R. Plestid | UKY & FNAL | in collaboration with O. Tomalak & R.H.J. Hill

$$\bar{\nu}\, A \to \ell^+\, p\, A' \qquad\qquad \big[\, \bar{\nu}\, n \to \ell^+\, p \,\big]_{V(x)}$$

Coulomb

NEUTRINO NUCLEUS SCATTERING AND

COULOMB CORRECTIONS

MOTIVATION

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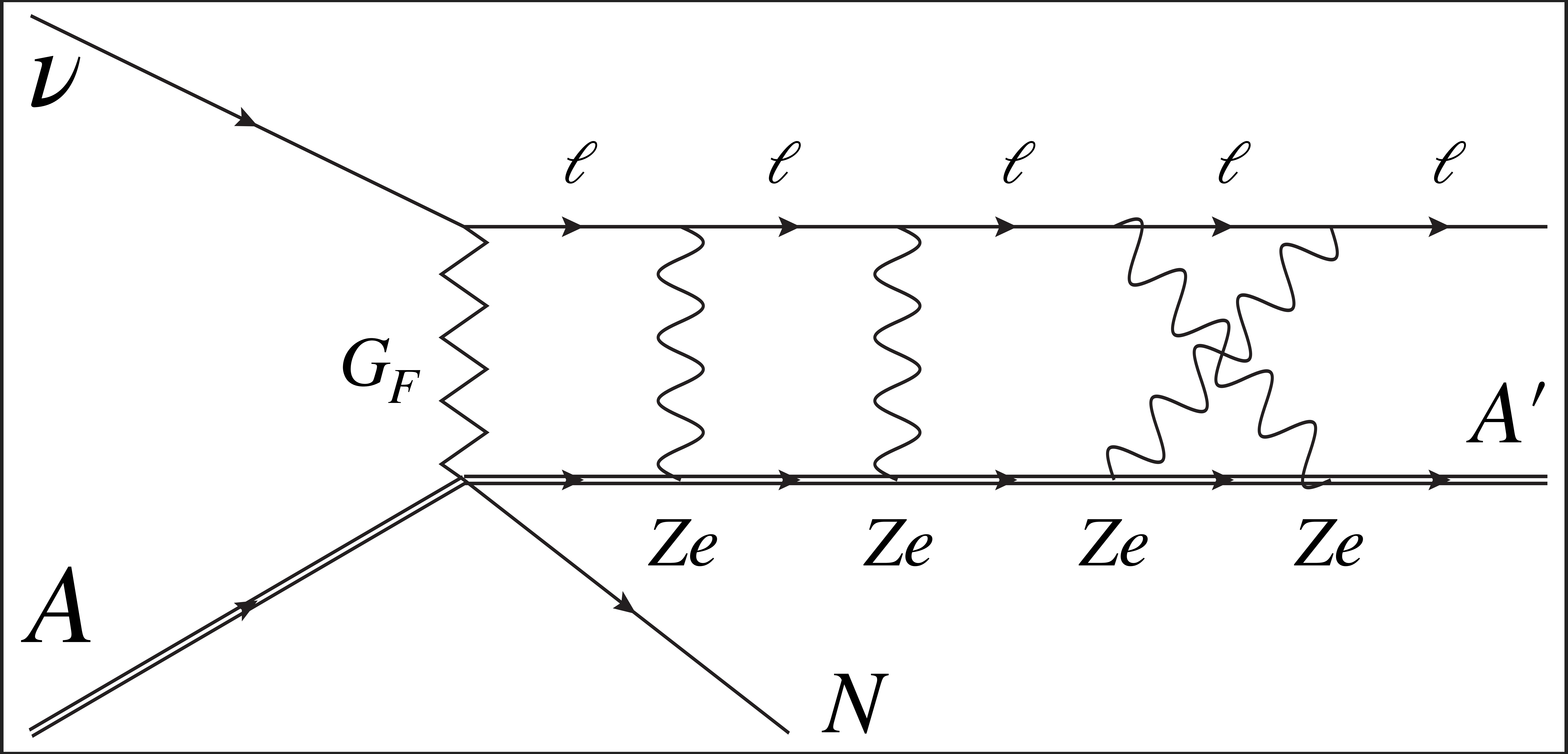
- Coulomb corrections are theoretically "easy", but computationally expensive. Avoid introducing 9-dim integrals. Similar to optical potential
- **This talk:** Work towards a systematically improvable *analytic* treatment of Coulomb corrections. Aim towards consistent power counting and reliable error estimates.

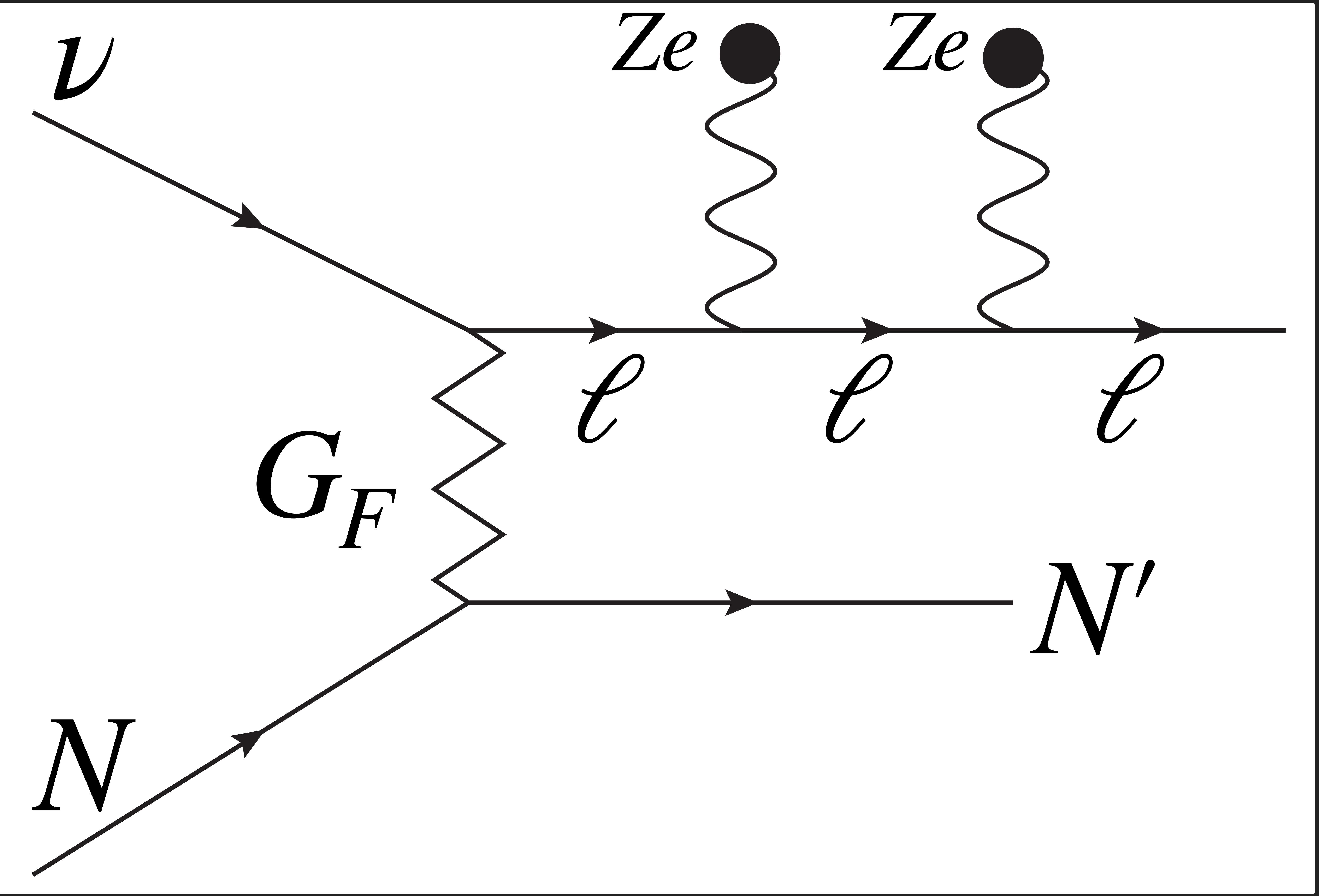

CAUTION
WORK IN
PROGRESS

INTRODUCTION

SOFT PHOTON EXCHANGE WITH NUCLEUS

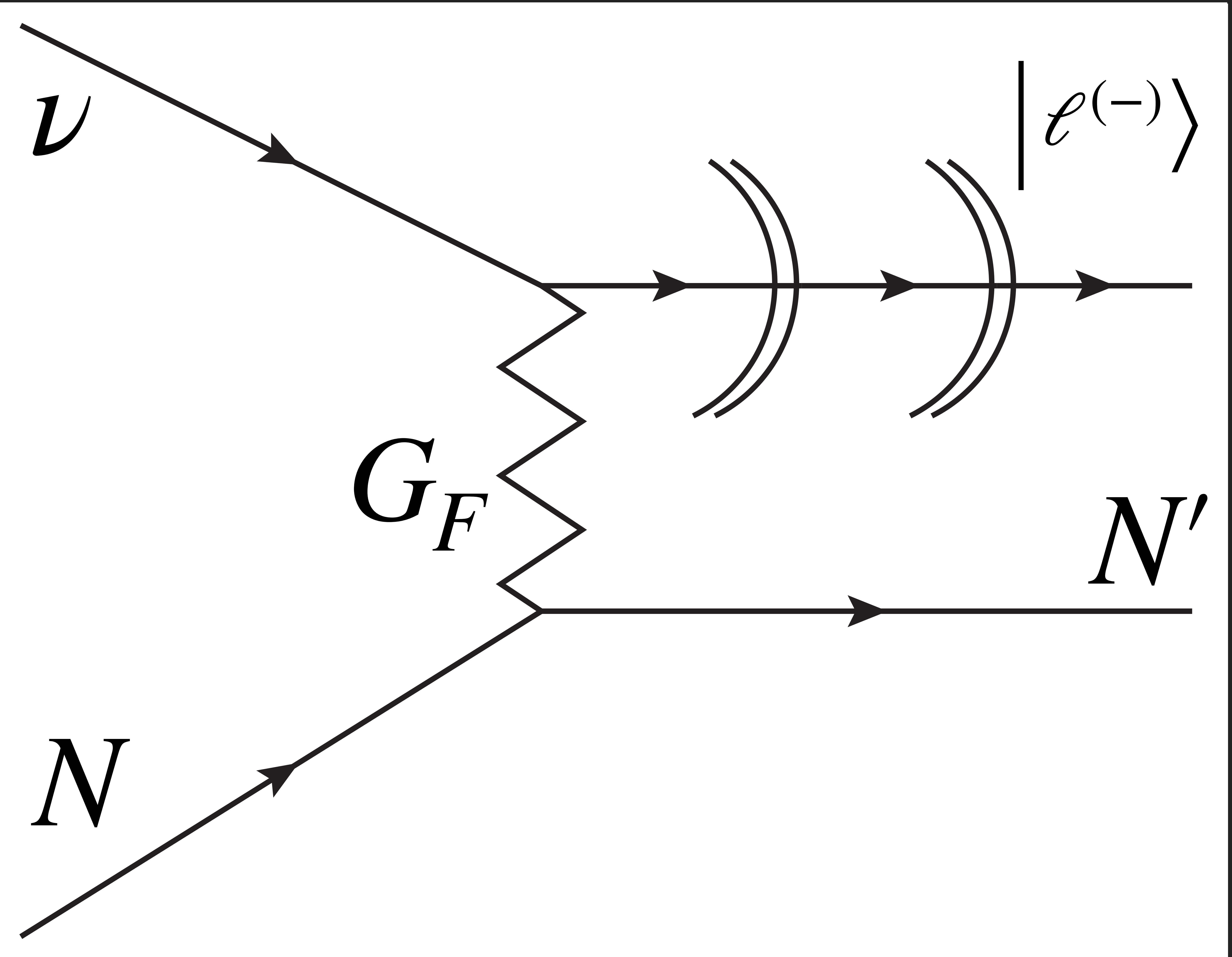


COULOMB FIELD OF A NUCLEUS



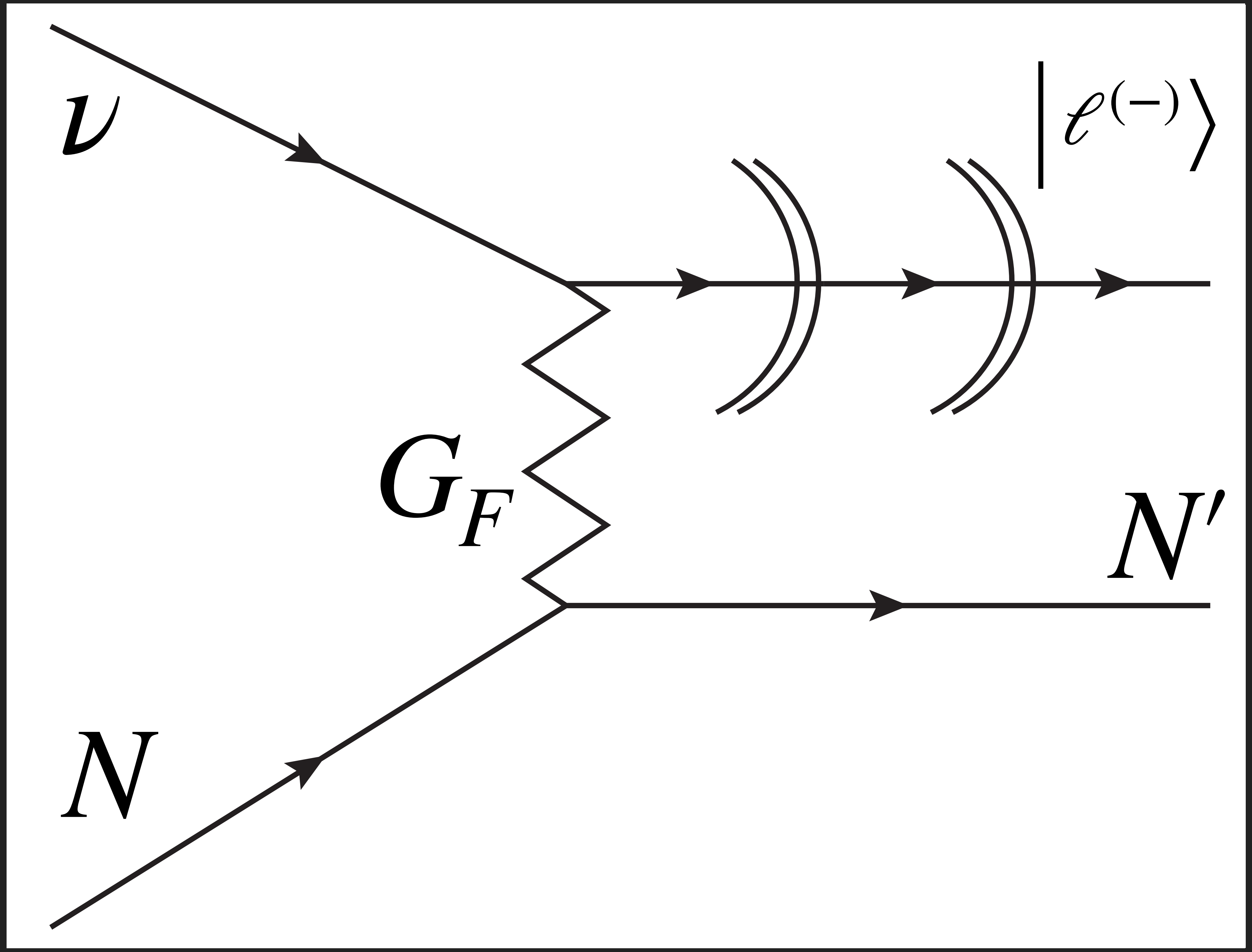
- Spectator nucleus becomes a background field.
- Coulomb field distorts lepton.
- In this talk we will ignore nucleon FSI.

DISTORTED WAVE BORN SERIES



- Use out-state solution of Coulomb scattering problem.
- S-matrix does not conserve momentum.
- Loss of plane wave leads to loss of $(2\pi)^3\delta^{(3)}(\Sigma P)$.

DISTORTED WAVE BORN SERIES



- Use out-state solution of Coulomb scattering problem.
- Loss of plane wave leads to loss of $(2\pi)^3\delta^{(3)}(\Sigma P)$

$$e^{ik'x}\bar{u}_{k'}\,\gamma_\mu(1-\gamma_5)u_k e^{-ikx} \to \bar{\mathcal{U}}_{k'}(x)\,\gamma_\mu(1-\gamma_5)u_k e^{-ikx}$$

DISTORTED WAVE MATRIX ELEMENTS

$$S = (2\pi)^4 \delta^{(4)}(\Sigma P) \mathrm{i}\mathscr{M} \quad \longrightarrow \quad S = 2\pi\delta(\Sigma E)\mathrm{i}\mathrm{M}$$

$$L_{\mu\nu} \to \int\int \mathrm{d}^3x \; \mathrm{d}^3y \; \mathrm{Tr}\left[L_\mu(x) L_\nu(y)\right]$$

$$\mathrm{d}\Pi(2\pi)^4\delta^{(4)}(\Sigma P)\,|\,\mathscr{M}\,|^2 \to \mathrm{d}\Pi\;(2\pi)\delta(\Sigma E)\,|\,\mathrm{M}\,|^2(\Sigma P)$$

DISTORTED WAVE MATRIX ELEMENTS

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$$\mathrm{d}\Pi (2\pi)^4 \delta^{(4)}(\Sigma P) |\mathscr{M}|^2 \to \mathrm{d}\Pi\ (2\pi)\delta(\Sigma E) |\mathrm{M}|^2 \underline{(\Sigma P)}$$

$$\boxed{\text{Extra Integrals} = 3 + 3 + 3 = 9}$$

EXISTING LITERATURE

EXISTING LITERATURE

arXiv:nucl-th/9711045

PHYSICAL REVIEW C VOLUME 57, NUMBER 4 APRIL 1998

Approximate treatment of lepton distortion in charged-current neutrino scattering from nuclei

Jonathan Engel
Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27599-3255
(Received 18 November 1997)

The partial-wave expansion used to treat the distortion of scattered electrons by the nuclear Coulomb field is simpler and considerably less time-consuming when applied to the production of muons and electrons by low- and intermediate-energy neutrinos. For angle-integrated cross sections, however, a modification of the ‘‘effective-momentum’’ approximation seems to work so well that for muons the full distorted-wave treatment is usually unnecessary, even at kinetic energies as low as 1 MeV and in nuclei as heavy as lead. The method does not work as well for electron production at low energies, but there a Fermi function often proves perfectly adequate. Scattering of electron neutrinos from muon decay on iodine and of atmospheric neutrinos on iron is discussed in light of these results. [S0556-2813(98)04804-3]

PACS number(s): 25.30.Pt, 11.80.Fv

- This is the ***only*** paper on Coulomb corrections for neutrino-nucleus scattering (to the best of my knowledge).

ENGEL & EMA / MEMA

arXiv:nucl-th/9711045

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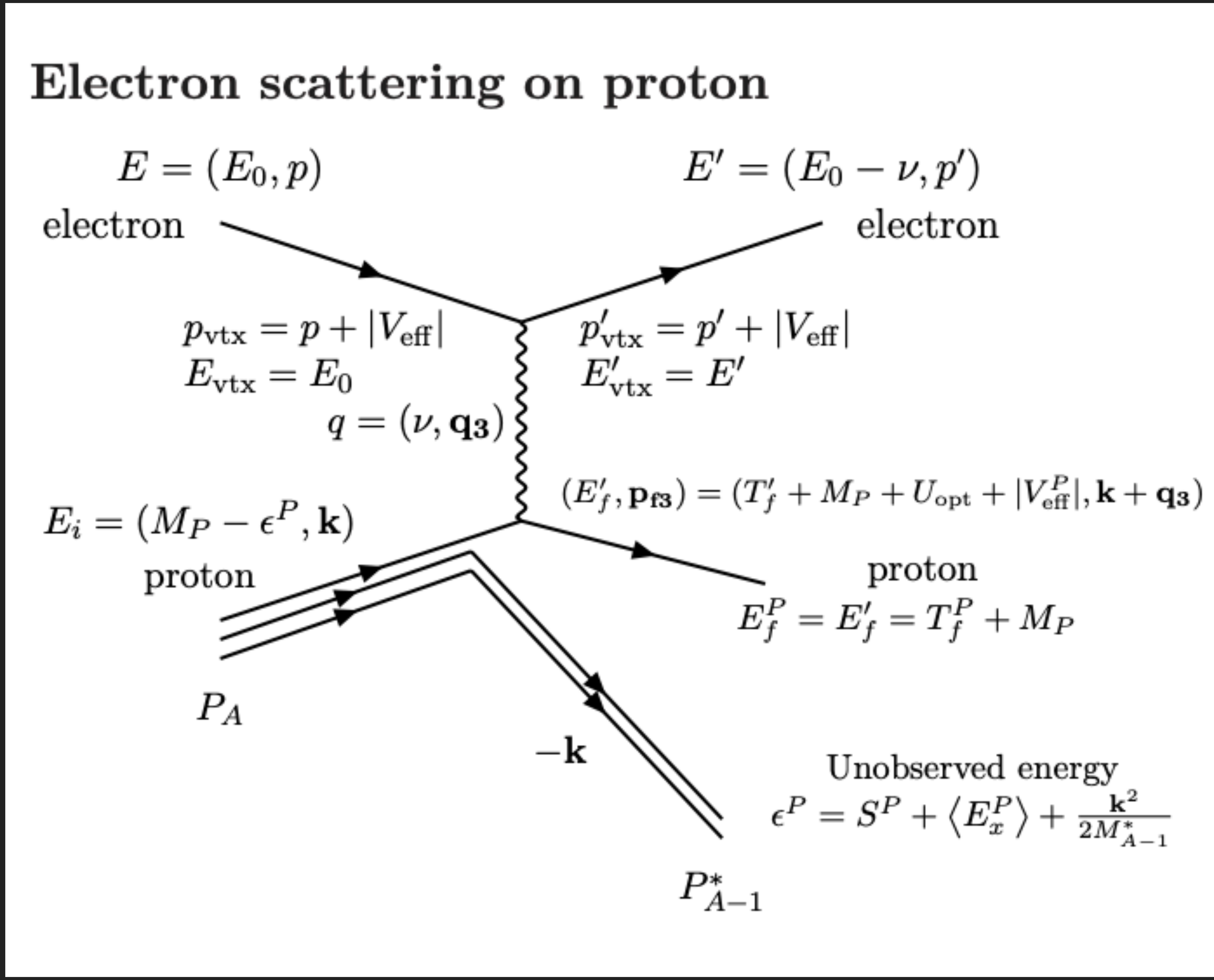
- Advocates for a effective momentum approximation.
- Validates against toy model with vector current.

OPTICAL POTENTIALS



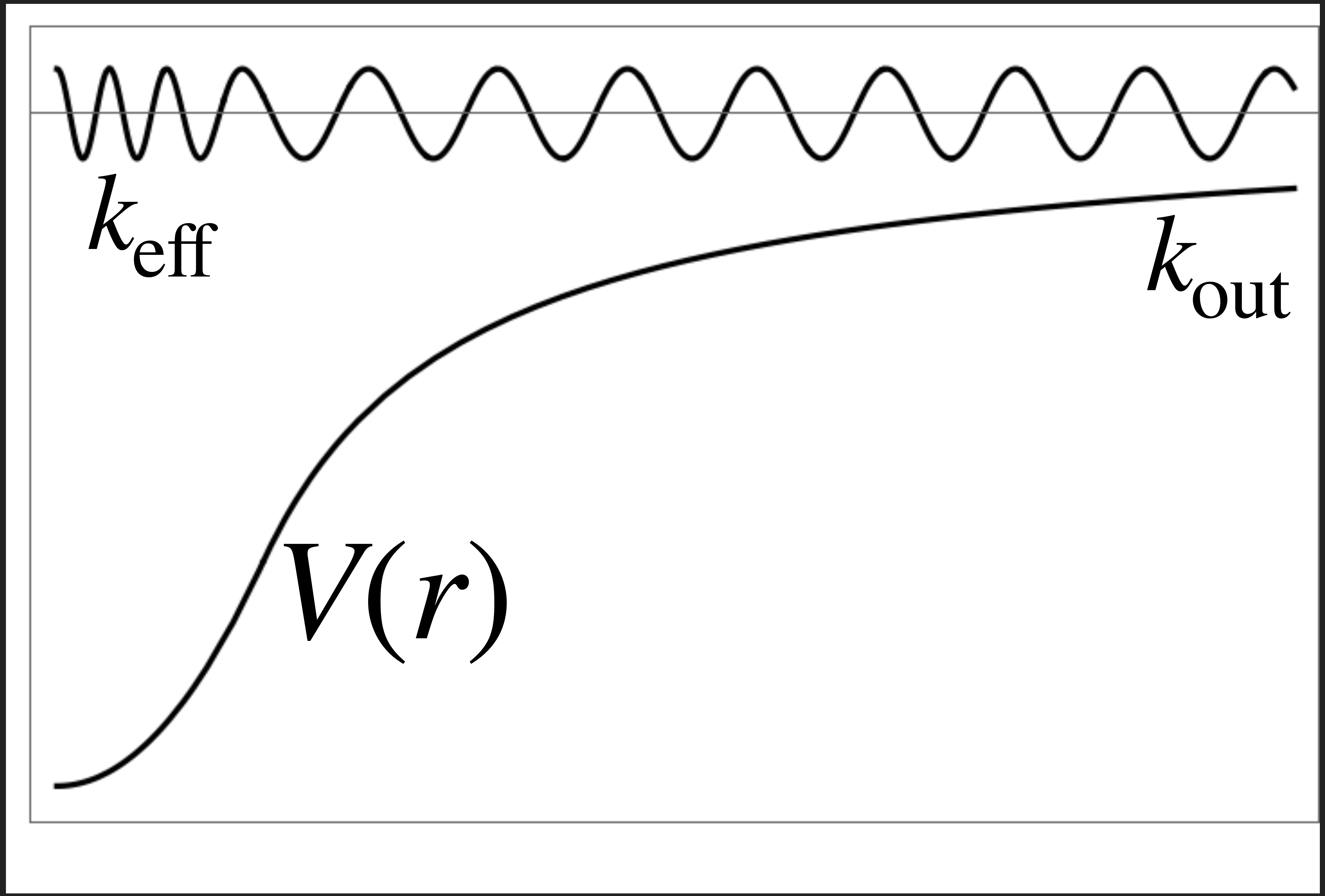
- Related work on optical potentials, e.g. Bodek & Cai arXiv:2004.00087

ELECTRON SCATTERING

- Extensive literature in electron scattering.
- Tjon & Wallace (2006) arXiv:nucl-th/0610115
- Yennie Boos & Ravenhall Phys. Rev. 137 (1965)
- Yennie, Ravenhall, & Willson Phys. Rev. 95 (1954)

EFFECTIVE MOMENTUM

arXiv:nucl-th/9711045



- Effective momentum near nucleus.
- Re-scaled wave amplitude by $\sqrt{kE/k_{\mathrm{eff}}E_{\mathrm{eff}}}$.
- Effective momentum still conserved in phase space.

- Advocates for a effective momentum approximation
- This is what is inside GENIE.

EIKONAL APPROXIMATION

EIKONAL APPROXIMATION — DIRAC EQUATION

$$\mathscr{U}_k^{(\pm)}(x) = \mathrm{e}^{-\mathrm{i}\omega t}\ \mathrm{e}^{\mathrm{i}kx}\mathrm{e}^{\mathrm{i}\chi^{(\pm)}(x)}\ u_\beta(k)$$

Solve Dirac equation with Coulomb field iteratively

$$\chi^{(\pm)} = \chi_0^{(\pm)} + \frac{1}{E}\chi_1^{(\pm)} + \frac{1}{E^2}\chi_2^{(\pm)} + \dots$$

EIKONAL APPROXIMATION — DIRAC EQUATION

$$\chi_0^{(+)} = -\frac{1}{v}\int_{-\infty}^{z} \mathrm{d}z \; V(z,b) \quad (\text{for } \hat{z}\cdot\hat{k} = 1)$$

Solve Dirac equation with Coulomb field iteratively

$$\chi^{(\pm)} = \chi_0^{(\pm)} + \frac{1}{E}\chi_1^{(\pm)} + \frac{1}{E^2}\chi_2^{(\pm)} + \ldots$$

TOY NUCLEAR MODEL

ANTI-NEUTRINO + BOUND PROTON $\longrightarrow$ ANTI-LETPTON + FREE NEUTRON

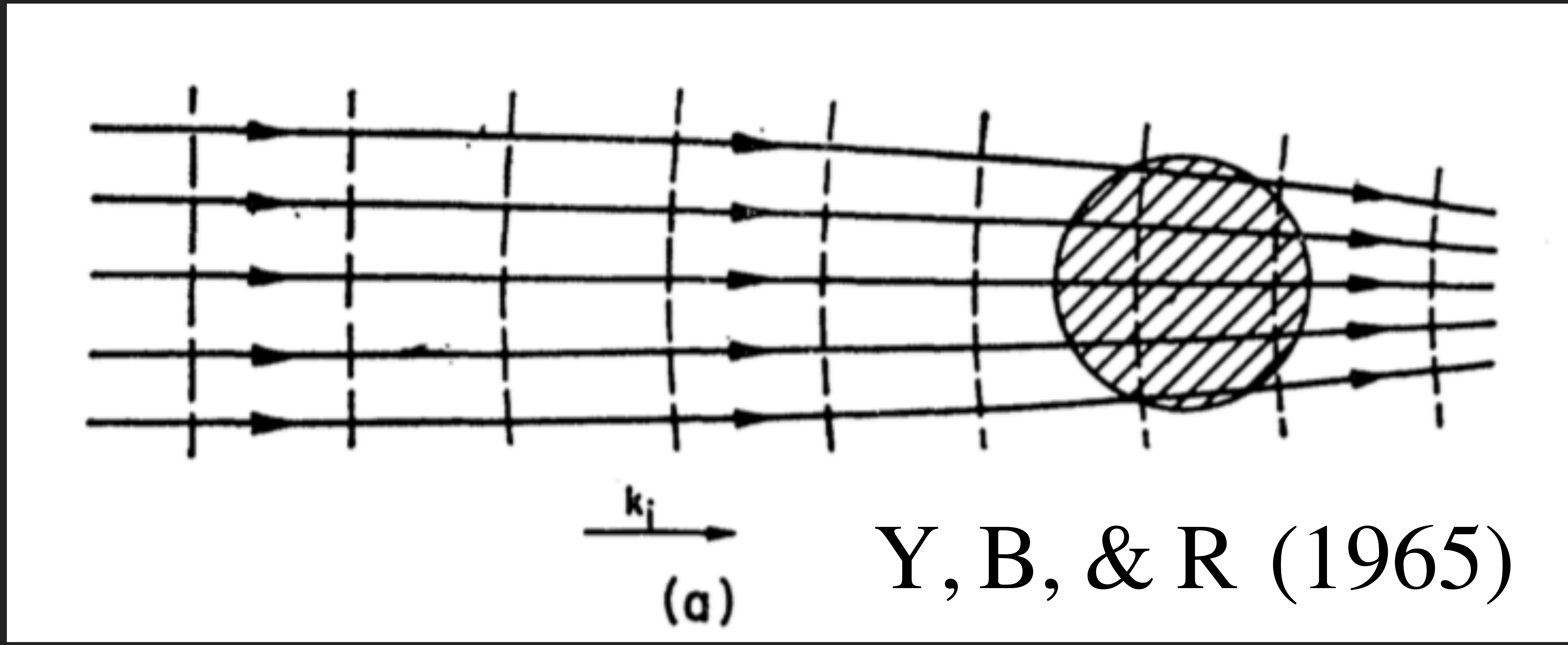
$$|\,\bar{\nu}\,\rangle \;+\; |\,\phi\,\rangle \rightarrow |\,\ell^{+}_{\mathrm{out}}\,\rangle \;+\; |\,n\,\rangle$$

$$\phi(p) \sim \frac{1}{r_A^3}\mathrm{e}^{-r_A^2 p^2} \qquad \mathrm{e}^{\mathrm{i}\chi_0(x)}$$

ANTI-NEUTRINO + BOUND PROTON $\longrightarrow$ ANTI-LETPTON + FREE NEUTRON

$$e^{i\chi_0(x)}$$

Focussing in transverse plane



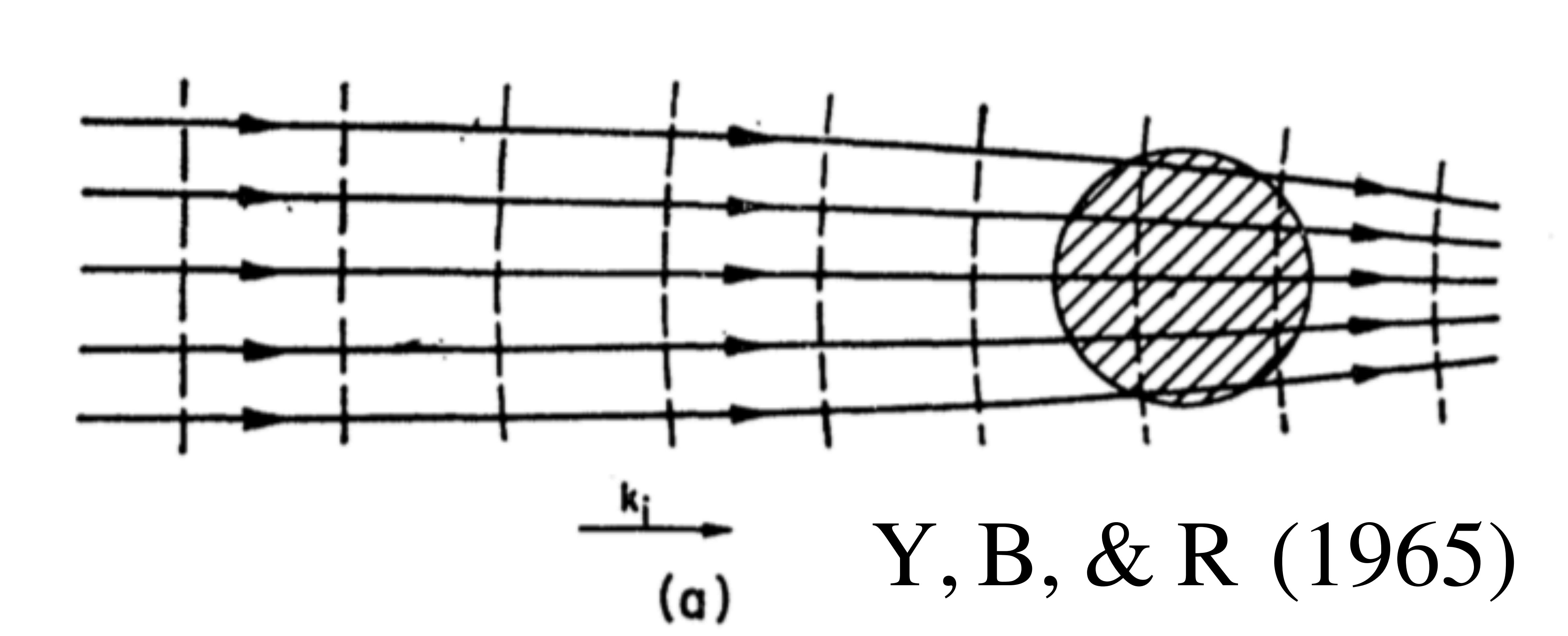
$$\sigma_\perp^2 = \frac{Z\alpha}{4\pi}\left\langle\frac{1}{r^2}\right\rangle$$

$$\chi_0(x) \approx \delta k \times z \; + \; \frac{1}{2}\sigma_\perp^2 \times b^2 + \ldots$$

ANTI-NEUTRINO + BOUND PROTON $\rightarrow$ ANTI-LETPTON + FREE NEUTRON

$$e^{i\chi_0(x)}$$

Focussing in transverse plane



Y, B, & R (1965)

$$\sigma_\perp^2 = \frac{Z\alpha}{4\pi}\left\langle \frac{1}{r^2} \right\rangle$$

$$\chi_0(x) \approx \delta k \times z \;+\; \frac{1}{2}\sigma_\perp^2 \times b^2 + \ldots$$

ANTI-NEUTRINO + BOUND PROTON $\longrightarrow$ ANTI-LETPTON + FREE NEUTRON

$$\text{Hierarchy } \frac{1}{E_\nu} \ll r_A \lesssim \frac{1}{\sigma_\perp}$$

$$\phi(p) \sim \exp[-r_A^2 p^2]$$

ANTI-NEUTRINO + BOUND PROTON $\longrightarrow$ ANTI-LETPTON + FREE NEUTRON

Hierarchy $\dfrac{1}{E_\nu} \ll r_A \lesssim \dfrac{1}{\sigma_\perp}$ $\qquad\qquad$ $\phi(p) \sim \exp[-r_A^2 p^2]$

$$\mathrm{d}\sigma \sim \mathrm{d}\sigma_{\mathrm{PW}} \;/\,.\; k_z \to k_z^{\mathrm{eff}} \;/\,.\; \delta^{(2)}(P_\perp) \to \mathrm{e}^{-\underline{P_\perp^2/\sigma_\perp^2}}$$

Transverse Momentum Fluctuations

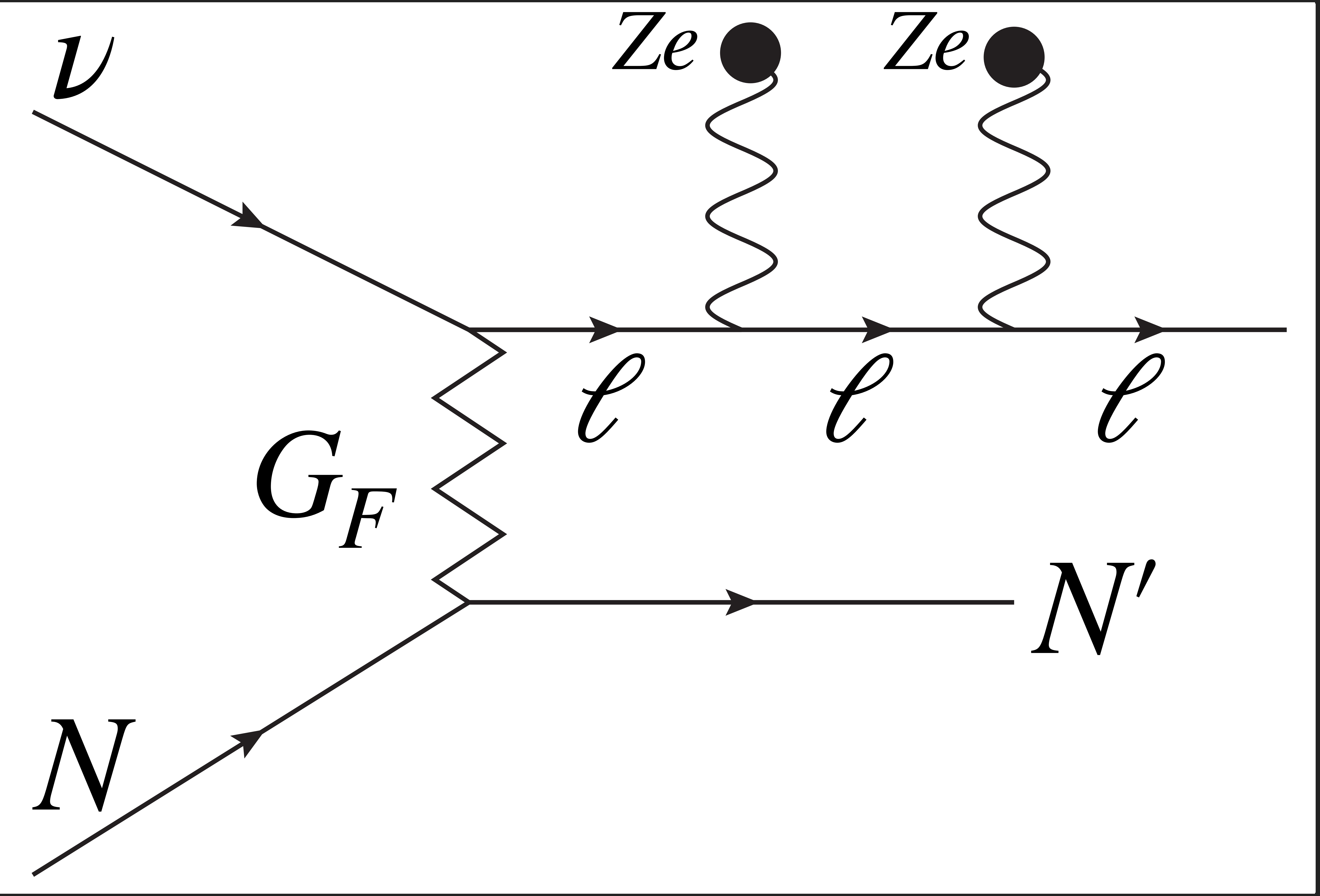
CONCLUSIONS

SUMMARY

- Analytic treatment possible using Eikonal expansion + hierarchy of scales (expansion in $1/E_\nu r_A$) .
- Only one distorted wave allows for analytic calculations (in contrast to electron scattering)
- Effective momentum approximation appears at leading order with calculable corrections.
- Coulomb field induces transverse momentum fluctuations.

EXTRA SLIDES

COULOMB FIELD OF A NUCLEUS



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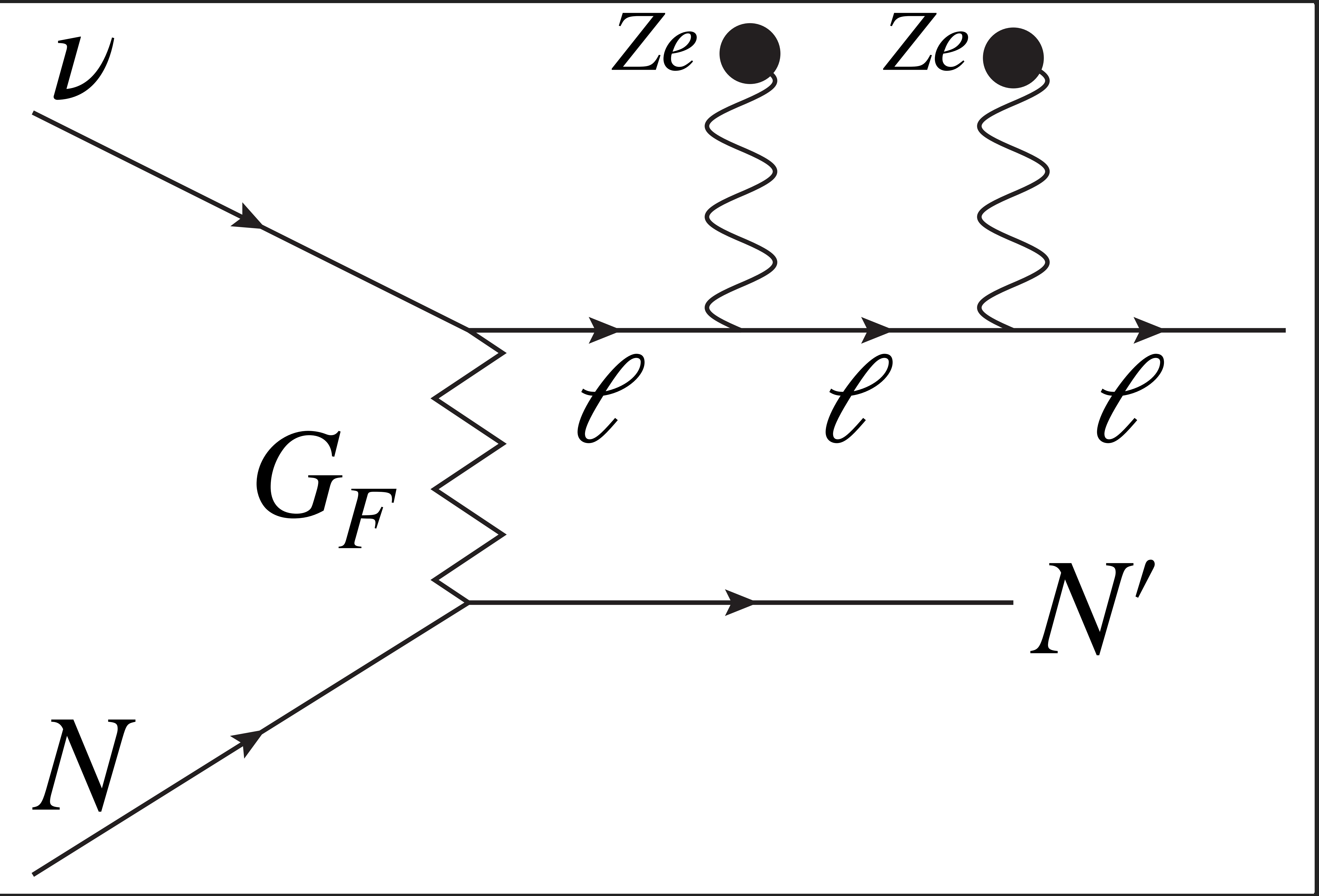

- Spectator nucleus becomes a background field.

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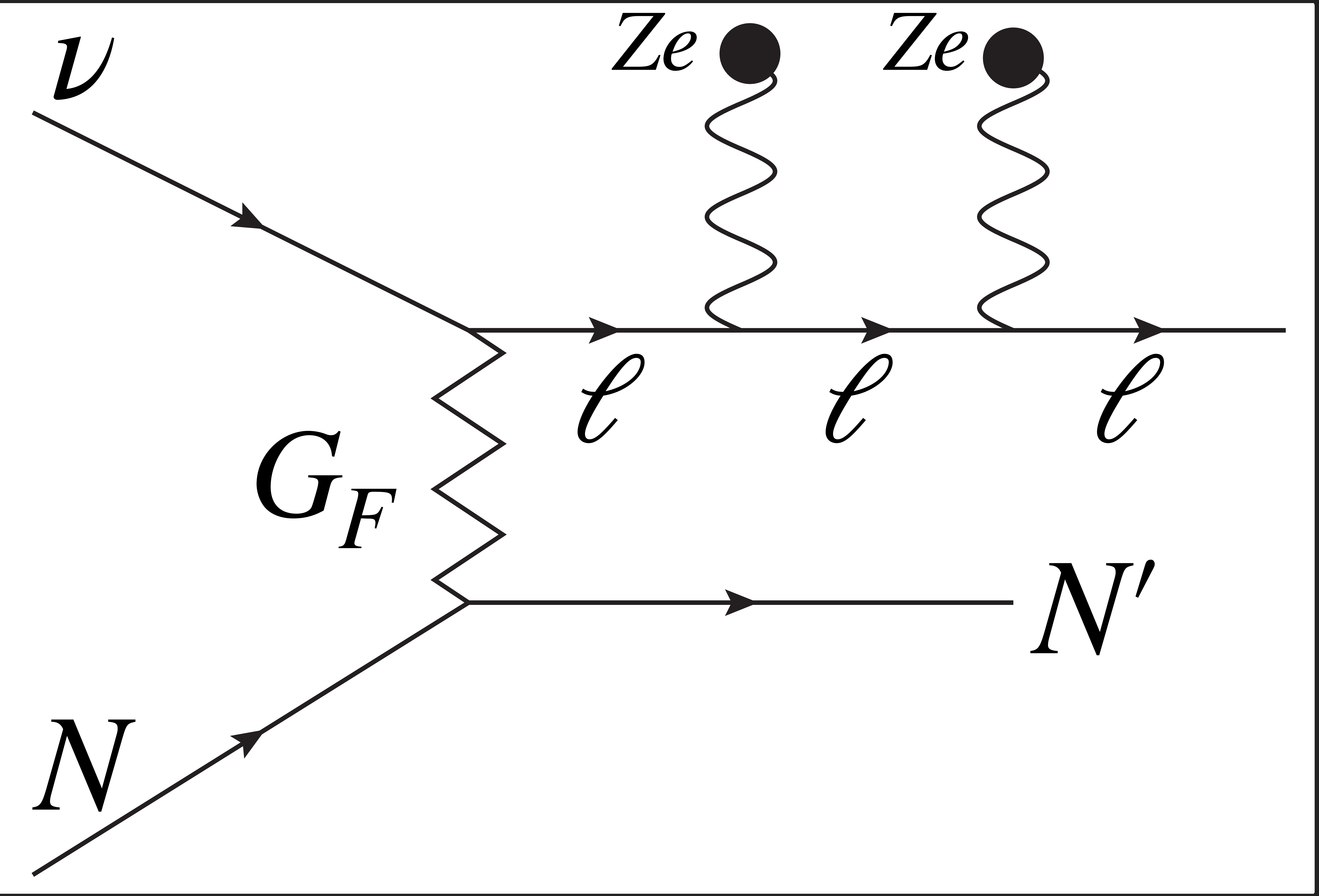

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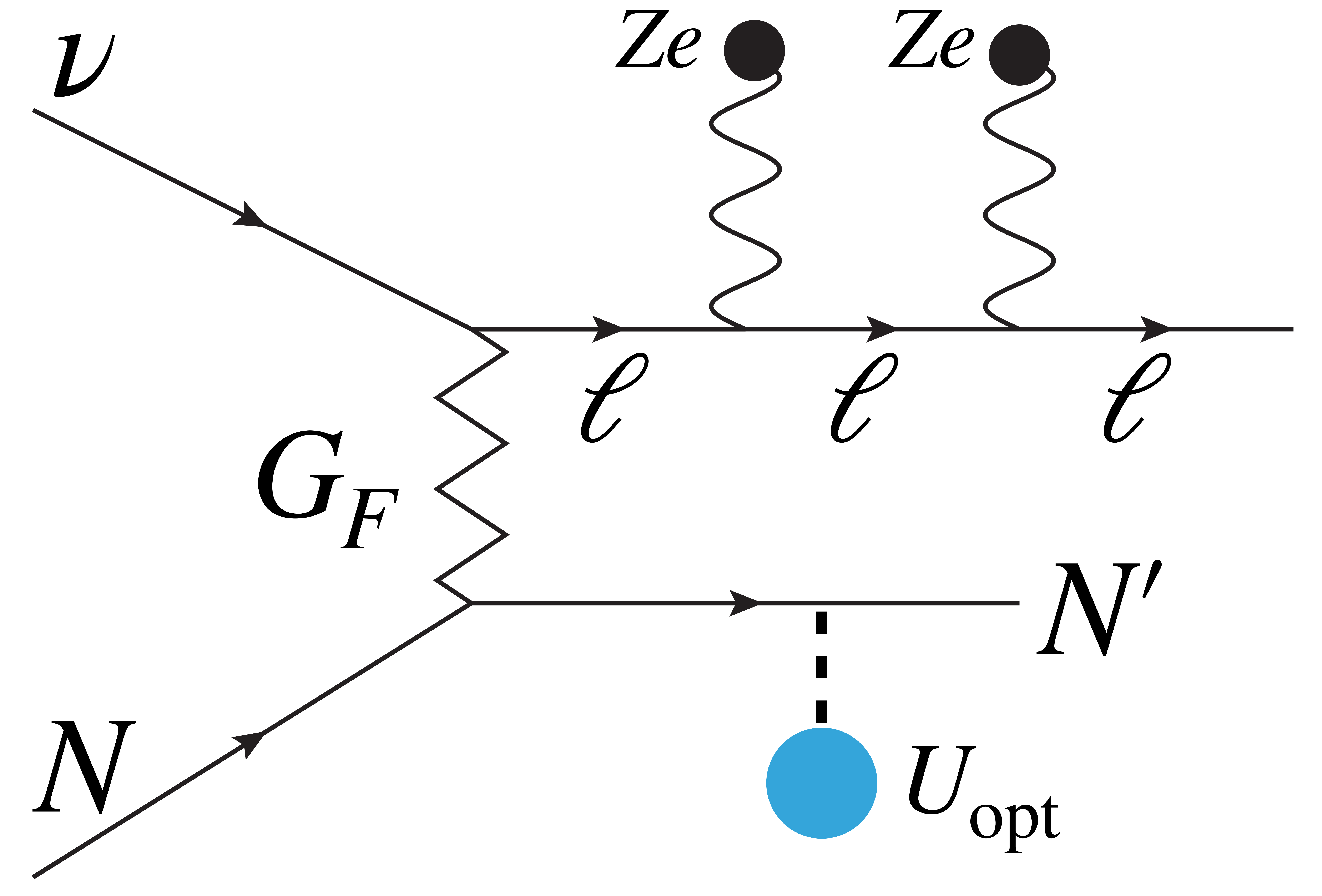
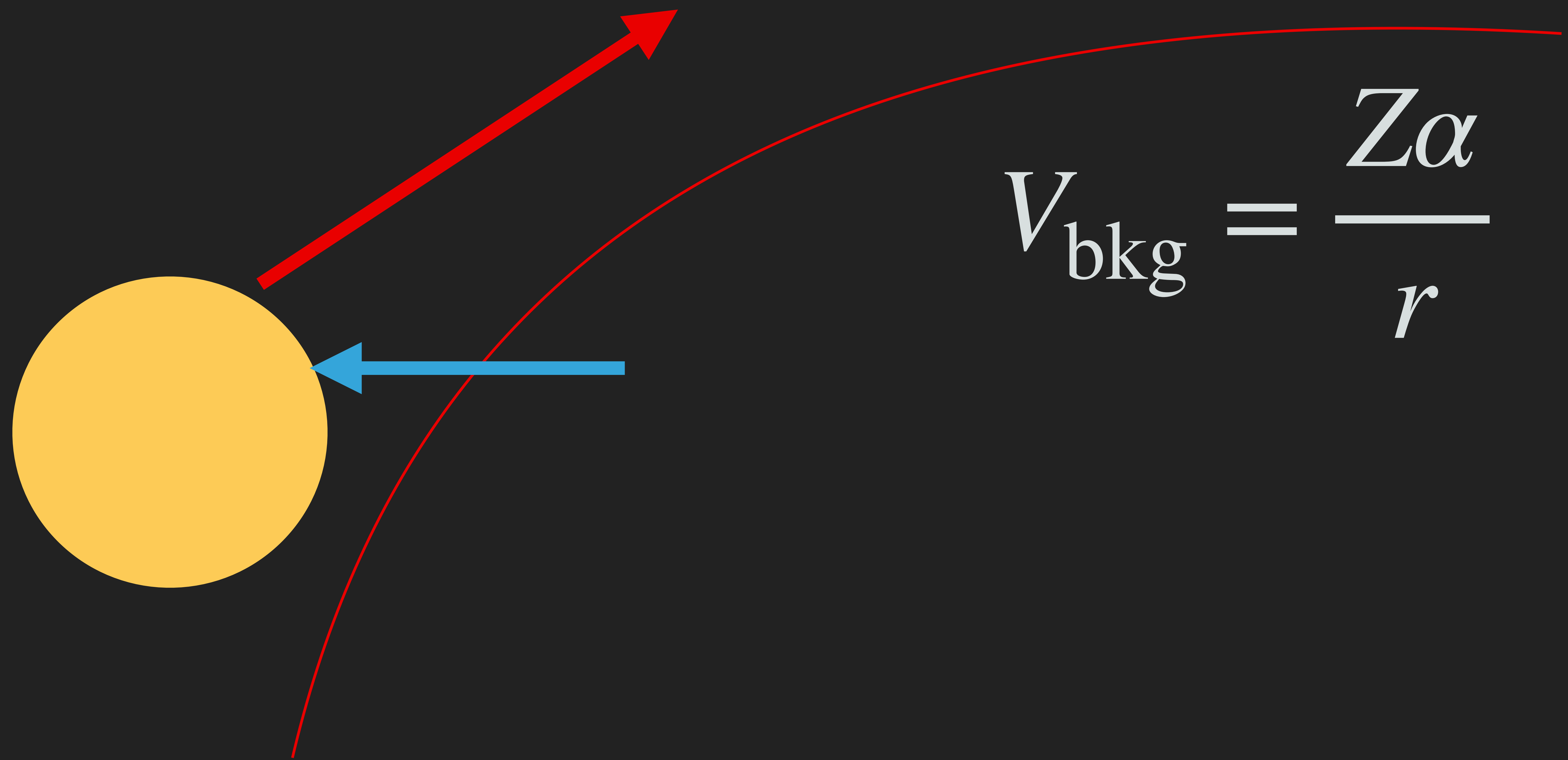


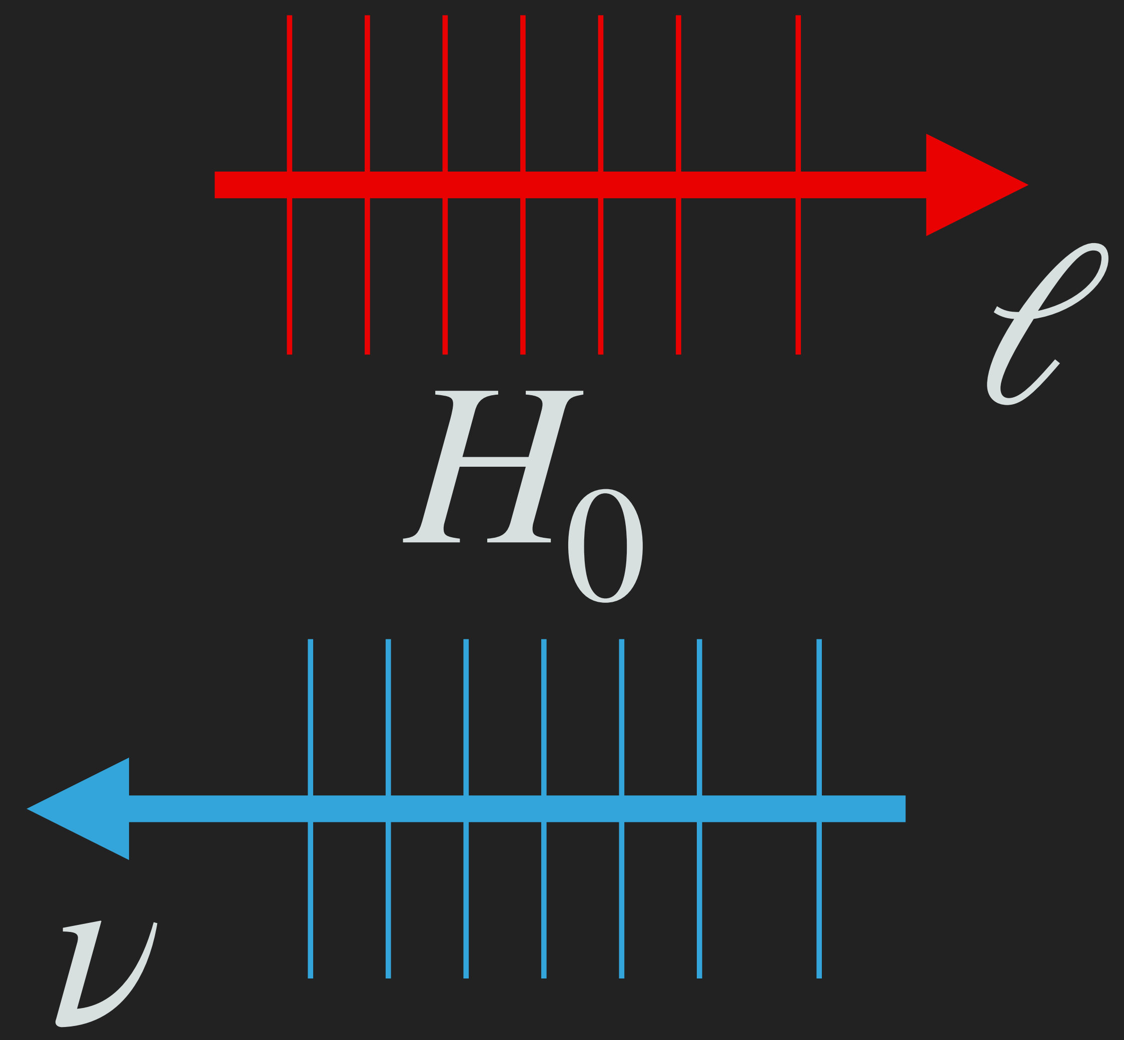
- Spectator nucleus becomes a background field.
- Coulomb field distorts lepton.
- One can also imagine adding nuclear optical potential.

COULOMB FIELD OF A NUCLEUS

$$H = H_0 + V_{\text{bkg}} + H_{\text{int}}$$



$$V_{\text{bkg}} = \frac{Z\alpha}{r}$$



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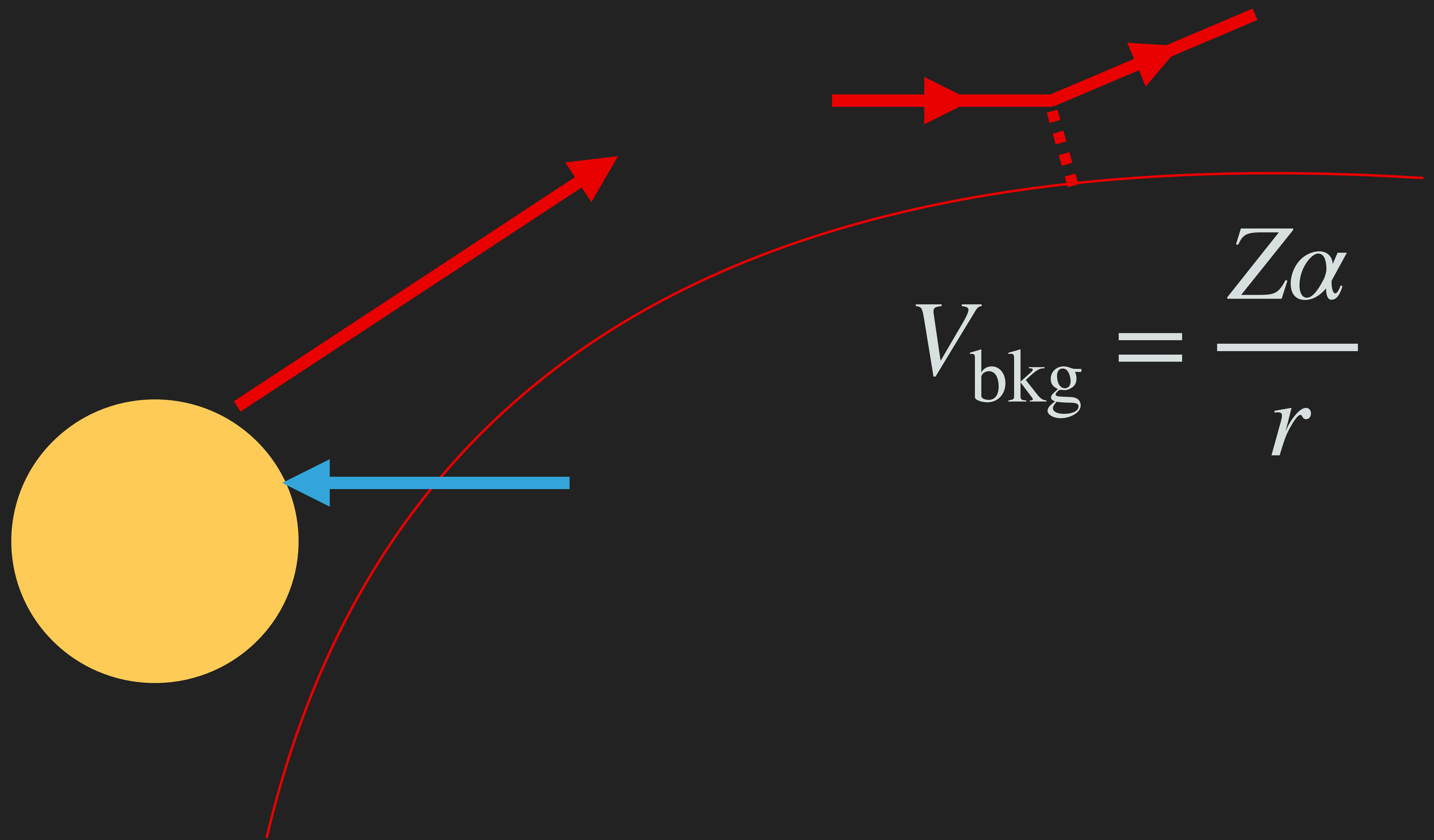

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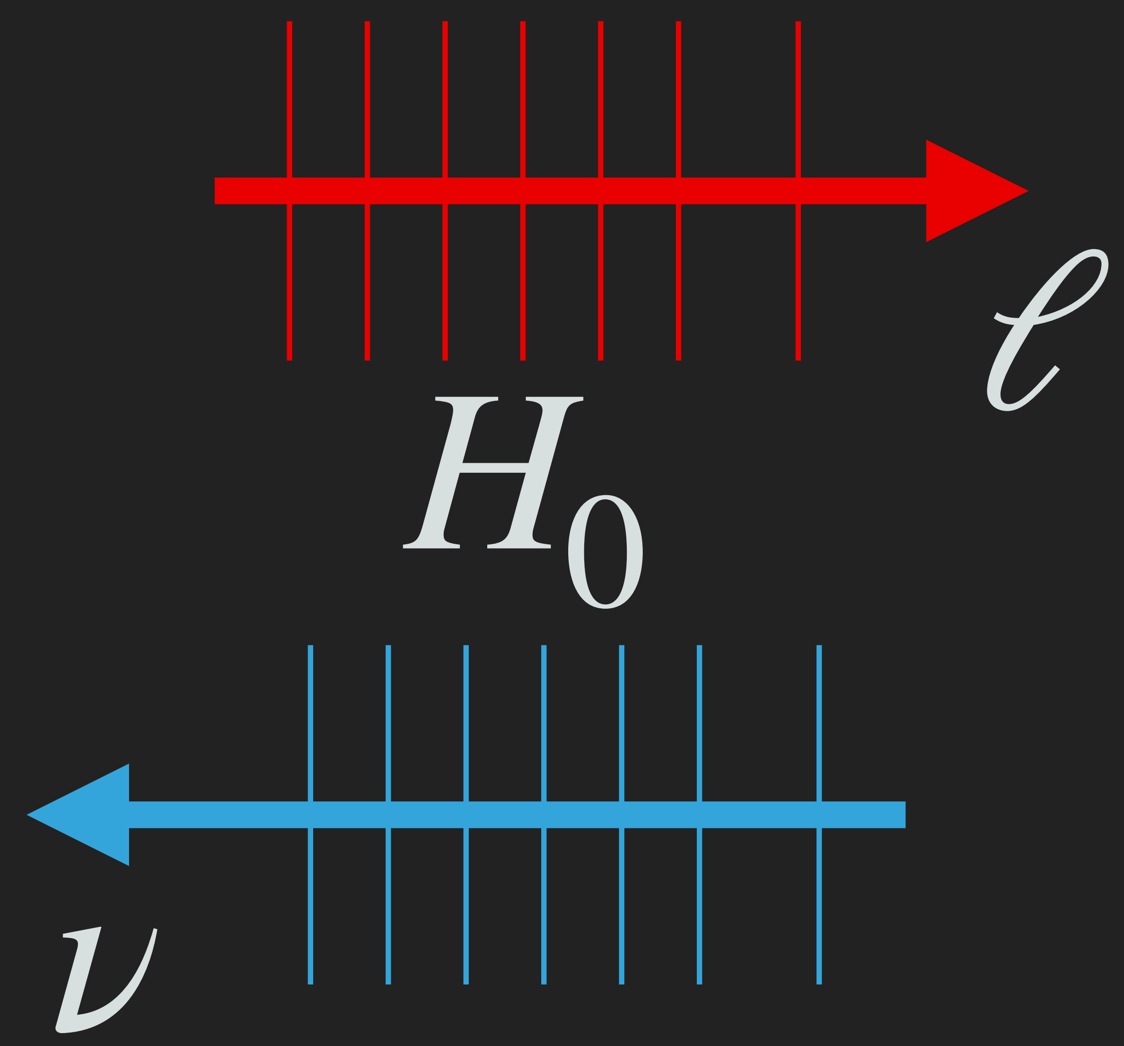

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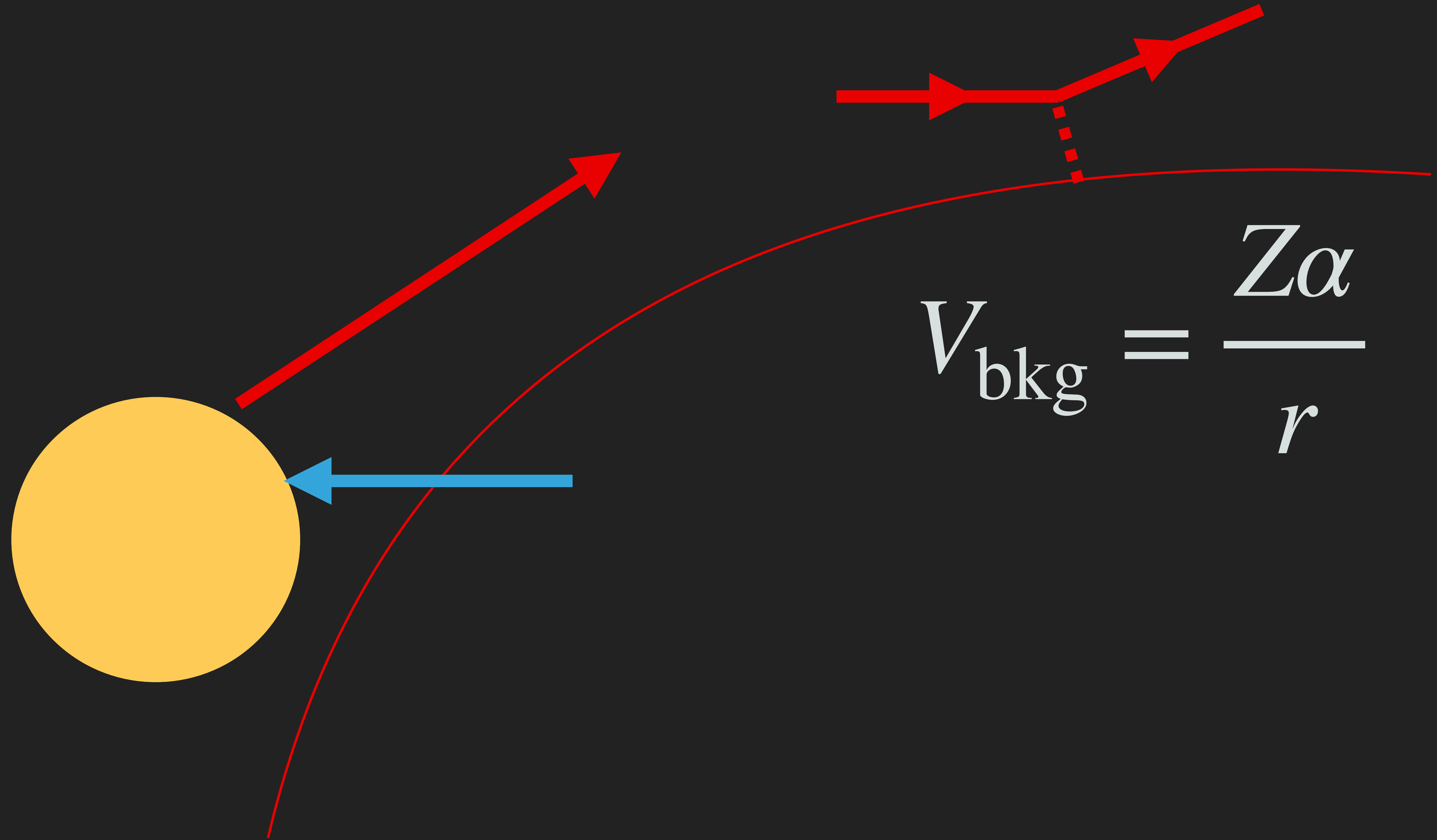

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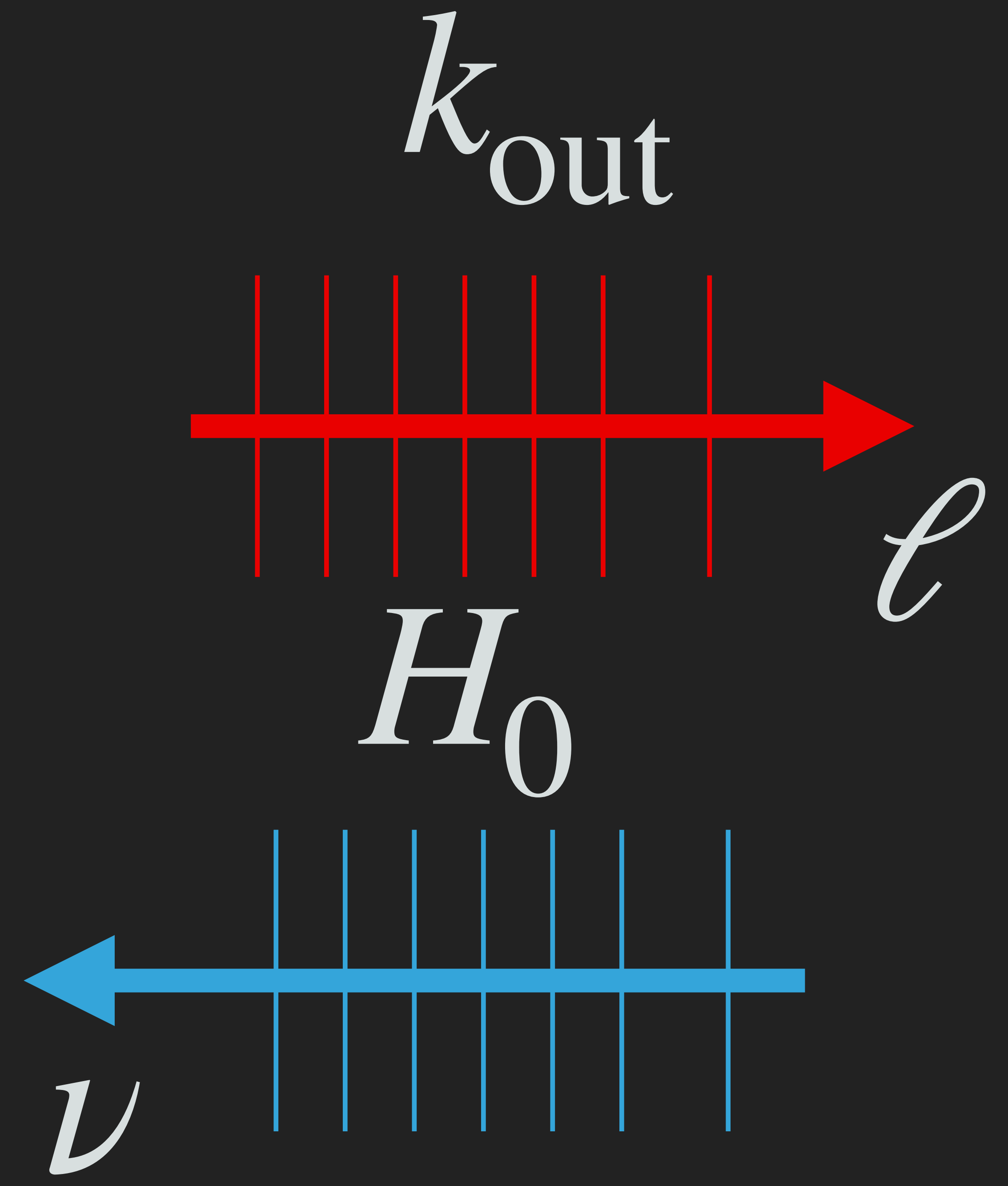

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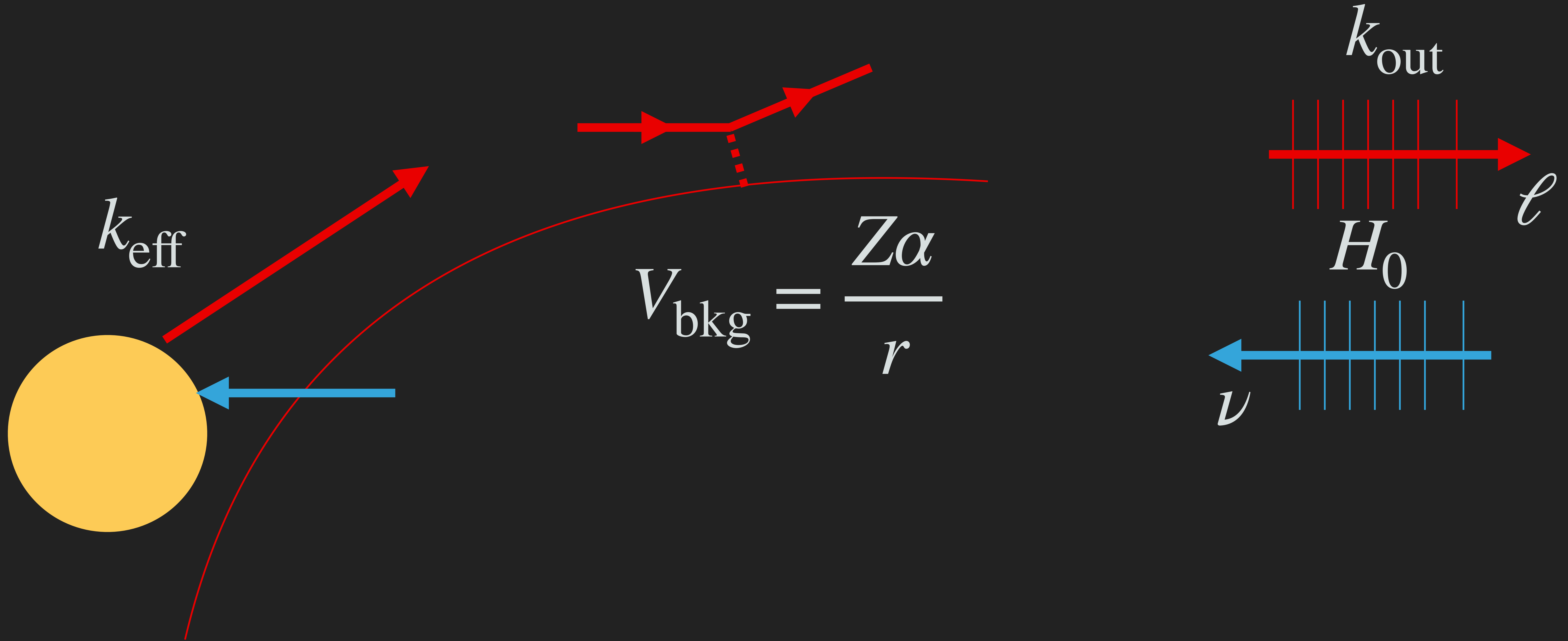
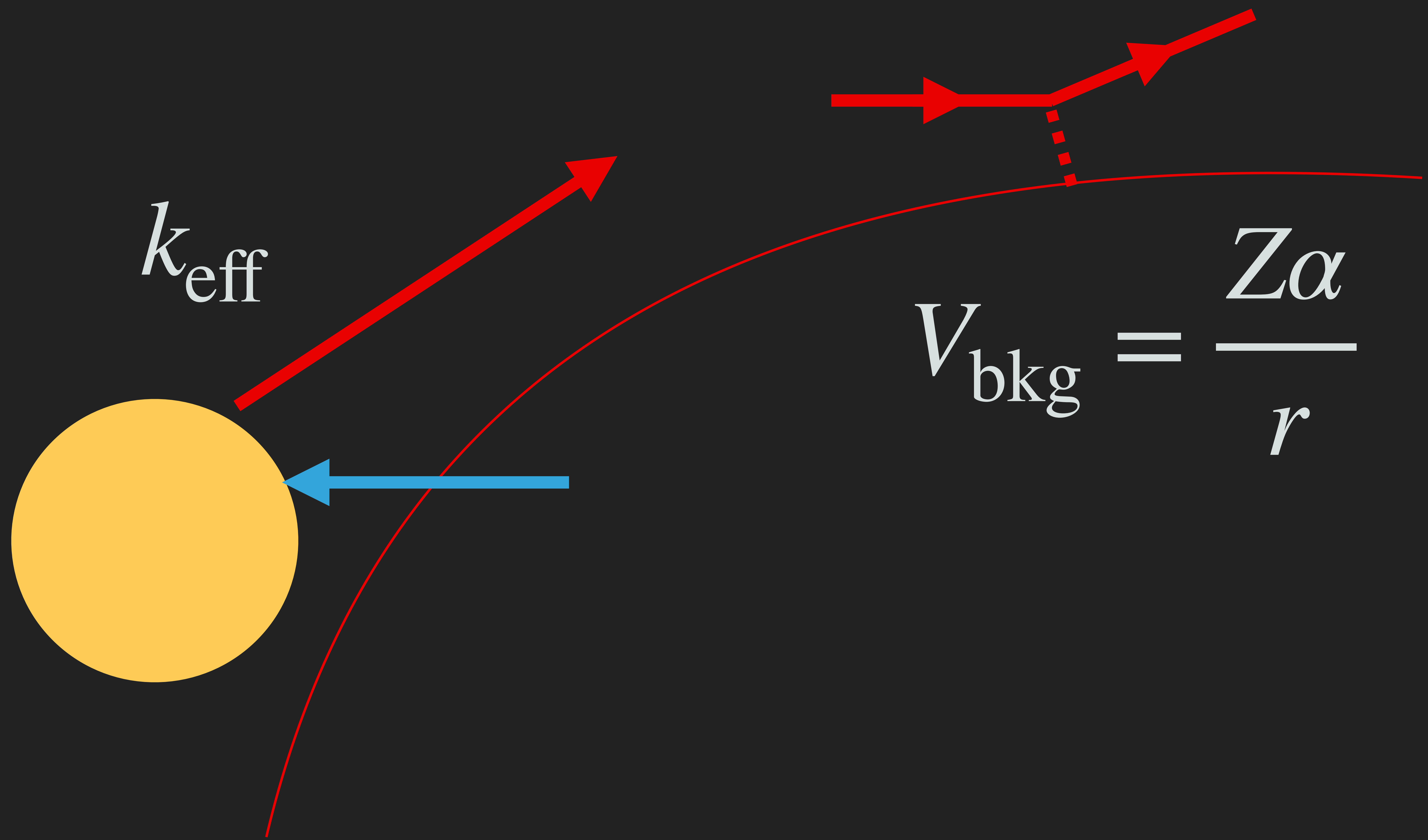


$$V_{\text{bkg}} = \frac{Z\alpha}{r}$$

COULOMB FIELD OF A NUCLEUS

$$k_{\text{eff}} \neq k_{\text{out}}$$

$$H = H_0 + V_{\text{bkg}} + H_{\text{int}}$$



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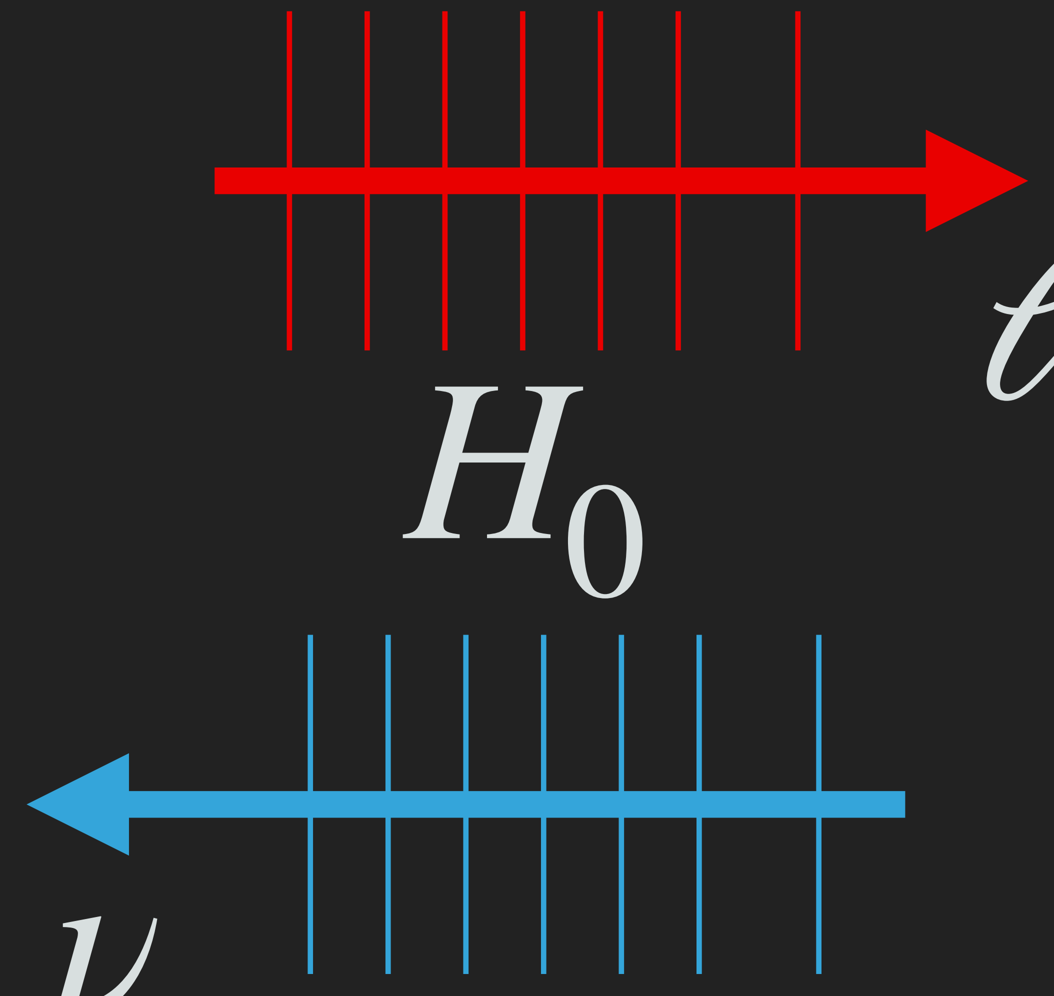
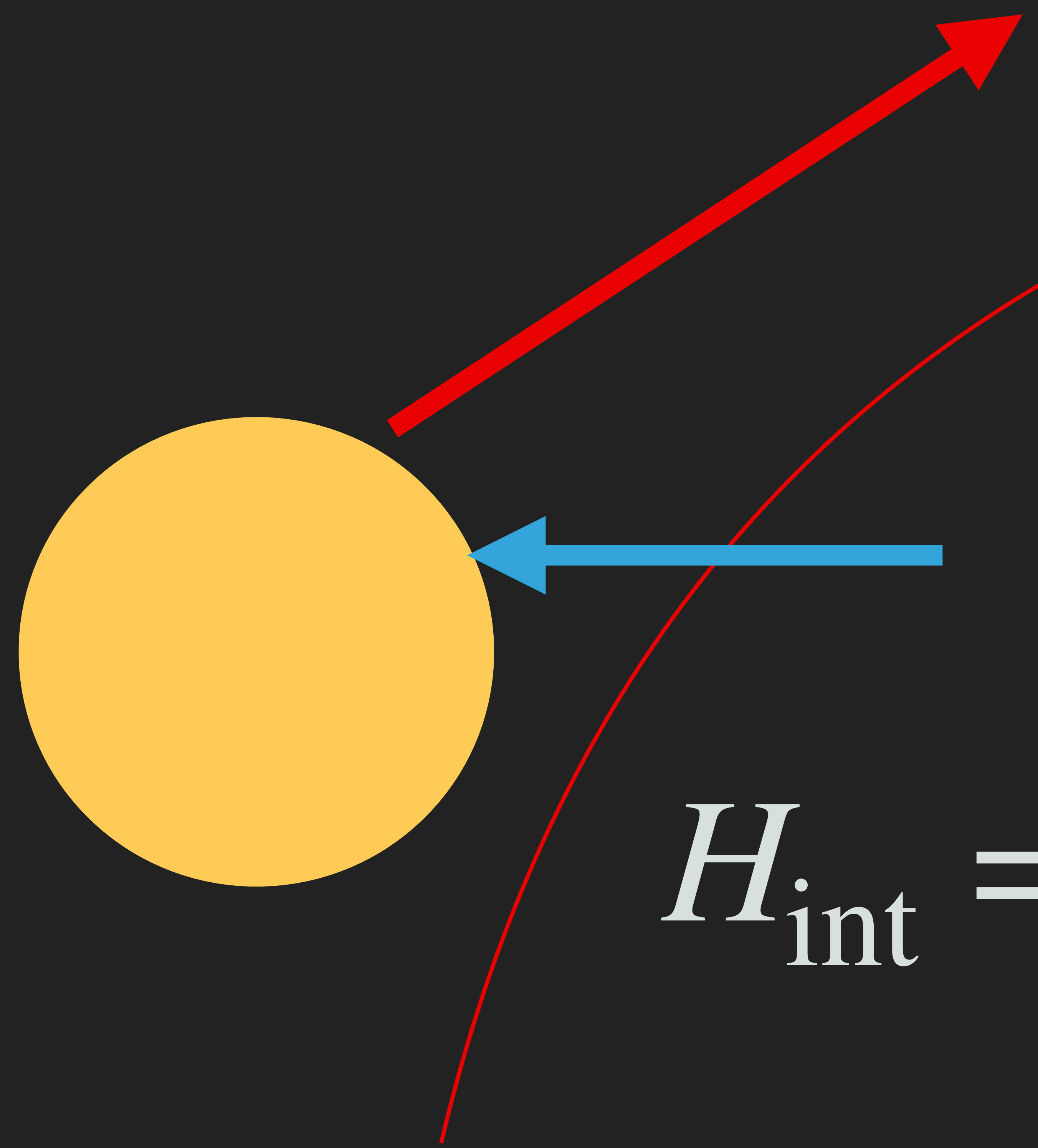


COULOMB FIELD OF A NUCLEUS

$$H = H_0 + V_{\text{bkg}} + H_{\text{int}}$$



$$V_{\text{bkg}} = \frac{Z\alpha}{r}$$

$$H_{\text{int}} = G_F \int \mathrm{d}^3x \; J^+_\mu J^-_\nu \eta^{\mu\nu}$$

- Nucleus is “infinitely” heavy: recoilless.
- Can model as static Coulomb field.
- Lepton wavefunction distorted by Coulomb field.

DISTORTED WAVE BORN APPROXIMATION

$$H = \underbrace{H_0 + V_{\text{bkg}}}_{\text{exact}} + \underbrace{H_{\text{int}}}_{\text{pert. th.}}$$

$$H_0 + \underbrace{H_{\text{int}}}_{\text{pert. th.}} \quad \Big| \quad S = \langle \phi^{(+)} | \psi^{(-)} \rangle \approx \langle \phi_0 | H_{\text{int}} | \psi_0 \rangle$$

$$H_{\text{bkg}} + \underbrace{H_{\text{int}}}_{\text{pert. th.}} \quad \Big| \quad S = \langle \phi^{(+)} | \psi^{(-)} \rangle \approx \langle \phi^{(+)}_{\text{bkg}} | H_{\text{int}} | \psi^{(-)}_{\text{bkg}} \rangle$$

DISTORTED WAVE BORN APPROXIMATION — STATIC POTENTIAL

$$H = \underbrace{H_0 + V_{\text{bkg}}}_{\text{exact}} + \underbrace{H_{\text{int}}}_{\text{pert. th.}}$$

$$\underbrace{H_0 + H_{\text{int}}}_{\text{pert. th.}} \quad \Big| \; S = J_\mu(q)\; \bar{u}_{k'}\; \Gamma^\mu\; u_k \;\; (2\pi)^4 \delta^{(4)}(k' - k - q)$$

$$\underbrace{H_{\text{bkg}} + H_{\text{int}}}_{\text{pert. th.}} \quad \Big| \; S = J_\mu(q) \left[\int \mathrm{d}^3x\; \bar{\mathcal{U}}_{k'}(x)\; \Gamma_\mu\; \mathcal{U}_k(x)\; \mathrm{e}^{iqx} \right] \; (2\pi)\delta(\Sigma E)$$

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ENERGY CONSERVED

$$H_{\text{bkg}} + \underbrace{H_{\text{int}}}_{\text{pert. th.}} \quad \Big| \quad S = J_\mu(q) \left[\int \mathrm{d}^3x\ \bar{\mathcal{U}}_{k'}(x)\ \Gamma_\mu\ \mathcal{U}_k(x)\ \mathrm{e}^{iqx} \right]\ (2\pi)\delta(\Sigma E)$$

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MOMENTUM NOT CONSERVED

STEP 1: AMPLITUDE WITH DISTORTED WAVES

$$\langle n\ell \,|\, H_{\rm int} \,|\, B\nu \rangle = (2\pi \mathrm{i})\delta(E_B + E_\nu - E_n - E_\ell)\ \mathrm{M}$$

$$|\, B\rangle = \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, \phi(p) \,|\, p\rangle$$

$$\mathrm{M} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, \phi(p) \langle n\ell \,|\, H_{\rm int} \,|\, \nu p\rangle$$