# GPDs from hard exclusive reactions with gamma-meson pair production

Towards improved hadron femtography with hard exclusive reactions Workshop

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Based on works with S. Wallon, L. Szymanowski, B. Pire, R. Boussarie, G. Duplančić, K. Passek-Kumerički

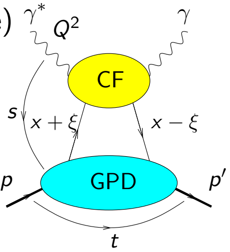
DVCS: exclusive process (non forward amplitude)

Fourier transf.: $t \leftrightarrow$ impact parameter

$(x,\ t) \Rightarrow$ 3-dimensional structure

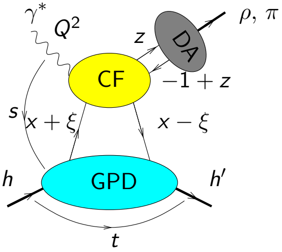
Coefficient Function $\otimes$ Generalized Parton Distribution
(hard) (soft)

[Müller et al. '91 - '94; Radyushkin '96; Ji '97]



DVMP: $\gamma$ replaced by $\rho,\ \pi, \cdots$

GPD (soft) $\otimes$ CF (hard) $\otimes$ Distribution Amplitude (soft)




[Collins, Frankfurt, Strikman '97; Radyushkin '97]

proofs valid only for some restricted cases

Quark GPDs at twist 2 [Diehl]

without helicity flip (chiral-even $\Gamma$ matrices): 4 chiral-even GPDs:
(Note: $\Delta = p' - p$)

$$
\begin{aligned}
F^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+ q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0} \\
&= \frac{1}{2P^+}\left[ H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\,\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right],
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\tilde{F}^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+\gamma_5\, q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0} \\
&= \frac{1}{2P^+}\left[ \tilde{H}^q(x,\xi,t)\, \bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\, \bar{u}(p') \frac{\gamma_5\, \Delta^+}{2m} u(p) \right].
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$H^q \xrightarrow{\xi=0,t=0}$ PDF $q$ $\qquad$ $\tilde{H}^q \xrightarrow{\xi=0,t=0}$ polarized PDF $\Delta q$

with helicity flip (chiral-odd $\Gamma$ matrices): 4 chiral-odd GPDs:

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, i\,\sigma^{+i}\, q(\tfrac{1}{2}z)\ |p\rangle\Big|_{z^+=0,\, z_\perp=0}$$

$$= \frac{1}{2P^+}\bar{u}(p')\left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\, \frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\, \frac{\gamma^+P^i - P^+\gamma^i}{m}\right] u(p)\,,$$

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$$H_T^q \xrightarrow{\xi=0,t=0} \text{quark transversity PDFs } \delta q$$

*Note*: $\tilde{E}_T^q(x,-\xi,t) = -\tilde{E}_T^q(x,\xi,t)$

# Why consider a gamma-meson pair?
## Understanding transversity

- Transverse spin content of the proton:

$$\begin{array}{ccc} |\uparrow\rangle_{(x)} & \sim & |\rightarrow\rangle + |\leftarrow\rangle \\ |\downarrow\rangle_{(x)} & \sim & |\rightarrow\rangle - |\leftarrow\rangle \\ \text{spin along } x & & \text{helicity states} \end{array}$$

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- Transversity GPDs can thus be accessed through chiral-odd $\Gamma$ matrices.
- Since (in the massless limit) QCD and QED are chiral-even ($\gamma^\mu$, $\gamma^\mu\gamma^5$), the chiral-odd quantities ($1$, $\gamma^5$, $[\gamma^\mu, \gamma^\nu]$) which one wants to measure should appear in pairs.

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## Can we probe transversity GPDs in meson production?

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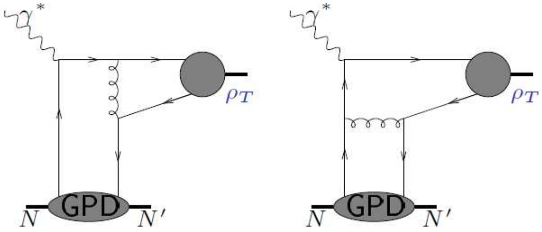
- lowest order diagrammatic argument:




$$\gamma^\alpha[\gamma^\mu, \gamma^\nu]\gamma_\alpha = 0$$

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## Go to higher twist?

- This vanishing only occurs at twist 2
- At twist 3 this process does not vanish [Ahmad, Goldstein, Liuti], [Goloskokov, Kroll]

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- This vanishing only occurs at twist 2
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- However processes involving twist 3 DAs may face problems with factorization (end-point singularities)

  can be made safe in the high-energy $k_T-$factorization approach

  [Anikin, Ivanov, Pire, Szymanowski, Wallon]

# Why consider a gamma-meson pair?

## A convenient alternative solution

- Circumvent this using 3-body final states [Ivanov, Pire, Szymanowski, Teryaev], [Enberg, Pire, Szymanowski], [El Beiyad, Pire, Segond, Szymanowski, Wallon]
- Consider the process $\gamma N \to \gamma M N'$, $M =$meson. Collinear factorisation of the amplitude at large $M^2_{\gamma M}$.

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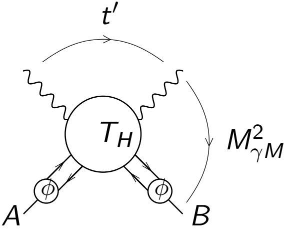
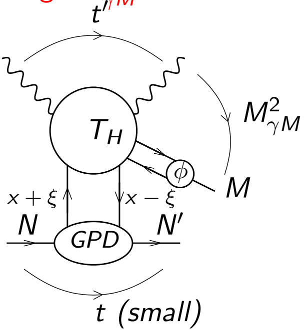



large angle factorization
à la Brodsky Lepage

$\longrightarrow$



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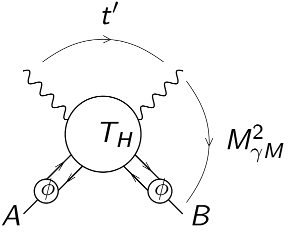


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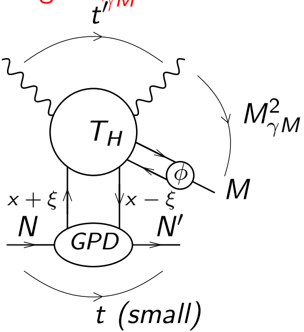
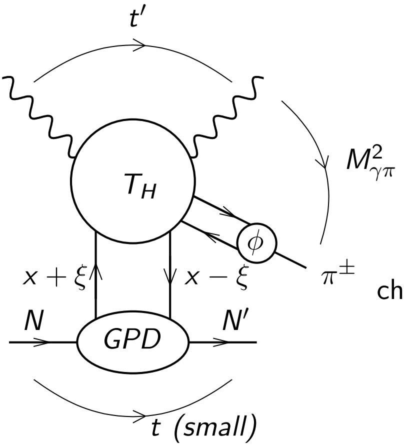



Also $MN \to \gamma\gamma N'$ [2205.07846] (Qiu's talk)

# Why consider a gamma-meson pair?

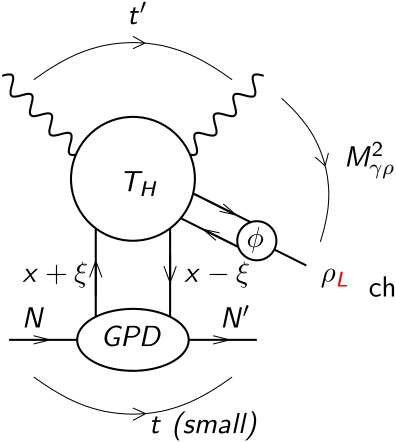
## Chiral-even GPDs using $\pi^\pm\gamma$ production




chiral-even twist 2 DA

chiral-even twist 2 GPD

[G. Duplančić, K. Passek-Kumerički, B. Pire, L. Szymanowski, S. Wallon]



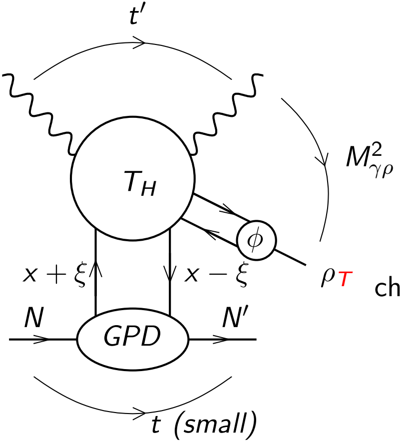

chiral-even twist 2 DA

chiral-even twist 2 GPD

[R. Boussarie, B. Pire, L. Szymanowski, S. Wallon]

# Why consider a gamma-meson pair?

## Chiral-odd GPDs using $\rho_T \gamma$ production

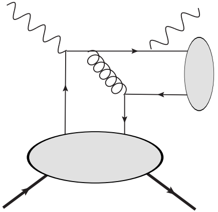



chiral-odd twist 2 DA

chiral-odd twist 2 GPD

[R. Boussarie, B. Pire, L. Szymanowski, S. Wallon]

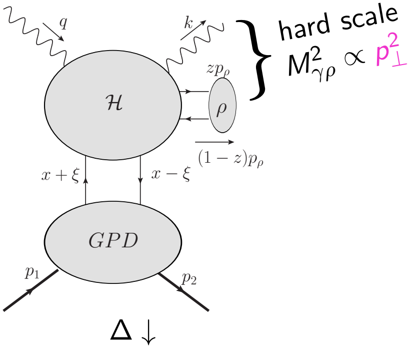
How does it work?



Typical non-zero diagram for a transverse $\rho$ meson

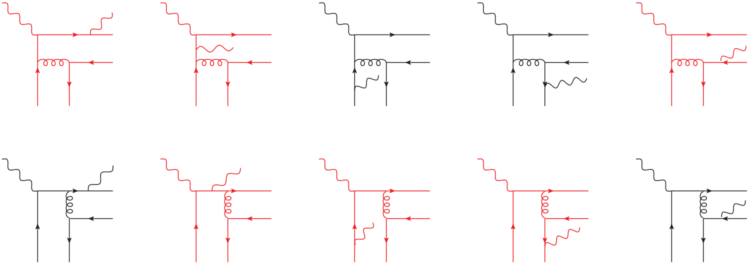
the $\sigma$ matrices (from either the DA or the GPD) do not kill it anymore!

$$\gamma^{(*)}(q) + N(p_1) \to \gamma(k) + \rho^0(p_\rho, \varepsilon_\rho) + N'(p_2)$$




Useful Mandelstam variables: $t = (p_2 - p_1)^2$, $u' = (p_\rho - q)^2$

A total of 20 diagrams to compute



- The other half can be deduced by $q \leftrightarrow \bar{q}$ (anti)symmetry depending on $C$-parity in $t-$channel
- Red diagrams cancel in the chiral-odd case

We parameterise the GPDs in terms of *double distributions (Radyushkin-type parametrisation)*

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For polarized PDFs (and hence transversity PDFs), two scenarios are proposed for the parameterization:

- "standard" scenario, with flavor-symmetric light sea quark and antiquark distributions.
- "valence" scenario with a completely flavor-asymmetric light sea quark densities.

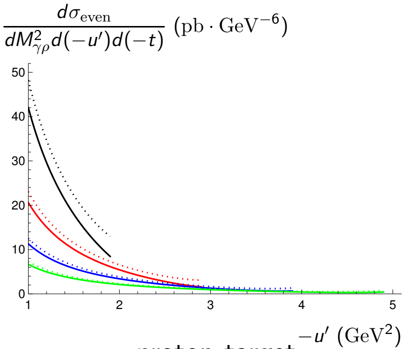
- We take the simplistic asymptotic form of the DAs

$$\phi_\pi(z) = \phi_{\rho\|}(z) = \phi_{\rho\perp}(z) = 6z(1-z)\,.$$

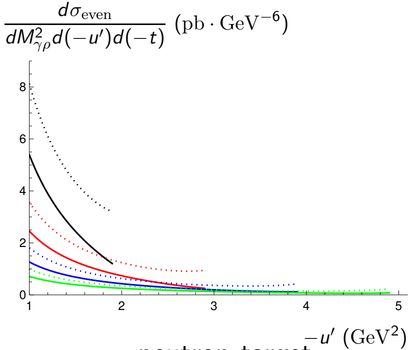
- A non asymptotical wave function can be also investigated (preliminary):

$$\phi_{sing}(z) = \frac{8}{\pi}\sqrt{z(1-z)}\,.$$

## Fully-differential cross-sections: $\rho_L^0$ (Chiral even)




proton target




neutron target

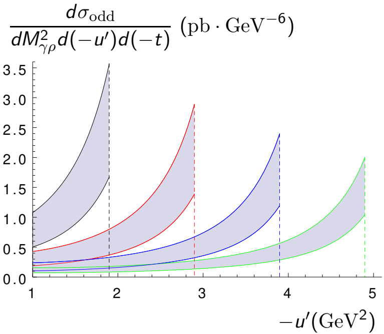
$$S_{\gamma N} = 20\ \text{GeV}^2, \text{ at } -t = (-t)_{\min}$$

$$M^2_{\gamma\rho} = 3, 4, 5, 6\ \text{GeV}^2$$

dotted: "standard" model

solid: "valence" model

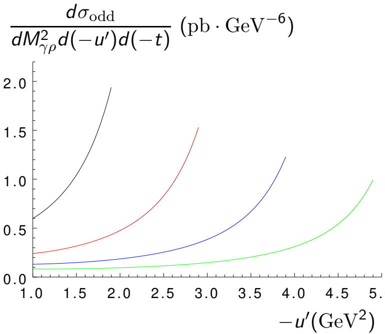
## Fully-differential cross-sections: $\rho^0_T$ (Chiral odd)




proton target

“valence” and “standard” models,

each of them with $\pm 2\sigma$ [S. Melis]




neutron target

“valence“ model only

$$S_{\gamma N} = 20 \ \mathrm{GeV}^2 \text{ at } -t = (-t)_{\mathrm{min}}$$

$$M^2_{\gamma\rho} = 3, 4, 5, 6 \ \mathrm{GeV}^2$$

large angle scattering: $M^2_{\gamma\rho} \sim -u' \sim -t' \gg \Lambda^2_{\mathrm{QCD}}$

$\Longrightarrow$ $-u' > 1\ \mathrm{GeV}^2$ and $-t' > 1\ \mathrm{GeV}^2$ and $(-t)_{\mathrm{min}} \leqslant -t \leqslant .5\ \mathrm{GeV}^2$

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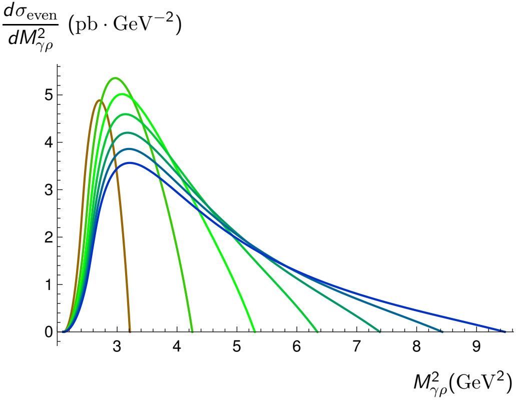
*See backup slides for more details, including information on the phase space evolution in the $(-t, -u')$ plane.*

# Results

## Single differential cross-section: $\rho_L^0$ (Chiral even)



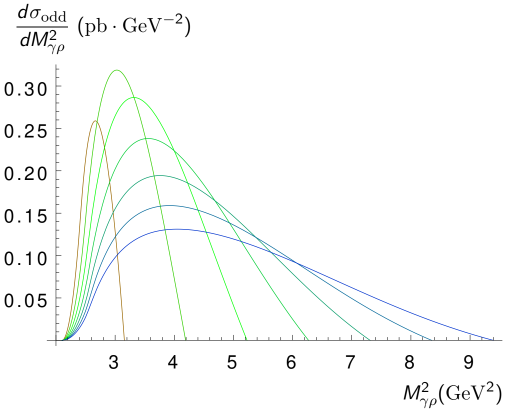

proton target, "valence" scenario

$S_{\gamma N} = 8, 10, 12, 14, 16, 18, 20\ \text{GeV}^2$

typical JLab kinematics

# Results

## Single differential cross-section: $\rho_T^0$ (Chiral odd)

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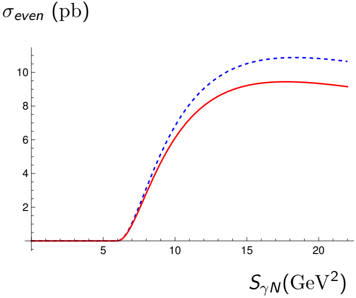
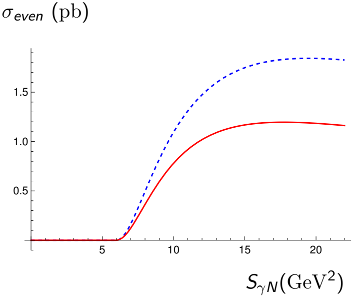
## Integrated cross-section: Valence vs Standard: $\rho_L^0$ (Chiral even)




proton target




neutron target

solid red: "valence" scenario

dashed blue: "standard" one

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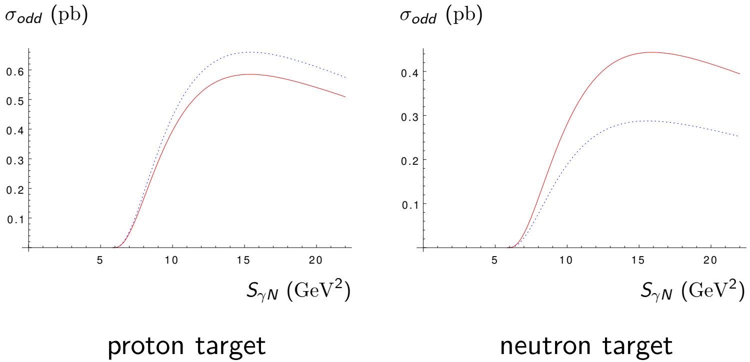


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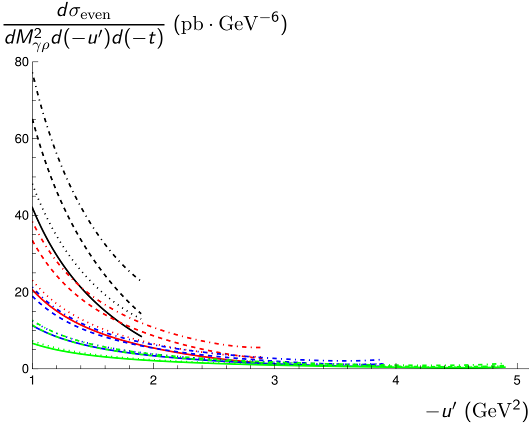
## Fully differential cross-section: Singular DA vs Asymptotical DA: $\rho_L^0$, Chiral-even




dashed dotted:
Sing. DA, “standard” model

dashed:
Sing. DA, “valence” model

dotted:
Asymp. DA, “standard” model

solid:
Asymp. DA, “valence” model

$S_{\gamma N} = 20$ GeV$^2$, at $-t = (-t)_{\rm min}$
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# Results (Preliminary)

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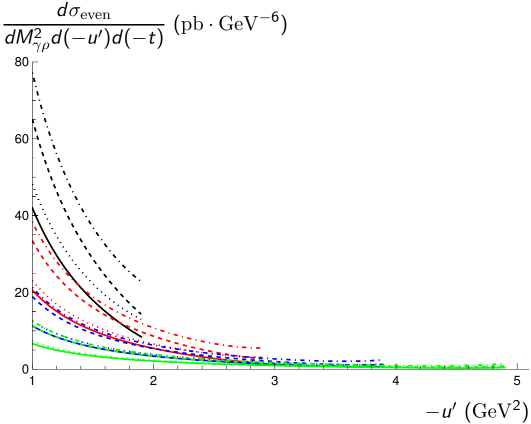


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$S_{\gamma N} = 20\ \mathrm{GeV}^2$, at $-t = (-t)_{\min}$
$M^2_{\gamma\rho} = 3, 4, 5, 6\ \mathrm{GeV}^2$

$\Longrightarrow$ sizable effect, larger than the one due to uncertainties on polarized PDFs

Good statistics: For example, at JLab Hall B:

- untagged incoming $\gamma \Rightarrow$ Weizsäcker-Williams distribution

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- with an expected luminosity of $\mathcal{L} = 100\ \mathrm{nb}^{-1} s^{-1}$, for 100 days of run:
  - $\rho^0_L$ (on $p$) : $\approx 2.4 \times 10^5$
  - $\rho^0_L$ (on $n$) : $\approx 4 \times 10^4$
  - $\rho^0_T$ : $\approx 7.5 \times 10^3$ (Chiral-odd)
  - $\rho^+_L$ : $\approx 1.4 \times 10^5$
  - $\rho^-_L$ : $\approx 1.6 \times 10^5$
  - $\pi^+$ : $\approx 1.8 \times 10^5$
  - $\pi^-$ : $\approx 1.3 \times 10^5$

# Prospects at experiments

## EIC, LHC at UPC,...

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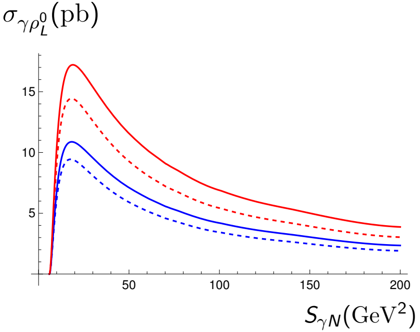
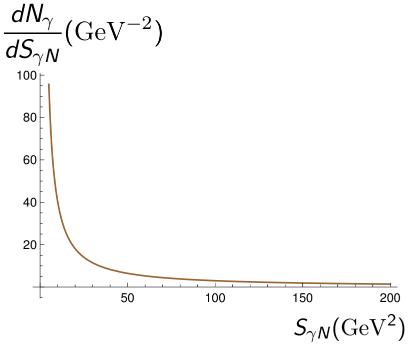
$\Longrightarrow$ *Low luminosity not being compensated by larger photon flux.*

- Photon-flux enhanced by a factor of $Z^2$, but drops rapidly with increasing centre-of-mass energy.

## Why counting rates lower UPCs at LHC?

- Photon-flux enhanced by a factor of $Z^2$, but drops rapidly with increasing centre-of-mass energy.



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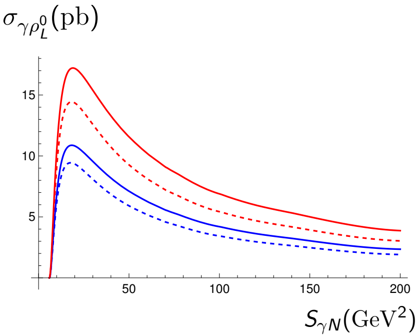


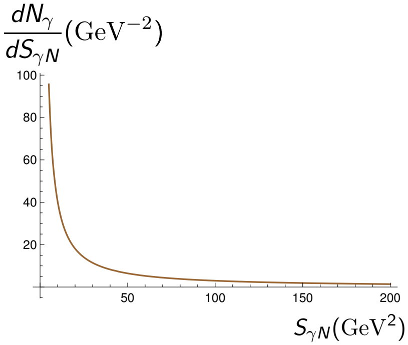


- LHC great for high energy, but JLab better in terms of luminosity.
- Still, LHC gives us access to the small $\xi$ region of GPDs!

# Outlook

- Use non-asymptotical DA, $\phi_M(z) = \frac{8}{\pi}\sqrt{z(1-z)}$, (instead of $\phi_M(z) = 6z(1-z)$) to model the outgoing meson $M$: suggested by AdS/QCD correspondence, dynamical chiral symmetry breaking. [Ongoing]

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- Investigate polarisation asymmetries of the initial $\gamma$. [Ongoing]
- Adjust kinematics for searches at EIC, LHC at UPC, LHeC and COMPASS. [Ongoing]
- Add Bethe-Heitler component (photon emitted from incoming lepton)
  - zero in chiral-odd case.
  - suppressed in chiral-even case, but could estimate their contributions.

# Outlook

- Compute NLO corrections.
  - Cancellation of IR divergences: Indication that QCD factorisation works.

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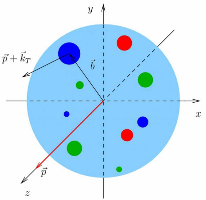
- Consider twist-3 contributions.
- Generalise to electroproduction ($Q^2 \neq 0$) (and include Bethe-Heitler contributions).

The END

# BACKUP SLIDES

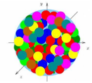
6D/5D

Wigner distributions for hadrons




$W(x,\vec{b},k_T)$

*Experimentally inaccessible directly*



perturbative Regge limit

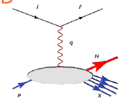
uPDFs (gluons)

Unintegrated parton distributions

$\int d^3\vec{b}$

=

3D




TMDs

$f(x,k_T)$

Semi-inclusive processes

Transverse momentum dependent distributions

$\int d^2k_T \int d\,b_z$

$f(x,b_T)$

Impact parameter distributions

$b_T \Leftrightarrow \Delta$

$H(x,0,t)$

$t=-\Delta^2$

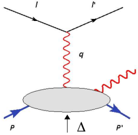
$\int d^2k_T \int Fourier(\vec{b})$

$\xi=0$

GPDs

$H(x,\xi,t)$

generalised parton distributions




exclusive processes

$\int d^2k_T$

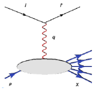
$\int d^2b_T$

$t=0$

$\int dx$

$\int dx\ x^{n-1}$

1D

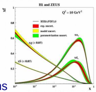


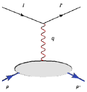

inclusive and semi-inclusive processes

PDFs

$f(x)$

parton distributions



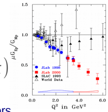




elastic processes

FFs

$G_{E,M}(t)$

form factors



GFFs

generalized form factors

lattices

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left(-\frac{1}{2}z^-\right) \gamma^+ \psi \left(\frac{1}{2}z^-\right) | p_1, \lambda_1 \rangle$$
$$= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H^q(x, \xi, t) \gamma^+ + E^q(x, \xi, t) \frac{i\sigma^{\alpha+}\Delta_\alpha}{2m} \right] u(p_1, \lambda_1)$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left(-\frac{1}{2}z^-\right) \gamma^+ \gamma^5 \psi \left(\frac{1}{2}z^-\right) | p_1, \lambda_1 \rangle$$
$$= \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ \tilde{H}^q(x, \xi, t) \gamma^+ \gamma^5 + \tilde{E}^q(x, \xi, t) \frac{\gamma^5 \Delta^+}{2m} \right] u(p_1, \lambda_1)$$

- Take the limit $\Delta_\perp = 0$.
- In that case and for small $\xi$, the dominant contributions come from $H^q$ and $\tilde{H}^q$.

$$
\begin{aligned}
&\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left(-\frac{1}{2}z^-\right) i\sigma^{+i} \psi \left(\frac{1}{2}z^-\right) | p_1, \lambda_1 \rangle \\
= \;& \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H_T^q(x, \xi, t) i\sigma^{+i} + \tilde{H}_T^q(x, \xi, t) \frac{P^+\Delta^i - \Delta^+ P^i}{M_N^2} \right. \\
+ \;& \left. E_T^q(x, \xi, t) \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2M_N} + \tilde{E}_T^q(x, \xi, t) \frac{\gamma^+ P^i - P^+\gamma^i}{M_N} \right] u(p_1, \lambda_1)
\end{aligned}
$$

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- GPDs can be represented in terms of Double Distributions [Radyushkin]

$$H^q(x,\xi,t=0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \; \delta(\beta+\xi\alpha - x)\, f^q(\beta,\alpha)$$

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- ansatz for these Double Distributions [Radyushkin]:
  - chiral-even sector:

$$f^q(\beta,\alpha,t=0) = \Pi(\beta,\alpha)\, q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\, \bar{q}(-\beta)\, \Theta(-\beta)\,,$$
$$\tilde{f}^q(\beta,\alpha,t=0) = \Pi(\beta,\alpha)\, \Delta q(\beta)\Theta(\beta) + \Pi(-\beta,\alpha)\, \Delta\bar{q}(-\beta)\, \Theta(-\beta)\,.$$

  - chiral-odd sector:

$$f_T^q(\beta,\alpha,t=0) = \Pi(\beta,\alpha)\, \delta q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\, \delta\bar{q}(-\beta)\, \Theta(-\beta)\,.$$

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  - $\Pi(\beta,\alpha) = \frac{3}{4}\frac{(1-\beta)^2-\alpha^2}{(1-\beta)^3}$ : profile function

- simplistic factorized ansatz for the $t$-dependence:

$$H^q(x,\xi,t) = H^q(x,\xi,t=0) \times F_H(t)$$

with $F_H(t) = \frac{C^2}{(t-C)^2}$ a standard dipole form factor ($C = 0.71\mathrm{GeV}^2$)

# Sets of used PDFs

- $q(x)$ : unpolarized PDF [GRV-98]

  and [MSTW2008lo, MSTW2008nnlo, ABM11nnlo, CT10nnlo]

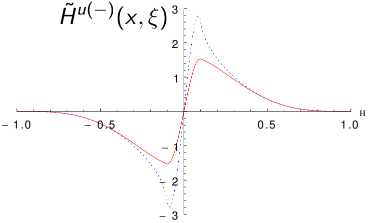
- $\Delta q(x)$ polarized PDF [GRSV-2000]

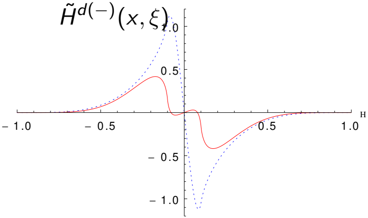
- $\delta q(x)$ : transversity PDF [Anselmino *et al.*]

Effects are not significant! But relevant for NLO corrections!

Typical kinematic point: $\xi = .1 \;\leftrightarrow\; S_{\gamma N} = 20\ \mathrm{GeV}^2$ and $M^2_{\gamma\rho} = 3.5\ \mathrm{GeV}^2$

$$\tilde{H}^{q(-)}(x,\xi,t) = \tilde{H}^{q}(x,\xi,t) - \tilde{H}^{q}(-x,\xi,t) \quad [C = -1]$$




"valence" and "standard": two GRSV Ansätze for $\Delta q(x)$

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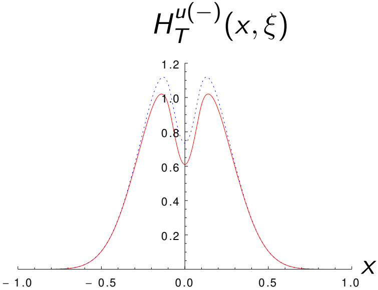
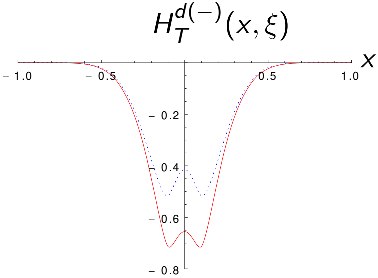
$$H_T^{q(-)}(x,\xi,t) = H_T^q(x,\xi,t) + H_T^q(-x,\xi,t) \quad [C=-1]$$




"valence" and "standard": two GRSV Ansätze for $\Delta q(x)$

$\Rightarrow$ two Ansätze for $\delta q(x)$

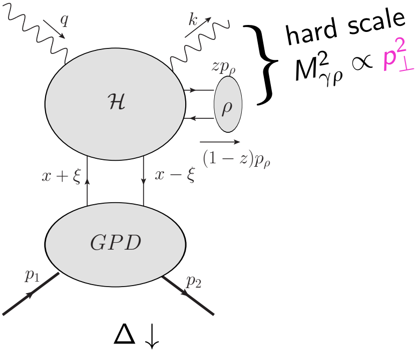
- Helicity conserving (vector) DA at twist 2: $\rho_L$

$$\langle 0|\bar{u}(0)\gamma^\mu u(x)|\rho^0(p,s)\rangle = \frac{p^\mu}{\sqrt{2}} f_\rho \int_0^1 du\ e^{-iup\cdot x}\phi_\parallel(u)$$

- $\rho_T$ DA at twist 2:

$$\langle 0|\bar{u}(0)\sigma^{\mu\nu} u(x)|\rho^0(p,s)\rangle = \frac{i}{\sqrt{2}}(\epsilon_\rho^\mu p^\nu - \epsilon_\rho^\nu p^\mu) f_\rho^\perp \int_0^1 du\ e^{-iup\cdot x}\ \phi_\perp(u)$$

- Work in the limit of:
  - $\Delta_\perp \ll p_\perp$
  - $M^2,\ m_\rho^2 \ll M_{\gamma\rho}^2$



- Work in the limit of:
  - $\Delta_\perp \ll p_\perp$
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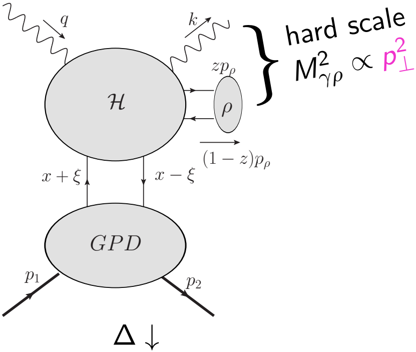
- initial state particle momenta:

$$q^\mu = n^\mu,$$
$$p_1^\mu = (1+\xi)\, p^\mu + \frac{M^2}{s(1+\xi)} n^\mu$$



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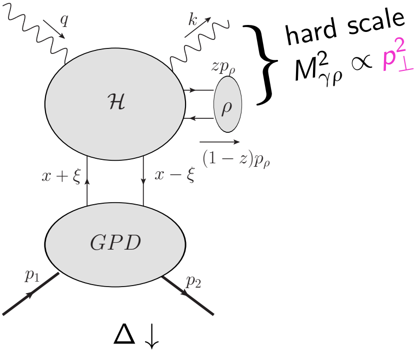
- final state particle momenta:



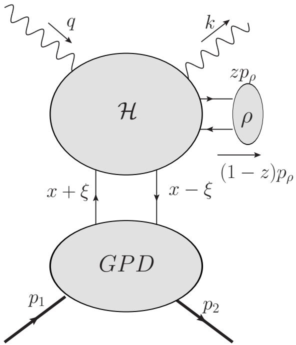

$$p_2^\mu = (1-\xi)\, p^\mu + \frac{M^2+\vec{p}_t^{\,2}}{s(1-\xi)} n^\mu + \Delta_\perp^\mu$$

$$k^\mu = \alpha\, n^\mu + \frac{(\vec{p}_t - \vec{\Delta}_t/2)^2}{\alpha s} p^\mu + p_\perp^\mu - \frac{\Delta_\perp^\mu}{2},$$

$$p_\rho^\mu = \alpha_\rho\, n^\mu + \frac{(\vec{p}_t + \vec{\Delta}_t/2)^2 + m_\rho^2}{\alpha_\rho s} p^\mu - p_\perp^\mu - \frac{\Delta_\perp^\mu}{2},$$

$$\mathcal{A} \propto \int_{-1}^{1} dx \int_{0}^{1} dz \; T(x,\xi,z) \; H(x,\xi,t) \; \Phi_\rho(z)$$

- $z$ integration performed analytically using an asymptotic DA $\propto z(1-z)$
- GPD models plugged into expression for amplitude and the integral performed w.r.t. $x$ numerically.



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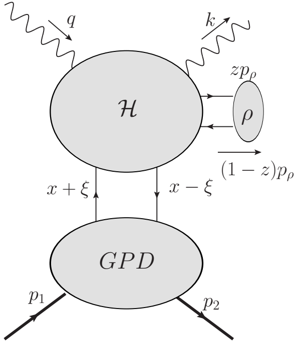
- GPD models plugged into expression for amplitude and the integral performed w.r.t. $x$ numerically.
- Differential cross section:

$$\left.\frac{d\sigma}{dt\, du'\, dM^2_{\gamma\rho}}\right|_{-t=(-t)_{min}} = \frac{|\overline{\mathcal{A}}|^2}{32 S^2_{\gamma N} M^2_{\gamma\rho} (2\pi)^3}\,.$$

- Kinematic parameters: $S_{\gamma N}$, $M^2_{\gamma\rho}$ and $-u'$
  Recall: $u' = (p_\rho - q)^2$, $t = (p_2 - p_1)^2$



large angle scattering: $M^2_{\gamma\rho} \sim -u' \sim -t'$

$\Longrightarrow$ $-u' > 1\ \mathrm{GeV}^2$ and $-t' > 1\ \mathrm{GeV}^2$ and $(-t)_{\min} \leqslant -t \leqslant .5\ \mathrm{GeV}^2$

# Results

## Phase space integration: Evolution in $(-t, -u')$ plane

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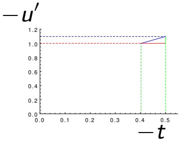
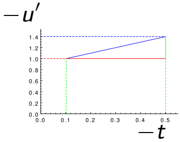
$\Longrightarrow -u' > 1 \text{ GeV}^2$ and $-t' > 1 \text{ GeV}^2$ and $(-t)_{\min} \leqslant -t \leqslant .5 \text{ GeV}^2$

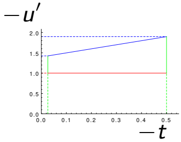
example: $S_{\gamma N} = 20 \text{ GeV}^2$



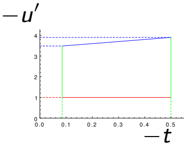

$M_{\gamma\rho} = 2.2 \text{ GeV}^2$



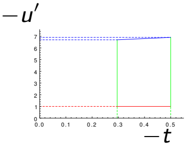

$M^2_{\gamma\rho} = 2.5 \text{ GeV}^2$



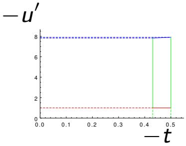

$M_{\gamma\rho} = 3 \text{ GeV}^2$




$M_{\gamma\rho} = 5 \text{ GeV}^2$

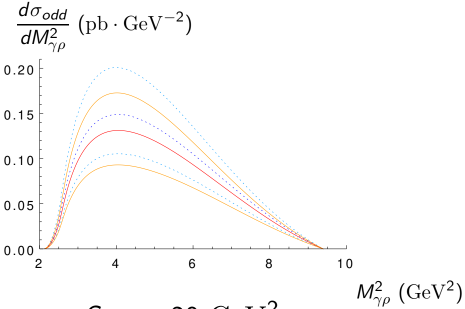



$M_{\gamma\rho} = 8 \text{ GeV}^2$




$M_{\gamma\rho} = 9 \text{ GeV}^2$

## Single differential cross-section: Valence vs Standard: $\rho_T$ (Chiral odd)

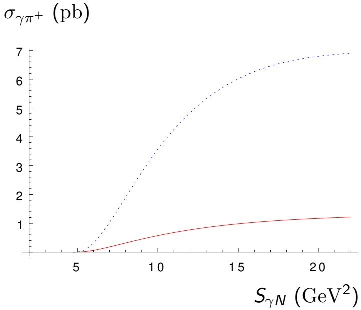



$$S_{\gamma N} = 20 \text{ GeV}^2$$

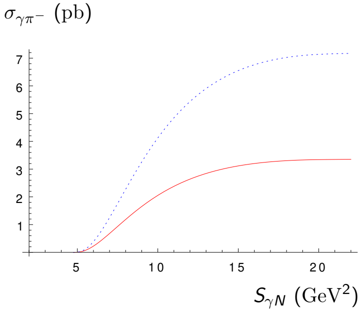
Various ansätze for the PDFs $\Delta q$ used to build the GPD $H_T$:

- *dotted curves*: “standard” scenario
- solid curves: “valence” scenario
- deep-blue and red curves: central values
- light-blue and orange: results with $\pm 2\sigma$.

Integrated cross-section: Valence vs Standard: $\pi^{\pm}$ (Chiral even)




proton target




neutron target

solid red: “valence” scenario

dashed blue: “standard” one