Pion DA from lattice using the leading-twist expansion of the quasi-DA matrix element

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ANL

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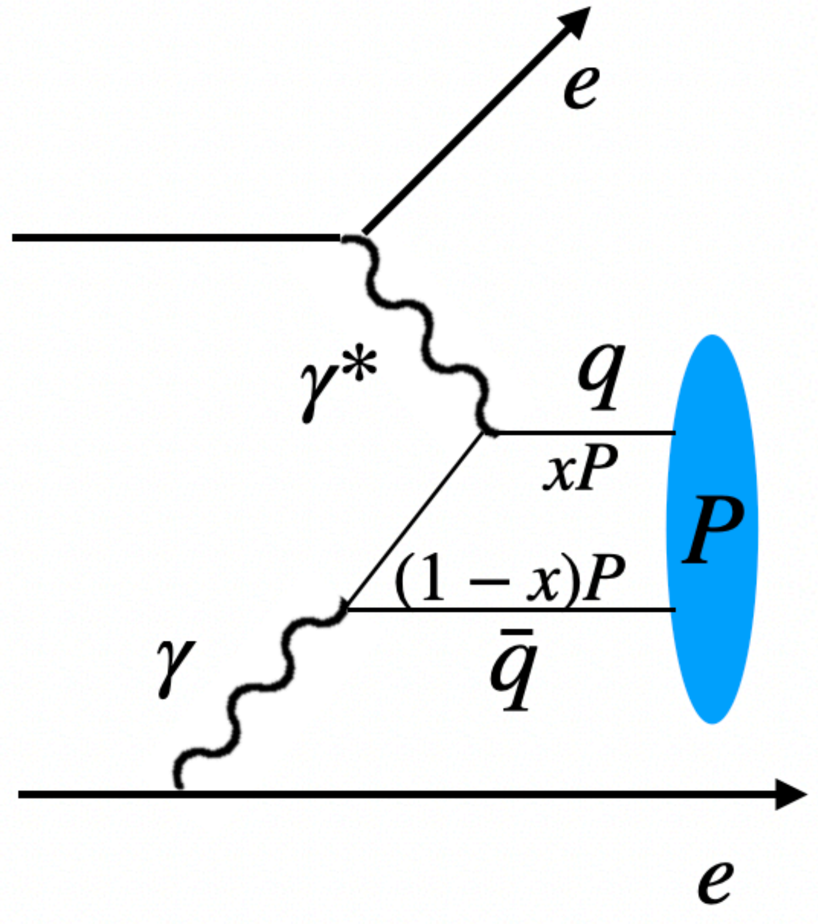
Exclusive 2022, Virginia Tech, Jul 18 – 22, 2022

Pion distribution amplitudes

Pion DAs captures the overlap of the pion with a state of two collinear valence quark carrying momentum fraction x and $(1-x)$.

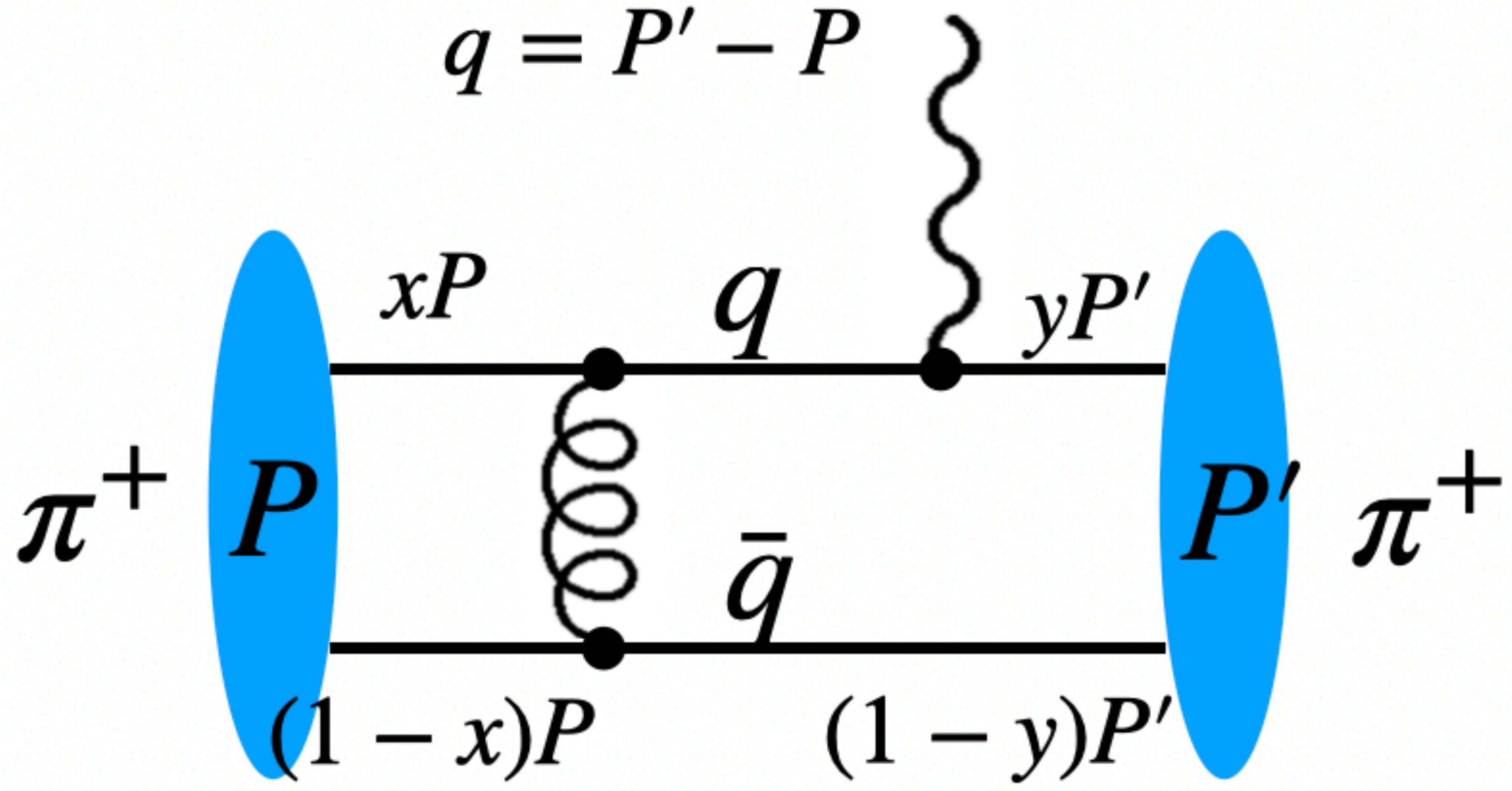
They are nonperturbative fundamental input to the collinear factorization for exclusive QCD processes:

- Gravitational form factors
- GPDs
- …



- Pion-photon transition form factor

$\gamma\gamma^* \rightarrow q\bar{q} \rightarrow \pi^0$



- Electromagnetic form factor

$\gamma^*\pi^+ \rightarrow \pi^+$

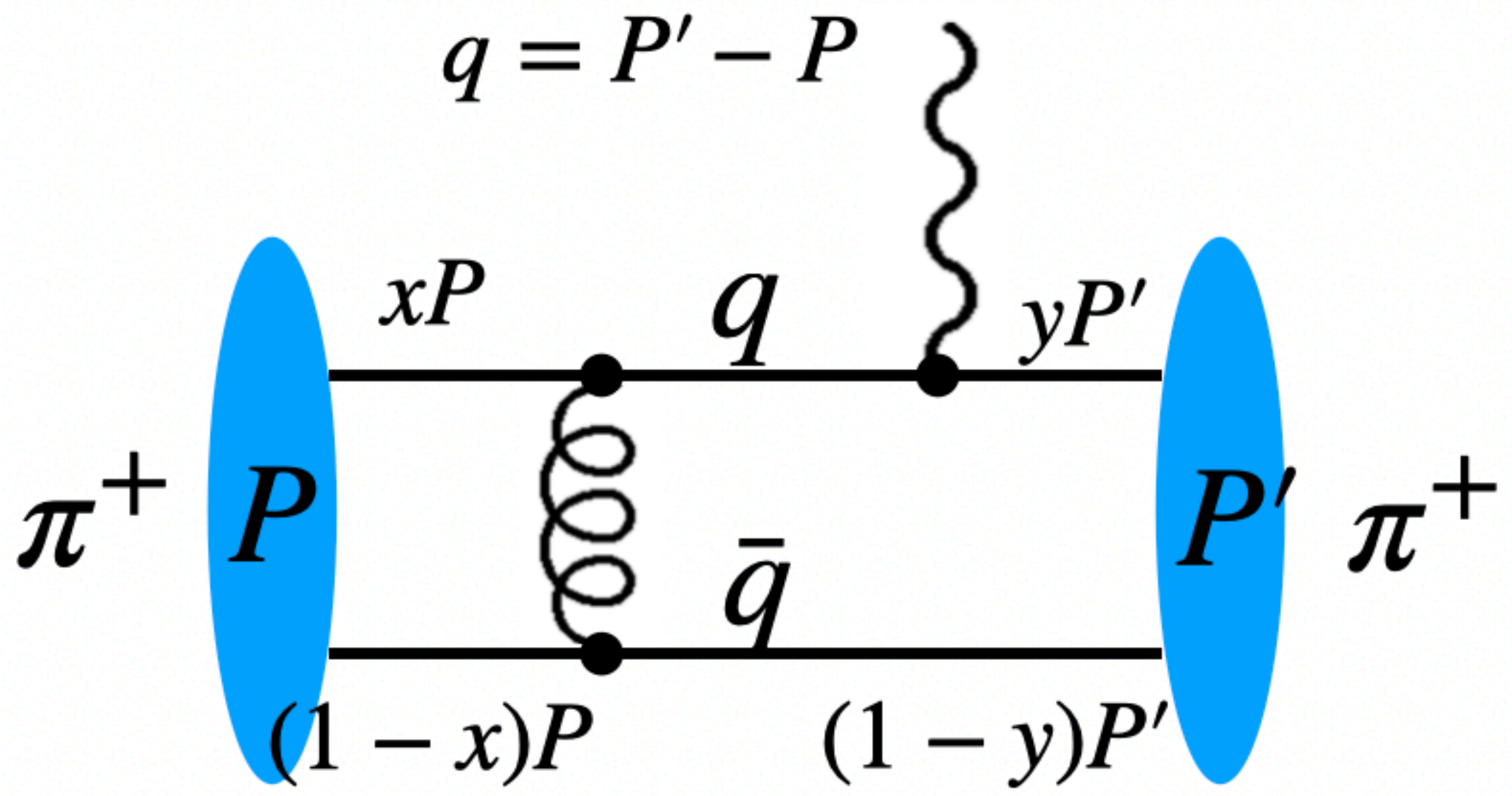
Pion distribution amplitudes

Through QCD factorization, infrared divergences in radiative corrections to a process are absorbed into DAs, and the remnant is calculable at the parton level in perturbation theory at large momentum transfer.

$$F_\pi(Q^2) = \mathcal{N} \int_0^1 \int_0^1 dxdy\ \phi^*(v, \mu_F^2)$$
$$\times\, T_F^V(u, v, Q^2, \mu_R^2, \mu_F^2)\phi(u, \mu_F^2) + \mathcal{O}(\frac{1}{Q^2})$$

Process dependent hard kernel from perturbation theory

Process independent non-perturbative DAs

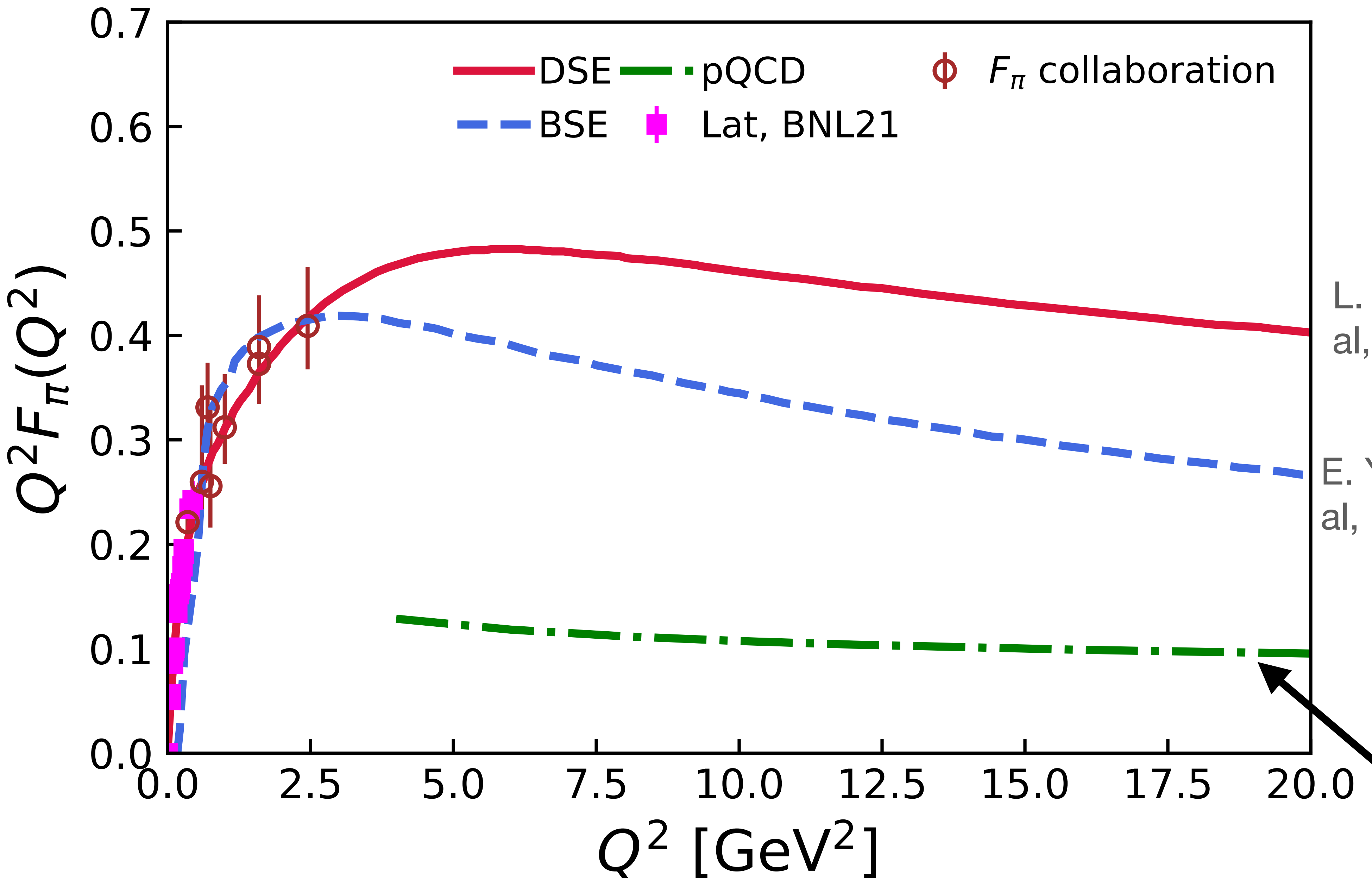


- Electromagnetic form factor

$\gamma^*\pi^+ \to \pi^+$

Pion distribution amplitudes

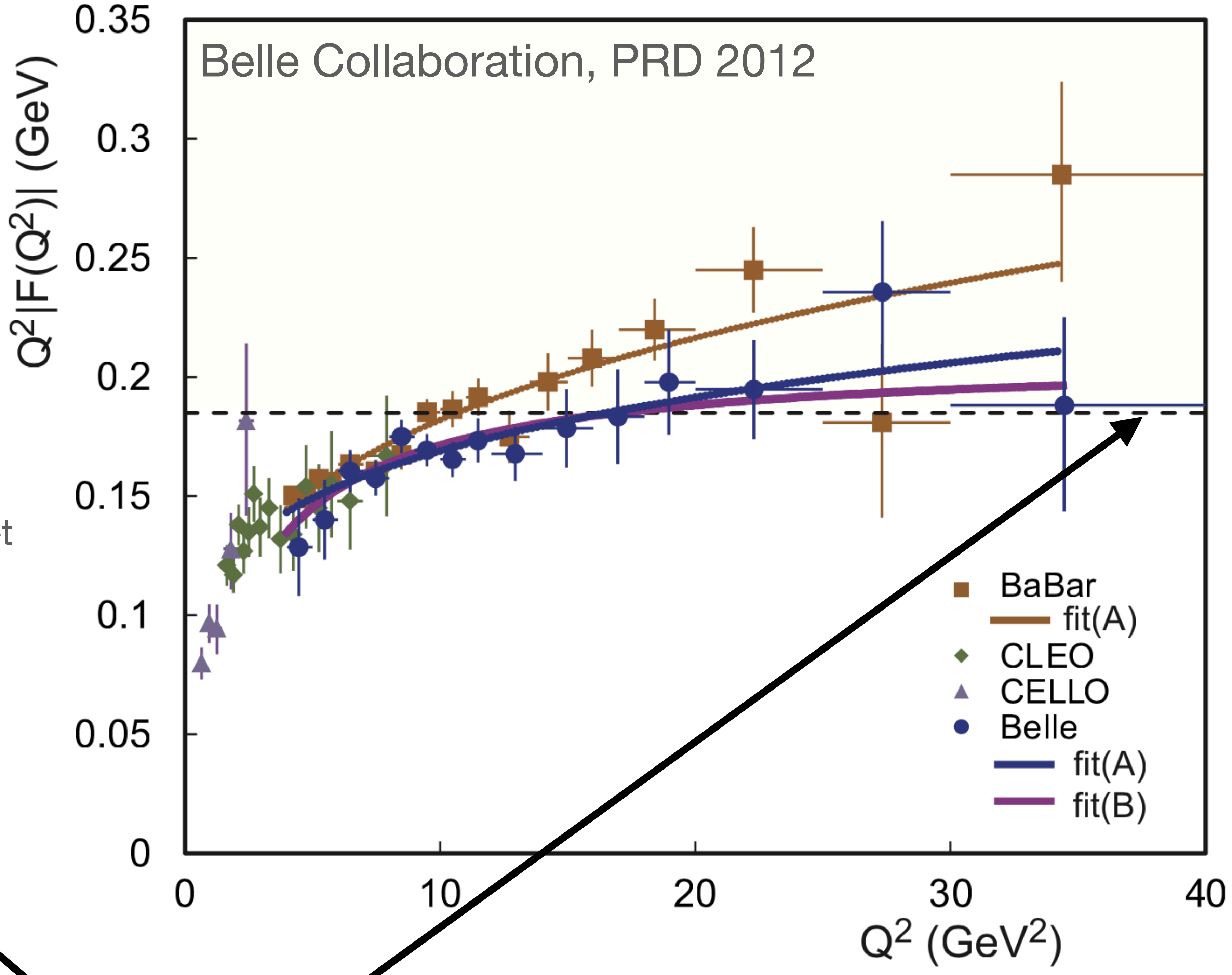
Pion electromagnetic form factor



L. Chang, et al, PRL2013

E. Ydrefors, et al, PLB2021

Pion-photon transition form factor



- Current pQCD prediction using asymptotic DA is challenged by existing data

$$\phi(x, \mu \to \infty) = 6x(1-x)$$

- How different is $\phi(x, \mu)$ from its asymptotic form when $\mu^2 \sim Q^2$? And how much does it contribute to the form factor?

Pion distribution amplitudes

The twist-2 pion DA is defined by the Fourier transform of the light-front correlation:

$$\phi(x,\mu) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} I(\lambda,\mu), \text{ with } \lambda = P^+ z^-$$

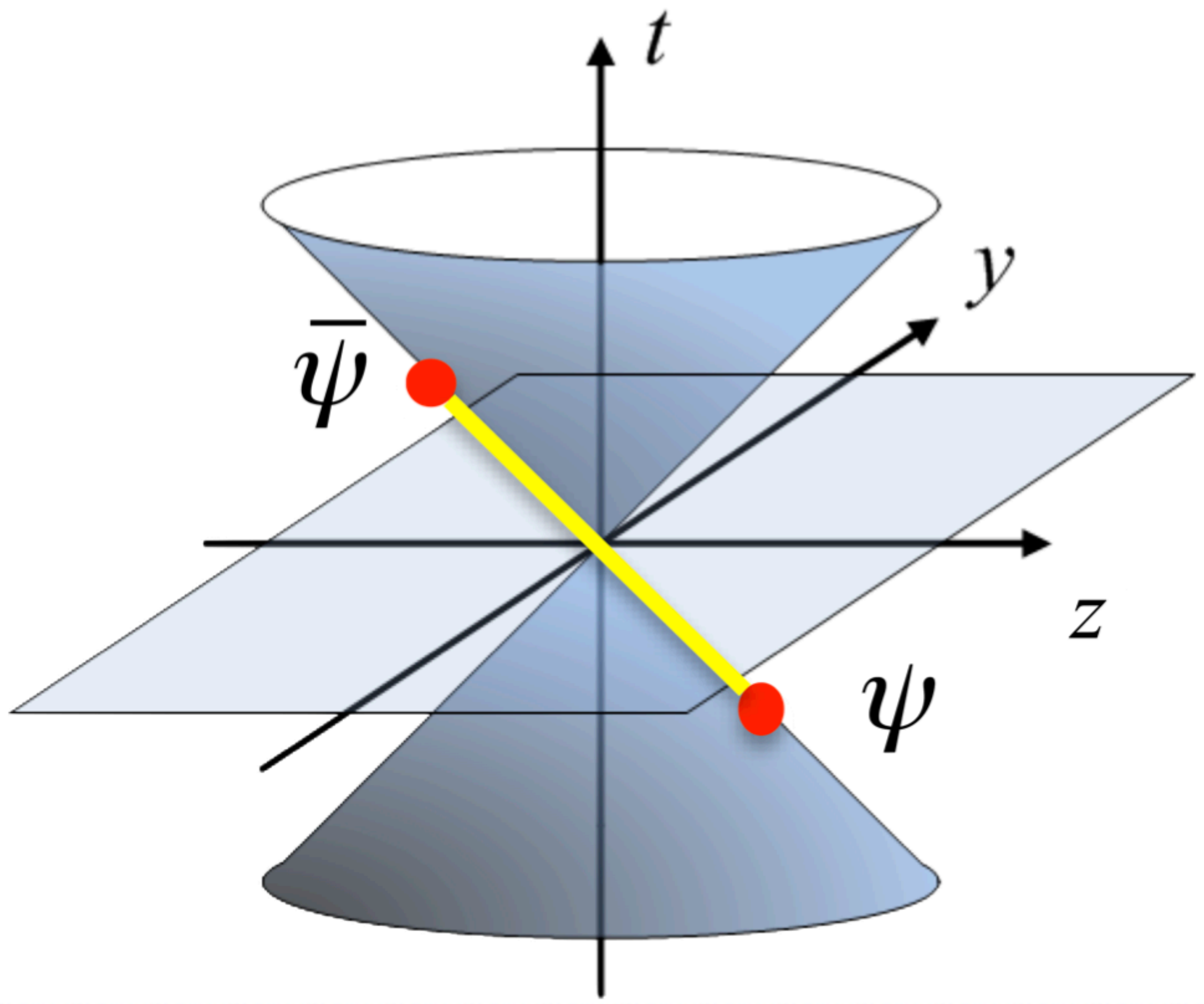
A. V. Radyushkin, 1977
G. P. Lepage and S. J. Brodsky, PLB 1979
G. P. Lepage and S. J. Brodsky, PRD 1980

with

$$i f_\pi P^+ I(\lambda,\mu) = \langle 0 | \bar{d}(-z^-/2)\gamma^+\gamma_5 W_+ u(z^-/2) | \pi^+; P\rangle$$

Though lattice QCD is a non-perturbative technique, the unequal time separation is a sign problem for Euclidean lattice.

$$z + ct = 0, \quad z - ct \neq 0$$



Equal-time matrix elements

Lattice computation of DA:

- Mellin or Gegenbauer Moments from leading-twist local operators.

  RQCD, PLB2017
  RQCD, JHEP2019

- Short distance expansion of current-current matrix elements.

  V. Braun and D. Müller, EPJC 2008
  A. J. Chambers, et al, PRL 2017
  HOPE Collaboration, PRD2022

- Large-momentum effective theory: x-space matching of quasi-DA.

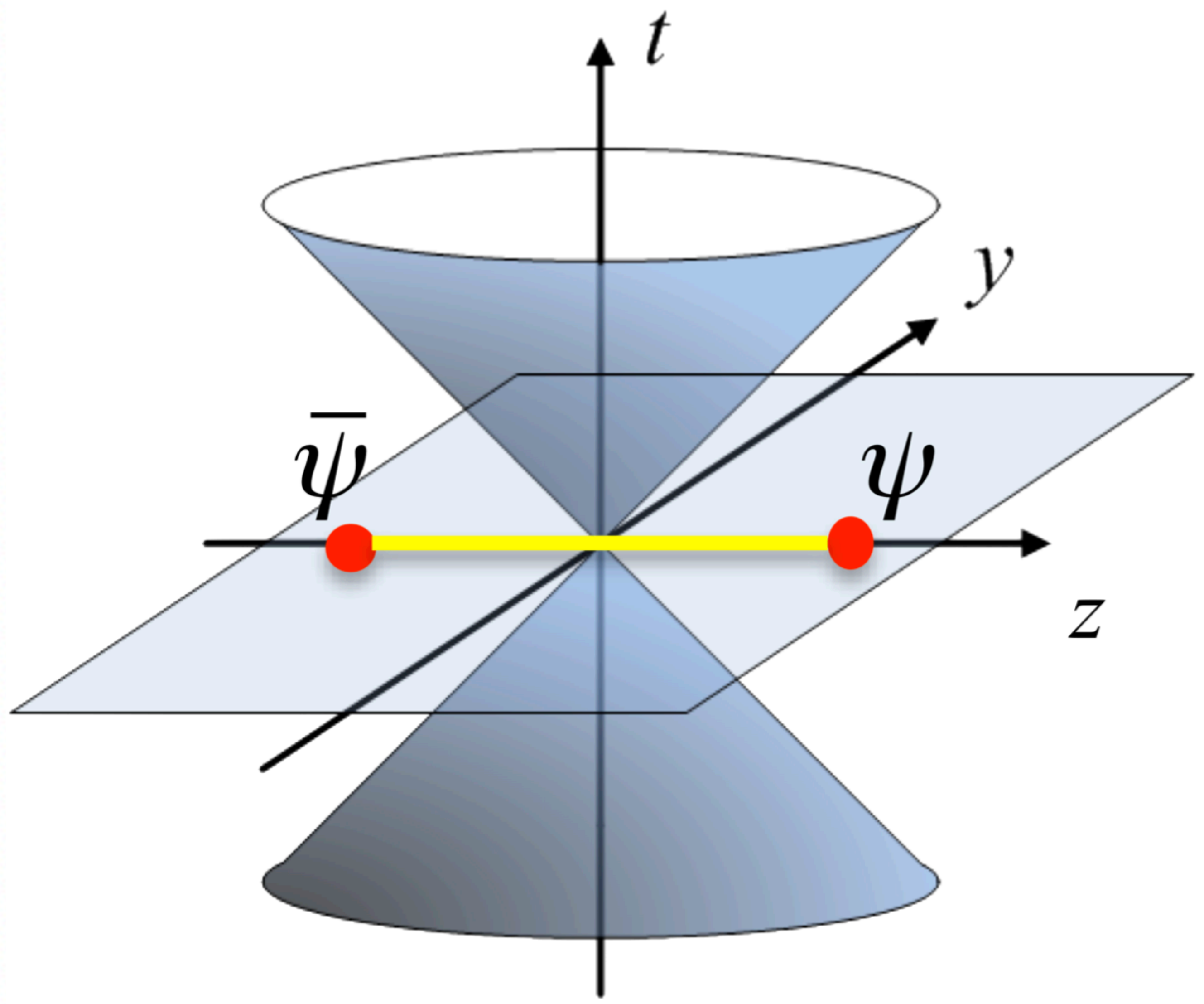
  X. Ji, PRL 2013
  X. Ji, et al, RevModPhys 2021
  LPC, arXiv: 2201.09173

- Leading-twist expansion of the quasi-DA matrix elements in position space or the pseudo-DA approach.

  A. V. Radyushkin, PRD 2017
  A. V. Radyushkin, Int.J.Mod.Phys.A 2020

- …

$t = 0, \quad z \neq 0$



quasi-DA matrix elements

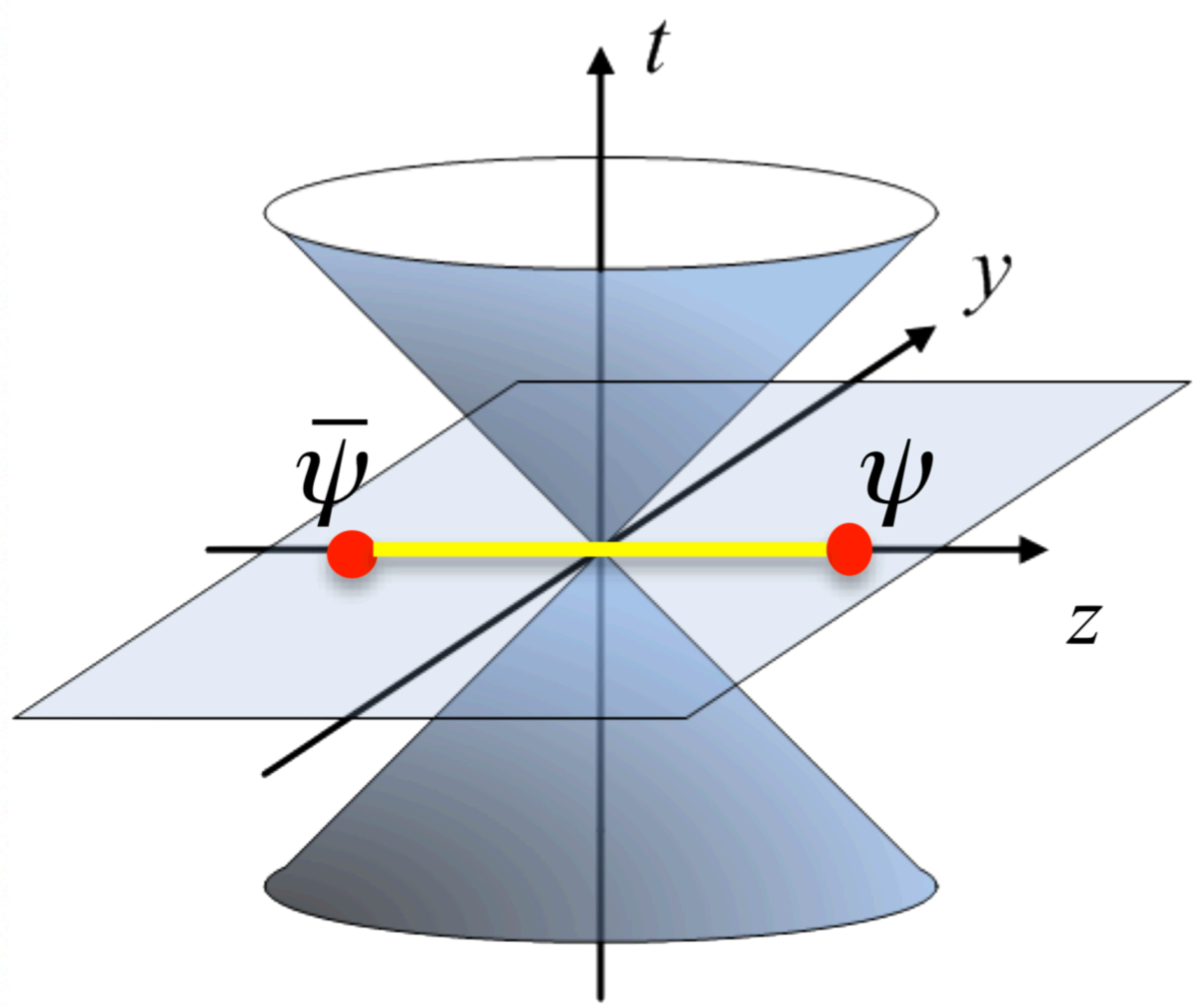
$$iP_z h(z, P_z) = \langle 0 | \bar{d}(-z_3/2)\gamma_z\gamma_5 W_{z_3} u(z_3/2) | \pi^+; P\rangle$$

Large-momentum effective theory

Quasi-DA:

$$\tilde{\phi}(x, P_z, \mu) = \int \frac{dz}{2\pi} e^{-ixzP_z} h^R(z, P_z, \mu)$$

$t = 0, \quad z \neq 0$



x-space factorization (matching):

perturbative kernel

$$\tilde{\phi}(x, P_z, \mu) = \int \frac{dy}{|y|} C(x, y, P_z, \mu) \phi(y, \mu)$$

Light-cone DA

$$+ \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{(1-x)^2 P_z^2}, \frac{\Lambda^2_{QCD}}{x^2 P_z^2}\right)$$

quasi-DA matrix elements

$$iP_z h(z, P_z) = \langle 0 | \bar{d}(-z_3/2) \gamma_z \gamma_5 W_{z_3} u(z_3/2) | \pi^+; P \rangle$$

- Predicting the x dependence of DA (also for PDFs, GPDs and so on) with controlled systematics.

See Yong Zhao's talk for more details.

X. Ji, et al, RevModPhys 2021
LPC, arXiv: 2201.09173
X. Gao, et al, PRL 2022

Leading-twist expansion

OPE in terms of Mellin moments

A. V. Radyushkin, PRD 2019
Y. S. Liu, et al, PRD 2019

Wilson coefficients

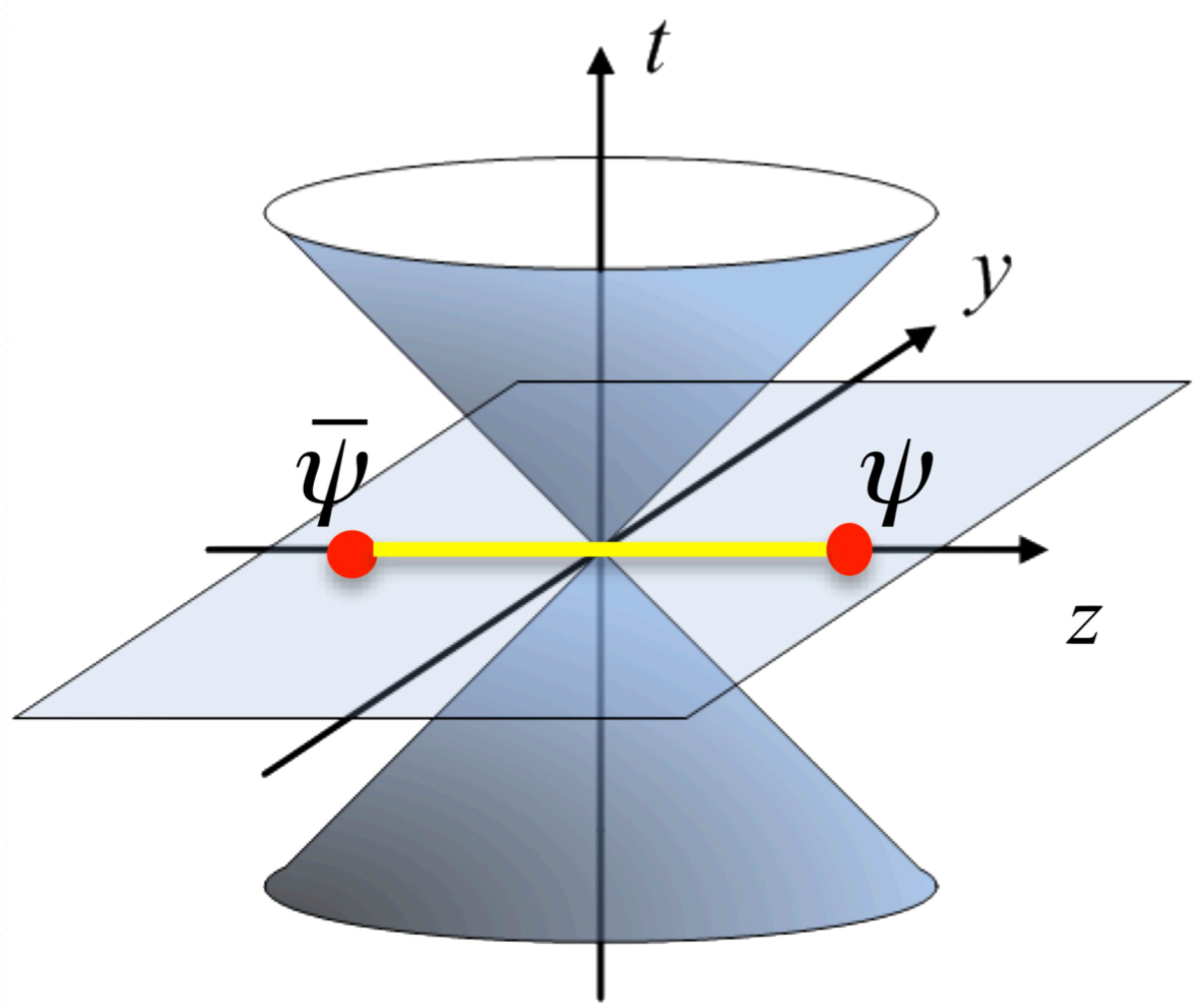
$$h^R(z, P_z, \mu) = \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2)\langle x^m\rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$

$\lambda = zP_z$

Mellin moments

- Under an ERBL evolution in μ, the different Mellin moments mix, which is also reflected in the non-vanishing off-diagonal nature of $C_{n,m}(\mu^2 z^2)$.

$t = 0, \quad z \neq 0$



quasi-DA matrix elements

$$iP_z h(z, P_z) = \langle 0 | \bar{d}(-z_3/2)\gamma_z\gamma_5 W_{z_3} u(z_3/2) | \pi^+; P\rangle$$

Leading-twist expansion

OPE in terms of Mellin moments

Large momentum P_z is the key!

A. V. Radyushkin, PRD 2019
Y. S. Liu, et al, PRD 2019

$$h^R(z, P_z, \mu) = \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2)\langle x^m\rangle(\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

$\lambda = zP_z$

Wilson coefficients

Mellin moments

- Under an ERBL evolution in μ, the different Mellin moments mix, which is also reflected in the non-vanishing off-diagonal nature of $C_{n,m}(\mu^2 z^2)$.

- The twist-2 expansion is limited in small z due to the power correction.
- Similar to the polynomial function, larger $\lambda = zP_z$ is needed to get access to higher moments.
- h^R of pion are pure real due to iso-spin symmetry, in other words, the odd Mellin moments of pion DA is 0.

Leading-twist expansion

OPE in terms of Mellin moments

A. V. Radyushkin, PRD 2019
Y. S. Liu, et al, PRD 2019

$$h^R(z, P_z, \mu) = \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2) \langle x^m \rangle (\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$

$\lambda = zP_z$

Wilson coefficients: $C_{n,m}(z^2\mu^2)$

Mellin moments: $\langle x^m \rangle (\mu)$

- Under an ERBL evolution in μ, the different Mellin moments mix, which is also reflected in the non-vanishing off-diagonal nature of $C_{n,m}(\mu^2 z^2)$.

The conformal OPE

V. Braun and D. Müller, EPJC 2008

$$h^R(z, P_z, \mu) = \sum_{n=0} a_n(\mu) \mathcal{F}_n(\lambda/2, z^2\mu^2; \alpha_s) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$

with Gegenbauer moments:

$$a_n(\mu) = \frac{4(n+3/2)}{3(n+1)(n+2)} \int_0^1 dx\, \phi(x,\mu) C_n^{3/2}(2x-1)$$

At LL accuracy, QCD is conformal, $a_n(\mu)$ evolves multiplicatively with the anomalous dimension $\gamma_n^{(0)}$

Lattice calculation

Lattice setup:

➡ Clover-fermion on 2+1f HISQ gauge ensembles

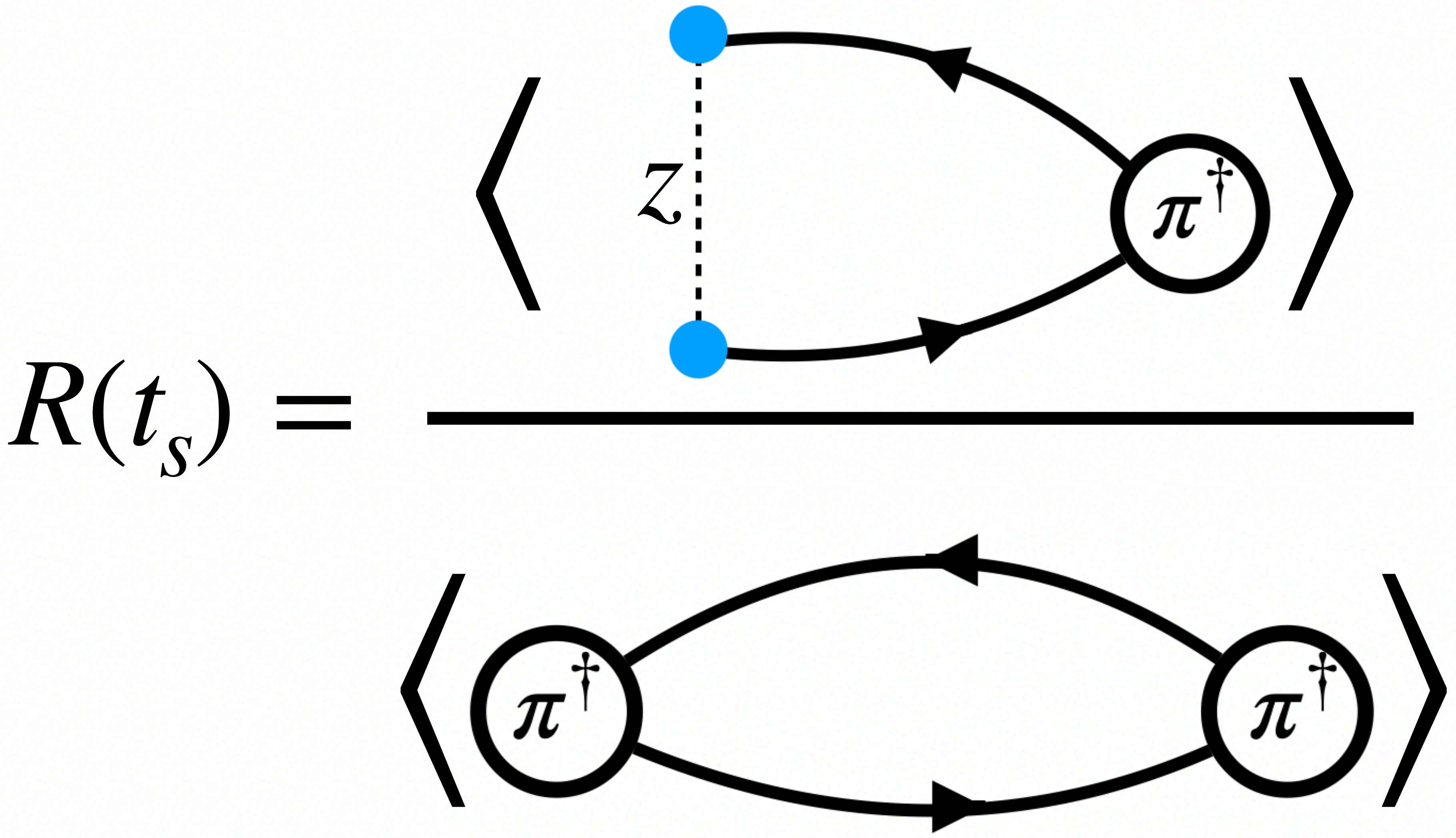
➡ $64^3 \times 64$, a = 0.076 fm, $m_\pi = 140$ MeV

➡ 8 momentum from 0 to 1.78 GeV using boosted smearing $P_3 = \frac{2\pi}{L_s a} n_3 \approx 0.254 \times n_3$ GeV

➡ 1-HYP smearing for Wilson line

➡ 350 configurations $\times$ 100 inversion sources

$\langle \bar{d}(-z_3/2)\gamma_z\gamma_5 W_{z_3} u(z_3/2)\pi_S^\dagger(P)\rangle$

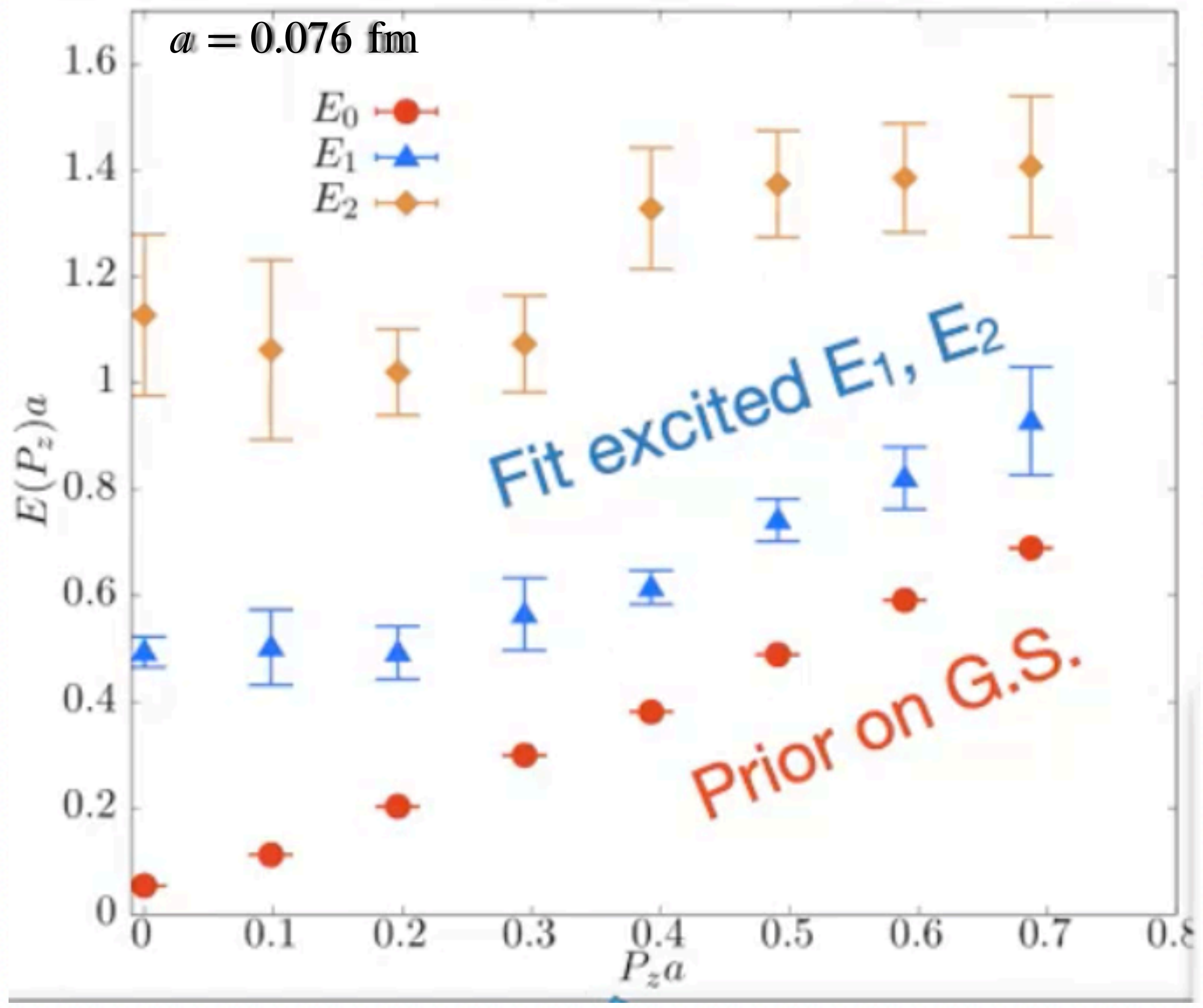


$\langle \pi_S(P)\pi_S^\dagger(P)\rangle$

$$R(t_s) \xrightarrow{t_s\to\infty} P_z h^B(z, P_z)/Z_0$$

$$Z_0 = \langle \pi; P_z | \pi^\dagger(P_z) | 0\rangle$$

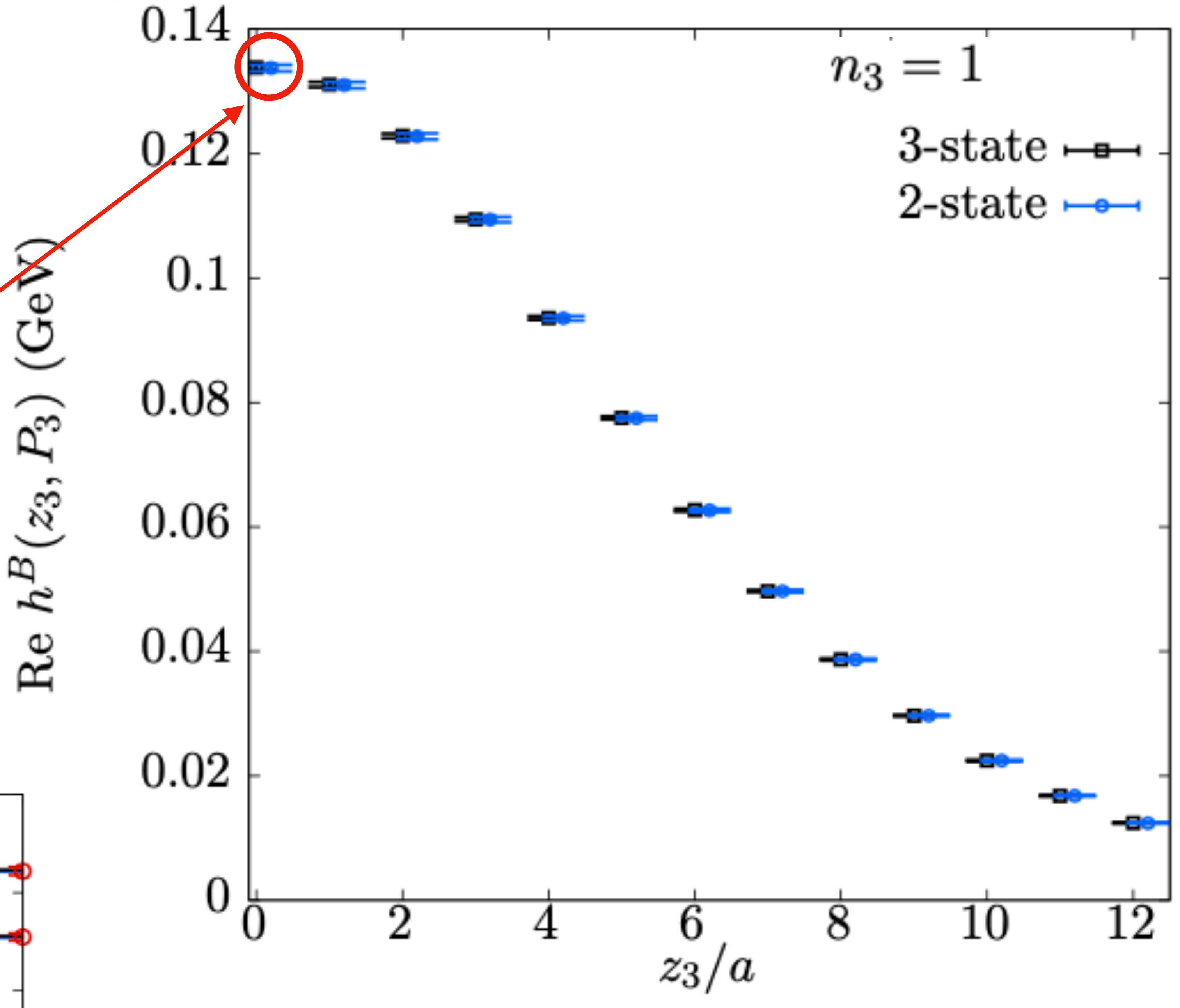
Bare quasi-DA matrix elements



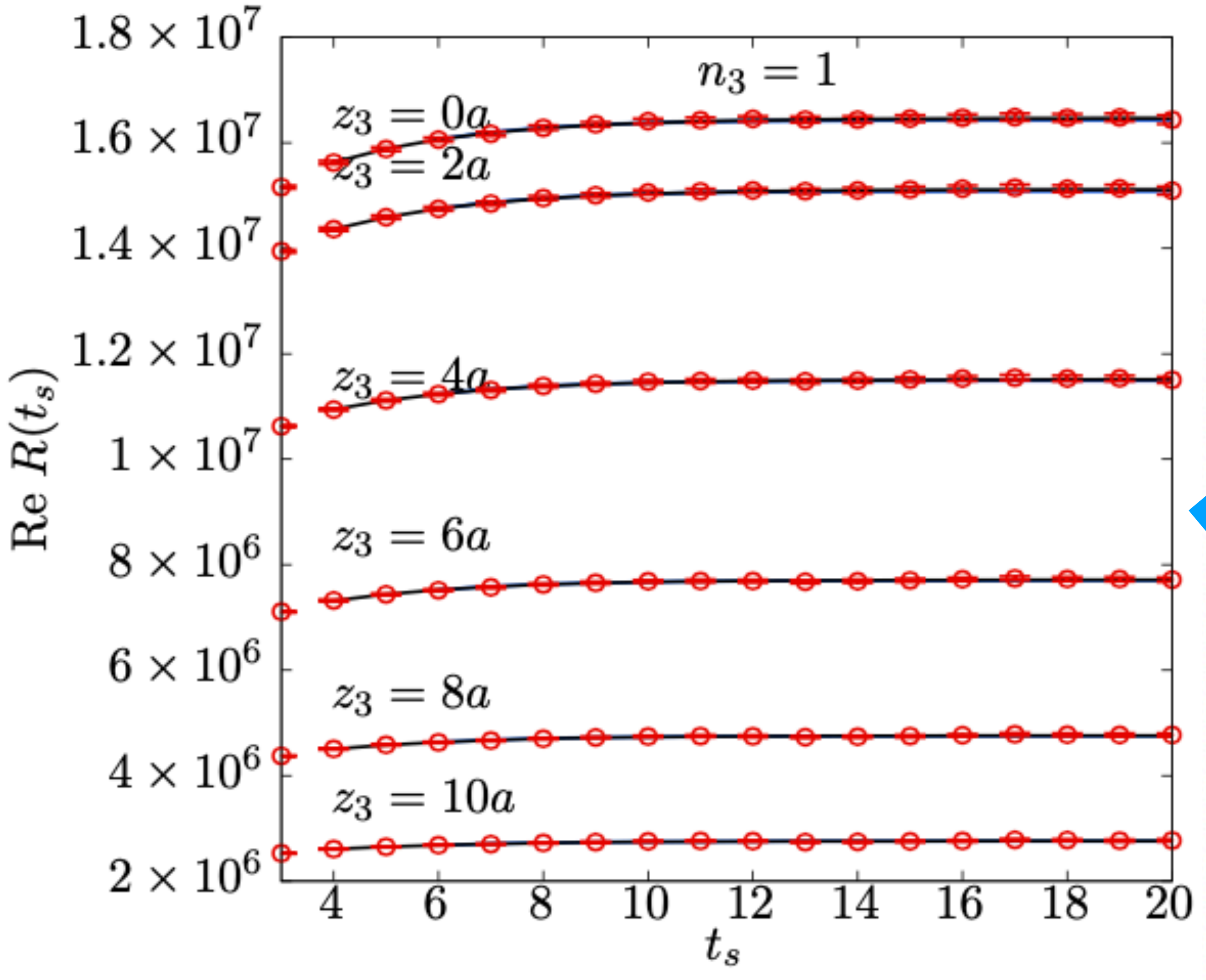
$$h^B(z = 0,\ P_z) = f_\pi / Z_A$$

$f_\pi = 130.0(4)$ MeV

A good cross check



$$h^B(z,\ P_z)$$



$$R(t_s) \xrightarrow{t_s \to \infty} P_z h^B(z,\ P_z)/Z_0$$

Ratio-scheme renormalization

The operator can be multiplicatively renormalized

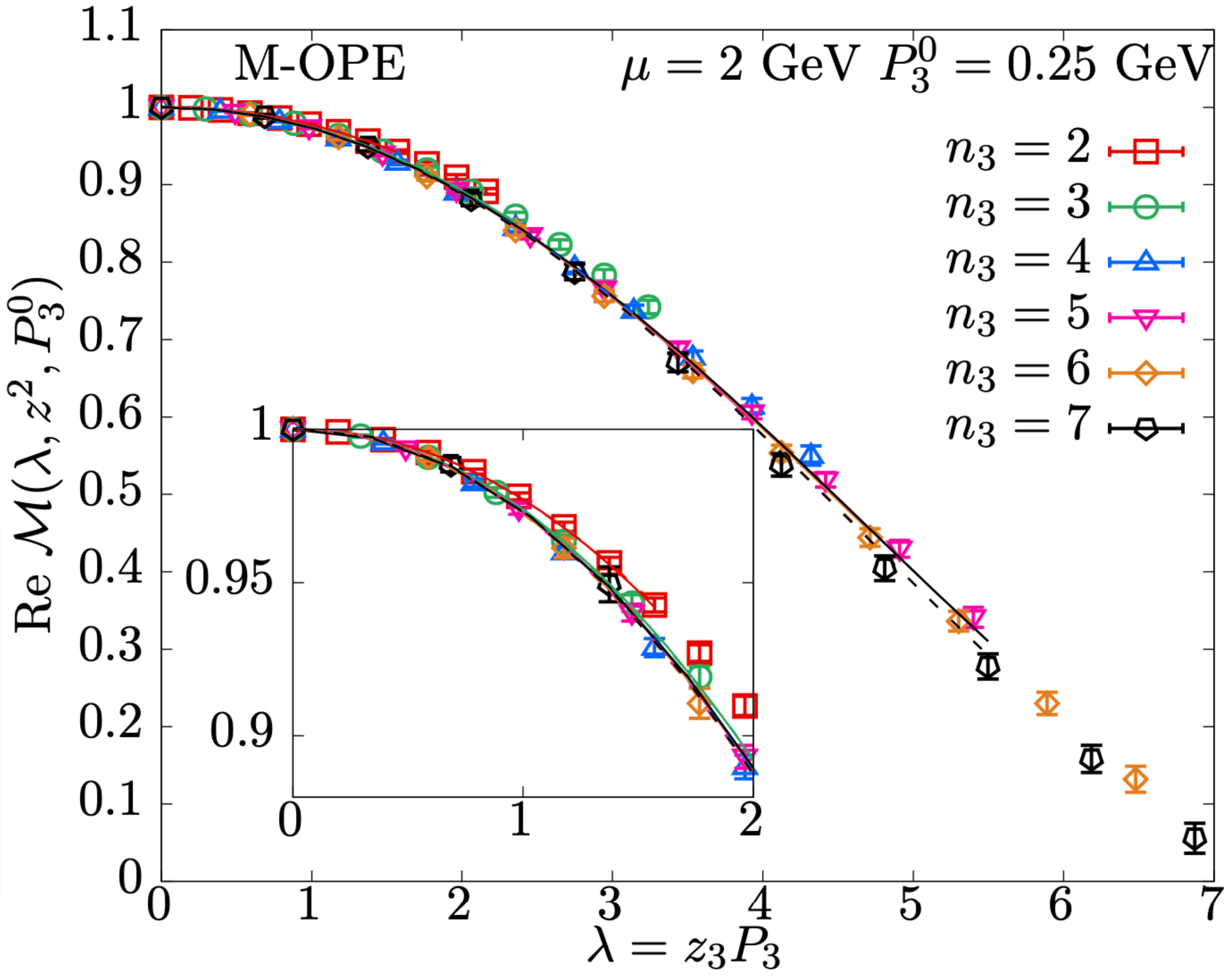
$$\tilde{O}_\Gamma(z,\mu) = Z_{\psi,z} e^{\delta m|z|} \tilde{O}_\Gamma(z,\epsilon)$$

- Hadron state independent.
- Construct the RG-invariant ratio.
- Impose the normalization condition at $z = 0$.

$$M(\lambda, z^2, P_z, P_z^0) \equiv \left(\frac{h^B(z,P_z)}{h^B(z,P_z^0)}\right)\left(\frac{h^B(0,P_z^0)}{h^B(0,P_z)}\right)$$

$$= \left(\frac{h^R(z,P_z)}{h^R(z,P_z^0)}\right)\left(\frac{h^R(0,P_z^0)}{h^R(0,P_z)}\right)$$

$$P_3 = \frac{2\pi}{L_s a} n_3 \approx 0.254 \times n_3 \text{ GeV}$$



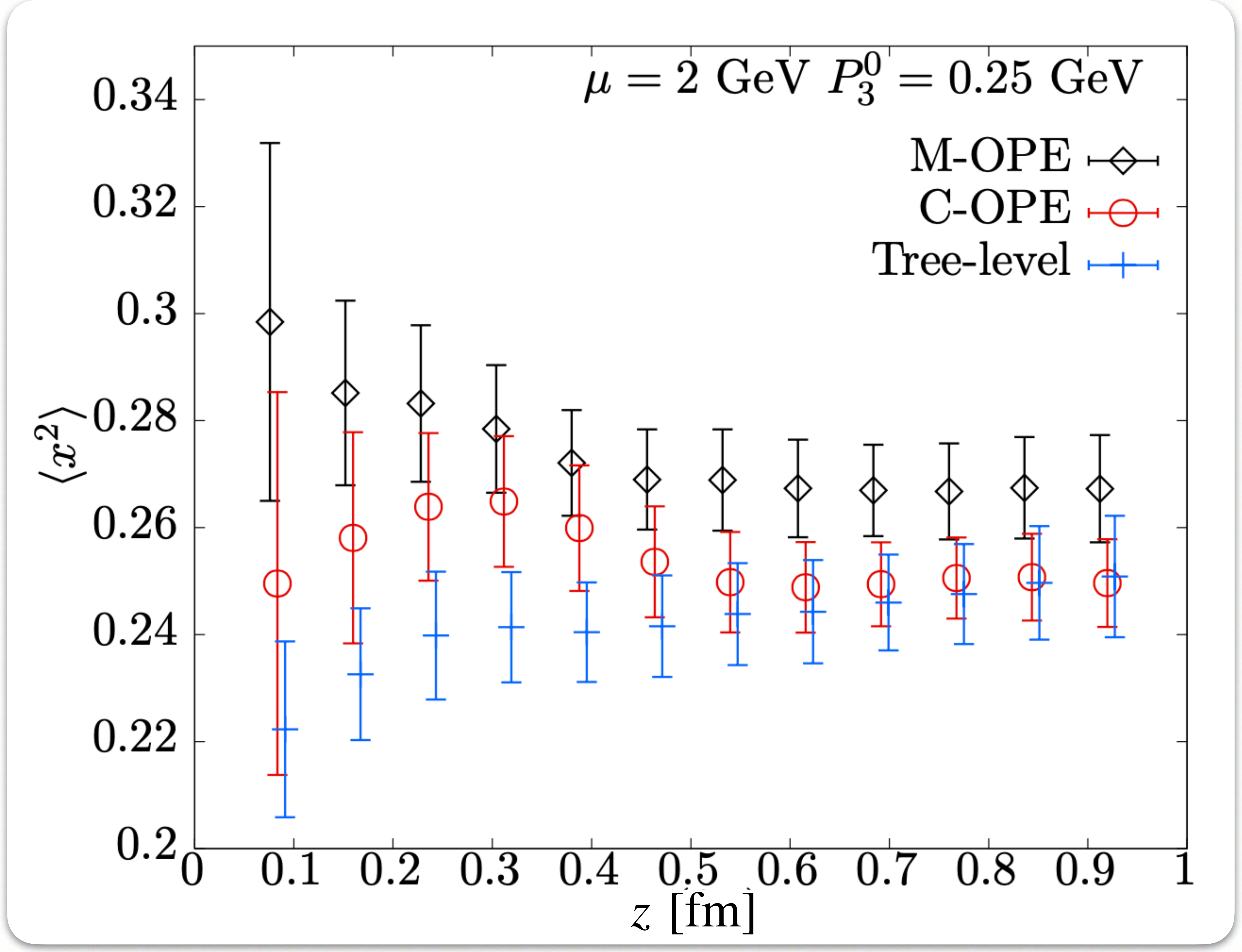
- Insert the twist-2 OPE formula.

$$h^{\text{tw2}}(z, P_z, \mu) = \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2)\langle x^m \rangle(\mu)$$

$$\lambda = zP_z$$

Moments from leading-twist approximation

$\langle x^2 \rangle$ extracted from fixed z



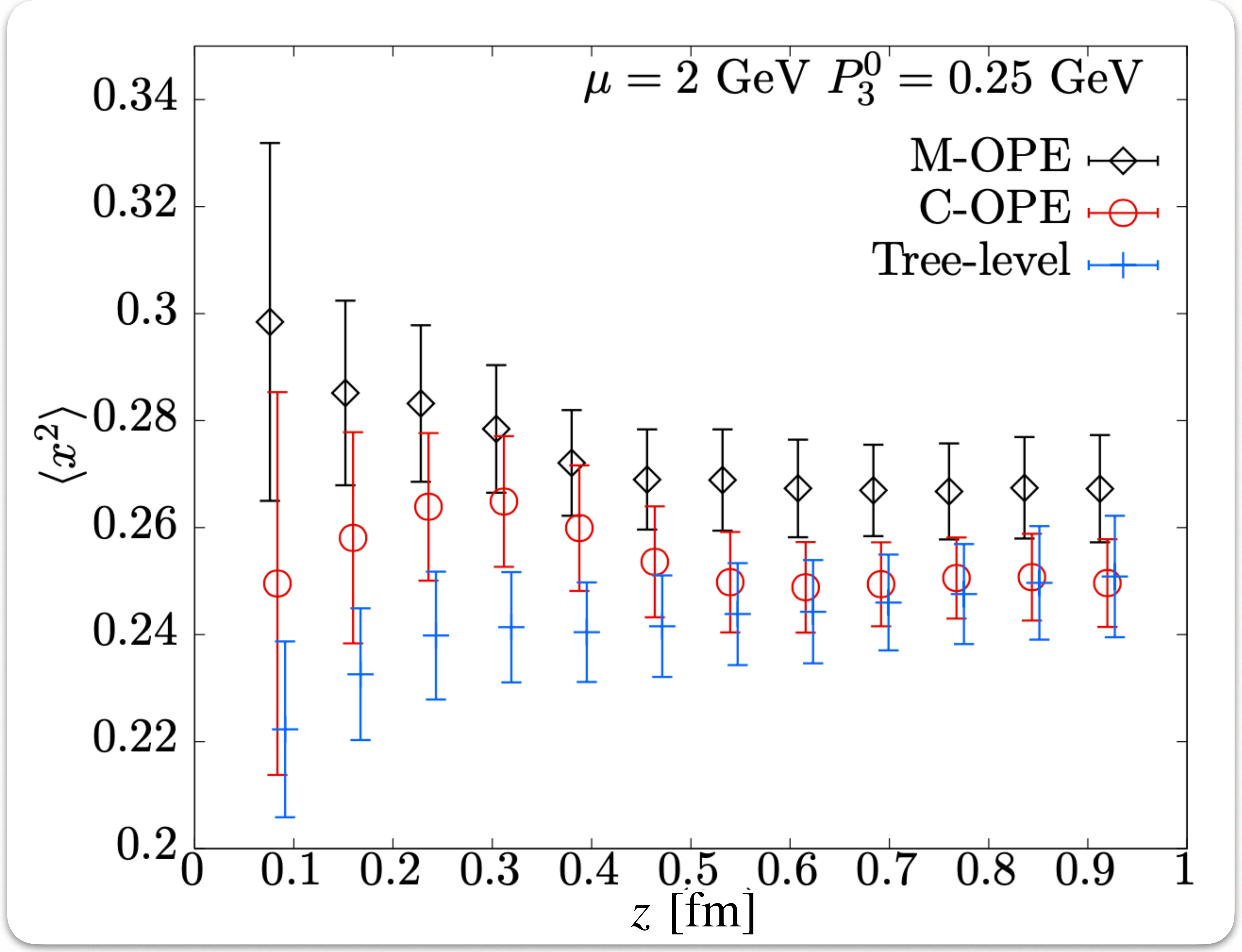
- The tree-level ($\alpha_s = 0$) result is approximately plateaued, showing the effect of z evolution to be mild at $\mu = 2$ GeV.
- $\langle x^2 \rangle$ can be determined at small z from both Mellin-OPE and Conformal-OPE, and remains to be z independent within the errors in long range.
- $\langle x^2 \rangle$ from M-OPE is about 3% higher than that from C-OPE, could be due to the remnant finite $\mathcal{O}(\alpha_s)$ corrections that are missing from C-OPE.

$$h^R(z, P_z, \mu) \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2)\langle x^m \rangle(\mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

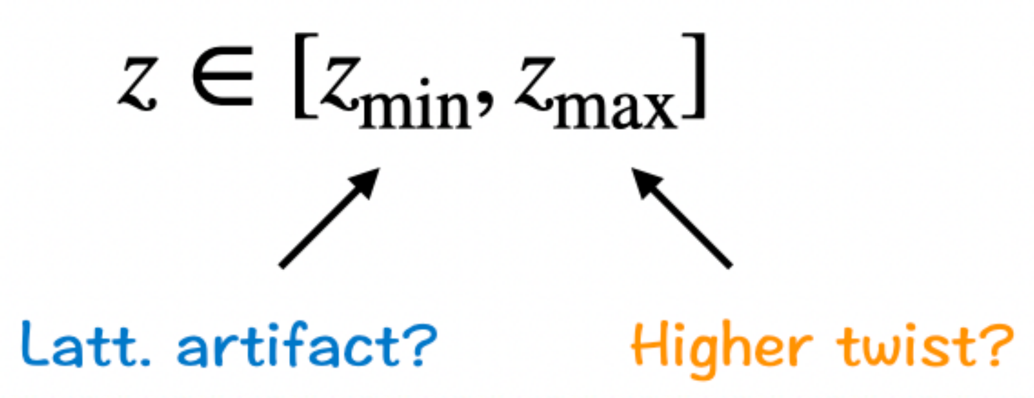
$$\lambda = zP_z$$

Moments from leading-twist approximation

$\langle x^2 \rangle$ extracted from fixed z



Combine z fit to stabilize the fit using

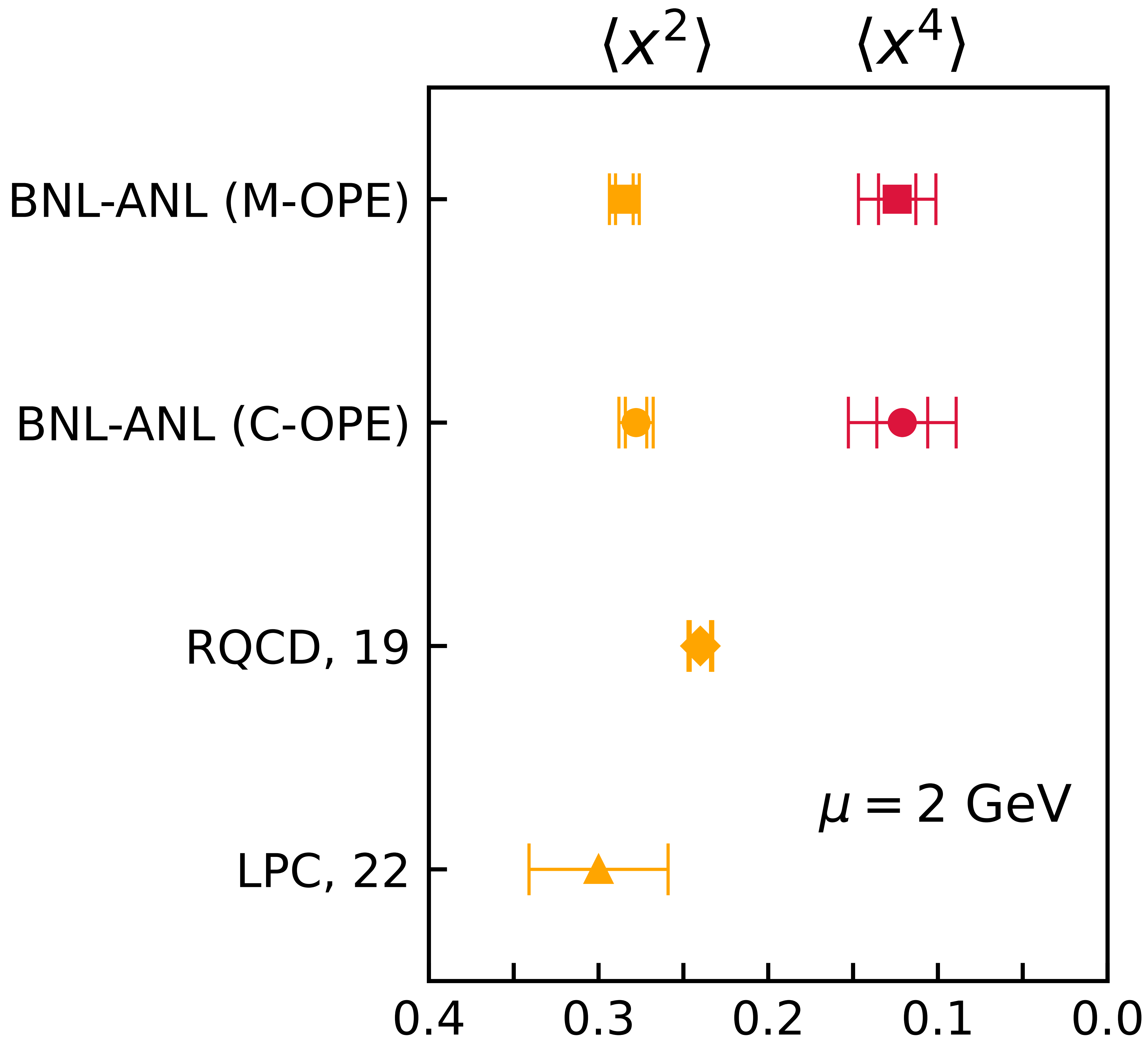


- We vary $z_{\min}$ ($2a$ and $3a$) and $z_{\max}$ (0.4 fm to 0.7 fm) to estimate the systematic errors.

$$h^R(z, P_z, \mu) \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2)\langle x^m\rangle(\mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

$$\lambda = zP_z$$

Moments from leading-twist expansion



RQCD 19: **local twist-2 operator**.

JHEP 08 (2019) 065, JHEP 11 (2020)

LPC 22: x-space **LaMET matching of quasi-DA** using Hybrid renormalization**.**

arXiv:2201.09173

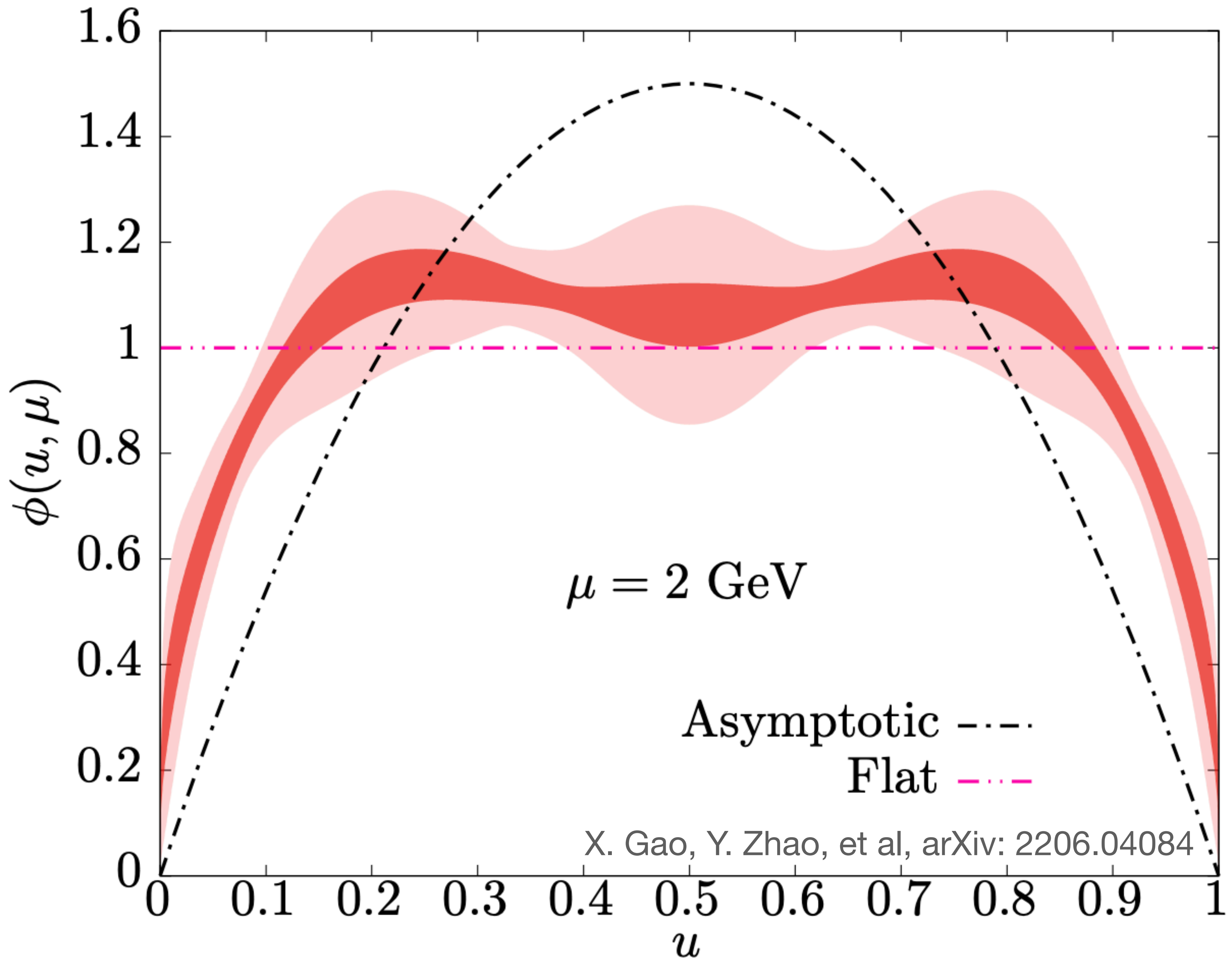
- Our results are consistent with one from x-space LaMET matching of quasi-DA approach within the errors.
- About 2.4-σ larger from the estimate using the local operator approach.
- Need to investigate the remaining systematical uncertainties such as the effect of finite lattice spacing, and will do the x-space LaMET matching using hybrid renormalization.

Pion DA from model determination

We inserted a generalized parametrization for the DA into the OPE formula

$$\phi(x) = \mathcal{N} x^{\alpha}(1-x)^{\alpha} \sum_{n=0}^{N_G+1} s_n C_{2n}^{1/2+\alpha}(1-2x)$$

- Shown systematic errors come from the variation of z, N_G.
- Basically equivalent to model the higher moments by the lower moments that data is sensitive to.
- Overall flat DA can be observed over a range of $u \in [0.2, 0.8]$ with sharp fall offs, u^{α} and $(1-u)^{\alpha}$ with $\alpha \approx 0.3$.



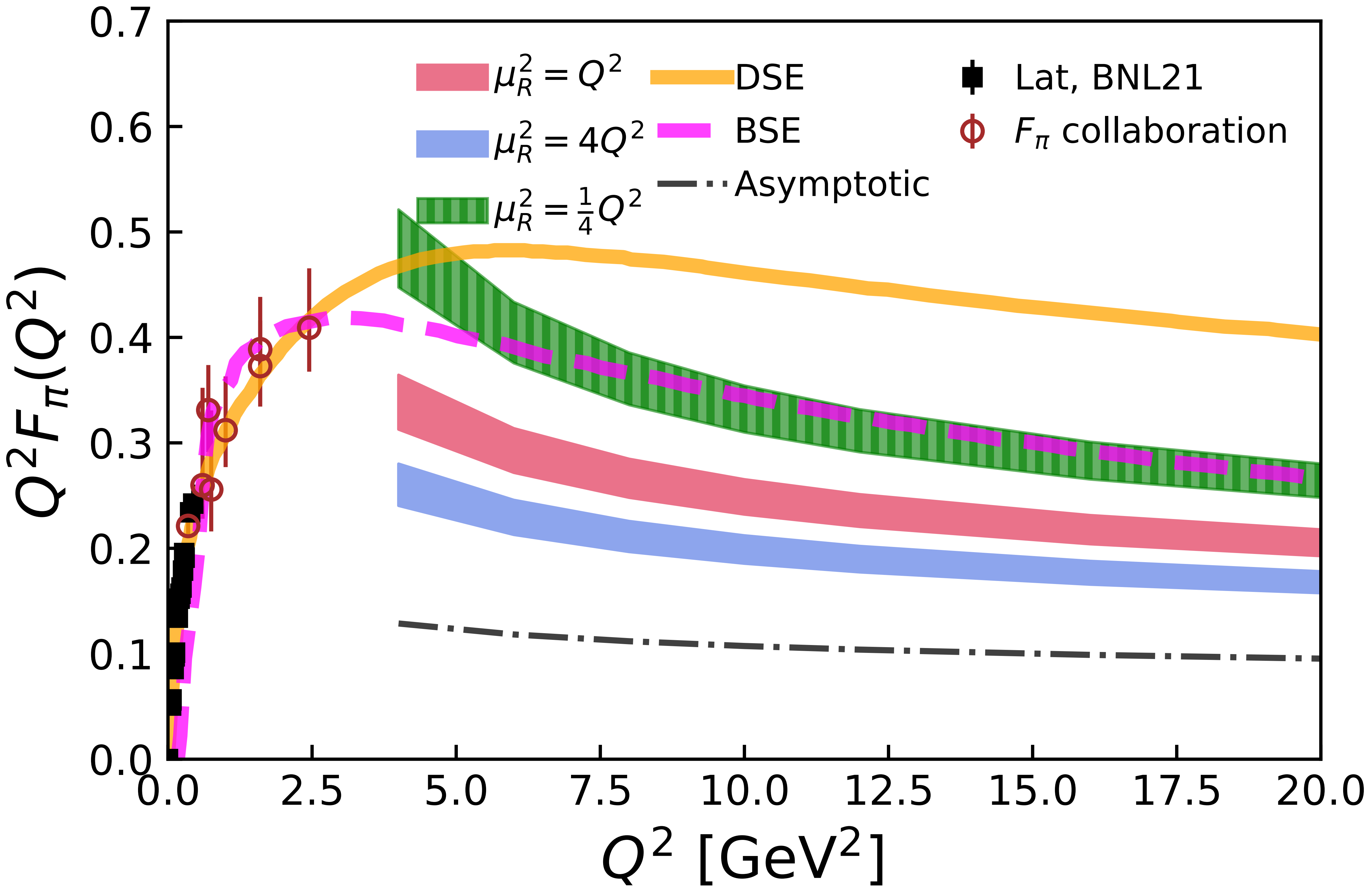
Form factors from pion DA

Electromagnetic form factor

B. Melic, et al, PRD 1999

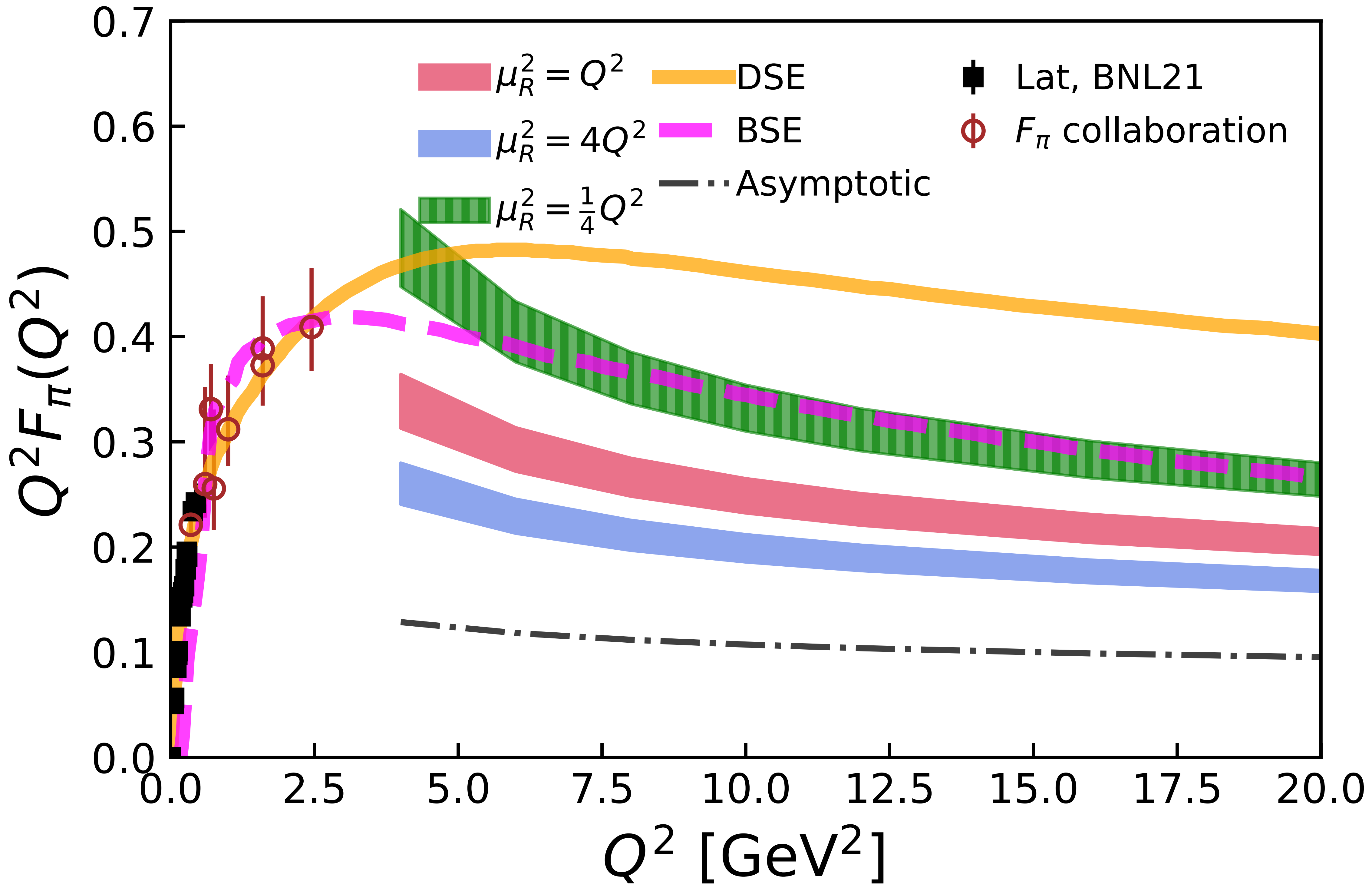
$$F_\pi(Q^2) = \mathcal{N} \int_0^1 \int_0^1 dxdy\ \phi^*(v, \mu_F^2) \times T_F^V(u, v, Q^2, \mu_R^2, \mu_F^2)\phi(u, \mu_F^2) + \mathcal{O}(\frac{1}{Q^2})$$

- We take the one-loop kernels and evolve our model fit result from the initial scale $\mu_0 = 2$ GeV to μ_F.
- We choose $\mu_R^2 = \mu_F^2 = Q^2$ as the central value of the scale, and vary the renormalization scale μ_R by a factor of 2 to estimate the perturbation uncertainty.



Form factors from pion DA

- Our prediction using the LO kernel is systematically lower than the DSE and BSE calculations but higher than the asymptotic DA.
- The scale uncertainty is big when Q^2 is not large enough. NLO and higher-twist DA correction may make a significant contribution.
- Form factors at larger Q^2 are needed to clarify the issue, either from experiment such as Jlab (up to 6 GeV^2) and EIC (up to 40 GeV^2) or direct lattice calculation (up to ~ 10 GeV^2 will be available soon).



Form factors from pion DA

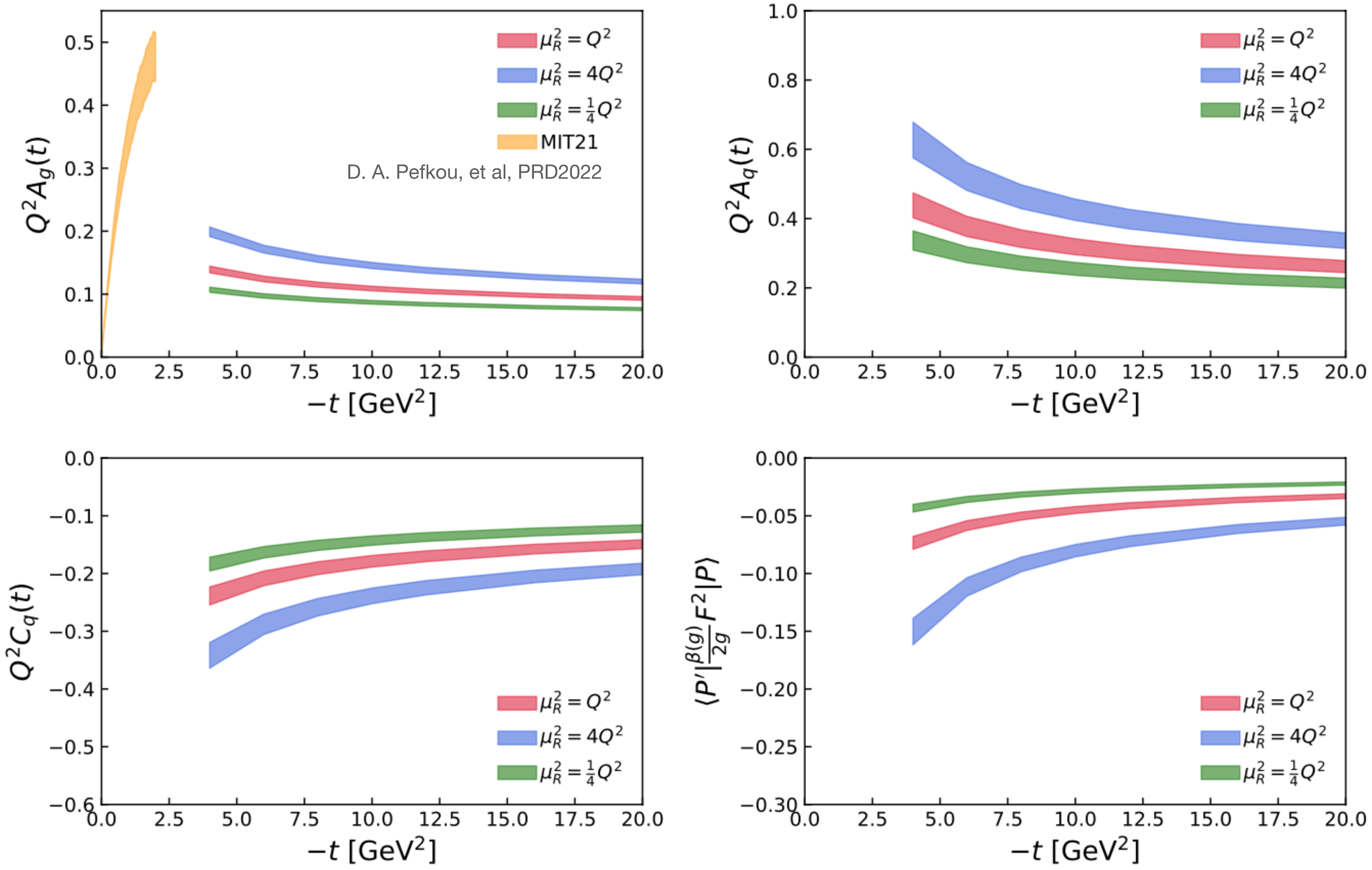
Gravitational form factors:

$$\langle P'|T_g^{\mu\nu}(\mu_R)|P\rangle = 2\bar{P}^\mu\bar{P}^\nu A_g^\pi(t,\mu_R)$$
$$+\frac{1}{2}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)C_g^\pi(t,\mu_R) + 2m^2 g^{\mu\nu}\overline{C}_g^\pi(t,\mu_R)$$

Using the same factorization convention

$$A_g^\pi(t,\mu_R) = \mathcal{N}\int \phi^*(v,\mu_F)$$
$$\times\, \mathscr{A}_g^\pi(u,v,t,\mu_R,\mu_F)\phi(u,\mu_F)\,,$$

X. B. Tong, J. P. Ma and F. Yuan, PLB 2021
X. B. Tong, J. P. Ma and F. Yuan, arXiv: 2203.13493



- Similar to EMFF, the pQCD contribution to pion GFFs $A_g^\pi(t)$ using twist-2 DA is low.

Summary and outlook

- ✓ We presented a lattice QCD study of the quasi-DA matrix element in real-space using the leading-twist OPE method for the first time.
- ✓ We extracted the moments model independently and present the x -dependence of the pion DA based on fits to Ansatze.
- ✓ From the Ansatze-based pion DA, we calculate the pQCD contribution to the form factors with large Q^2 using the leading-twist LO convolutions.
- ➡ We plan to extend the current work to study the Kaon DA and quantify the effects of explicit SU(3) flavor symmetry breaking.
- ➡ We are computing pion and kaon electromagnetic form factors with large Q^2 up to 10 GeV^2.

Thanks for your attention