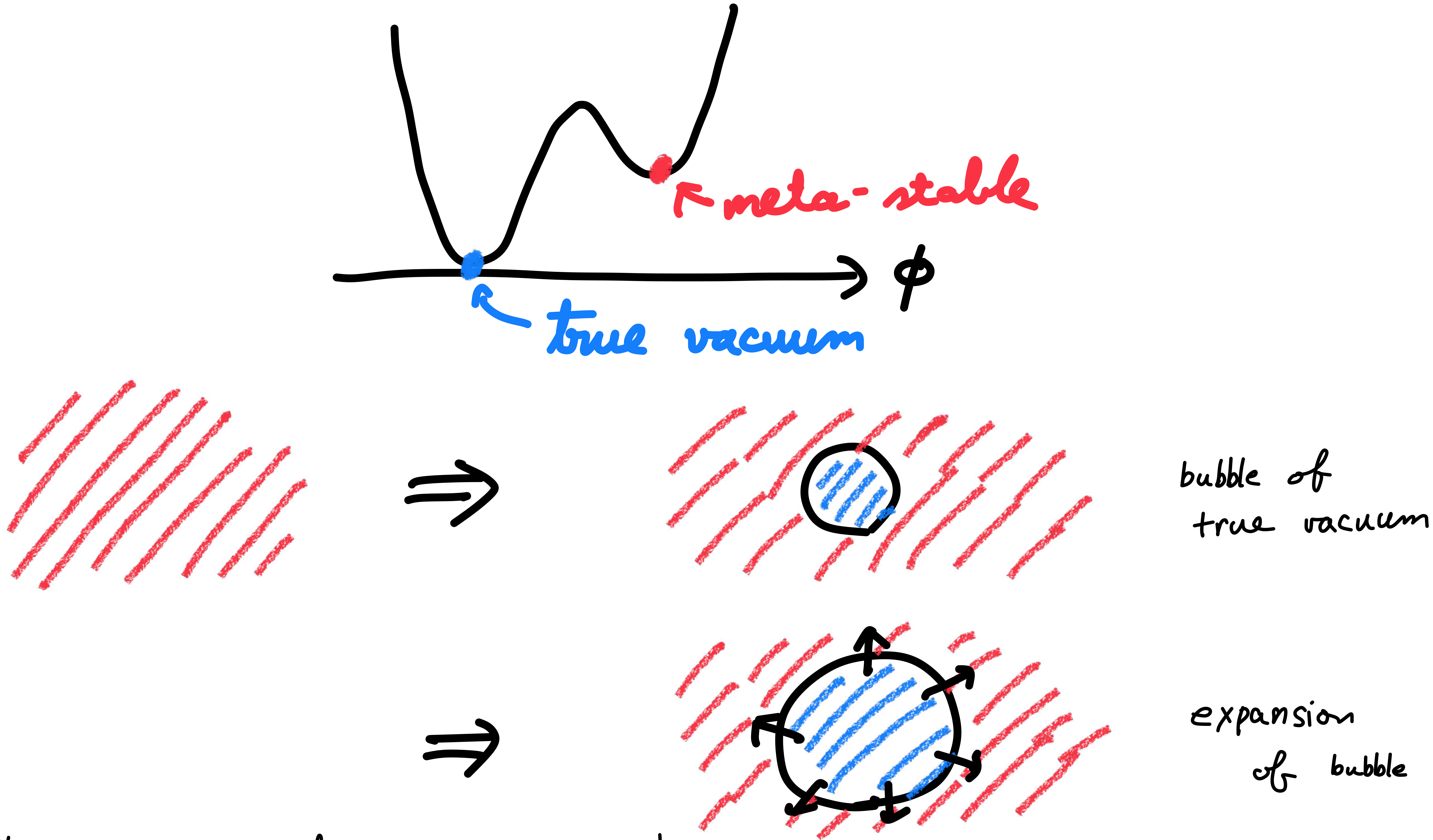
Modified instanton sum in 4d gauge theories

Yuya Tanizaki (YITP, Kyoto)

1912.01033 w/ Mithat Ünsal (NCSU)

Decomposition 2021

Vacuum decay



This may sound to be general.
But, this never occurs if QFT enjoys "decomposition".

Example : 2d Maxwell theory

$$\mathcal{L} = \frac{1}{2e^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01}$$ (in real time)

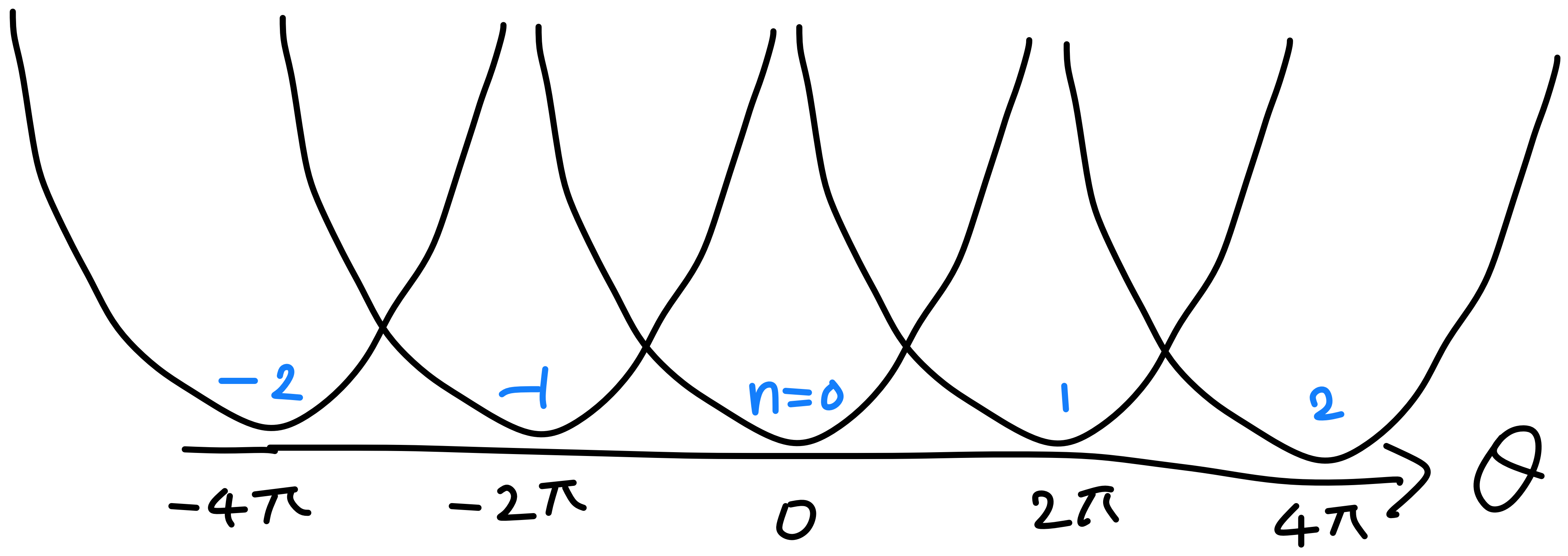
Quantization (in temporal gauge)

$$\begin{cases} \Pi = \dfrac{\partial \mathcal{L}}{\partial \dot{a}_1} = \dfrac{1}{e^2}\dot{a}_1 + \dfrac{\theta}{2\pi} & \text{(canonical momentum)} \\ H = \dfrac{e^2}{2}\left(\Pi - \dfrac{\theta}{2\pi}\right)^2 & \text{(Hamiltonian)} \end{cases}$$

w/ Gauss law $\partial_1 \Pi = 0$.

Eigenstates (on $S^1 \times \mathbb{R}_{time}$)

$$\begin{cases} \Psi_n[A_1] = \exp\left(i n \oint_0^L dx\, a_1(x)\right) \\ E_n = \dfrac{e^2}{2}\left(n - \dfrac{\theta}{2\pi}\right)^2 \end{cases}$$



Ψ_n is an eigenstate. So, it never decays to the lowest energy state!!

Just an accident for a free theory ??

Or, any other reasonings ??

$\hookrightarrow$ 1-form symmetry in (1+1) dim.

Symmetry generator of $U(1)^{[1]}$

$$U_\alpha(x) = \exp\left(i\alpha\, \Pi(x)\right).$$

This is a topological point-like operator.

$$\left(\begin{array}{ll} \partial_1 U_\alpha(x) = (\partial_1 \Pi)\, U_\alpha(x) = 0 & (\text{Gauss law}) \\ \partial_t U_\alpha(x) \sim [H, U_\alpha(x)] = 0 & (H \sim \Pi^2) \end{array}\right) \text{ — } (*)$$

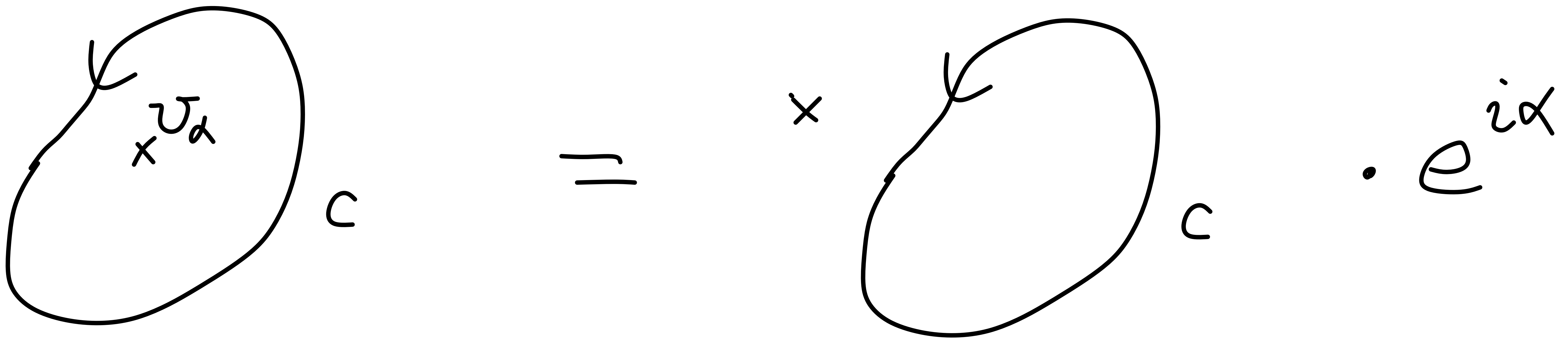
Each state $|n\rangle = \Psi_n$ is an eigenstate of this operator :

$$U_\alpha(x)\,|n\rangle = e^{i\alpha n}\,|n\rangle.$$

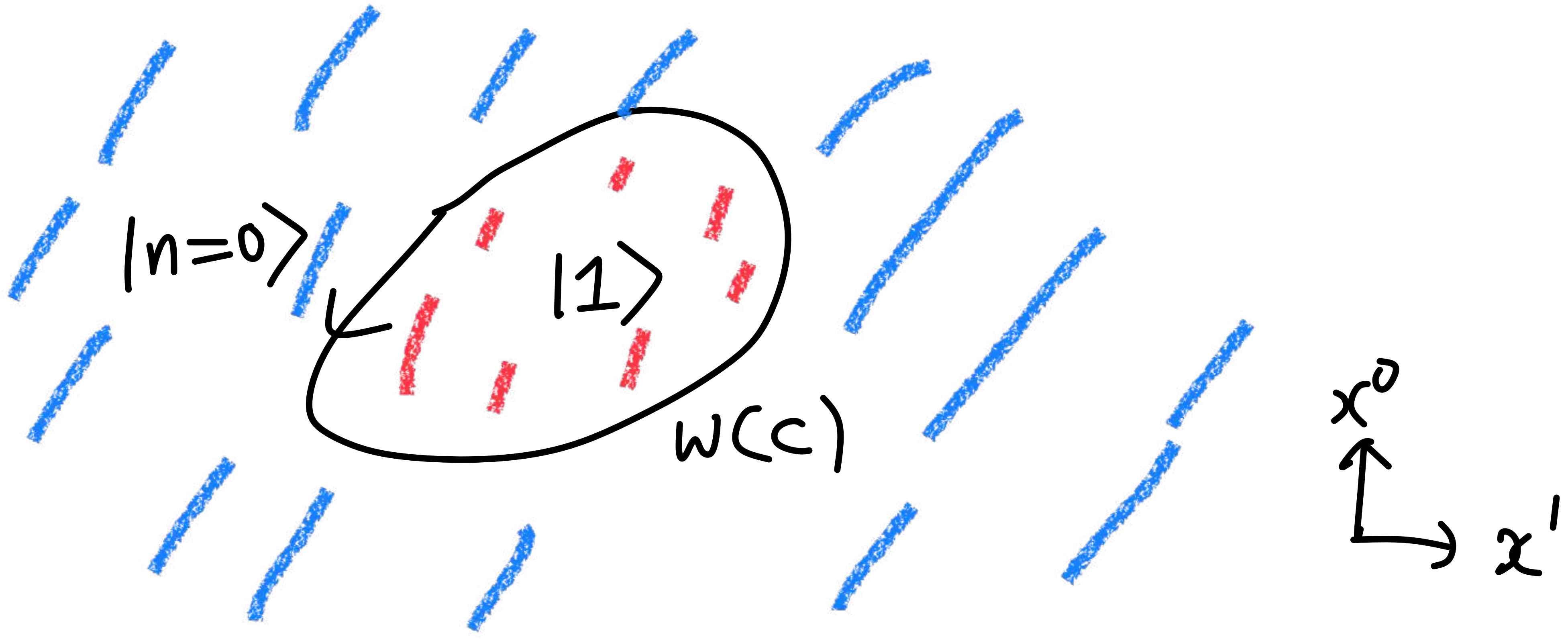
Conservation law $(*)$ tells that $U_\alpha(x)$ should be constant in spacetime !!

Wilson loop connects $|n\rangle$ and $|n\pm 1\rangle$.

$$W(C) = \exp\left(i \oint_C a\right)$$



$\Longrightarrow$



But, local operators in QFT have no ways to change n.

So, no decays.

Generalization to interacting theories

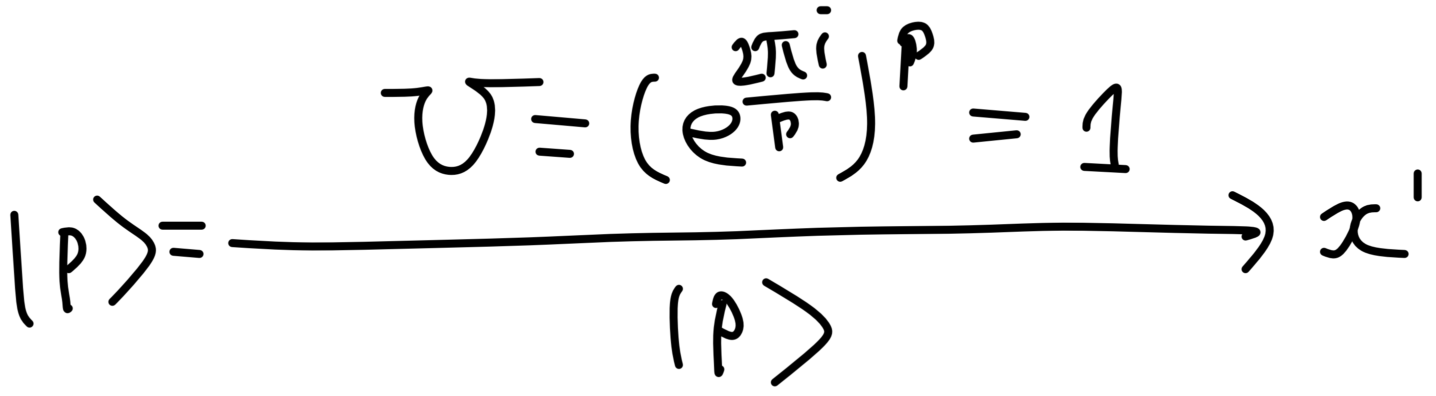
2d $U(1)$ gauge theory w./ charge-p matter $(p>1)$:

$$\mathcal{L} = \frac{1}{2e^2}F_{01}^2 + \frac{\theta}{2\pi}F_{01} + \bar{\Psi}\, i\gamma^\mu(\partial_\mu + ip\, a_\mu)\Psi + m\bar{\Psi}\Psi .$$

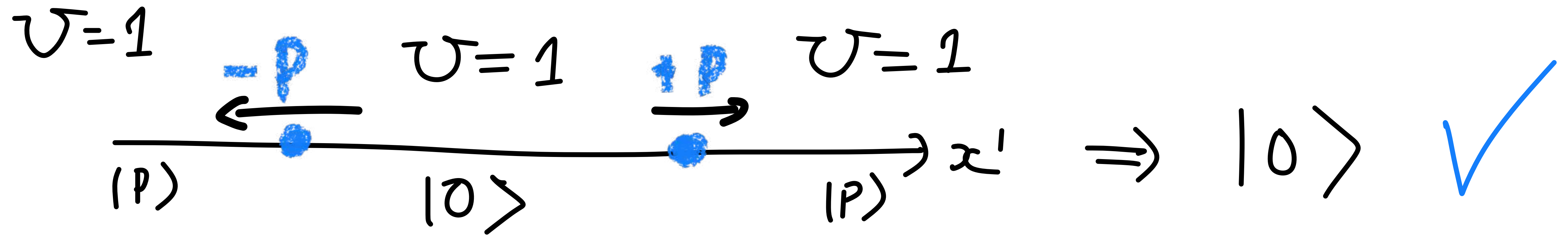
$U(1)^{[1]}$ is explicitly broken. But, it still enjoys $\mathbb{Z}_p^{[1]} \subset U(1)^{[1]}$, generated by

$$U(x) \sim \exp\left(\frac{2\pi i}{p}\Pi(x)\right).$$

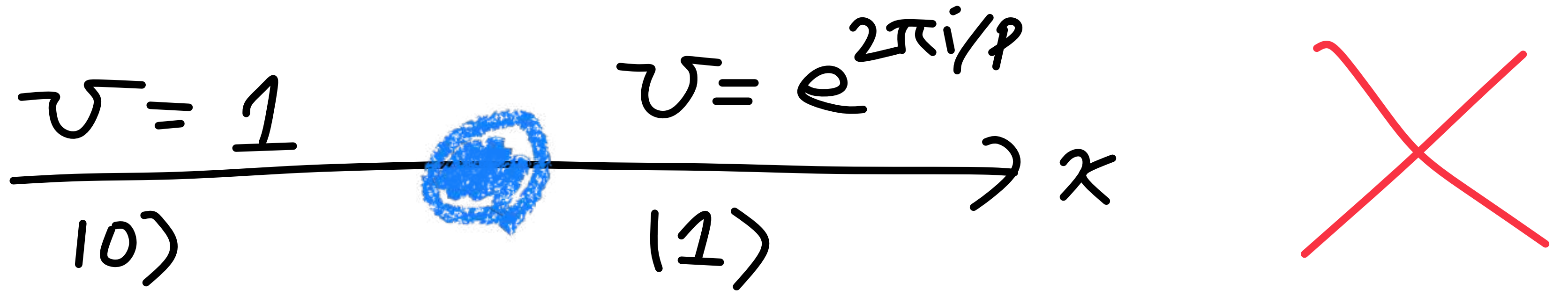
$\Rightarrow$ Some vacuum decay is possible, such as $|n\pm p\rangle \Rightarrow |n\rangle$.



$\Rightarrow$



Still, $|n=1\rangle \Rightarrow |n=0\rangle$ is NOT allowed:



Generalization to higher dim

So far, everything is about 2dim QFTs.

Higher dimensions ??

$\Rightarrow$ $(d-1)$-form symmetry. (YT, Ünsal, 2019)

There should be a topological and point-like operator $U(x)$.

Universe

"Decompose" the Hilbert space by eigenvalues of $U(x)$: $(\mathbb{Z}_p^{[d-1]})$

$$\mathcal{H} = \bigoplus_{k=0}^{p-1} \mathcal{H}_k . \qquad \left(\mathcal{H}_k = \left\{ |\psi\rangle \in \mathcal{H} \;\middle|\; U(x)|\psi\rangle = e^{i\frac{2\pi}{p}k} |\psi\rangle \right\} \right)$$

Let's call each $\mathcal{H}_k$ as universe.

Vacuum decay / domain walls between different $\mathcal{H}_k$ are not possible.

4 dim Yang-Mills theory

$$S = \frac{1}{g^2}\int_{M_4} \mathrm{tr}(F \wedge * F) + i\int_{M_4} \frac{\theta}{8\pi^2}\,\mathrm{tr}(F \wedge F).$$

Let's quantize this theory on compact spacetime M_4:

$$Z = \int \mathcal{D}a \, \exp(-S).$$

as $\pi_3(SU(N)) = \mathbb{Z}$, while $\pi_1 = \pi_2 = 0$, there are nontrivial bundles w/

$$n = \frac{1}{8\pi^2}\int_{M_4} \mathrm{tr}(F \wedge F) \in \mathbb{Z}.$$

Q. Do we have to sum up all these distinct sectors?

A. since '70s Yes. This is required for locality/cluster decomposition.

A. No. Modified instanton sum is also consistent.

Modified instanton sum in 4d YM

Let us declare that we only sum up restricted instanton sectors

$$\frac{1}{8\pi^2}\int_{M_4} \mathrm{tr}(F\wedge F) \in p\mathbb{Z}.$$

This is consistent with locality.

Indeed, there is a local Lagrangian !! (Seiberg, 2010)

$$\mathcal{L} = \frac{1}{g^2}\mathrm{tr}(F\wedge * F) + i\frac{\theta}{8\pi^2}\mathrm{tr}(F\wedge F)$$

$$+ 2\pi i \frac{\chi^{(0)}}{2\pi}\wedge\left(\underline{\frac{1}{8\pi^2}\mathrm{tr}(F\wedge F) - \frac{p}{2\pi}dc^{(3)}}\right)$$

$$\begin{cases} \chi^{(0)} : 2\pi\text{-periodic scalar.} \\ c^{(3)} : U(1)\ 3\text{-form gauge field.} \end{cases}$$

EOM of $\chi^{(0)}$: $\underline{\frac{1}{8\pi^2}\mathrm{tr}(F\wedge F) = \frac{p}{2\pi}dc^{(3)}}$ $\Rightarrow$ $\int \frac{1}{8\pi^2}\mathrm{tr}(F^2) \in p\mathbb{Z}$.

3-form symmetry & universe

Instead of EOM of $\chi^{(0)}$, consider EOM of $c^{(3)}$:

$$d\chi^{(0)} = 0.$$

$\Rightarrow$ $U(x) = \exp(i\chi^{(0)}(x))$ is a topological, point-like operator,

so is a generator of 3-form symmetry.

Moreover,

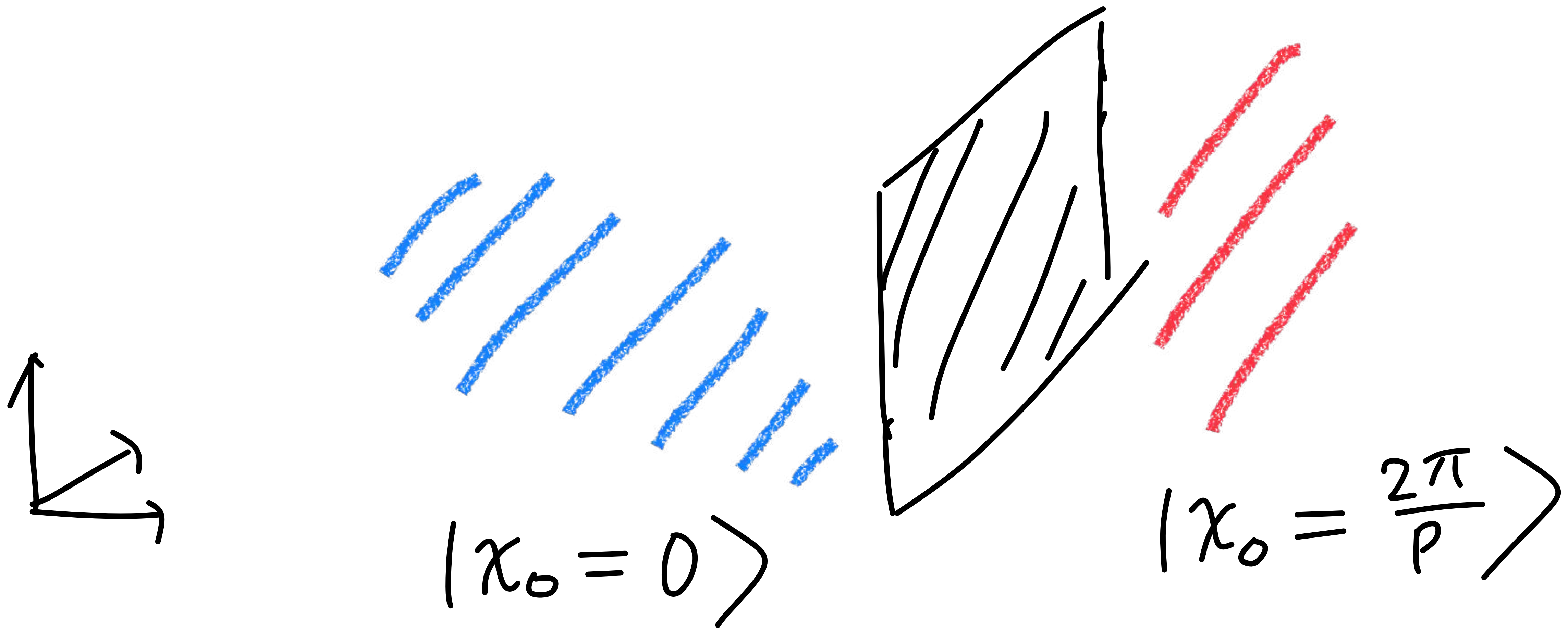
$$\int \mathcal{D}c^{(3)}\, e^{ip\int \chi^{(0)} \wedge dc^{(3)}} = \sum_n e^{ipn\chi^{(0)}} = \delta\left(\chi^{(0)} \in \frac{2\pi}{p}\mathbb{Z}\right).$$

$\Rightarrow$ $U(x)^p = 1$. Thus, $U(x)$ generates $\mathbb{Z}_p^{[3]}$:

$$\mathcal{H} = \bigoplus_{k=0}^{p-1} \mathcal{H}_k \quad \text{w/} \quad \mathcal{H}_k = \left\{ |\psi\rangle \;\middle|\; e^{i\chi^{(0)}_{(x)}}|\psi\rangle = e^{\frac{2\pi i}{p}k}|\psi\rangle \right\}.$$

Different universes can be connected only if Wilson "wall" operator is inserted:

$$e^{i\int_{M_3} C^{(3)}}$$



When $|x_0 = 0\rangle$, $|x_0 = \frac{2\pi}{p}\rangle$ have different energies,
this is a would-be domain wall for vacuum decays.

But, it is never dynamically generated due to 3-form sym.

1-form + 3-form symmetry $\Rightarrow$ 4-group symmetry

$SU(N)$ YM theory also enjoys $\mathbb{Z}_N^{[1]}$, acting on Wilson loops.

However, gauging of $\mathbb{Z}_N^{[1]}$ is not possible:

B : $\mathbb{Z}_N$ 2-form gauge field $\quad F \Rightarrow F - B$.

Then, $Q_{top} = \frac{1}{8\pi^2}\int \mathrm{tr}\left((F-B)^2\right) = \underbrace{-\frac{N}{8\pi^2}\int B\wedge B}_{\in \frac{1}{N}\mathbb{Z}.}$ mod 1.

However, EOM of $\chi^{(0)}$ says $\quad Q_{top} = \frac{p}{2\pi}\int dc^{(3)} \in p\,\mathbb{Z}$.

To make it consistent, we must also introduce

D : $\mathbb{Z}_{pN}$ 4-form gauge field.

w/

$$\frac{p}{2\pi} D = -\frac{N}{8\pi^2} B\wedge B \quad \text{mod } 1$$

't Hooft anomaly / global inconsistency

In modified YM, as $\frac{1}{8\pi^2}\int \mathrm{tr}(F^2) \in p\mathbb{Z}$,

$\theta \sim \theta + \frac{2\pi}{p}$.

Introducing the background 2-form & 4-form gauge field (B, D),

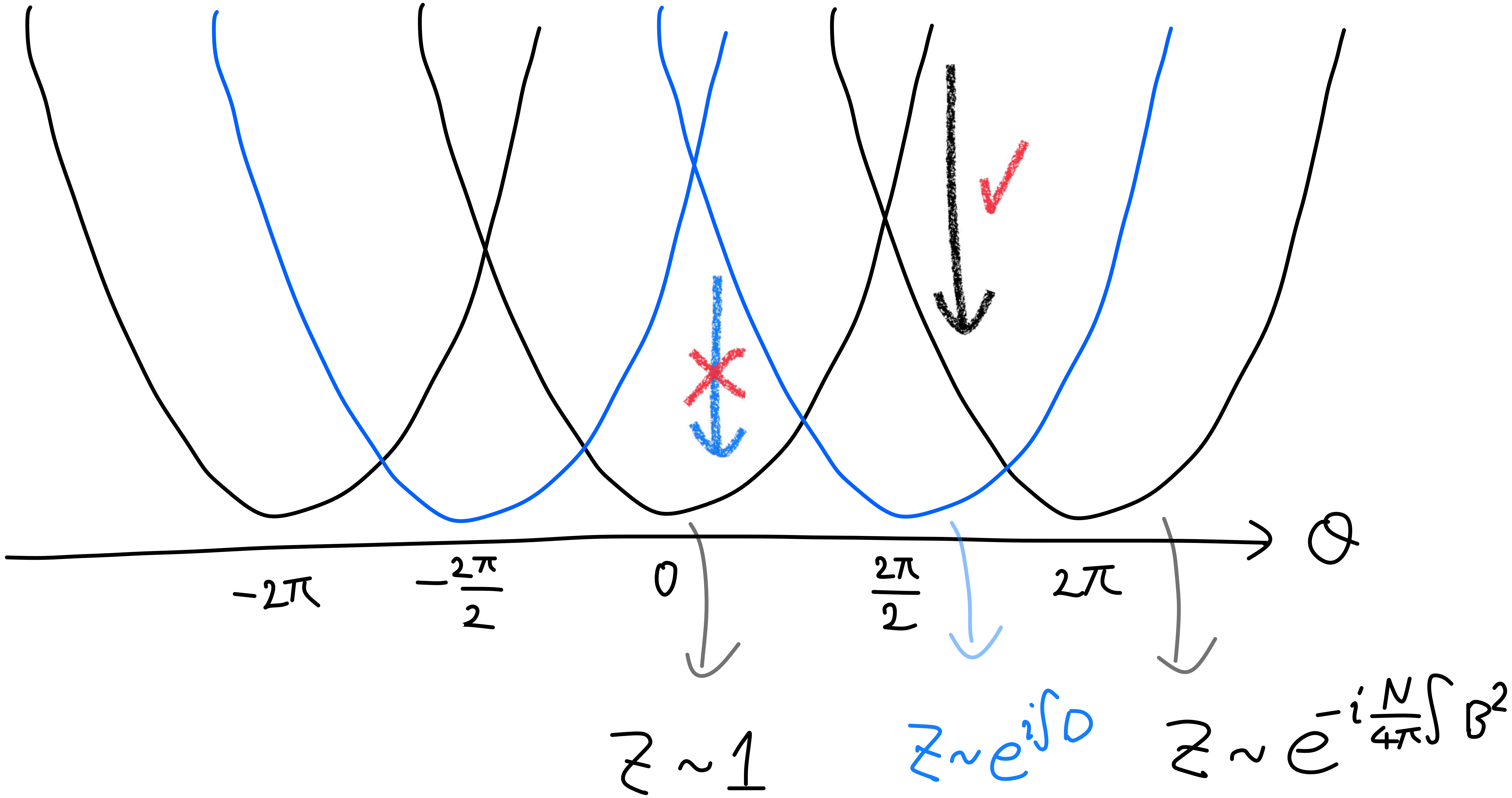
$$Z_{\theta + \frac{2\pi}{p}}[B, D] = \underbrace{e^{i\int D}}_{\text{"anomaly" bet. different universes}} \times Z_\theta[B, D].$$

Thanks to the constraint, D can be eliminated for $\theta \longrightarrow \theta + 2\pi$:

$$Z_{\theta+2\pi}[B, D] = e^{ip\int D} \times Z_\theta[B, D]$$

$$= \underbrace{e^{-i\frac{N}{4\pi}\int B\wedge B}}_{\text{"anomaly" within the same universe}} \times Z_\theta[B, D].$$

$\underline{p=2}$ $\quad \mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1$



Fermions & chiral symmetry w/ modified instanton sum

Let's add a massless Dirac fermion.

Usually, its chiral symmetry is absent due to ABJ anomaly:

$$\mathcal{D}\bar{\Psi}\,\mathcal{D}\Psi \to \exp\left(2i\alpha \frac{1}{8\pi^2}\int \mathrm{tr}(F\wedge F)\right)\mathcal{D}\bar{\Psi}\,\mathcal{D}\Psi$$

for $\Psi \to e^{i\alpha\gamma_5}\Psi$, $\bar{\Psi} \to \bar{\Psi}e^{i\alpha\gamma_5}$.

$\Rightarrow$ This is symmetry only if $\alpha \in \pi\mathbb{Z}$ for the usual instanton sum, so only the fermion parity (vector symmetry!) survives.

In modified instanton sum, $\frac{1}{8\pi^2}\int \mathrm{tr}(F^2) \in p\mathbb{Z}$, $\alpha \in \frac{2\pi}{2p}\mathbb{Z}$ gives symmetry.

$\Rightarrow (\mathbb{Z}_{2p})_{\text{axial}}$ is a good symmetry.

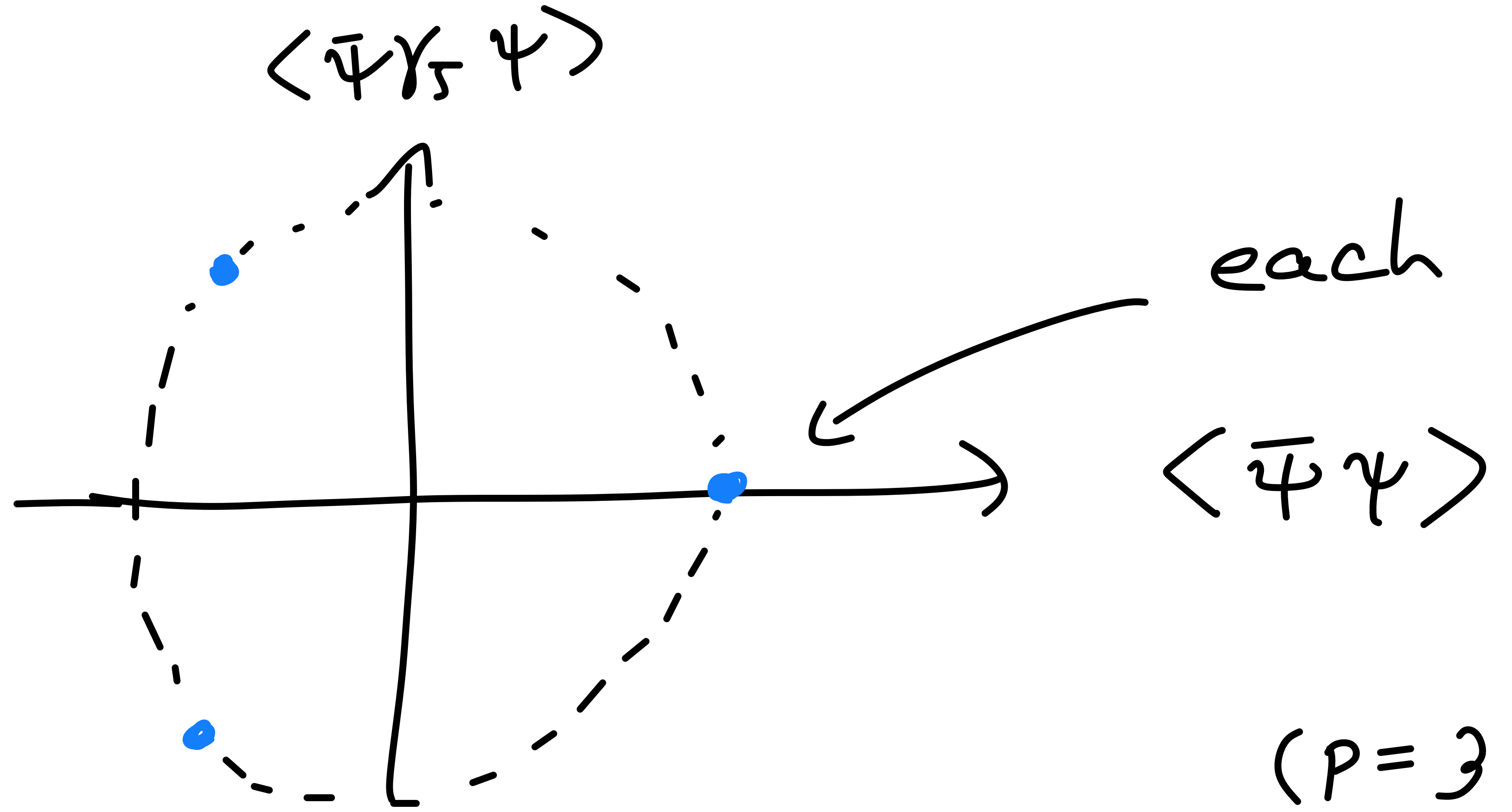
Spontaneous breaking $(\mathbb{Z}_{2p})_{axial} \to (\mathbb{Z}_2)_F$

$(\mathbb{Z}_{2p})_{axial}$ & $\mathbb{Z}_p^{[3]}$ has a mixed anomaly:

$$S_{5\,dim} = \frac{i}{2\pi} \int_{M_5} (2A_{axial}) \wedge D.$$

$\Rightarrow$ To match the 't Hooft anomaly, we need SSB

$$(\mathbb{Z}_{2p})_{axial} \longrightarrow (\mathbb{Z}_2)_F.$$



each vacuum is described by usual 1-flavor QCD.

$(p=3)$

Summary

- Decomposition of QFT is a general phenomenon, when the d-dim. QFT has a $(d-1)$-form symmetry.
  - $U(x)$: a topological, point-like op.
  - Universe is distinguished by eigenvalues of $U(x)$.
- In 4d YM, $\mathbb{Z}_p^{[3]}$ appears by restricting instanton sectors into $Q_{top} \in p\mathbb{Z}$.
- For $SU(N)$ pure YM, $\mathbb{Z}_N^{[1]}$ and $\mathbb{Z}_p^{[3]}$ form a 4-group structure:

$$p\, D_{4\text{-form}} = -\frac{N}{4\pi} B_{2\text{-form}}^2 \pmod 1.$$

- It has a mixed anomaly w/ θ-periodicity and/or chiral symmetry.