Lifetimes of Near-Eternal False Vacua

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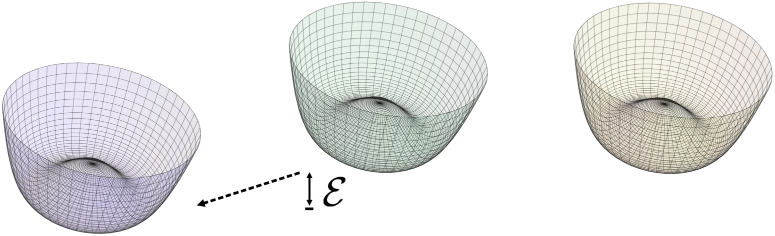
Based on `2012.10555`,

with Aleksey Cherman



Outline

This talk:

Gaining physical intuition about theories with exact ‘decomposition’
by breaking the higher form symmetries responsible

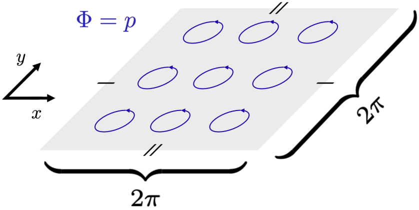
1d: Landau problem on a torus

2d: Charge-p Schwinger model

4d: QCD with a modified instanton sum

A Simple Quantum Mechanical System

Particle on a torus in a constant B field: “Landau problem”



In Landau gauge, $\vec{A} = \frac{p}{2\pi} x\,\hat{y}$:

$$L = \frac{1}{2g}(\dot{x}^2 + \dot{y}^2) + \frac{p}{2\pi} x\,\dot{y}$$

Relevant global symmetries:

$$(\mathbb{Z}_p)_x : x \to x + \tfrac{2\pi}{p}, \qquad (\mathbb{Z}_p)_y : y \to y + \tfrac{2\pi}{p}$$

$$(\mathbb{Z}_2)_P : x \to -x,\ y \to -y$$

Mixed Anomaly

Lagrangian: $$L = \frac{1}{2g}(\dot{x}^2 + \dot{y}^2) + \frac{p}{2\pi} x\, \dot{y}$$

Canonical momenta: $p_x = \frac{\dot{x}}{g}, \quad p_y = \frac{\dot{y}}{g} + \frac{p}{2\pi}\, x$ (note: $p_y \sim p_y + p$)

Hamiltonian: $$H = \frac{g}{2}\left[\hat{p}_x^2 + \left(\hat{p}_y - \frac{p}{2\pi}\hat{x}\right)^2\right]$$

Symmetry operators:

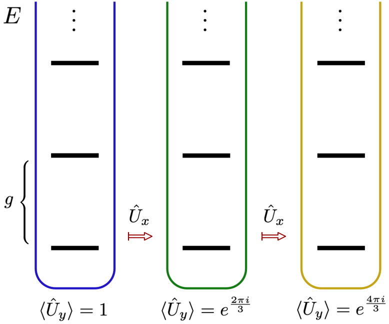
$$\hat{U}_x = e^{i\frac{2\pi}{p}\hat{p}_x}\, e^{-i\hat{y}}, \quad \hat{U}_y = e^{i\frac{2\pi}{p}\hat{p}_y}, \quad (\hat{U}_x)^p = (\hat{U}_y)^p = 1$$

$\hat{U}_x$ charged under $(\mathbb{Z}_p)_y$ $\implies \hat{U}_x\hat{U}_y = e^{2\pi i/p}\,\hat{U}_y\hat{U}_x$

Central extension of $(\mathbb{Z}_p)_x \times (\mathbb{Z}_p)_y \implies$ mixed 't Hooft anomaly

Mixed Anomaly

Hilbert space 'decomposes' into charge sectors: $\mathcal{H} = \bigoplus_{k=0}^{p-1} \mathcal{H}_k$



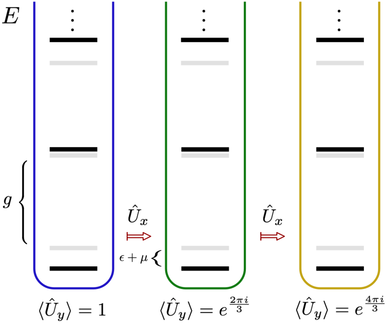
Mixed anomaly ensures sectors are degenerate:

smallest representation of $\hat{U}_x \hat{U}_y = e^{2\pi i/p}\, \hat{U}_y \hat{U}_x$ is p-dimensional (clock&shift)

Mixed Anomaly

$(\mathbb{Z}_p)_x \times (\mathbb{Z}_p)_y$-symmetric perturbations:

$$L = \frac{1}{2g}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{p}{2\pi} x\,\dot{y} + \epsilon\cos(px) + \mu\cos(py)$$



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Mixed Anomaly

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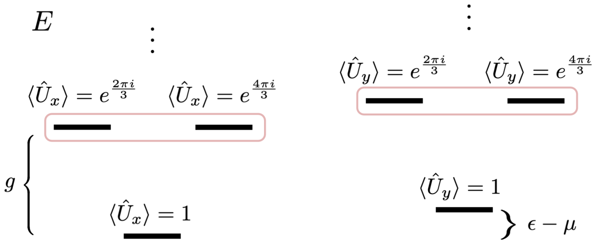


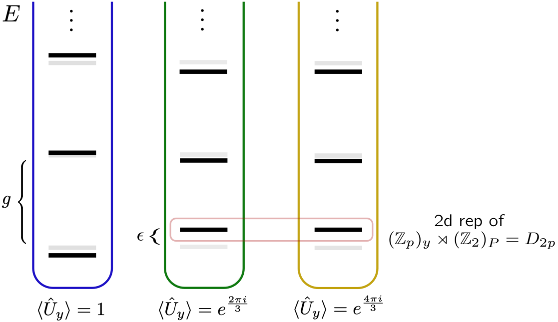
No mixed anomaly $\Longrightarrow$ ordinary representations of symmetries

Two copies of "T_p" model studied in
(Aitken, Cherman, Ünsal, '18)

Breaking the Symmetries

Break $(\mathbb{Z}_p)_x$:

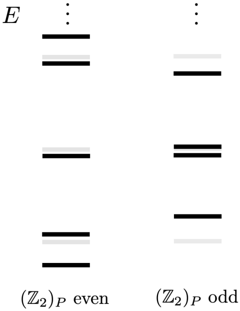
$$L = \frac{1}{2g}(\dot{x}^2 + \dot{y}^2) + \frac{p}{2\pi} x\,\dot{y} + \epsilon\cos(x)$$



Ground state unique, but Hilbert space still 'decomposes' (somewhat trivially!)

Breaking the Symmetries

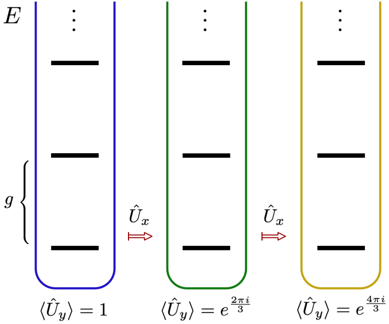
$$\boxed{\text{Break } (\mathbb{Z}_p)_x \text{ and } (\mathbb{Z}_p)_y\text{:}}\; L = \frac{1}{2g}(\dot{x}^2 + \dot{y}^2) + \frac{p}{2\pi} x\,\dot{y} + \epsilon\cos(x) + \mu\cos(y)$$

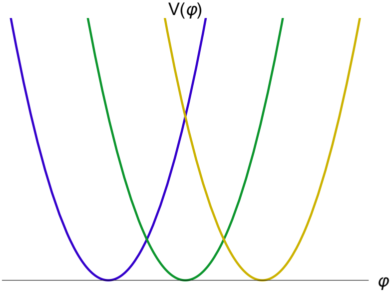


States are completely split—no selection rule prohibiting parity-even states from decaying to the unique ground state

Generalizing to Decomposition in QFTs

Ground states in QM → Lorentz-invariant vacua in QFT





0-form global symmetry → $(d-1)$-form global symmetry

Charge-p Schwinger Model

$1+1$d QED:

$U(1)$ gauge field + (massless) Dirac fermion

Komargodski, Sharon,
Thorngren, Zhou '17
Anber, Poppitz '18
Misumi, Tanizaki, Ünsal '19
Komargodski, Ohmori,
Roumpedakis, Seifnashri '20, ...

$$\mathcal{L} = \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\gamma^\mu(\partial_\mu - i p\, a_\mu)\psi + \frac{i\theta}{2\pi}\epsilon^{\mu\nu}\partial_\mu a_\nu \qquad \boxed{p \in \mathbb{Z}}$$

- $Q_{\text{top}} = \dfrac{1}{2\pi}\displaystyle\int_{M_2} \epsilon^{\mu\nu}\partial_\mu a_\nu \in \mathbb{Z}$, so that $\theta \sim \theta + 2\pi$
- $[e^2] = 2$, photon has $d-2=0$ physical polarizations

Discrete chiral symmetry:

- $U(1)_A$ is anomalous: $\psi \to e^{i\alpha\gamma_5}\psi$, $\mathcal{D}\bar{\psi}\mathcal{D}\psi \to \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{i2p\alpha\, Q_{\text{top}}}$
  $\implies \alpha = \frac{2\pi k}{2p}$, $k = 0, 1, \ldots, p-1$
- $U(1)_A \to \mathbb{Z}_{2p}$, faithfully $\mathbb{Z}_p^{(0)} = \mathbb{Z}_{2p}/\mathbb{Z}_2$

Bosonization and 1-Form Symmetry

Abelian bosonization (Coleman, '75)

$$\boxed{\mathcal{L} = \frac{1}{2e^2}(da)^2 + \frac{1}{8\pi}(d\varphi)^2 + \frac{ip}{2\pi}\varphi\, da} \quad \varphi \sim \varphi + 2\pi$$

- 0-form chiral ⟶ shift symmetry $\mathbb{Z}_p^{(0)} : \varphi \to \varphi + \frac{2\pi}{p}$
- 1-form symmetry $\mathbb{Z}_p^{(1)} : e^{i \oint_C a} \to e^{2\pi i/p}\, e^{i \oint_C a}$

EOM of a_μ: $d \underbrace{\left(\frac{p}{2\pi}\varphi - \frac{i}{e^2} \star da\right)}_{\equiv\, \star j^{(2)}} = 0 \quad \Longrightarrow$

$U_\alpha(x) = e^{i\alpha \star j^{(2)}(x)}$

is topological (independent of x)

and *well-defined* if $\alpha = \frac{2\pi k}{p}$

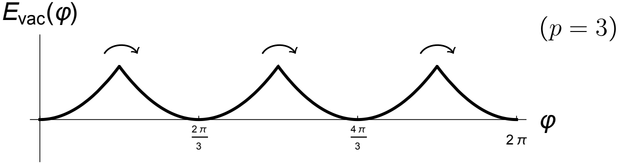
$$\langle U_{\frac{2\pi}{p}}(x)\, e^{i \oint_C a}\rangle = e^{2\pi i/p}\langle e^{i \oint_C a}\rangle \text{ if } x \text{ links } C$$

$$\boxed{U_{\frac{2\pi}{p}}(x) = e^{\frac{2\pi}{p}\frac{1}{e^2}\star da(x)}\, e^{i\varphi(x)} \text{ is the symmetry operator for } \mathbb{Z}_p^{(1)}}$$

$U_\alpha(x)$ charged under $\mathbb{Z}_p^{(0)} \Longrightarrow$ $\boxed{\mathbb{Z}_p^{(0)} \times \mathbb{Z}_p^{(1)} \text{ mixed anomaly}}$

Scalar Effective Potential

Naive picture: vacuum energy density is **everywhere finite**

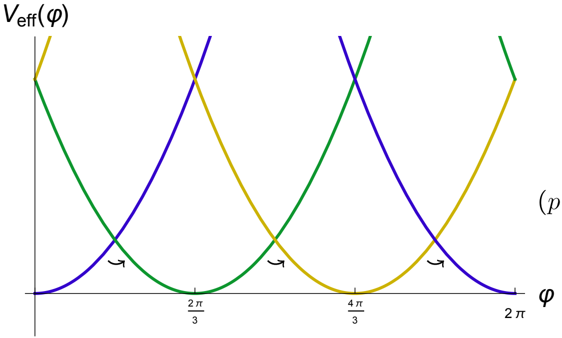


Discrete shift (chiral) symmetry spontaneously broken

Finite derivative $\Longrightarrow$ finite DW tension?

Scalar Effective Potential

Actual picture: many branches



$(p = 3)$

DW must implement transition between branches

Not possible in theory with *just* a charge-p fermion

$\langle U_{\frac{2\pi}{3}}(x) \rangle = 1, e^{2\pi i/3}, e^{4\pi i/3}$ in the 3 distinct branches

Connection to Mixed Anomaly

More formally:

No dynamical domain wall in chiral limit $\Longleftrightarrow$ Finite-volume path integral with $\mathbb{Z}_p^{(0)}$-twisted b.c.s '$\tilde{Z}$' vanishes

Usually expect $\tilde{Z}$ to be dominated by DW, finite action in finite volume $\tilde{Z} \sim e^{-TV}$

$$\tilde{Z}(L,\beta) = \int\limits_{\varphi(\beta)=\varphi(0)+\frac{2\pi}{p}} \mathcal{D}a\,\mathcal{D}\varphi\, e^{-\int_{S^1_L\times S^1_\beta}\mathcal{L}} = Z\left(\begin{matrix}\text{background}\\ \mathbb{Z}_p^{(0)}\ \text{gauge field}\end{matrix}\right)$$

$$\Big\downarrow\ \begin{matrix}\mathbb{Z}_p^{(1)}\ \text{transformation:}\\ a \to a + \epsilon^{(1)},\\ \oint_L \epsilon^{(1)} = \frac{2\pi}{p}\end{matrix}$$

mixed 't Hooft anomaly between $\mathbb{Z}_p^{(0)}$ and $\mathbb{Z}_p^{(1)}$

$$= e^{\frac{2\pi i}{p}}\, Z\left(\begin{matrix}\text{background}\\ \mathbb{Z}_p^{(0)}\ \text{gauge field}\end{matrix}\right) = 0$$

‘Universes’

Finite-volume path integral with $\mathbb{Z}_p^{(0)}$-twisted boundary conditions and *any local operator insertion* vanishes

$$\int_{\varphi(\beta)=\varphi(0)+\frac{2\pi}{p}} \mathcal{D}a\,\mathcal{D}\varphi\,\mathcal{O}(x)\,e^{-\int_{T^2}\mathcal{L}} = 0$$

$m = 0$: states with different $\langle e^{i\varphi}\rangle$ are not mixed by local operators

Generic m: states with different $\langle U_{\frac{2\pi}{p}}\rangle$ are not mixed by local operators

- only non-local operators $e^{i\int a}$ can connect the states
- stronger than superselection rule: applies at finite volume

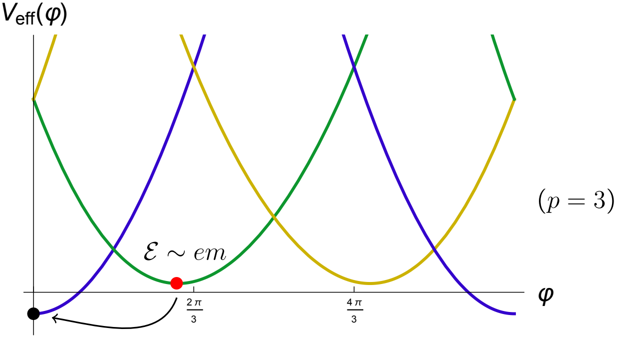
Noted in 2d Ando, Hellerman, Henriques, Pantev, Sharpe '07
Sharpe '14, '19; Anber, Poppitz '18; Aminov '19

Recently such sectors have been dubbed ‘universes’

Tanizaki, Ünsal '19;
Komargodski, Ohmori, Roumpedakis, Seifnashri '20

Scalar Effective Potential

Turn on a mass: $m\,\bar{\psi}\psi \to -em\,\cos(\varphi) \in \mathcal{L}$

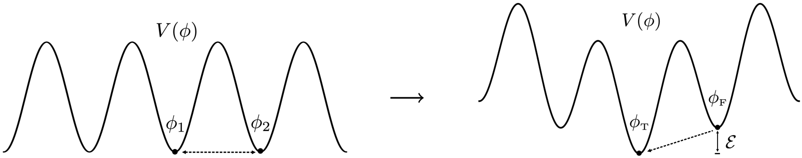


$\langle U_{\frac{2\pi}{3}}(x)\rangle = 1, e^{2\pi i/3}, e^{4\pi i/3}$ in the 3 distinct branches ‘Universes’

Mass term lifts the degeneracy between universes

What is the fate of the false vacua?

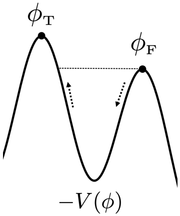
The Fate of the False Vacuum



Euclidean picture (Coleman, '77)

$$\Gamma \sim \exp\left(-S_{\text{bounce}}\right) \sim \exp\left(-\# \frac{T^d}{\mathcal{E}^{d-1}}\right)$$

$d =$ spacetime dimension



$T =$ tension (energy per unit area in 4d)

of domain wall (DW) between $\phi_1 \leftrightarrow \phi_2$

$\Gamma \to 0$ as $T \to \infty$ with $\mathcal{E}$ held fixed

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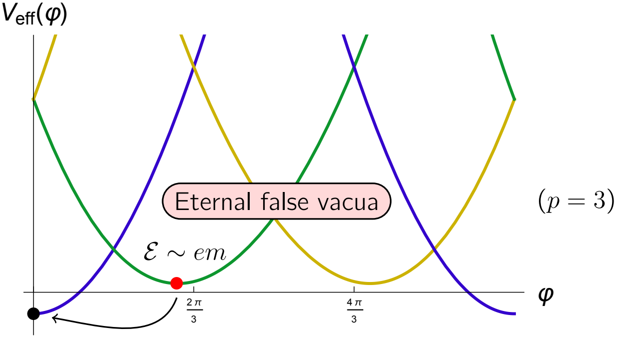

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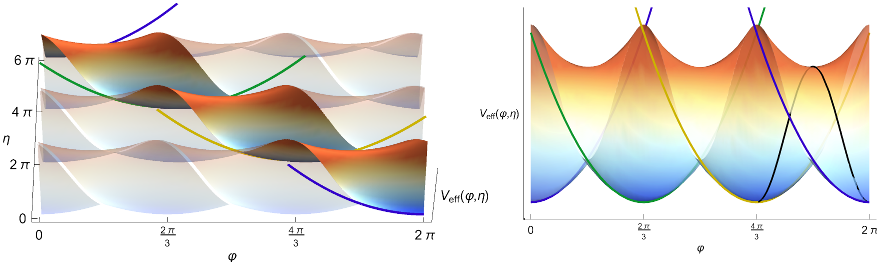
Breaking the 1-Form Symmetry

Add a charge-1 fermion with mass $M \gg m, e$

$\Longleftrightarrow$ bosonized scalar η with $V(\eta) = -M^2 \cos(\eta)$



Domain walls are sine-gordon kinks, $T \sim M$

DW $=$ worldline of the heavy charge-1 fermion

Breaking the 1-Form Symmetry

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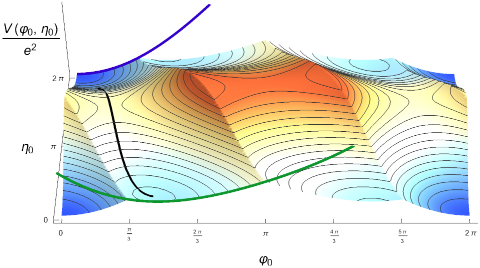
$\Longleftrightarrow$ bosonized scalar η with $V(\eta) = -M^2\cos(\eta)$



$$\Gamma \sim \exp\left(-\frac{T^2}{\mathcal{E}}\right) \sim \exp\left(-\frac{M^2}{em}\right)$$

arbitrarily large ‘UV’ scale

‘natural’ scales of the theory

Modifying the Instanton Sum in QCD

Possible to restrict instanton number Q_{top}

- $Q_{\text{top}} = 0$ without violating locality: couple to an **axion**

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + f^2 |d\chi|^2 + \frac{i}{8\pi^2}\,\chi\,\text{tr}F \wedge F$$

Integral over zero mode χ_0
gives delta function constraint:
$Q_{\text{top}} = 0$

- $Q_{\text{top}} \in p\mathbb{Z}$ without violating locality + without new propagating dof: couple to a **TQFT** (Seiberg '10; Tanizaki, Ünsal '19)

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + i\,\chi\left(\frac{1}{8\pi^2}\text{tr}F \wedge F - p\,\frac{1}{2\pi}dc^{(3)}\right)$$

$\chi \sim \chi + 2\pi$

$c^{(3)} =$ 3-form $U(1)$ gauge field:

$\int \frac{dc^{(3)}}{2\pi} \in \mathbb{Z}$

- Local constraint: $\frac{1}{8\pi^2}\text{tr}F \wedge F = \frac{p}{2\pi}dc^{(3)} \quad \Longrightarrow \quad Q_{\text{top}} = \frac{p}{2\pi}\int dc^{(3)} \in p\,\mathbb{Z}$
- Theta periodicity shorter $\theta \sim \theta + \frac{2\pi}{p}$, no change to $\mathcal{X}_{\text{top}}$

Symmetries and Universes

$$\mathcal{L} = \frac{1}{2g^2}\mathrm{tr}F \wedge \star F + \bar{\psi}_i(\not{D} + m)\psi_i + \frac{i\theta}{8\pi^2}\mathrm{tr}F \wedge F + i\,\chi\left(\frac{1}{8\pi^2}\mathrm{tr}F \wedge F - \frac{p}{2\pi}\,dc^{(3)}\right)$$

$\mathbb{Z}^{(0)}_{2N_f p}$: $\psi_i \to e^{i\frac{2\pi}{2N_f p}\gamma_5}\psi_i$, $\chi \to \chi - \frac{2\pi}{p}$, order parameters for $\mathbb{Z}^{(0)}_p$: $\det \bar{\psi}_i\psi_j$, $e^{i\chi}$

$\mathbb{Z}^{(3)}_p : e^{i\oint_{M_3} c^{(3)}} \to e^{2\pi i/p}\, e^{i\oint_{M_3} c^{(3)}}$

$U(1)^{(2)} : j^{(3)} = \star d\chi$

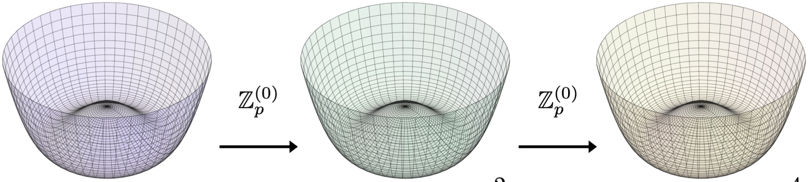
- EOM of $c^{(3)} : d\chi = 0 \quad \Longrightarrow \quad \chi =$ constant
- Sum over topological charges of $c^{(3)}$: $\chi = \frac{2\pi k}{p}, \quad k = 0, 1, \ldots, p-1$
- Path integral over χ: sum over k

$\Longrightarrow$ $e^{i\chi}$ is a topological (constant) operator:
it is the symmetry operator for $\mathbb{Z}^{(3)}_p$

Universes labelled by expectation values of $\langle e^{i\chi}\rangle = e^{i2\pi k/p}$

Vacua

Assume that when N_f is not too large, massless QCD breaks continuous and discrete chiral symmetries:



$$\arg\langle \det \bar{\psi}_{L,i}\psi_{R,j}\rangle = 0 \qquad \arg\langle \det \bar{\psi}_{L,i}\psi_{R,j}\rangle = \frac{2\pi}{3} \qquad \arg\langle \det \bar{\psi}_{L,i}\psi_{R,j}\rangle = \frac{4\pi}{3}$$

$$\langle e^{i\chi}\rangle = 1 \qquad \langle e^{i\chi}\rangle = e^{-2\pi i/3} \qquad \langle e^{i\chi}\rangle = e^{-4\pi i/3}$$

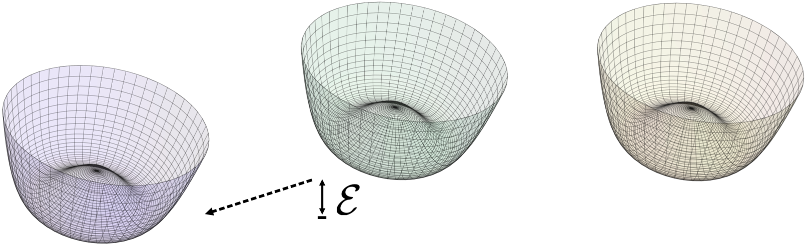
Dynamical DWs are not possible: different $\langle e^{i\chi}\rangle \Longleftrightarrow$ different universes

Probe DWs are possible after inserting volume operator $e^{i\int_{M_3} c^{(3)}}$

Eternal False Vacua

Add $m\,\bar{\psi}_i\psi_i$ to Lagrangian: breaks $\mathbb{Z}^{(0)}_{2N_f p} \to \mathbb{Z}^{(0)}_2$

$SU(N_f)_A$ softly broken: unique vacuum



False vacua with $\mathcal{E} \sim m\,\Lambda^3_{\text{QCD}}$ are protected by $\mathbb{Z}^{(3)}_p$ symmetry

Near-Eternal False Vacua

Could break $\mathbb{Z}_p^{(3)}$ by including dynamical charged 2-branes or dynamical axion

More natural to imagine that $\mathbb{Z}_p^{(3)}$ is emergent

IR

$$i\chi \wedge \left(\frac{1}{8\pi^2} \mathrm{tr} F \wedge F - \frac{p}{2\pi} dc^{(3)} \right)$$

$\mathbb{Z}_p^{(3)}$

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UV

$$f^2|d\chi|^2 + i\chi \wedge \frac{1}{8\pi^2}\mathrm{tr}F \wedge F + \mu^4(1-\cos(p\,\chi))$$

$\cancel{\mathbb{Z}_p^{(3)}}$

↑

IR

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$$\uparrow$$

$$f^2|d\chi|^2 + i\chi \wedge \frac{1}{8\pi^2}\mathrm{tr}F\wedge F + \mu^4(1-\cos(p\,\chi)) \qquad \cancel{\mathbb{Z}_p^{(3)}}$$

IR

$$i\chi \wedge \left(\frac{1}{8\pi^2}\mathrm{tr}F\wedge F - \frac{p}{2\pi}dc^{(3)}\right) \qquad \mathbb{Z}_p^{(3)}$$

$$\boxed{\Gamma \sim \exp\left(-\frac{(f\mu^2)^4}{(m\Lambda_{\mathrm{QCD}}^3)^3}\right)}$$

← UV scales

← IR scales

Higher-Group and Emergence

Higher groups arise when background gauge field for symmetry A cannot be introduced without a background field for symmetry B

Should be able to turn on A backgrounds at all stages of an RG flow
$\Longrightarrow$ If A is unbroken then B must be unbroken shown for continuous higher groups in (Córdova, Dumitrescu, Intriligator '18)

Higher-Group and Emergence

Imagine turning on $U(1)_B = U(1)/\mathbb{Z}_N$ background field
(which induces fractional topological charges) at different stages in the RG flow

IR

$$i\chi_0 \int \Big(\underbrace{\frac{1}{8\pi^2}\mathrm{tr}F\wedge F}_{\in\frac{\mathbb{Z}}{N}} - \underbrace{\frac{p}{2\pi}dc^{(3)}}_{\in p\mathbb{Z}} \Big)$$

$\mathbb{Z}_p^{(3)}$

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$\mathbb{Z}_p^{(3)}$

$D^{(4)} =$ (modified) 4-form $\mathbb{Z}_p^{(3)}$ gauge field (Tanizaki, Ünsal '19)

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UV

$$i\chi_0 \int \Big(\underbrace{\frac{1}{8\pi^2}\mathrm{tr} F \wedge F}_{\in \frac{\mathbb{Z}}{N}} - \underbrace{\frac{p}{8\pi^2}\,\mathrm{tr} G \wedge G}_{\in p\mathbb{Z}} \Big)$$

~~$\mathbb{Z}_p^{(3)}$~~

↑

IR

$$i\chi_0 \int \Big(\underbrace{\frac{1}{8\pi^2}\mathrm{tr} F \wedge F}_{\in \frac{\mathbb{Z}}{N}} - \underbrace{\frac{p}{2\pi} dc^{(3)}}_{\in p\mathbb{Z}} + \underbrace{\frac{p}{2\pi} D^{(4)}}_{\in \frac{\mathbb{Z}}{N}} \Big)$$

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UV

$$i\int \chi\wedge\left(\underbrace{\frac{1}{8\pi^2}\mathrm{tr}F\wedge F}_{\in\frac{\mathbb{Z}}{N}} - \underbrace{\frac{p}{8\pi^2}\,\mathrm{tr}G\wedge G}_{\in p\mathbb{Z}} + \underbrace{\frac{1}{2\pi}H^{(4)}}_{\in\frac{\mathbb{Z}}{N}}\right)$$

$\boxed{\cancel{\mathbb{Z}_p^{(3)}}}$

$H^{(4)} =$ (modified) field strength of 3-form $U(1)^{(2)}$ gauge field

Hidaka, Nitta, Yokokura '20
Brennan, Cordova '20

$$\text{IR} \longrightarrow \text{UV}$$

IR

$$i\chi_0\int\left(\underbrace{\frac{1}{8\pi^2}\mathrm{tr}F\wedge F}_{\in\frac{\mathbb{Z}}{N}} - \underbrace{\frac{p}{2\pi}dc^{(3)}}_{\in p\mathbb{Z}} + \underbrace{\frac{p}{2\pi}D^{(4)}}_{\in\frac{\mathbb{Z}}{N}}\right)$$

$\boxed{\mathbb{Z}_p^{(3)}}$

$D^{(4)} =$ (modified) 4-form $\mathbb{Z}_p^{(3)}$ gauge field (Tanizaki, Ünsal '19)

Summary and Future Work

- Mixed 0-form $(d-1)$-form anomalies imply the absence of dynamical DWs
- Explicitly breaking 0-form symmetry $\longrightarrow$ eternal false vacua
- Lifetimes exponentially suppressed by ratio of UV to IR scales

Open Questions

- physical implications of the higher-group
  - how is the anomaly matched across RG scales?
- are there interesting non-invertible local topological operators that give rise to universes?
  - 2D YM decomposes: $Z = \sum_R d_R^{2-2g} e^{-c_R A}$ and has local, non-invertible topological operators (Nguyen, Tanizaki, Ünsal, '21)