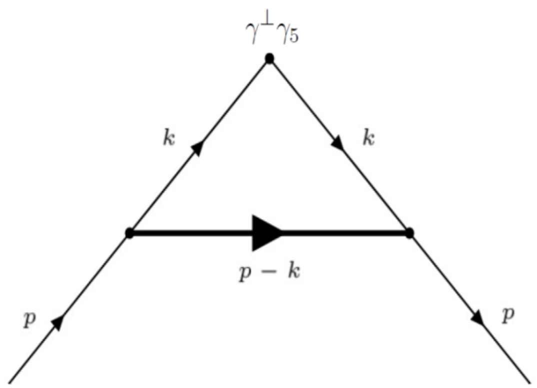


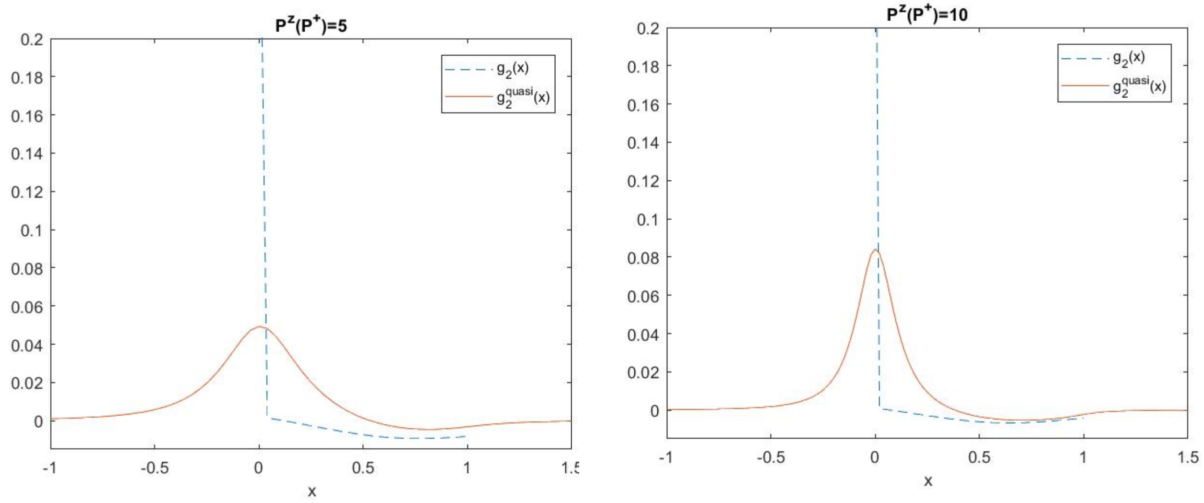
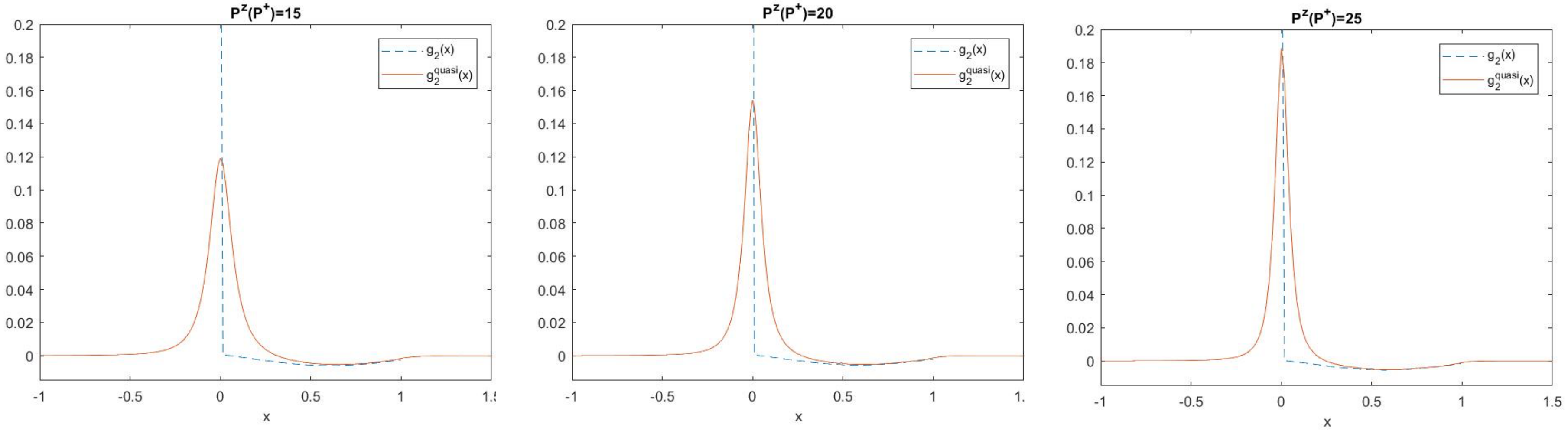
$P^z = P^+ = 25$

$g_2(\boldsymbol{x}),\ \ g_2{}^{quasi}(\boldsymbol{x})$



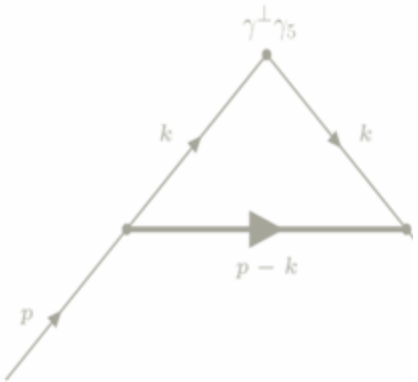
$g_2(x)$ & $g_2^{quasi}(x)$ in scalar di-quark model

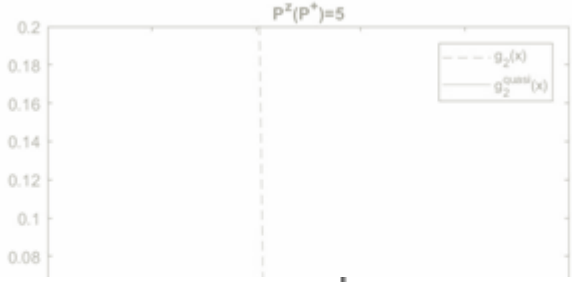


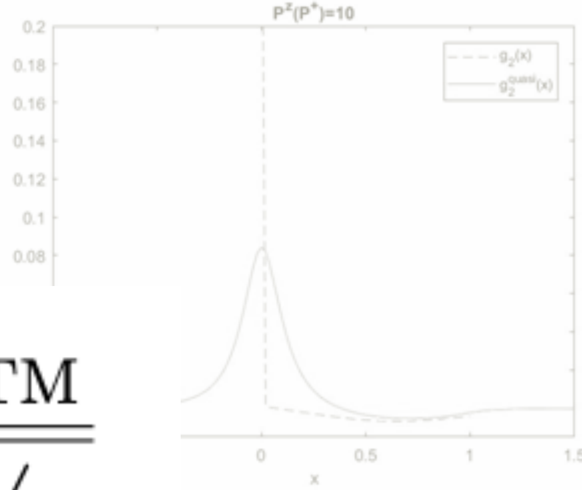



There is a singularity at x=0

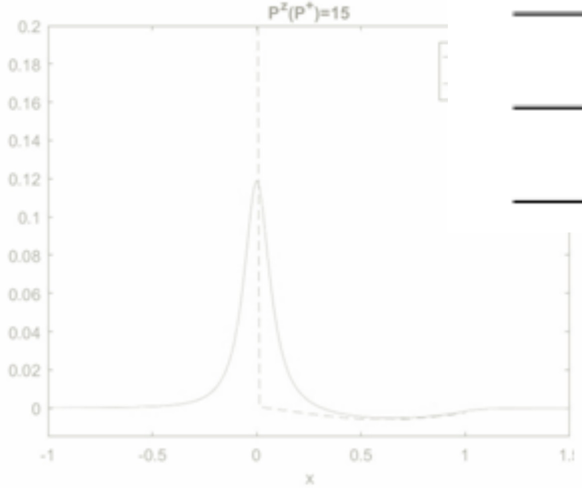
$g_2(x)$ & $g_2{}^{quasi}(x)$ in scalar di-quark model

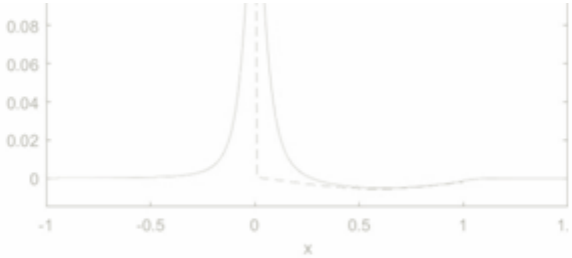


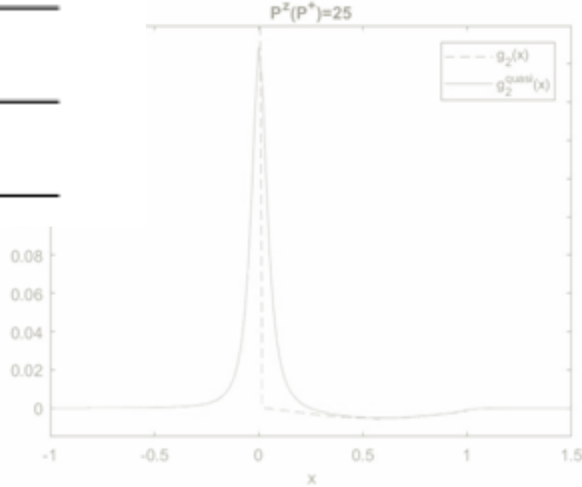




Twist-3 PDF	SDM	QTM
$e(x)$	√	√
$h_L(x)$	√	√
$g_2(x)$	√	×

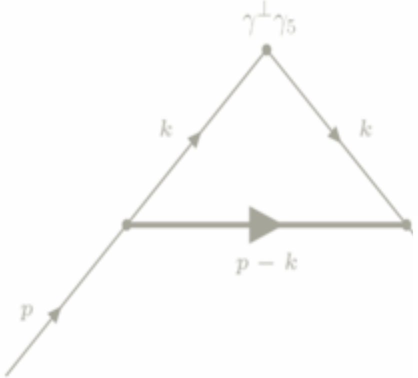


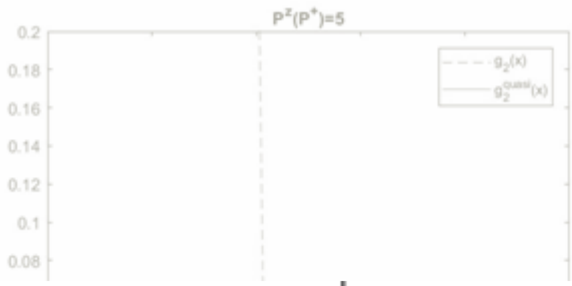


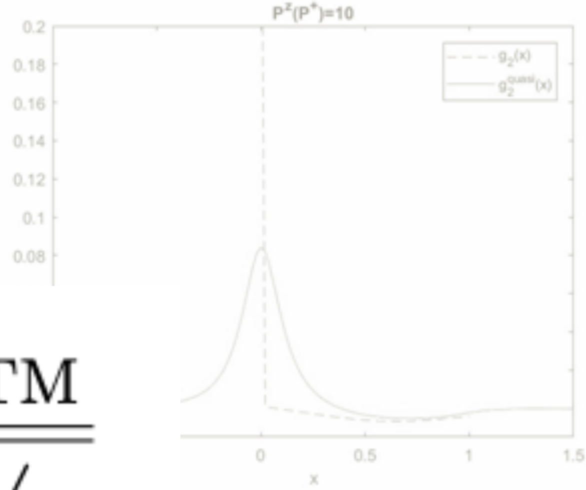


There is a singularity at x=0

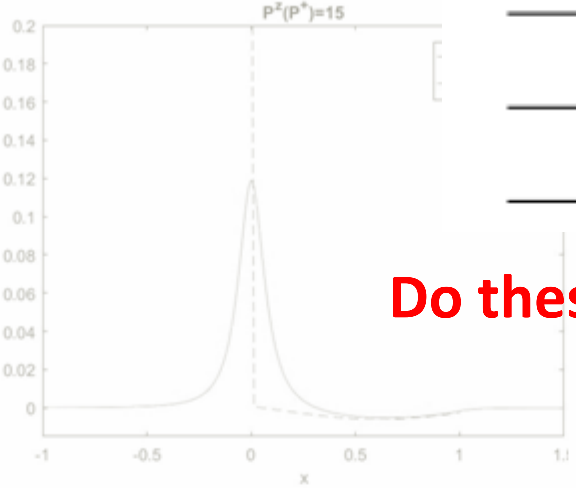
$g_2(x)$ & $g_2{}^{quasi}(x)$ in scalar di-quark model

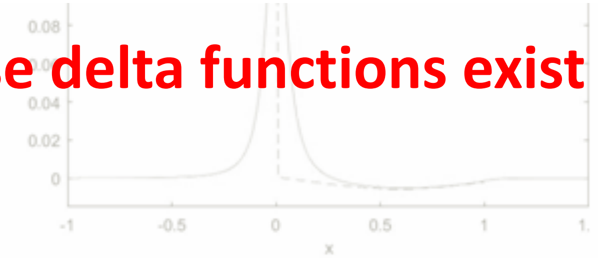


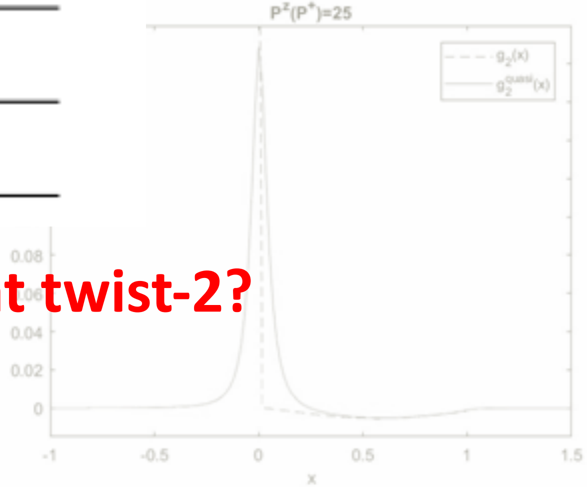




Twist-3 PDF	SDM	QTM
$e(x)$	√	√
$h_L(x)$	√	√
$g_2(x)$	√	×

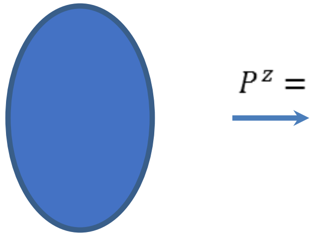






Do these delta functions exist at twist-2?

There is a singularity at x=0

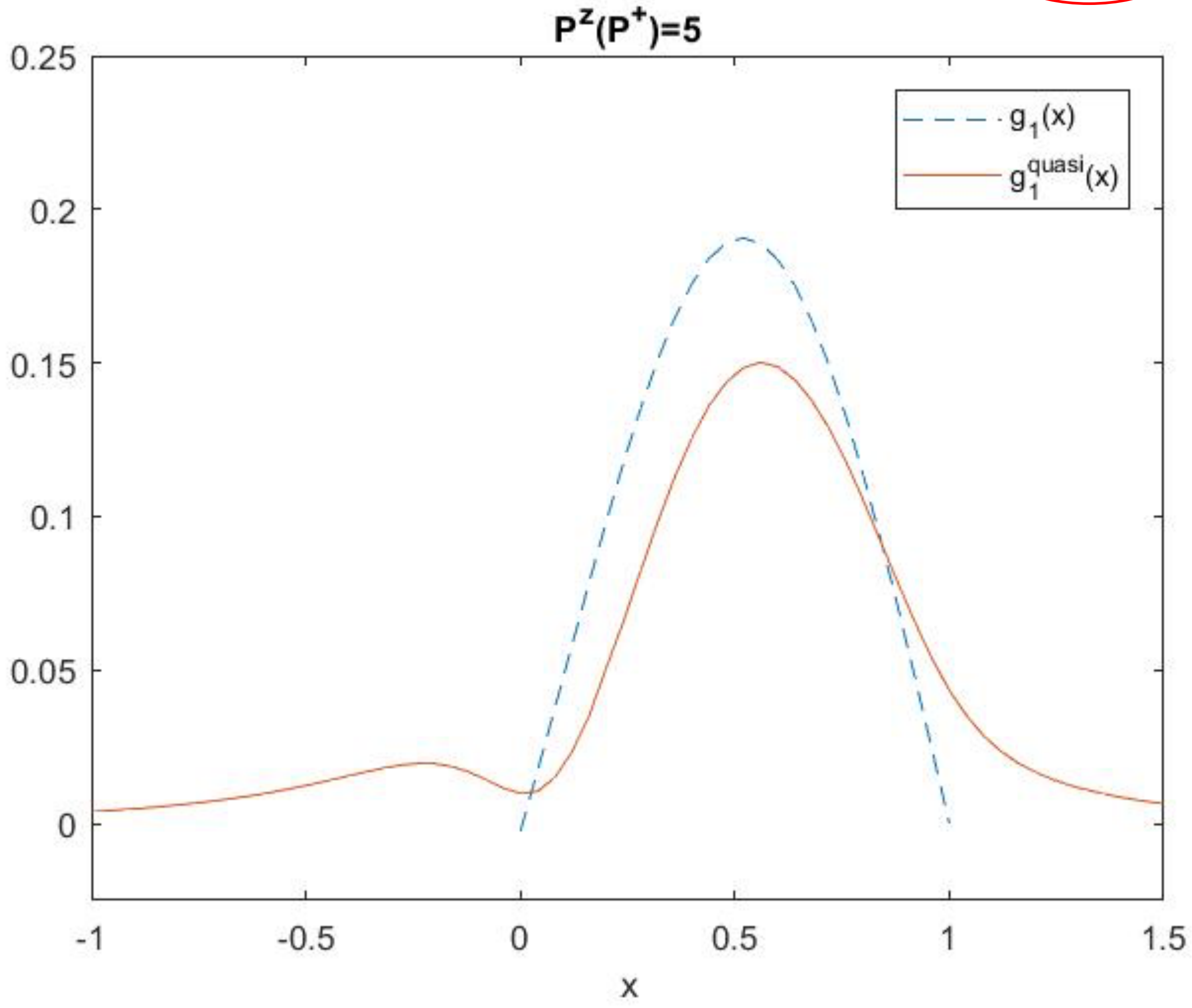


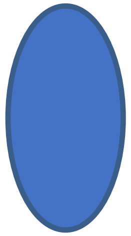
$P^z = P^+ = 5$

$$g_1(x),\ \ g_1{}^{quasi}(x)$$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

Twist-2	Twist-3
Independent of P^+	$1/P^+$



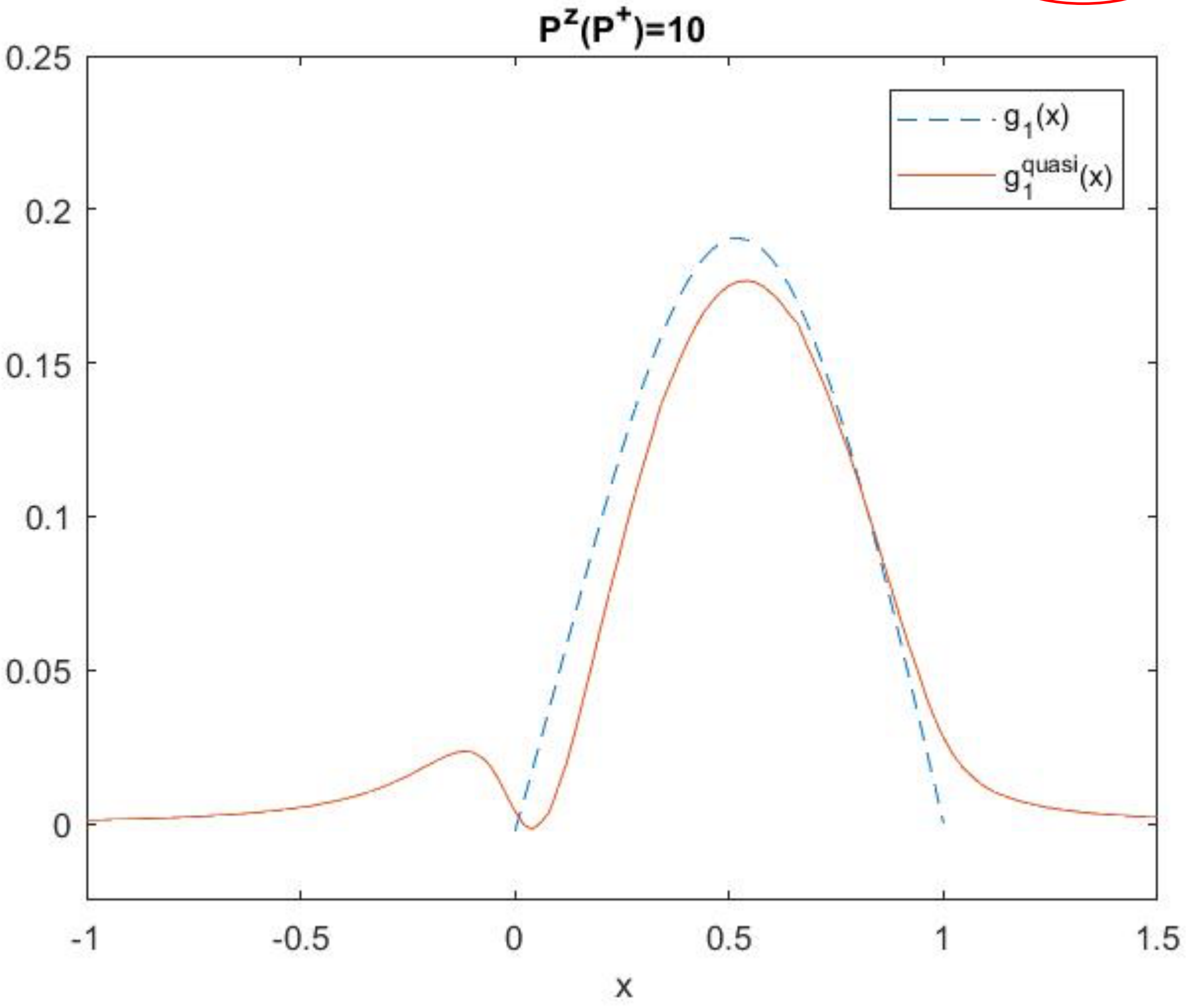


$P^z = P^+ = 10$

$g_1(x), \; g_1^{quasi}(x)$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

Twist-2	Twist-3
Independent of P^+	$1/P^+$

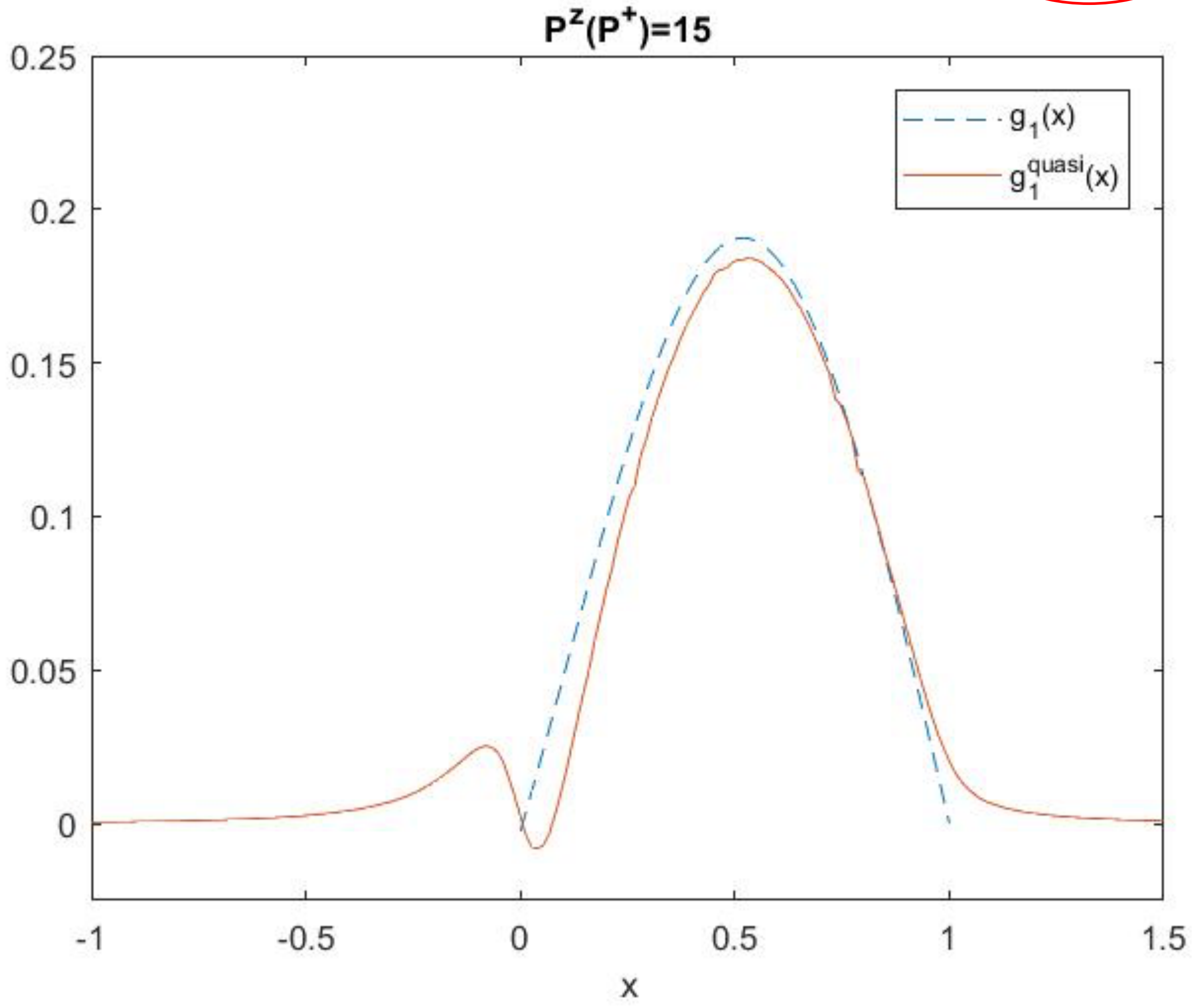


$P^z = P^+ = 15$

$g_1(x), \; g_1{}^{quasi}(x)$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

Twist-2	Twist-3
Independent of P^+	$1/P^+$

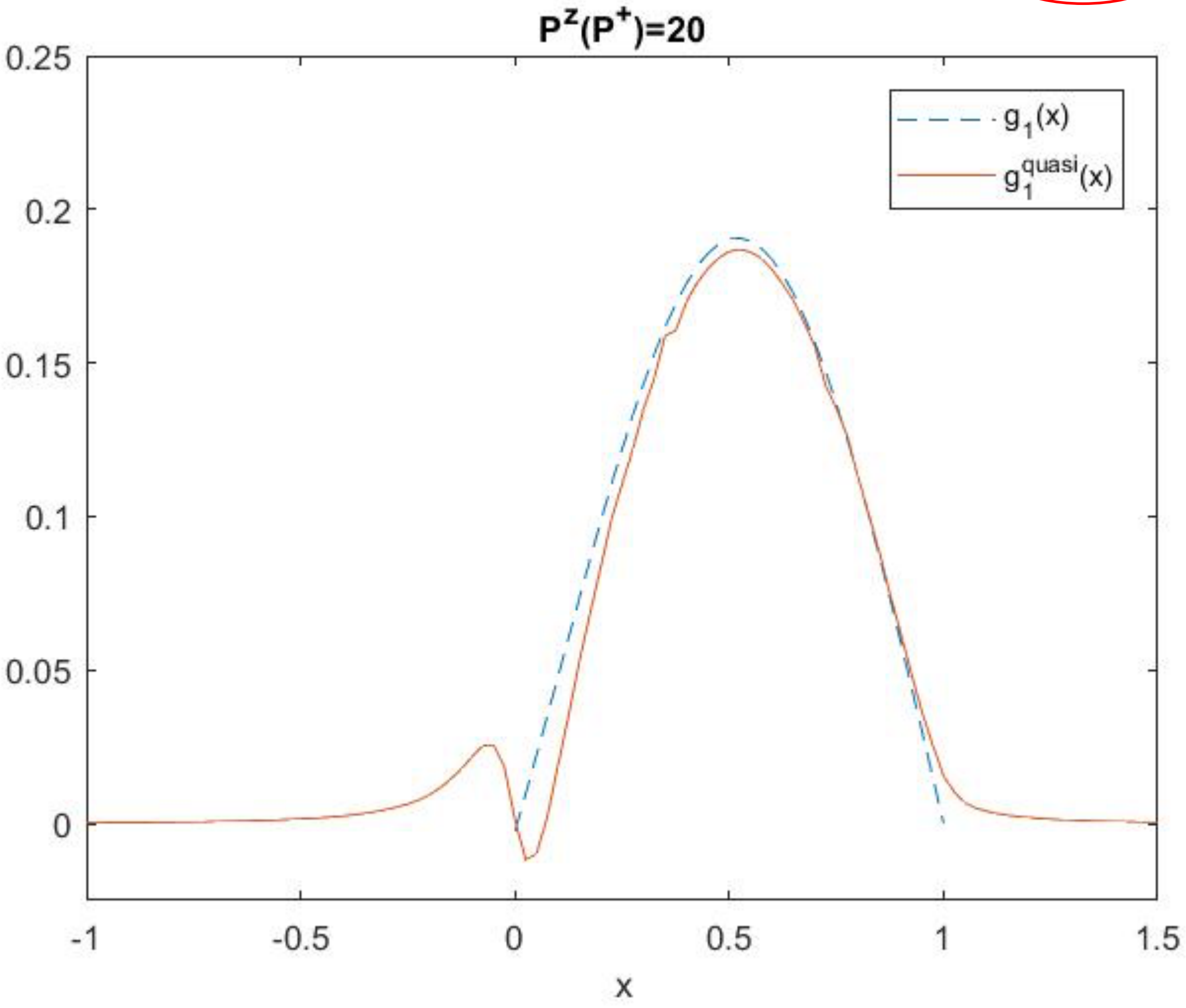


$P^z = P^+ = 20$

$g_1(x), \ g_1{}^{quasi}(x)$

Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

Twist-2	Twist-3
Independent of P^+	$1/P^+$



$P^z = P^+ = 20$

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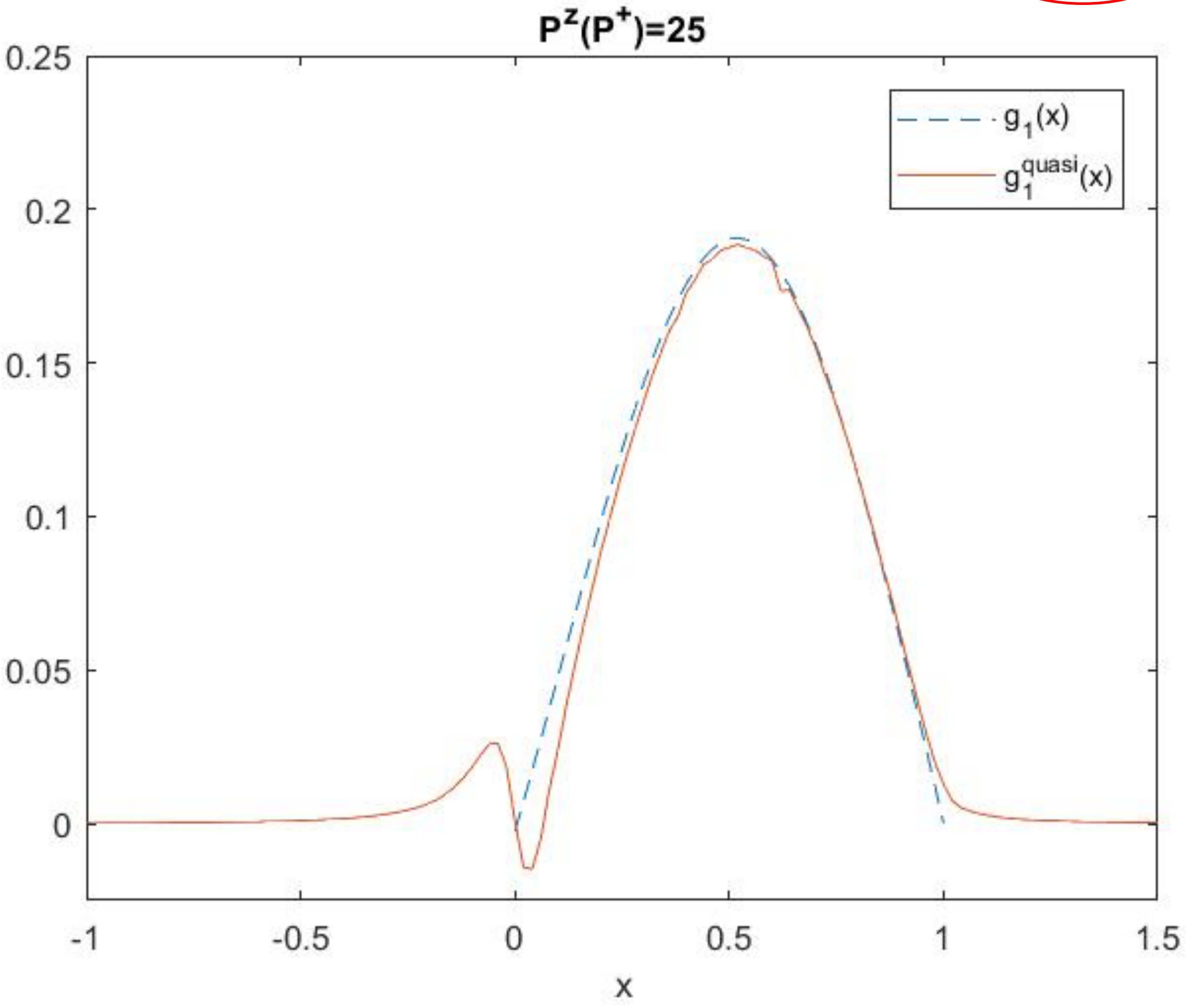
Behavior of twist-2 and twist-3 distributions under longitudinal momentum boost, P^+.

Twist-2	Twist-3
Independent of P^+	$1/P^+$



Singularities in twist-3 quark distributions

✓: There is a δ(x)
✗: There is no δ(x)

Twist-2 PDF	SDM	QTM
$f_1(x)$	×	×
$g_1(x)$	×	×
$h_1(x)$	×	×

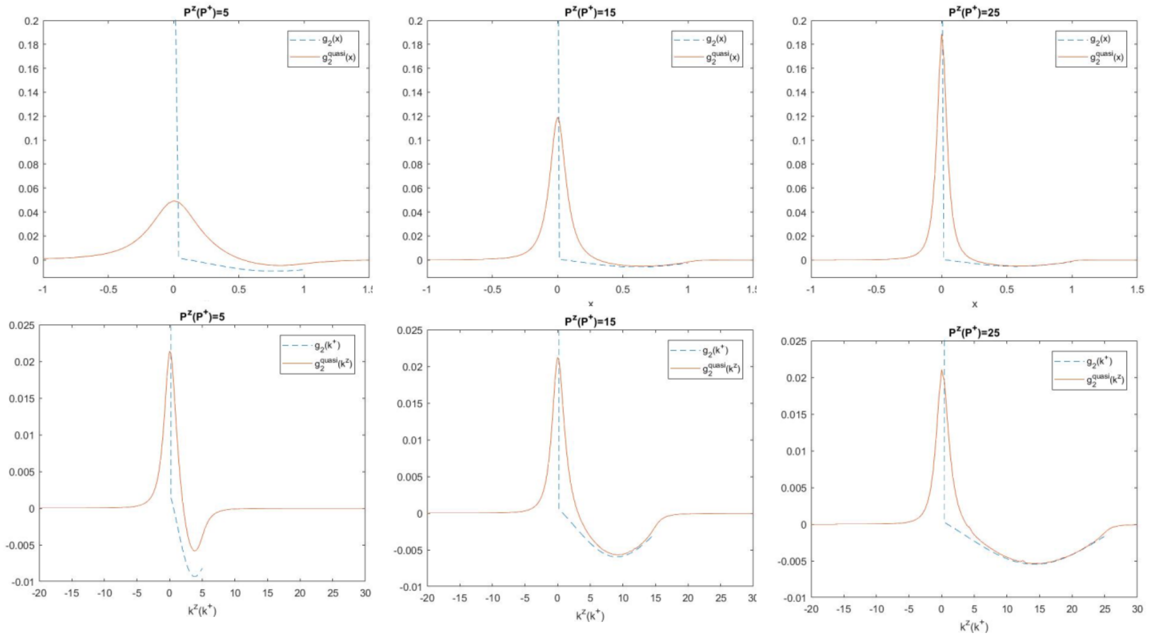
Twist-3 PDF	SDM	QTM
$e(x)$	√	√
$h_L(x)$	√	√
$g_2(x)$	√	×

Aslan, Burkardt,
Singularities in Twist-3 Quark Distributions, 2018.

Burkardt, Koike,
Violation of sum rules for twist three parton distributions in QCD, 2001.

- **At twist-3 there is something that does not exist in twist-2: There are delta functions.**

- **We identify these delta functions with momentum components in the nucleon state that do not scale as the nucleon is boosted to the infinite momentum.**



Decomposition of twist-3 $h_L(x) = h_L^{WW}(x) + h_L^m(x) + h_L^3(x)$

δ(x) term appears not only in h_L^m but also in h_L^3

Burkardt & Koike,
Violation of Sum Rules for Twist 3 Parton Distributions in QCD, 2001

Regularization of the singularities

The effects of different regularization schemes on the δ(x)

$\delta(x)$ remains	$\delta(x)$ is recovered
Transverse momentum cut off Dimensional regularization Adding form factors Adding axial diquark contribution	Pauli-Villars regularization

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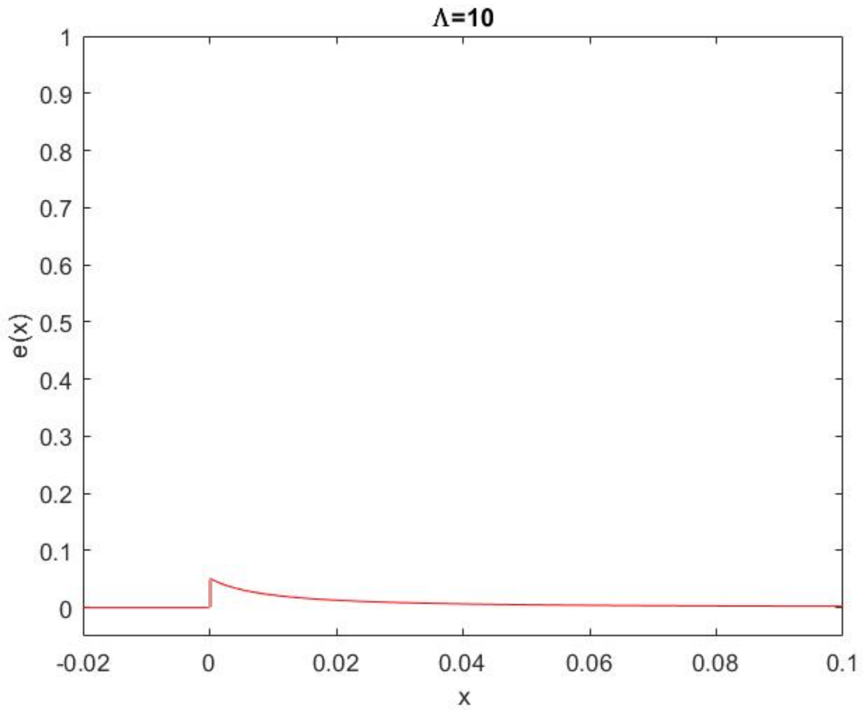
The effects of different regularization schemes on the δ(x)

$\delta(x)$ remains	$\delta(x)$ is recovered
Transverse momentum cut off	Pauli-Villars regularization
Dimensional regularization	
Adding form factors	
Adding axial diquark contribution	



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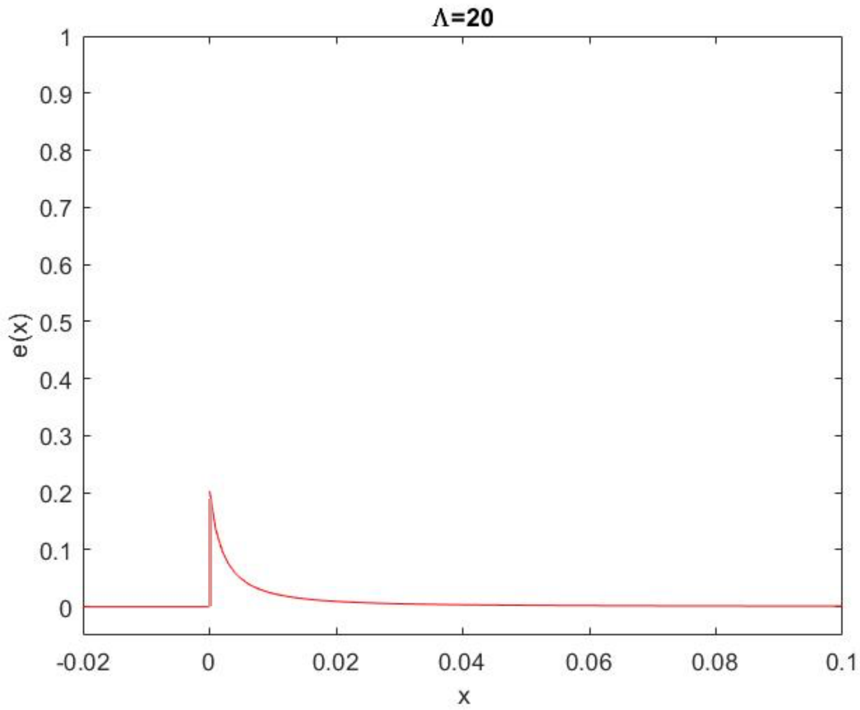

Regularization of the singularities

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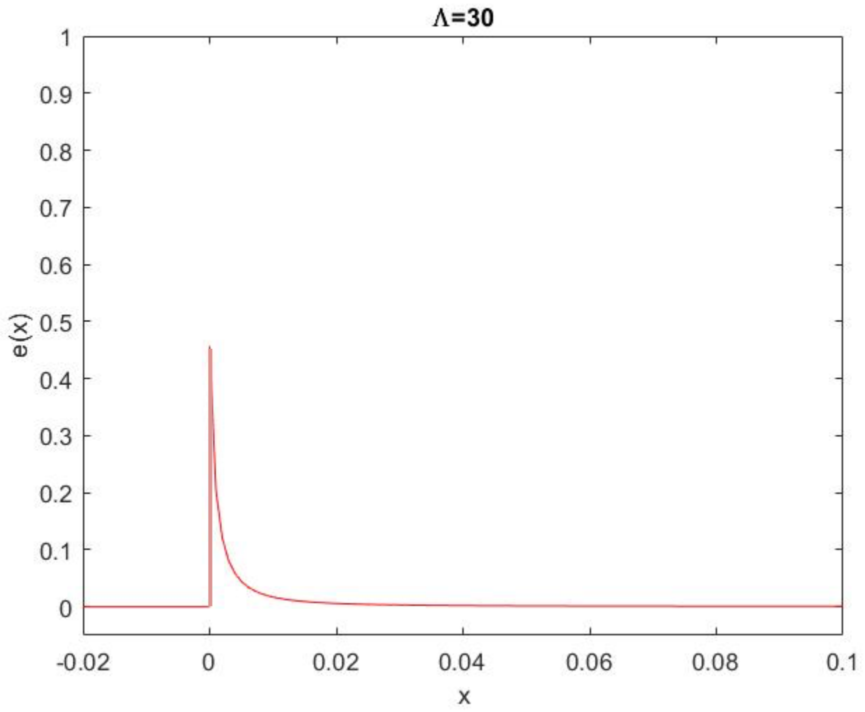
Adding form factors	
Adding axial diquark contribution	



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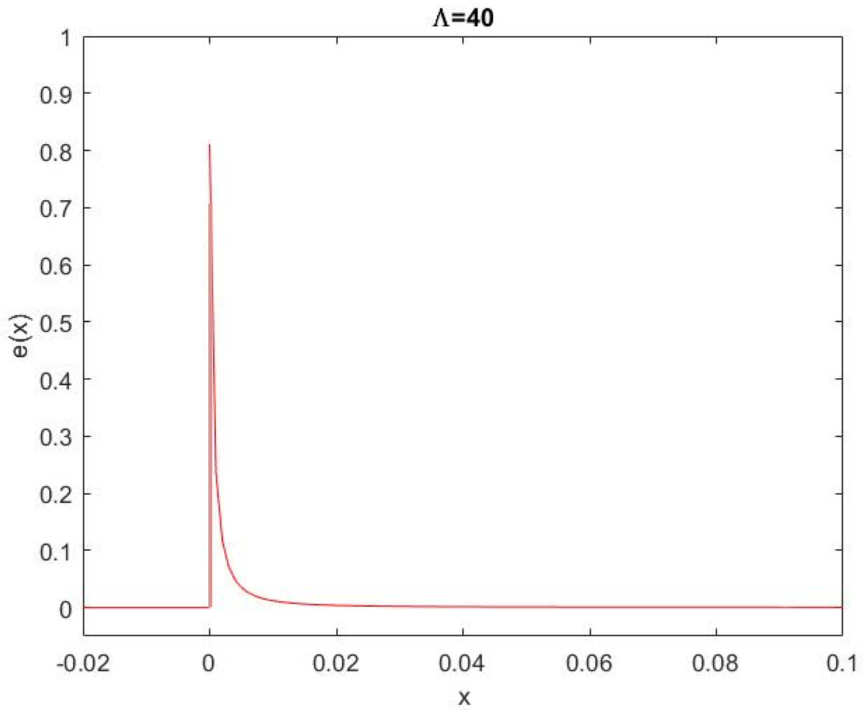

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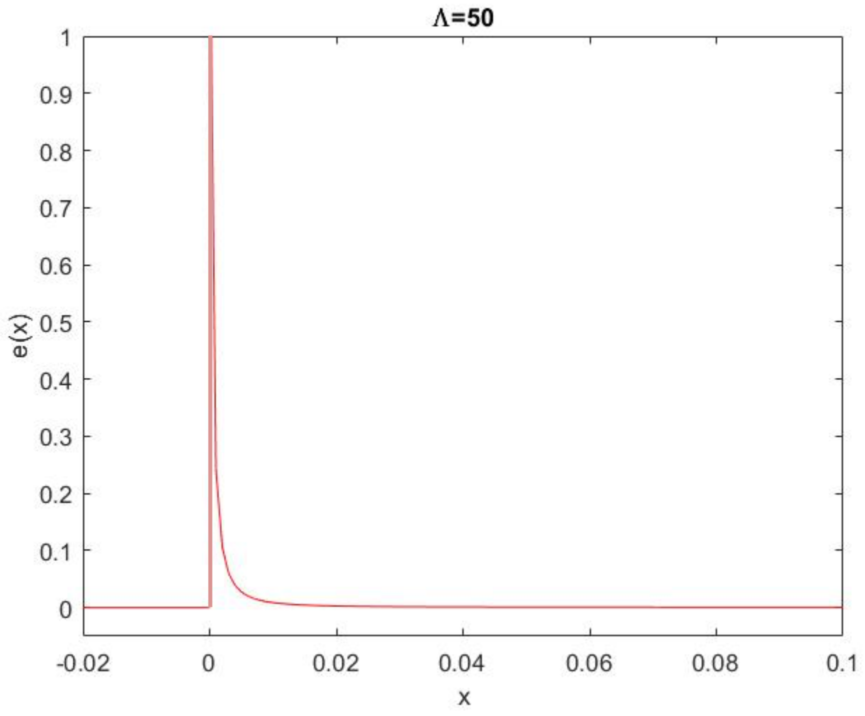


-- What happens if twist-3 distributions involve a $\delta(x)$?

-- Some sum rules are violated if we don't take it into account.

Lorentz invariance of twist-3 GPDs

$$\int_{-1}^{1} dx G_i(x,\xi,\Delta) = 0, \qquad \int_{-1}^{1} dx \widetilde{G}_i(x,\xi,\Delta) = 0.$$

$$\lim_{\epsilon\to 0}\int_{-1}^{\epsilon} dx G_i(x,\xi=0,\Delta) + \lim_{\epsilon\to 0}\int_{\epsilon}^{1} dx G_i(x,\xi=0,\Delta) \neq 0,$$

$$\lim_{\epsilon\to 0}\int_{-1}^{\epsilon} dx \widetilde{G}_i(x,\xi=0,\Delta) + \lim_{\epsilon\to 0}\int_{\epsilon}^{1} dx \widetilde{G}_i(x,\xi=0,\Delta) \neq 0.$$

In SDM the divergent part of G_2 was calculated as
$$G_2 = \begin{cases} -\dfrac{g^2}{4\pi^2}\dfrac{(1-x)}{(1-\xi^2)}\ln\Lambda_\perp & \text{for} \quad \xi < x \leq 1, \\ -\dfrac{g^2}{16\pi^2}\dfrac{(2x+\xi-1)}{\xi(1+\xi)}\ln\Lambda_\perp & \text{for} \quad -\xi \leq x \leq \xi, \\ 0 & \text{for} \quad -1 < x < \xi. \end{cases}$$

$$\int_{-1}^{1} dx\ G_2 = -\frac{g^2}{16\pi^2}\int_{-\xi}^{\xi} dx\ \frac{(2x+\xi-1)}{\xi(1+\xi)}\ln\Lambda_\perp - \frac{g^2}{4\pi^2}\int_{\xi}^{1} dx\ \frac{(1-x)}{(1-\xi^2)}\ln\Lambda_\perp = 0$$

The Lorentz invariance of G_2 is satisfied ✓

-- **What happens if twist-3 distributions involve a $\delta(x)$?**

-- **Some sum rules are violated if we don't take it into account.**

Lorentz invariance of twist-3 GPDs

$$\int_{-1}^{1} dx G_i(x,\xi,\Delta) = 0, \qquad \int_{-1}^{1} dx \widetilde{G}_i(x,\xi,\Delta) = 0.$$

$$\int_{-1}^{1} dx g_1(x) = \int_{-1}^{1} dx g_T(x)$$

$$\int_{-1}^{1} dx h_1(x) = \int_{-1}^{1} dx h_L(x)$$

$$\int_{-1}^{1} dx e(x) = \frac{1}{2M}\langle p|\overline{\psi}(0)\psi(0)|p\rangle = \frac{d}{dm}M$$

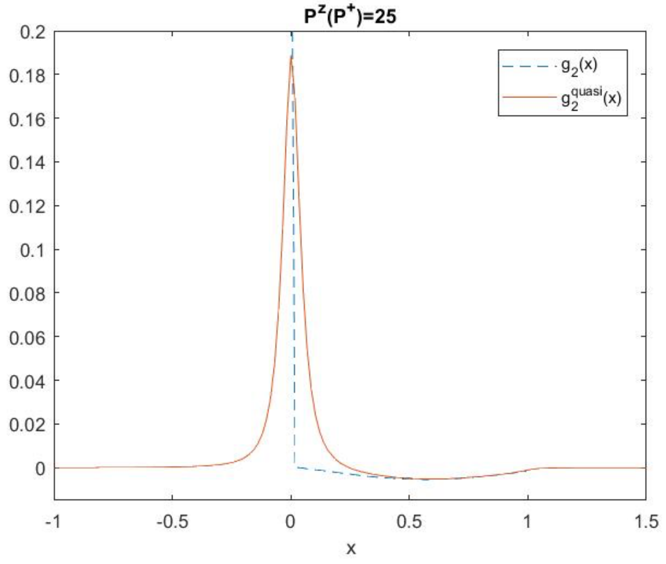
If one tries to confirm these sum rules experimentally by drawing conclusions from the behavior <u>near x=0</u> about the behavior <u>at x=0</u> they might claim that the sum rules are violated.

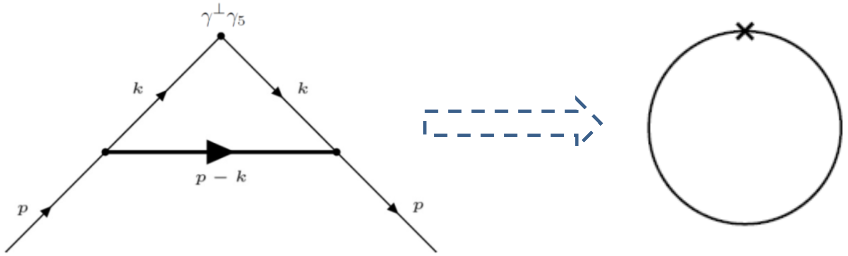
The origin of δ(x)

$$g_T(x) = ig^2 \int \frac{d^4k}{(2\pi)^4}\delta(k^+ - xP^+)\frac{(x+\frac{m}{M})(2k^-P^+ + mM)}{(k^2-m^2+i\epsilon^2)[(p-k)^2-\lambda^2+i\epsilon]}$$

$$k^- = \frac{M^2}{2p^+} - \frac{[(p-k)^2-\lambda^2]}{2(p^+-k^+)} - \frac{(k_\perp^2+\lambda^2)}{2(p^+-k^+)}$$

$$\int \frac{dk^-}{(k^2-m^2+i\epsilon)^2}$$





for $k^+ \neq 0$, $\int \frac{dk^-}{(k^2-m^2+i\epsilon)^2} = \int \frac{dk^-}{\left[2k^+\left(k^- - \frac{(k_\perp^2+m^2)}{2k^+} + \frac{i\epsilon}{2k^+}\right)\right]^2} = 0$

for all k^+ $\int dk^+dk^- \frac{1}{(k^2-m^2+i\epsilon)^2} = \int dk^+dk^- \frac{1}{(2k^+k^- - k_\perp^2 - m^2 + i\epsilon)^2}$

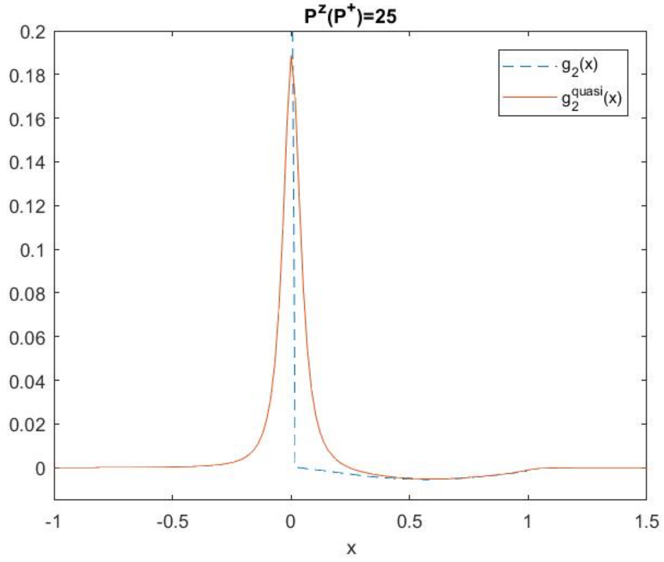
$$= \int d^2k_L \frac{1}{(k_L^2 - k_\perp^2 - m^2 + i\epsilon)^2} = \frac{i\pi}{k_\perp^2+m^2}$$

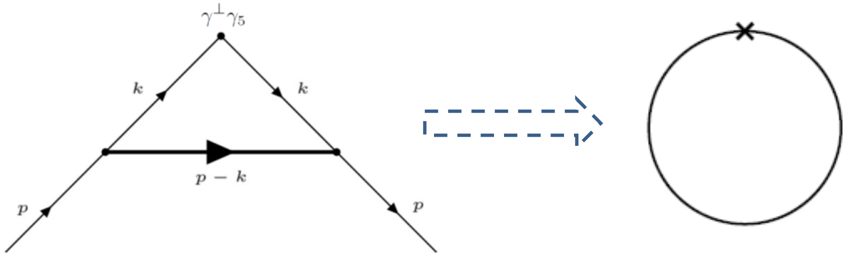
The origin of δ(x)

$$g_T(x) = ig^2 \int \frac{d^4k}{(2\pi)^4}\delta(k^+ - xP^+)\frac{(x+\frac{m}{M})(2k^- P^+ + mM)}{(k^2-m^2+i\epsilon^2)\left[(p-k)^2-\lambda^2+i\epsilon\right]}$$

$$k^- = \frac{M^2}{2p^+} - \frac{[(p-k)^2-\lambda^2]}{2(p^+-k^+)} - \frac{(k_\perp^2+\lambda^2)}{2(p^+-k^+)}$$

$$\int \frac{dk^-}{(k^2-m^2+i\epsilon)^2}$$





for $k^+ \neq 0$, $$\int \frac{dk^-}{(k^2-m^2+i\epsilon)^2} = \int \frac{dk^-}{\left[2k^+\left(k^- - \frac{(k_\perp^2+m^2)}{2k^+} + \frac{i\epsilon}{2k^+}\right)\right]^2} = 0$$

for all k^+ $$\int dk^+ dk^- \frac{1}{(k^2-m^2+i\epsilon)^2} = \int dk^+ dk^- \frac{1}{(2k^+k^- - k_\perp^2 - m^2 + i\epsilon)^2}$$

$$= \int d^2k_L \frac{1}{(k_L^2 - k_\perp^2 - m^2 + i\epsilon)^2} = \frac{i\pi}{k_\perp^2+m^2}$$

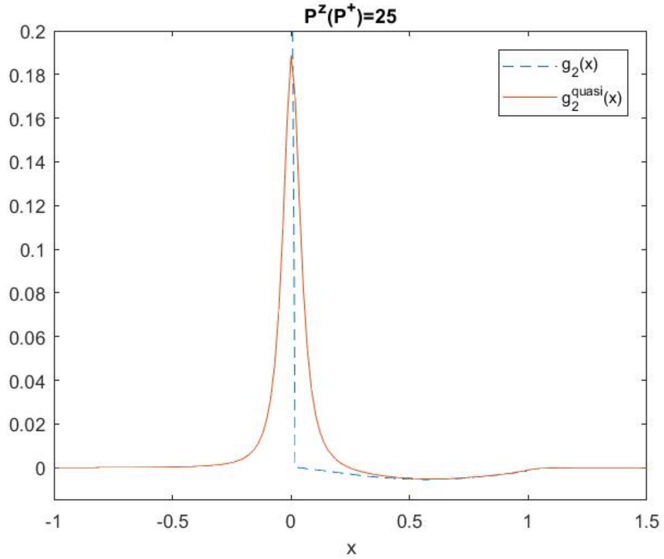
$$\int \frac{dk^-}{(k^2-m^2+i\epsilon)^2} = \frac{i\pi}{k_\perp^2+m^2}\delta(k^+).$$

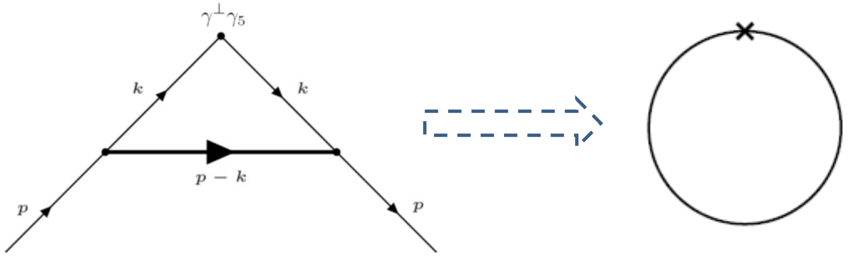
The origin of δ(x)

$$g_T(x) = ig^2 \int \frac{d^4k}{(2\pi)^4}\delta(k^+ - xP^+)\frac{(x+\frac{m}{M})(2k^-P^+ + mM)}{(k^2-m^2+i\epsilon^2)[(p-k)^2-\lambda^2+i\epsilon]}$$

$$k^- = \frac{M^2}{2p^+} - \frac{[(p-k)^2-\lambda^2]}{2(p^+-k^+)} - \frac{(k_\perp^2+\lambda^2)}{2(p^+-k^+)}$$

$$\int \frac{dk^-}{(k^2-m^2+i\epsilon)^2}$$





ZERO MODES

$$k^+ = 0$$

for $k^+ \neq 0$, $$\int \frac{dk^-}{(k^2-m^2+i\epsilon)^2} = \int \frac{dk^-}{\left[2k^+\left(k^- - \frac{(k_\perp^2+m^2)}{2k^+} + \frac{i\epsilon}{2k^+}\right)\right]^2} = 0$$

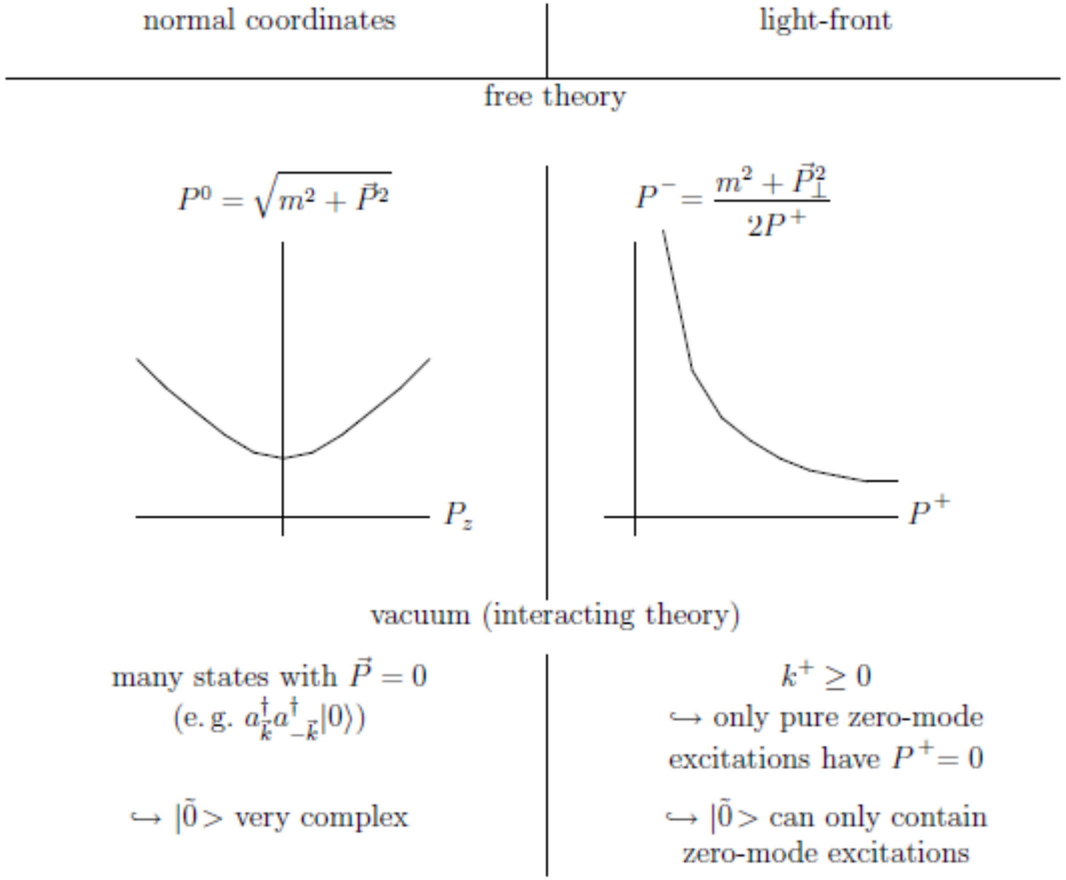
$$\int \frac{dk^-}{(k^2-m^2+i\epsilon)^2} = \frac{i\pi}{k_\perp^2+m^2}\delta(k^+).$$

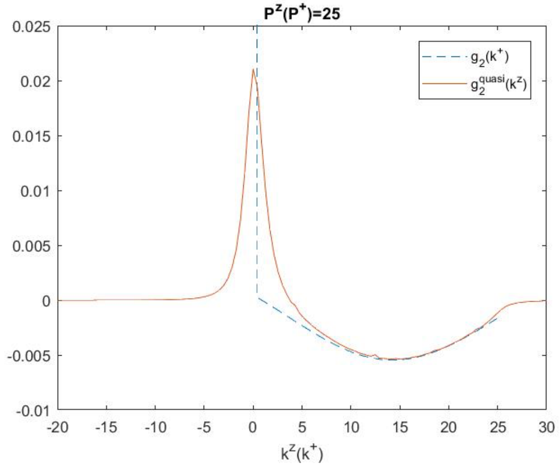
for all k^+ $$\int dk^+dk^- \frac{1}{(k^2-m^2+i\epsilon)^2} = \int dk^+dk^- \frac{1}{(2k^+k^- - k_\perp^2 - m^2 + i\epsilon)^2}$$

$$= \int d^2k_L \frac{1}{(k_L^2 - k_\perp^2 - m^2 + i\epsilon)^2} = \frac{i\pi}{k_\perp^2+m^2}$$

ZERO MODES and THE VACUUM

In LF framework zero modes are responsible for vacuum condensates.



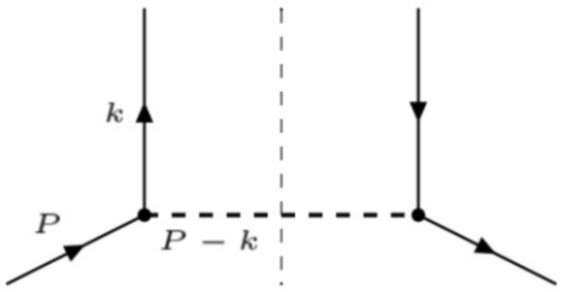


$$\delta(x) = 0 \rightarrow k^+ = 0$$

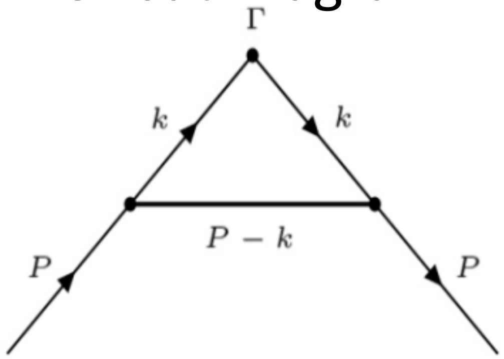
Matthias Burkardt, Light Front Quantization, 1995

CUT and UNCUT DIAGRAMS

Cut Diagram



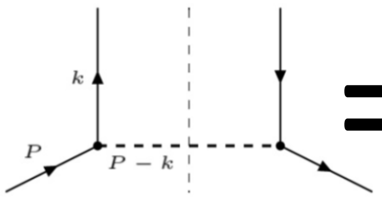
Uncut Diagram



There is no $\delta(x)$

There is a $\delta(x)$

Twist-2



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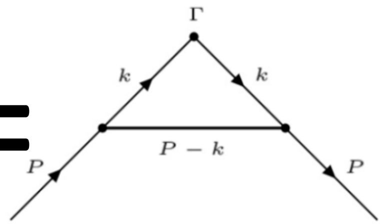
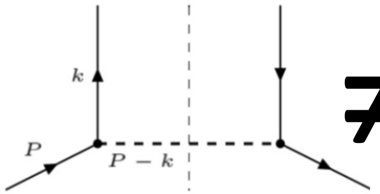


Twist-3



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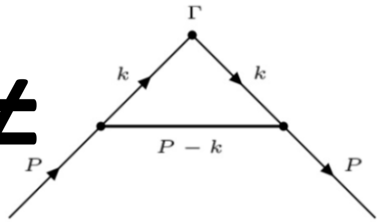


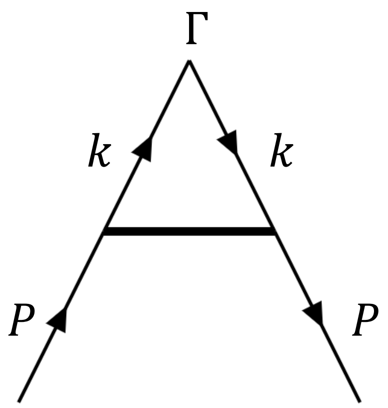
-- There is no difference between the two approaches at twist-2 level. Both methods are equivalent and yields identical PDFs. They also agree for 0<x<1,

so how can one method result in a violation of LIR and other does not?

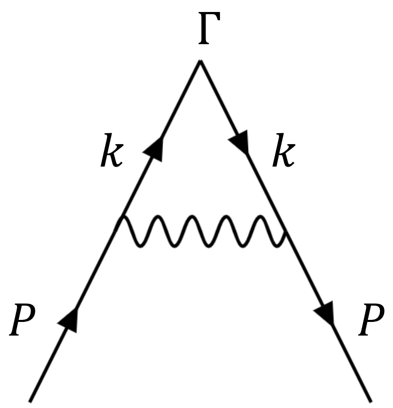
--The answer is in the appearance of δ(x) term when using the uncut diagrams which is not present in the cut diagrams.

Conclusions

SDM



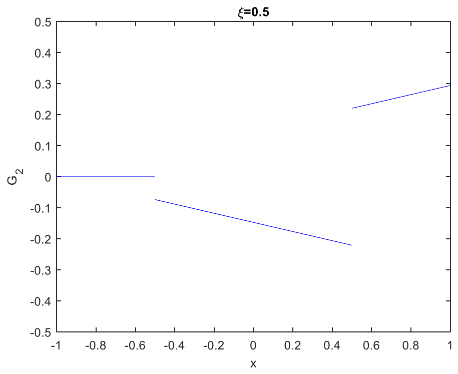
QTM

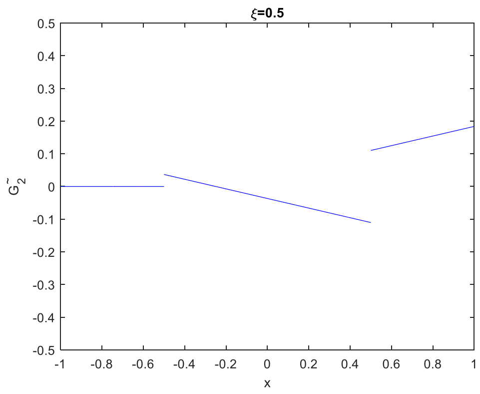


- Twist-3 GPDs have discontinuities

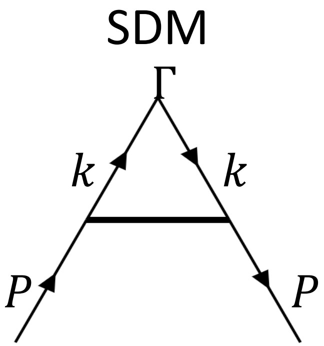
Twist-3 GPD (Vector)	Quark Target Model
G_1	✓
G_2	x
G_3	x
G_4	x

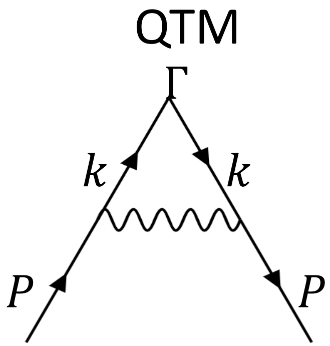
Twist-3 GPD (Axial V.)	Quark Target Model
$\tilde{G}_1$	✓
$\tilde{G}_2$	x
$\tilde{G}_3$	x
$\tilde{G}_4$	x





Conclusions





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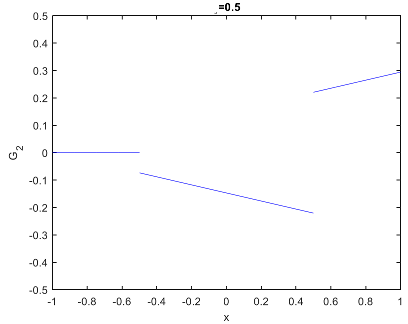
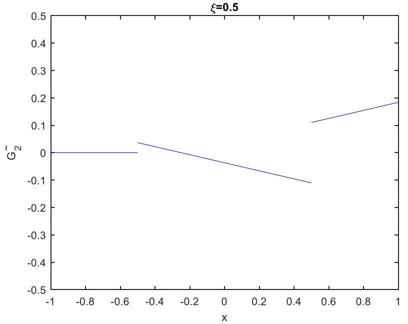
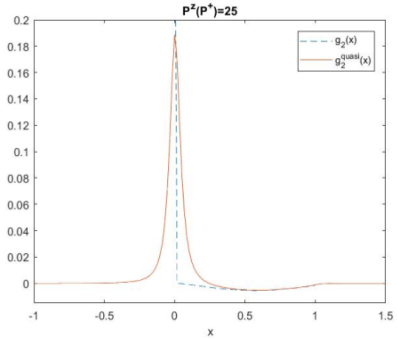


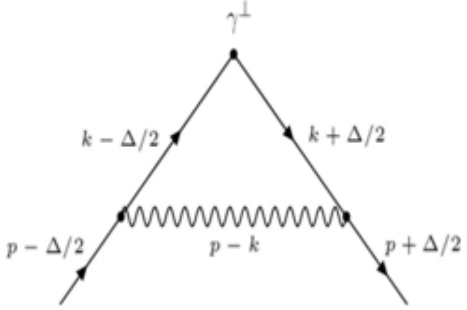


- ❑ Twist-3 PDFs contain a δ(x).

Twist-3 PDF	SDM	QTM
$e(x)$	√	√
$h_L(x)$	√	√
$g_2(x)$	√	×

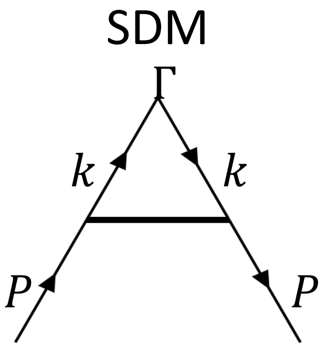


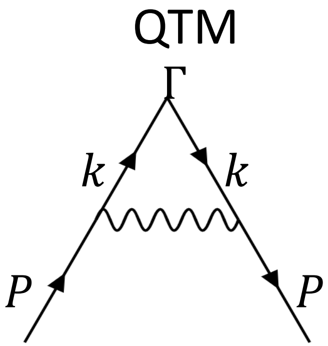
- ❑ The sum rules for twist-3 distributions are violated if we do not take the δ(x) into account
- ❑ δ(x) is related to the zero modes in the LF framework.
- ❑ Zero modes are generated by twist-3 evolution



Twist -3 evolution

Conclusions





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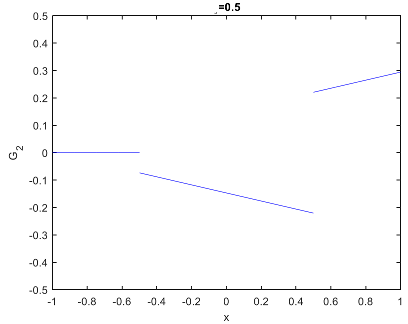

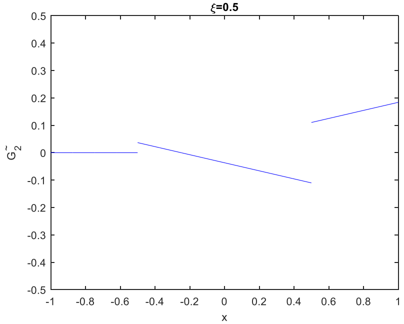

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$e(x)$	√	√
$h_L(x)$	√	√
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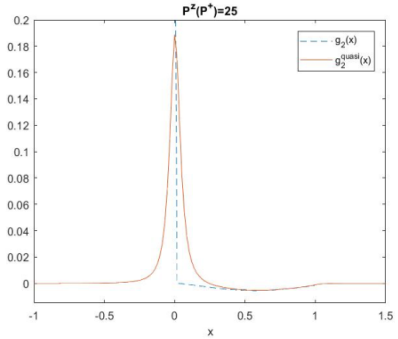

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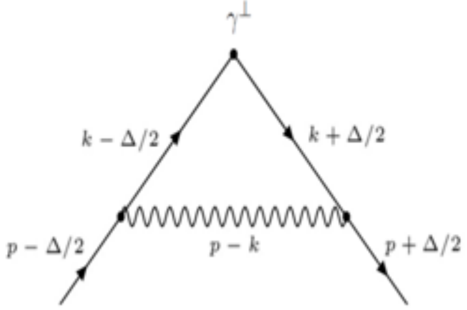
THANK YOU



Twist -3 evolution