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Water Oscillations in a U-Tube

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A U-tube or manometer consists of two vertical pipes connected at their bottom ends by another pipe. Fluid (say water) partly fills the manometer, sufficiently below the tops of the pipes that it does not spill out when it is set into oscillations. Assume the flow is laminar; in reality that requires the connection not be a single horizontal pipe because that would generate eddy swirls at the corners as the flow suddenly shifts from vertical to horizontal motion. Either one can ignore those swirls (by assuming the flow speed is small enough to minimize turbulence) or, better, one can make the connection via a smoothly varying arc-shaped pipe. Make all the other usual simplifying assumptions that the fluid is incompressible, irrotational, and inviscid. Further neglect air drag, so there is no mechanism to damp the oscillations of the fluid.

To set the fluid into oscillation, one can insert a piston into one vertical pipe, push the water down some distance, and then rapidly yank the piston out of that pipe. The problem is then: Under what circumstances are the fluid oscillations described by simple harmonic motion? For example, must the initial displacement by the piston (which equals the amplitude of the motion) be small? Clearly we need to model the physics to explore that question. What would be the best approach? For example, can one apply the standard Bernoulli equation?

So far you have probably assumed the cross-sectional area of the pipes (both the vertical ones and the one making the bottom connection, be it horizontal or arc-shaped) is constant, since I didn't say anything otherwise. But what happens if that is not the case? For example, the U-tube might be tapered from one top end around to the other top end.

Surprisingly (to me at least) even simple non-uniform cross-sectional variations do not appear to be amenable to analytic solution. I will first discuss the case of constant cross-section, which is analytically soluble; however I will show a serious error in previous treatments of this case in journals such as The Physics Teacher. I will then present an example of numerical calculations using Mathematica for a tapered tube. I will then end by briefly describing some interesting applications of such non-uniform cross-sections.

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