Exclusive double quarkonium production:

From low to medium x

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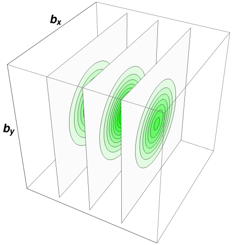
Foreword



- Exclusive processes are central to our understanding of partonic structure, which is encoded in GPDs of different flavors, helicity states

- Typical amplitude is a convolution

$$\mathcal{A} = \int dx \sum_a C_a\,(x,\xi)\, H_a(x,\xi,t) \qquad (1)$$

  - ▷ Summation over flavors, helicities (a) is implied
  - ▷ Coefficient functions are process-dependent

- Direct "deconvolution" (mathematical inversion of (1)) is NOT possible:
  - ▷ After we take into account NLO corrections, situation with deconvolution clearly becomes hopeless

⇒ Extraction of GPDs from (1) inevitably includes modeling, and requires inclusion of multiple channels to constrain better the GPDs

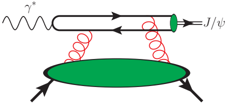
- ▷ Most widely used channels:
  - – Compton scattering (DVCS, TCS, DDVCS, ...)
  - – Meson Production (HEMP, DVMP, ...)
  - – ...

Why exclusive photoproduction of quarkonia?

(see QWG review, Eur.Phys.J. C71 (2011) 1534)

Single quarkonia photoproduction

- Clean probe of the hadronic structure
- Collinear factorization approach:
  - ▷ probe (gluon) GPDs of the protons



- Advantages:
  - ▷ Heavy mass $m_Q \gg \Lambda_{\rm QCD}$, "natural" hard scale, wide region of applicability of perturbative treatment

- Disadvantage: this process alone provides limited information
  - **Amplitude**: [PLB 440, 157; EPJC34, 297; PRD 85, 051502]
    (in collinear factorization approach)

$$\mathcal{A} \sim \int dx\, H_g\left(x, \xi, t\right) \left(\frac{1}{x-\xi} - \frac{1}{x+\xi}\right) \left(1 + \frac{\alpha_s}{2\pi} T^{(1)}\left(\frac{\xi \pm x}{2\xi}, z\right)\right)$$

$$\xi = \frac{x_B}{2 - x_B}, \quad x_B = \frac{Q^2 + M^2}{Q^2 + W^2}$$

▷Hard scales: Q^2, M^2, OK even for *photo*production
▷LO: Access to Compton form factors $\mathcal{H}$, $\mathcal{E}$, ...
▷NLO: lengthy expression for $T^{(1)}$, convolutes meson WFs with GPDs

Natural extension: photoproduction of quarkonia *pairs*

- Process:

$$\gamma^{(*)}+p\to M_1+M_2+p,\quad M_1,M_2=J/\psi,\,\eta_c,\,\chi_c,\dots$$

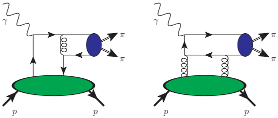
- **Advantages:**
  - ▷ Can study meson pairs with different J^P⇒should help to disentangle effects due to wave function from target-related effects
  - ▷ Can vary independently $(y,\boldsymbol{p}_\perp)$ of each quarkonium, form various new observables⇒ much more detailed information about the target (parton distributions, dipole amplitudes at $x\ll 1$)
- **Disadvantage:**
  - ▷Cross-sections are small
    - –Measurable at high-luminosity **EIC**, JLab@24 GeV, UPC@LHC, LHeC, FCC-he
    - –Focus on charmonia sector (larger cross-sections than for bottomonia).

Previous studies of meson *pair* production

▶ Studies in Bjorken regime:

[PLB 475, 147; PRD 63, 114001; NPA 679, 185 ...]

$$\gamma^{(*)} + p \to M_1 + M_2 + p, \quad M_1, M_2 = \pi^{\pm}, \pi^0 \ldots$$



Focused on light mesons

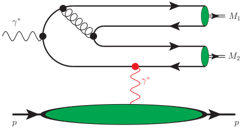
- ▷ mass $M \ll Q$, twist expansion
- ▷ If $\left(p_{M_1} + p_{M_2}\right)^2$ is small, contributions from feed-down channels ($\rho \to \pi\pi$, ...)

▷Sensitive to 2-pion distribution amplitudes (poorly known nonpert. contributions)

Not applicable when $M \sim Q \gg \Lambda_{\rm QCD}$

▶ Photoproduction of heavy quarkonia:

[PRD **101**, 034025; EPJC **49**, 675; **73**, 2335; **76**, 103; **80**, 806.]



- ▷ Focused on $J/\psi\, J/\psi$ channel
- ▷ Is dominated by photon-photon fusion (C-parity)
- ▷ Extra photon⇒additional $\mathcal{O}\left(\alpha_{\rm em}^2\right)$-suppression in cross-sections
- ▷ At smaller energies might get contributions from diagrams with odderon in t-channel

Our suggestion:

- ▶ Production of quarkonia pairs with *opposite* C-parity:

$$J/\psi\,\eta_c,\; J/\psi\,\chi_c,\; J/\psi\,\eta_b,\; B_c^+ B_c^- \ldots$$

(C-even exchanges in t-channel)

- ▷ All possible diagrams fall into two main classes (see right)
  - – Heavy flavor content of quarkonia determines if (A) or (B) contributes (same flavour or mixed)

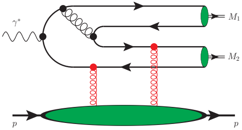
	type-A	type-B
$(J/\psi\,\eta_c)$, $(\Upsilon\,\eta_b)$, ...	✓	✓
$(B_c^+ B_c^-)$, $(B_c^{*+} B_c^-)$	✓	✗
$(J/\psi\,\eta_b)$, $(\Upsilon\,\eta_c)$, ...	✗	✓

- ▷ Hard part is dominated by gluon exchanges,
  ⇒cross-section is significantly larger than for $J/\psi\, J/\psi$
- ▶ Dominant contribution: quasireal photons with $Q^2 \approx 0$ ($Q^2 \ll M_Q^2$)
- ▷ Typical values of variables ξ, x_B

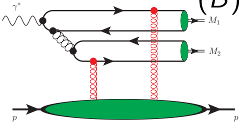
$$x_B = \frac{Q^2 + M_{12}^2}{Q^2 + W^2}, \qquad \xi = \frac{x_B}{2 - x_B}.$$

-for dominant contribution due to quasireal photon ($Q^2 \approx 0$) expect x_B, $\xi \ll 1$

(A)



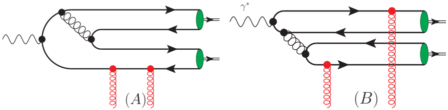
(B)



Summation over all possible connections of gluons to quark lines is implied

Framework for evaluations

- **Collinear factorization**
  - ▷ Evaluation straightforward, amplitude: $\mathcal{A} \sim \int dx\, H_g\left(x,\xi,t\right) C\left(x,\xi\right)$
  - ▷ Coefficient function $C(x,\xi)$ includes contributions of 21 diagrams of type-A and 6 diagrams of type-B



Summation over all possible connections of gluons to quark lines is implied

- Full expression for $C(x,\xi)$ is too lengthy, has a structure

$$C(x,\xi) = \sum_{\pm} \frac{N_{\pm}\left(x,\,\xi\right)}{x \mp \xi \pm i0}$$

where functions $N_{\pm}\left(x,\,\xi\right)$ remain finite for $x \to \pm\xi$

- **Caveat:** Evaluation done at LO.
  - ▷ At higher orders (NLO, ...) large contributions due to BFKL logs ($\sim \ln x$).
    - – Explicit evaluation of all those corrections is not feasible.

⇒For photoproduction ($Q^2 \sim 0$) have $x_B \sim M_{12}^2/W^2 \ll 1$, so the framework might be not reliable.

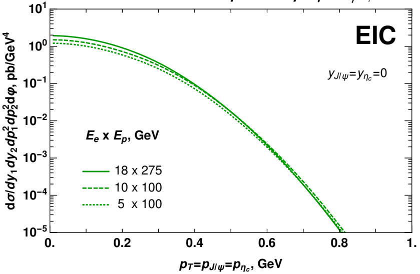
⇒ This approach is fine only for large $Q^2 \sim W^2$.

Preliminary results in collinear factorization

- Use Kroll-Goloskokov GPD for gluons
  - Photon energy:

$$q^+ = e^{y_1}\sqrt{M_1^2+p_{1\perp}^2}+e^{y_2}\sqrt{M_2^2+p_{2\perp}^2}$$

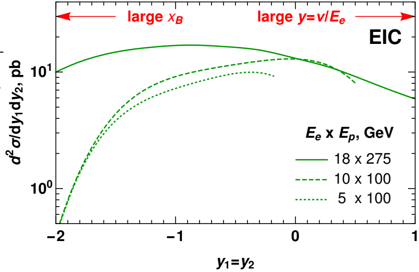
  - cross-sections for $ep \to ep\,\eta_c\,J/\psi$:



- Exponential suppression at large-p_T
  - due to implemented t-dependence in gluon GPD $H(x,\xi,t)$

$$H_g(x,\xi,t) \sim e^{B(x)\,t}$$

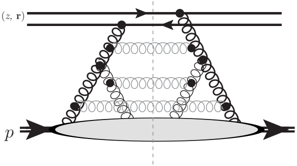
  - recall that $t \sim -4p_\perp^2 + \ldots$



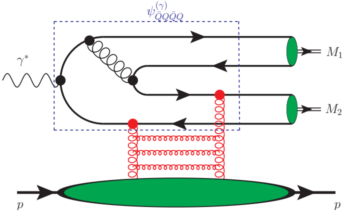
- Peaked at central rapidities $(y_1, y_2 \sim (-1, 0))$
  - At negative rapidities x_B, ξ increase; so gluon GPD decreases
  - At positive rapidities suppression due to leptonic factor, elasticity $y = q\cdot p/k\cdot p = \nu/E_e$ approaches unity

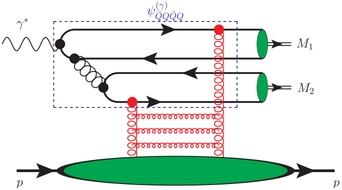
Framework for evaluations (II)

- **Color Dipole approach**
  - ▷Valid at high energies (small-$x, \xi \ll 1$)
  - ▷ Based on eikonal picture in the target rest frame
  - ▷Replaces individual partons ⇒ parton showers
  - ▷Interaction with target is described by (universal) dipole amplitude, which:
    - – satisfies Balitsky-Kovchegov equation
    - – effectively resums the fan-like diagrams as shown in the Figure



- ►Interactions of shower with heavy quarks is still suppressed by $\alpha_s(m_Q)$:





Eikonal picture ⇒ factorize amplitude into wave functions and dipole amplitudes

Amplitude of the process

Eikonal picture:

►Interactions with t-channel gluons are multiplicative in config. space

⇒The amplitude reduces to a convolutions of wave functions with a linear combination of color singlet dipole amplitudes

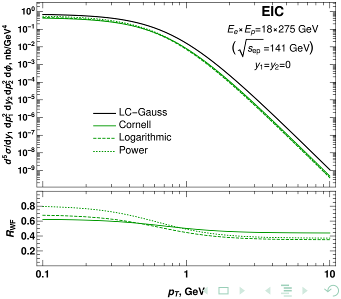
$$\mathcal{A} \sim \prod_{s=1}^{4}\left(\int d\alpha_s d^2 r_s\right)\sum_{ijklm}\psi_{M_1}\left(\alpha_{ij},\, \boldsymbol{r}_i-\boldsymbol{r}_j\right)\psi_{M_2}\left(\alpha_{kl},\, \boldsymbol{r}_k-\boldsymbol{r}_\ell\right)\otimes$$

$$\otimes c_m N\left(x,\, \mathfrak{r}_m, \mathfrak{b}_m\right)\psi^{(\gamma)}_{QQQQ}\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4\right) e^{i\left(\boldsymbol{p}_T^{(1)}\cdot\boldsymbol{r}_{ij}+\boldsymbol{p}_T^{(2)}\cdot\boldsymbol{r}_{k\ell}\right)}$$

where $\mathfrak{r}_m$, $\mathfrak{b}_m$ are some linear combinations of $\boldsymbol{r}_1...\boldsymbol{r}_4$, and c_m are color factors (exact structure of $\mathfrak{r}_m$, $\mathfrak{b}_m$, c_m) depends on diagram

► The wave function $\psi^{(\gamma)}_{QQQQ}$ is evaluated perturbatively, since $\alpha_s(m_c) \ll 1$

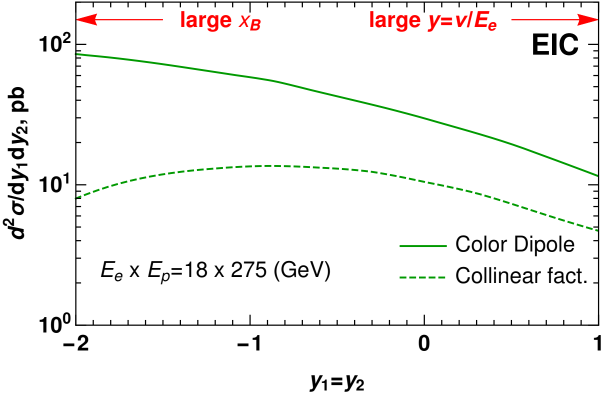
► The quarkonia WFs ψ_{M_1}, ψ_{M_2} are evaluated in potential models; comparable with LC-Gauss for J/ψ wave function



▷ In general results are close to each other, discrepancy $\sim \alpha_s(m_c) \sim 1/3$ (see right)

Color dipole vs. collinear factorization approach

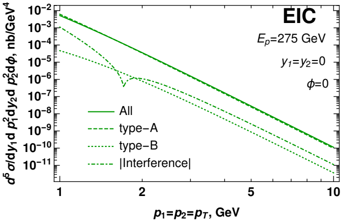
- Use Kroll-Goloskokov GPD for gluons, *b*-CGC for color dipole amplitude



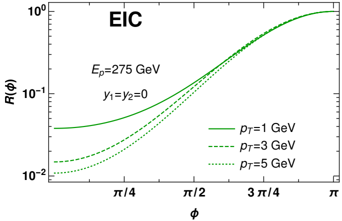
- At positive $y_1, y_2 > 0$:
  - the shape of rapidity dependence is similar
  - numerically predictions for cross-section differ by a factor of 2.
- At negative $y_1, y_2 < 0$:
  - the shape of rapidity dependence is completely different
  - approach the kinematics $x_B \gtrsim 10^{-2}$, where dipole model is less reliable.

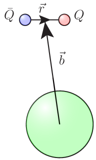
Analysis of cross-sections

- $J/\psi\,\eta_c$ has the largest cross-section, dominated by contributions of "type-A" diagrams, "type-B" is strongly suppressed
  - $\Rightarrow$ Important for understanding processes, which get contributions only from "type-B".
  - ▷ Strong p_T-dependence $\sim 1/p_T^n$. Dominant contribution from $p_T \lesssim 1\,\mathrm{GeV}$.



- The dependence on azimuthal angle ϕ between $\boldsymbol{p}_\perp^{J/\psi}$ and $\boldsymbol{p}_\perp^{\eta_c}$ has a peak at $\phi = \pi$ (back-to-back)
  - ▷ Minimizes momentum transfer to proton at fixed $|p_\perp^{J/\psi}|$ and $|p_\perp^{\eta_c}|$
  - ▷ $R(\phi)$ is cross-sections normalized to 1 at $\phi = \pi$ (eliminate $\sim 1/p_T^n$ suppression factor)
  - ▷ Sensitive probe of implemented dependence on dipole orientation in dipole amplitude (angle between $\vec{\boldsymbol{r}}, \vec{\boldsymbol{b}}$)



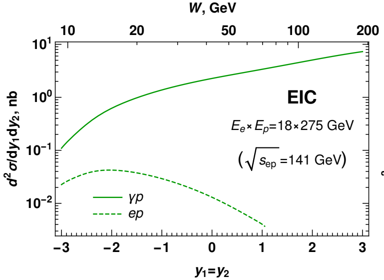


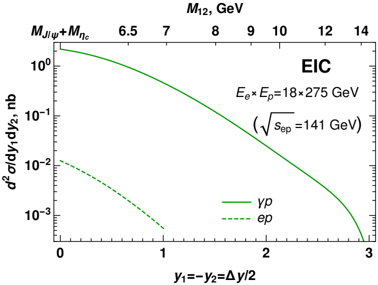
Phen. parametrizations (b-CGC, b-Sat):

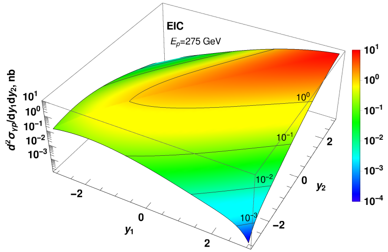
no φ-dependence

$N = N(x, |\boldsymbol{r}|, |\boldsymbol{b}|)$

Rapidity dependence







Rapidity sign:

- ▷ positive in direction of $e/\gamma^{(*)}$
- ▷ negative in direction of proton

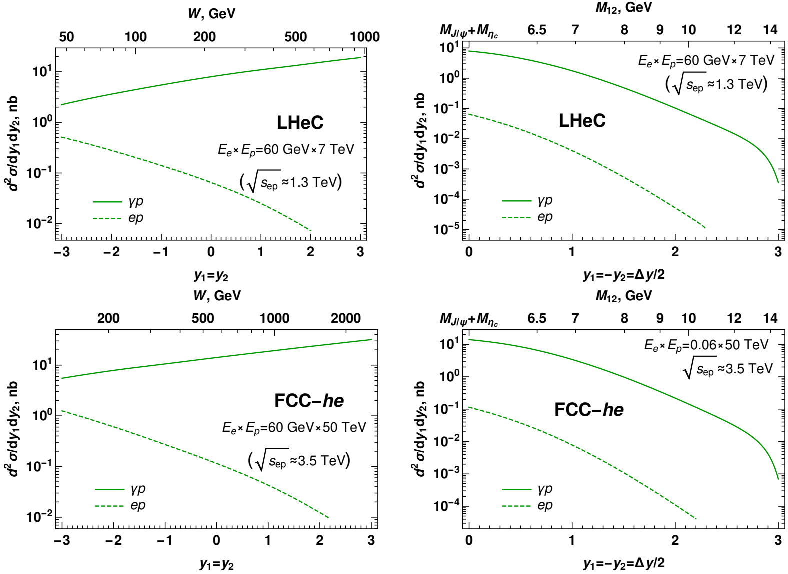
- ▶ The variable $W \equiv \sqrt{s_{\gamma p}}$ and

$$M_{12} = \sqrt{\left(p_{J/\psi} + p_{\eta_c}\right)^2}$$

is the invariant mass

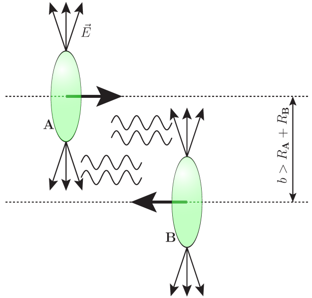
- ▷ In collider kinematics quarkonia pair are produced with small rapidity separation in forward $(e/\gamma^{(*)})$ direction
- ▷ For *ep* suppression at very forward direction due to leptonic prefactor

Predictions for other future *ep* colliders



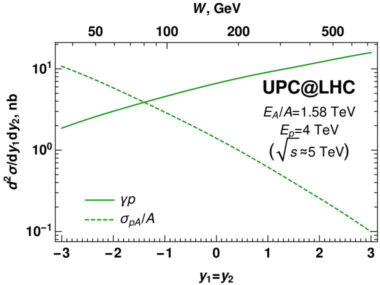
- Qualitatively the same behavior as for EIC
- Cross-section grows mildly with energy as $(W_{\gamma p})^\lambda$

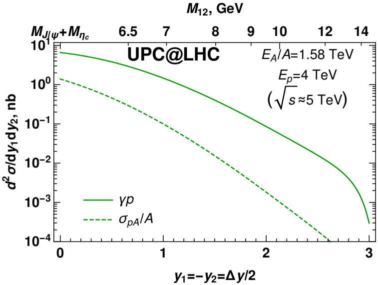
Studies in ultraperipheral *pp* and *pA* collisions @LHC



Ultraperipheral collisions:

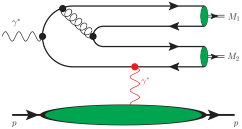
- Impact parameter $b > R_A + R_B$
- Proceed via exchange of quasireal photon ($Q^2 < 1\,\mathrm{GeV}^2$)
- Nuclear targets:
  - ▷ Enhancement by $\sim Z$(atomic number) in amplitude, $\sim Z^2$ in cross-section
  - ▷ Due to nuclear form factor $Q^2 \lesssim 1/R_A^2 \lesssim 0.1\,\mathrm{GeV}^2$
- Feasibility demonstrated at RHIC, LHC



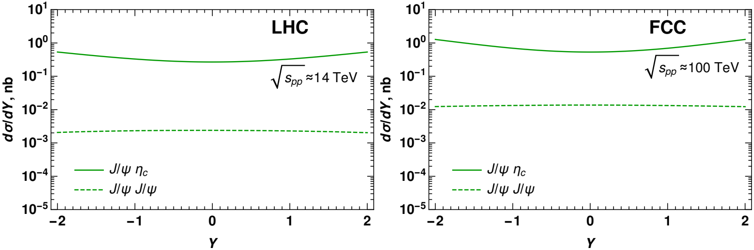


- Qualitatively the same behavior as for *ep* (see our publication for more details)

Our mechanism ($J/\psi\,\eta_c$) vs. $J/\psi\,J/\psi$ production



- $J/\psi\,J/\psi$ proceeds via $\gamma\gamma \to J/\psi\,J/\psi$ sub-process, extra suppression $\sim \mathcal{O}\left(\alpha_{\rm em}^2\right)$
  - ▷ Extra photon⇒additional $\mathcal{O}\left(\alpha_{\rm em}^2\right)$-suppression in cross-sections
- Comparison with predictions from [PRD 101 (2020) no.3, 034025] :

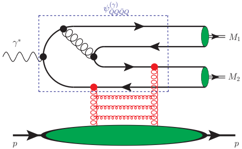


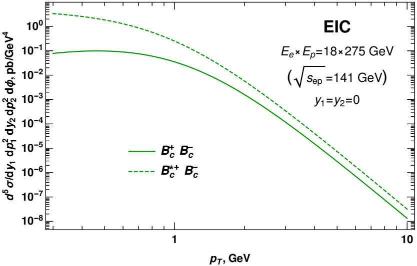
- Cross-section of suggested mechanism is larger by 2 orders of magnitude (not by factor $\sim \mathcal{O}\left(\alpha_{\rm em}^{-2}\right) \sim 10^4$ as naively expected)

Predictions for quarkonia with *b*-mesons

- All-bottom meson pairs (e.g. $\Upsilon(1S)\,\eta_b$) are similar to all-charm; numerically have much smaller cross-section

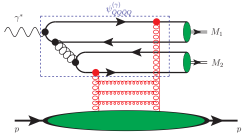
- Mixed pairs are more interesting, probe subsets of diagrams:
  - $B_c^+ B_c^-$ are sensitive only to type-A diagrams





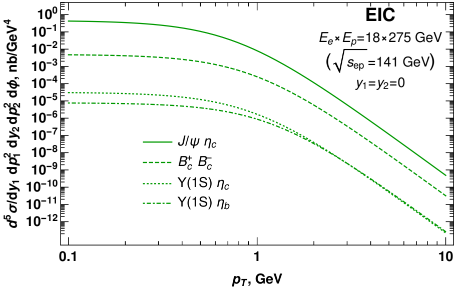
  - No restrictions from C-parity on internal quantum numbers, so can study both B_c and B_c^* in different combinations (probe of spin structure)
  - As of now, PDG2021 includes only $B_c^\pm$ with $J^P = 0^-$. Our mechanism could be used for clean (low-background) studies of possible $B_c^{*\pm}$ states.

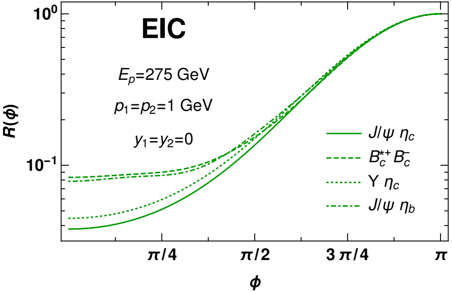
Predictions for quarkonia with *b*-mesons (II)



- Mixed hidden-charm hidden-bottom pairs get contributions only from type-*B* subsets of diagrams (see right)

- Results for cross-sections:





- Qualitatively similar behaviour for p_T, ϕ, y-dependence.
- Suppression with mass $\sim (\Lambda/\mu_1)^{2n}(\Lambda/\mu_2)^{2n}$, where μ_i is the reduced mass of the $\bar{Q}Q$ pair in mesons M_1, M_2
  - ▷For $\Upsilon(1S)\eta_c$ cross-section is much smaller than for $B_c^+ B_c^-$ since it gets contribution only from "small" type-*B* diagrams

Summary

- Exclusive production of opposite C-parity quarkonia ($J/\psi\,\eta_c$, $J/\psi\,\chi_c$...) might be used as a complementary source of information about the partonic structure of the target
  - Access to GPD H_g in collinear factorization approach
  - Access to dipole amplitude in color dipole approach

- Numerically the cross-section are sufficiently large for experimental studies, at least in all-charm sector
  - Bottomonia and $B_c^+ B_c^-$ pairs have smaller cross-sections, but are also theoretically interesting
  - The quarkonia pairs are produced predominantly with small and oppositely directed transverse momenta ($|p_\perp| \lesssim 1\,\mathrm{GeV}$) , small rapidity difference

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Thank You for your attention!