Importance of study of quasielastic hyperon production at DUNE energies

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Collaborators:

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March 17, 2021

Plan of the presentation

✠ Quasielastic Hyperon production with $\bar{\nu}_l$ and e^-

The results are presented only for $\bar{\nu}_\mu$ induced reactions on the free nucleon target.

- **Standard model calculation with SU(3) symmetry**
- **Effect of SU(3) breaking using Holstein-Fasseler et al. model**
- **Effect of second class currents with and without T invariance**
- **Effect of SU(3) breaking and second class currents**

Hyperon production

The observation of hyperons produced in the antineutrino ($\bar{\nu}_l + p \longrightarrow l^+ + Y; Y = \Lambda, \Sigma^{0,-}$) and electron ($e^- + p \longrightarrow \nu_e + Y$) induced processes may provide an opportunity to:

- **test the SU(3) symmetry, G invariance and T invariance.**
- **determine the $N - Y$ transition form factors.**
- **get some information about the second class currents.**

The measurement of the hyperon polarization may determine independently the form factors appearing in the weak hadronic current.

Using high luminosity electron beam at the JLab and MAMI, or antineutrino beam at DUNE using LArTPC detector, such studies could be possible.

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T-violation in weak interactions

- **The asymmetry in the angular distribution of pion ($\Lambda \to p\pi^-$ or $n\pi^0$ and $\Sigma^- \to n\pi^-$) determines the polarization component.**

- **The polarization measurement of the hyperons produced in the antineutrino and electron induced processes gives an alternative method to study T-violation.**

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- **The polarization measurement of the hyperons produced in the antineutrino and electron induced processes gives an alternative method to study T-violation.**

- **The measurement of P_T, which lies perpendicular to the reaction plane, gives information about T violation.**

Matrix element: $\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p')$

$$\mathcal{M} = \frac{G_F \sin\theta_c}{\sqrt{2}} l_\mu J^\mu \qquad l_\mu = \bar{u}(k')\gamma_\mu(1 \mp \gamma_5)u(k) \text{ for } e^-/\bar{\nu} \text{ processes}$$

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Vector operator

Axial vector operator

$$J^\mu = \bar{u}_{B'}(p')\left[V^\mu_{B'B}(p',p) - A^\mu_{B'B}(p',p)\right]u_B(p)$$

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$$V^\mu_{B'B}(p', p) = f_1^{B'B}(Q^2)\gamma_\mu + \frac{i\sigma^{\mu\nu} q_\nu}{M_B + M'_B} f_2^{B'B}(Q^2) + \frac{2q_\mu}{M_B + M'_B} f_3^{B'B}(Q^2)$$

$$A^\mu_{B'B}(p', p) = g_1^{B'B}(Q^2)\gamma_\mu\gamma_5 + i\sigma_{\mu\nu}\gamma_5 \frac{q^\nu}{M_B + M'_B} g_2^{B'B}(Q^2) + \frac{2q^\mu}{M_B + M'_B}\gamma_5 g_3^{B'B}(Q^2)$$

First class current form factors: $f_1(Q^2), f_2(Q^2), g_1(Q^2)$ and $g_3(Q^2)$

Second class current form factors: $f_3(Q^2)$ and $g_2(Q^2)$

Matrix element: $\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p')$

$$\mathcal{M} = \frac{G_F \sin\theta_c}{\sqrt{2}} l_\mu J^\mu \qquad l_\mu = \bar{u}(k')\gamma_\mu(1 \mp \gamma_5)u(k) \text{ for } e^-/\bar{\nu} \text{ processes}$$

Vector operator ← ; → Axial vector operator

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f_1: **Vector FF**; f_2: **Magnetic FF**; f_3: **Scalar FF**

$$A^\mu_{B'B}(p',p) = g_1^{B'B}(Q^2)\gamma_\mu\gamma_5 + i\sigma_{\mu\nu}\gamma_5 \frac{q^\nu}{M_B + M_B'} g_2^{B'B}(Q^2) + \frac{2q^\mu}{M_B + M_B'}\gamma_5 g_3^{B'B}(Q^2)$$

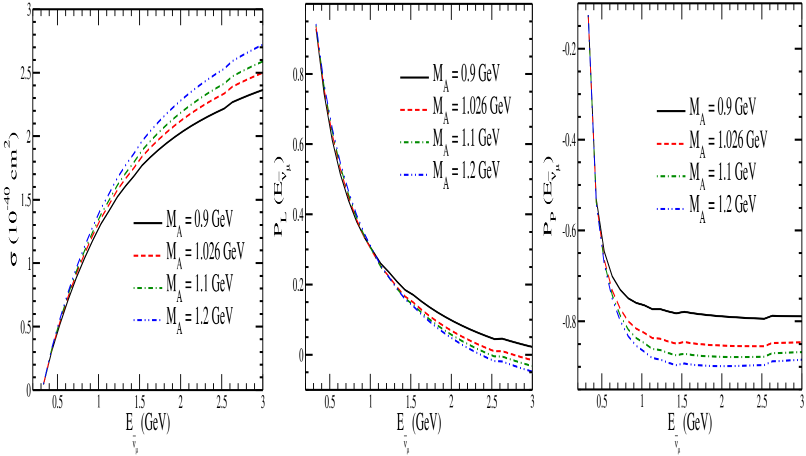
g_1: **Axial vector FF**; g_2: **Electric FF**; g_3: **Pseudoscalar FF**

First class current form factors: $f_1(Q^2), f_2(Q^2), g_1(Q^2)$ and $g_3(Q^2)$

Second class current form factors: $f_3(Q^2)$ and $g_2(Q^2)$

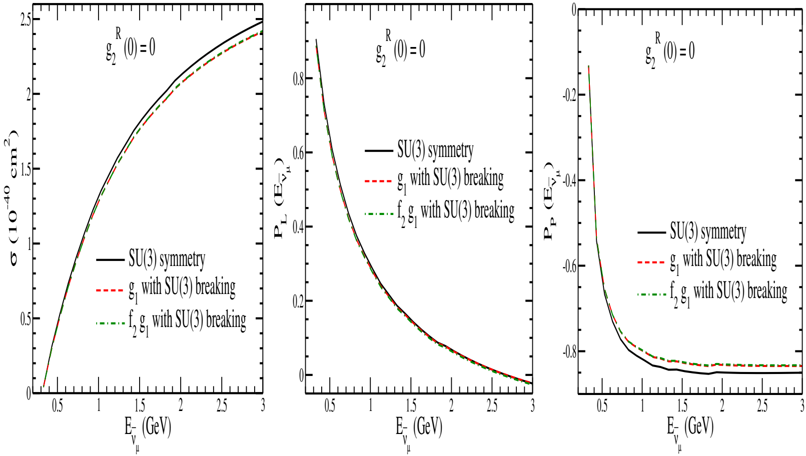
First class currents (contribution from $f_1(Q^2)$, $f_2(Q^2)$, $g_1(Q^2)$ and $g_3(Q^2)$): M_A variation

$$\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p')$$



First class currents with SU(3) breaking

$$\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p')$$



SU(3) breaking effects: **A. Faessler et al., PRD 78, 094005 (2008)**

Effect of second class currents with T invariance: $g_2^R(0)$ variation

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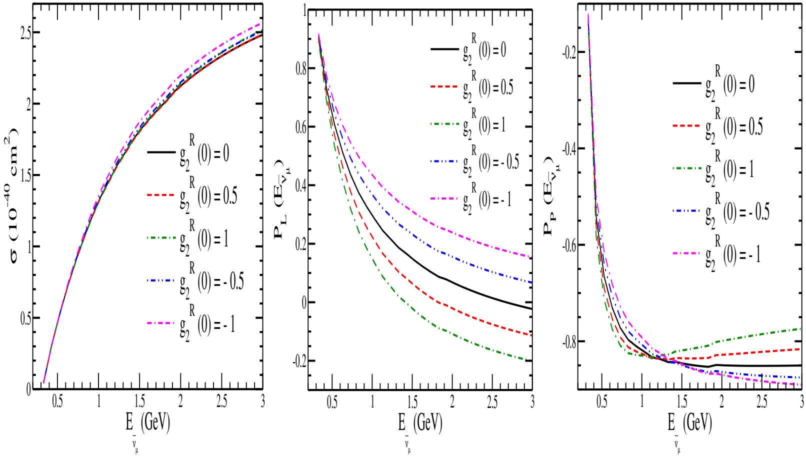

AF, MSA, SKS, Phys. Rev. D 98, 033005 (2018).

Effect of second class currents with T violation: $g_2^I(0)$ variation

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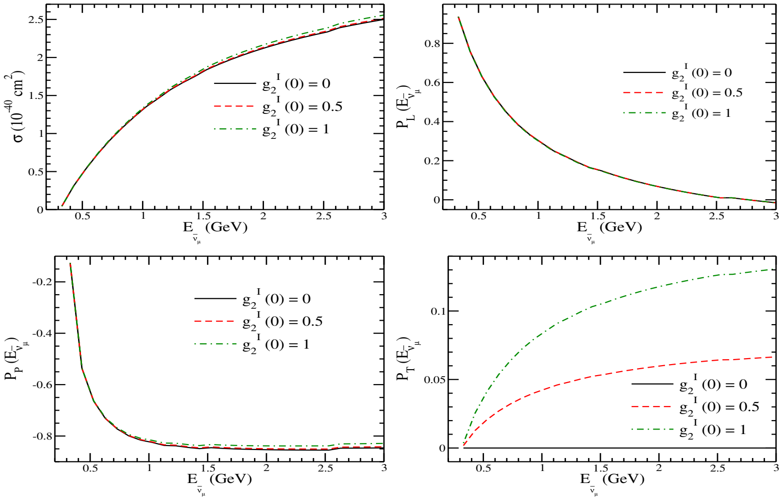
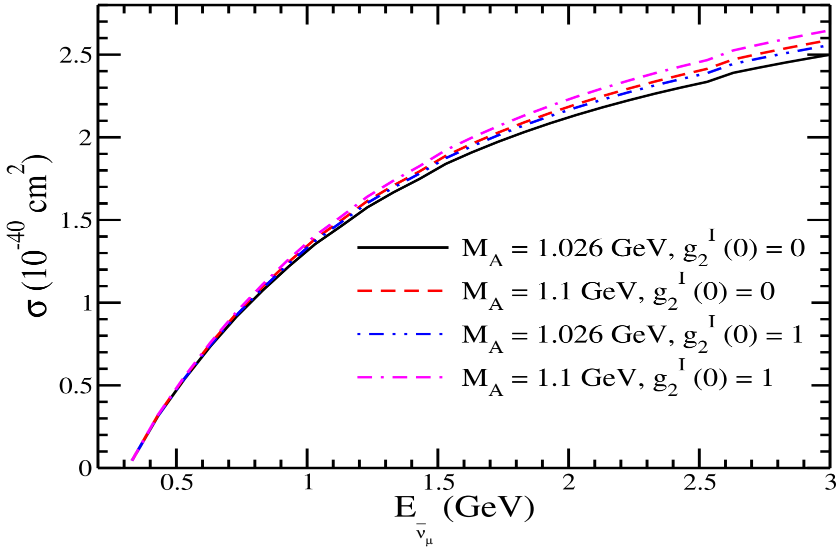


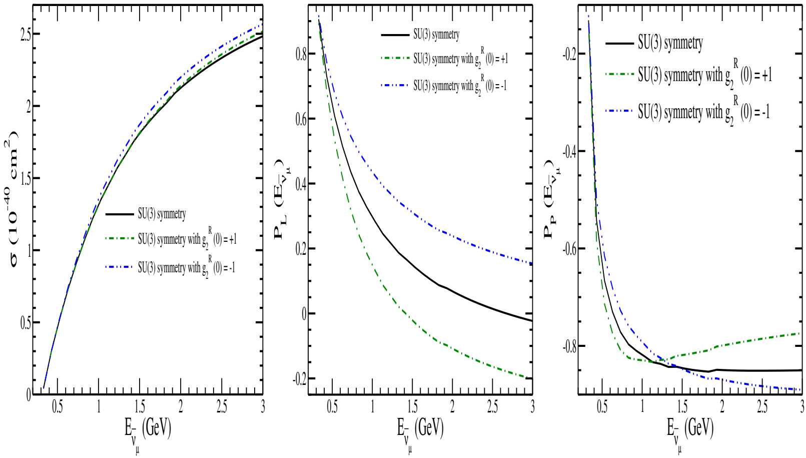
AF, MSA, SKS, Phys. Rev. D 98, 033005 (2018).

σ vs $E_{\bar{\nu}_\mu}$ for the antineutrino induced Λ production



Effect of second class currents with T invariance and SU(3) breaking

$$\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p')$$



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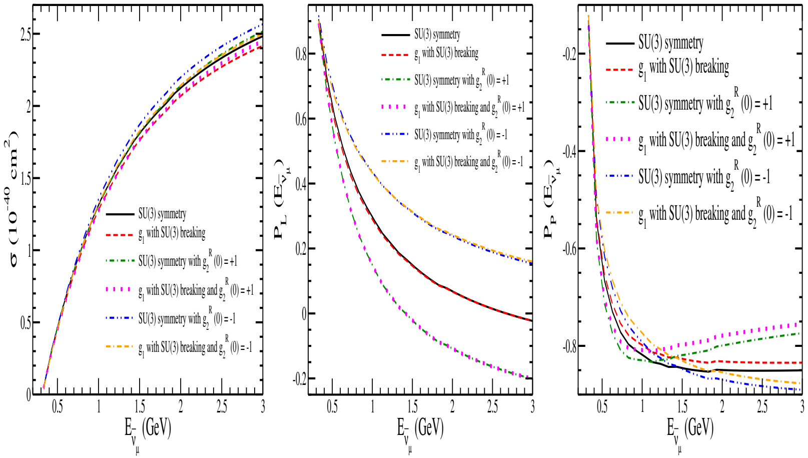
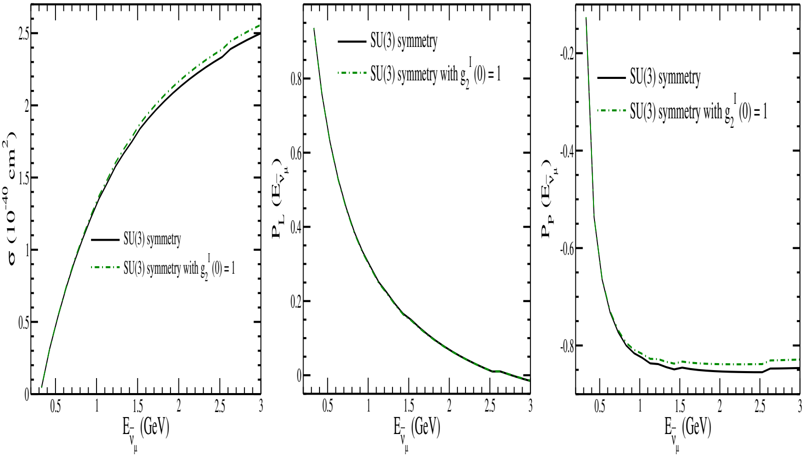


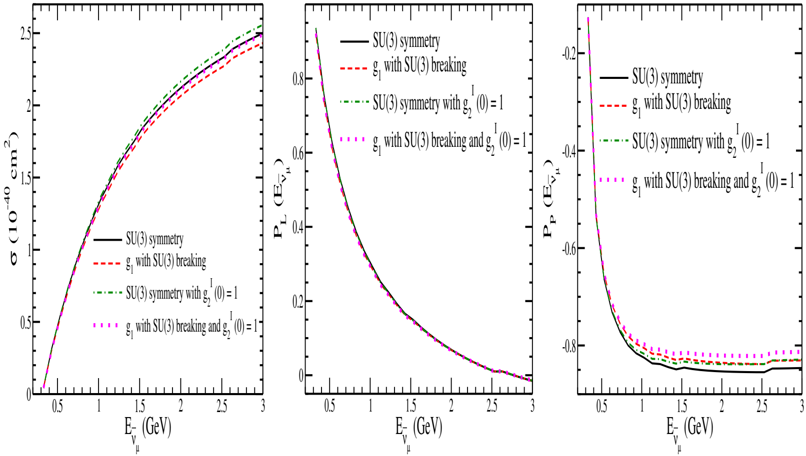
Effect of second class currents with T violation and SU(3) breaking

$$\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p')$$



Effect of second class currents with T violation and SU(3) breaking

$$\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p')$$



Conclusion

- **In the hyperon production processes induced by electrons and antineutrinos, it is feasible to study the effect of T violation.**

- **The presence of SCC form factor with T invariance taken to be negative in sign ($g_2^R(0) = -1$) and without T invariance compensates the effect of SU(3) breaking.**

Conclusion

- **In the hyperon production processes induced by electrons and antineutrinos, it is feasible to study the effect of T violation.**

- **The presence of SCC form factor with T invariance taken to be negative in sign ($g_2^R(0) = -1$) and without T invariance compensates the effect of SU(3) breaking.**

- **In the absence of SU(3) symmetry breaking effects, the presence of SCC with or without T invariance would lead to a smaller value of M_A.**

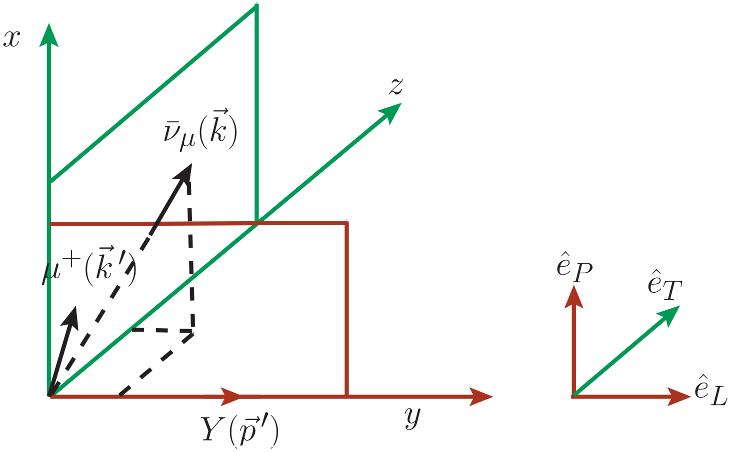
- **These results would be important for the experiments like MicroBooNE and DUNE, which use LArTPC detectors.**

- ★ SU(3) symmetry breaking effects: A. Faessler, ..., B. R. Holstein et al., Phys. Rev. D 78, 094005 (2008).
- ★ Antineutrino induced hyperon production:
  A. Fatima, M. Sajjad Athar and S. K. Singh, Front. in Phys. 7, 13 (2019),
  A. Fatima, M. Sajjad Athar and S. K. Singh, Phys. Rev. D 98, 033005 (2018),
  F. Akbar, M. Rafi Alam, M. Sajjad Athar and S. K. Singh, Phys. Rev. D 94, 114031 (2016).
- ★ Electron induced hyperon production:
  A. Fatima, M. Sajjad Athar and S. K. Singh, Eur. Phys. J. A 54, 95 (2018),
  F. Akbar, M. Sajjad Athar, A. Fatima and S. K. Singh, Eur. Phys. J. A 53, 154 (2017).
- ★ $\nu_\tau(\bar{\nu}_\tau)$ induced nucleon production:
  A. Fatima, M. Sajjad Athar and S. K. Singh, Phys. Rev. D 102, 113009 (2020).

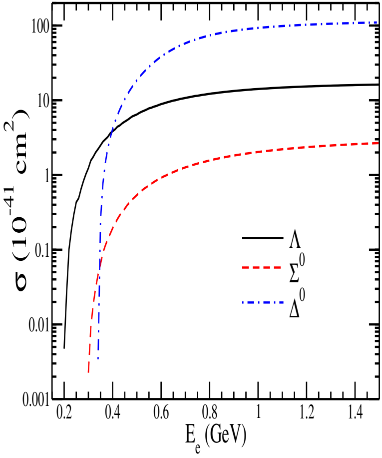
BACKUP

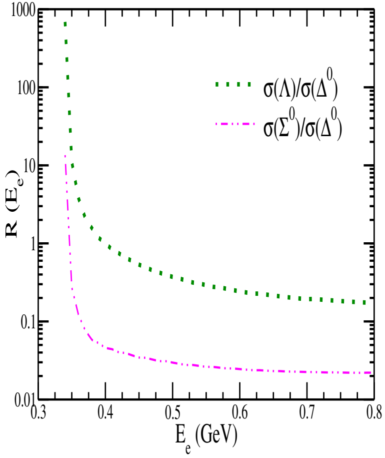
Polarization components of final hyperon

$$\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p')$$



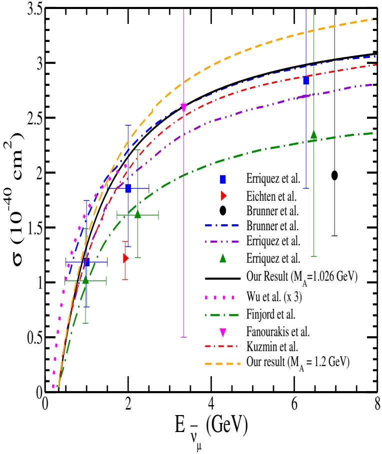
σ vs E_e for Λ, Σ^0 and Δ^0 productions induced by electrons

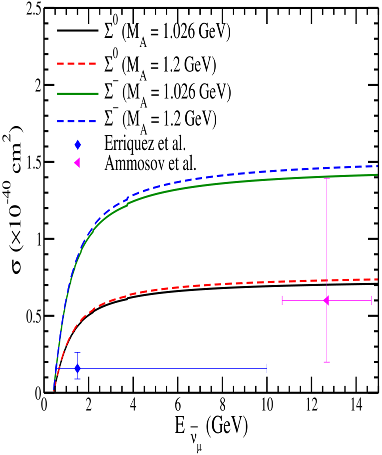




FA, MSA, A. Fatima, SKS, Eur. Phys. J. A 53, 154 (2017).

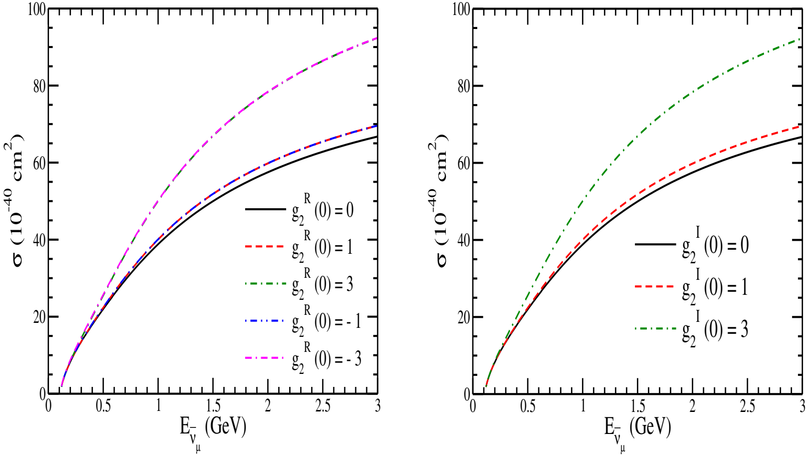
σ vs $E_{\bar{\nu}_\mu}$ for Λ, Σ^0 and Σ^- productions induced by antineutrinos





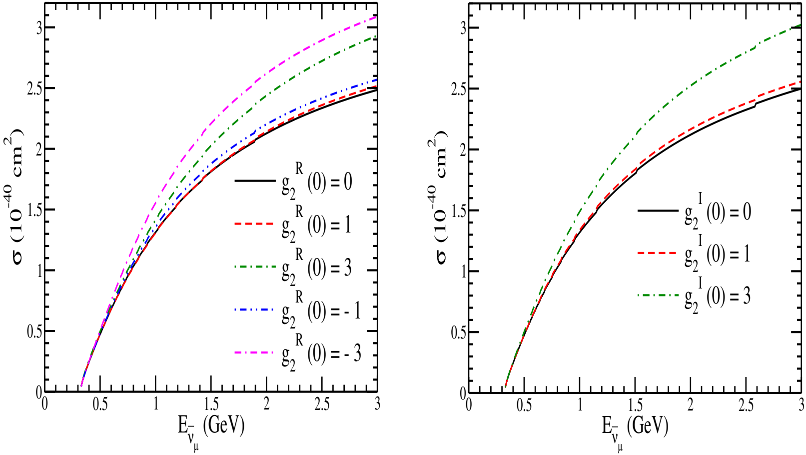
A. Fatima, MSA, SKS, Front. in Phys. 7, 13 (2019).

σ vs $E_{\bar{\nu}_\mu}$ for the process $\bar{\nu}_\mu + p \longrightarrow \mu^+ + n$



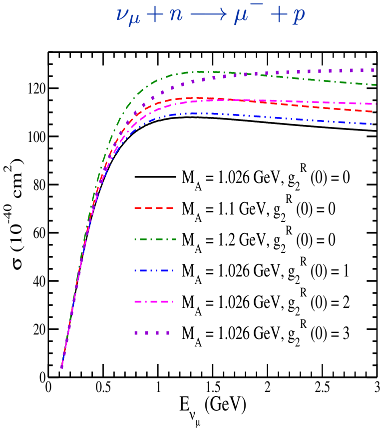
A. Fatima, MSA, SKS, Phys. Rev. D 98, 033005 (2018).

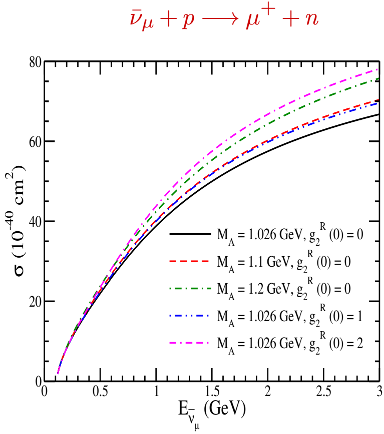
σ vs $E_{\bar{\nu}_\mu}$ for the process $\bar{\nu}_\mu + p \longrightarrow \mu^+ + \Lambda$



A. Fatima, MSA, SKS, Phys. Rev. D 98, 033005 (2018).

σ vs $E_{\bar{\nu}_\mu}$ for the process $\nu_\mu + n \longrightarrow \mu^- + p$ and $\bar{\nu}_\mu + p \longrightarrow \mu^+ + n$





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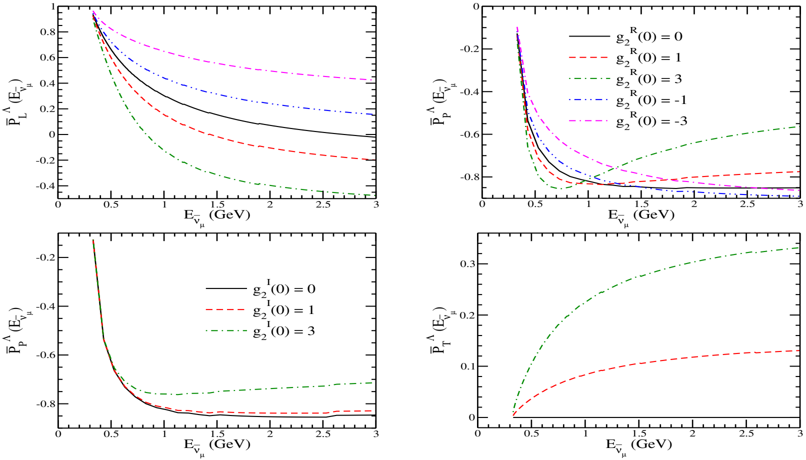
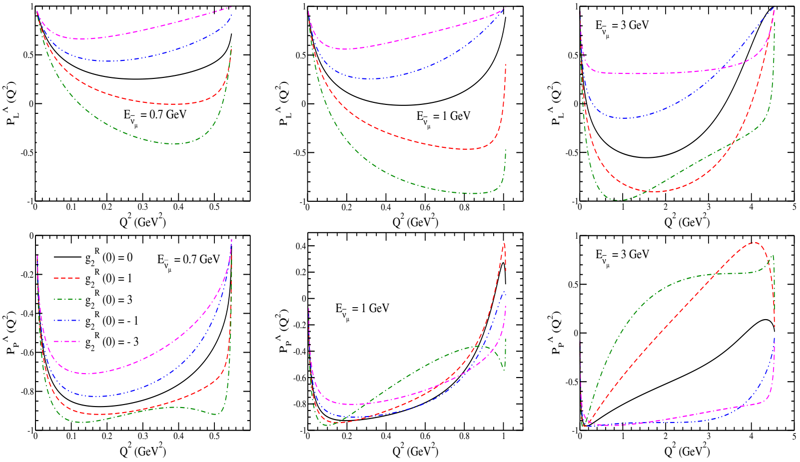
Polarization components vs $E_{\bar{\nu}_\mu}$ for the process $\bar{\nu}_\mu + p \longrightarrow \mu^+ + \Lambda$



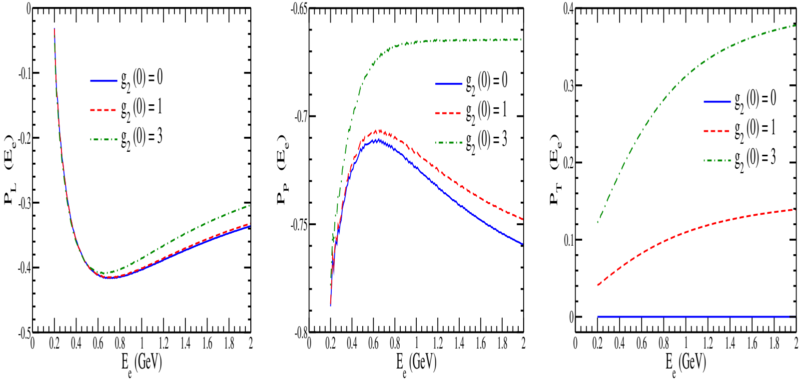
A. Fatima, MSA, SKS, Phys. Rev. D. 98, 033005 (2018).

Polarization components vs Q^2 for the process $\bar{\nu}_\mu + p \longrightarrow \mu^+ + \Lambda$



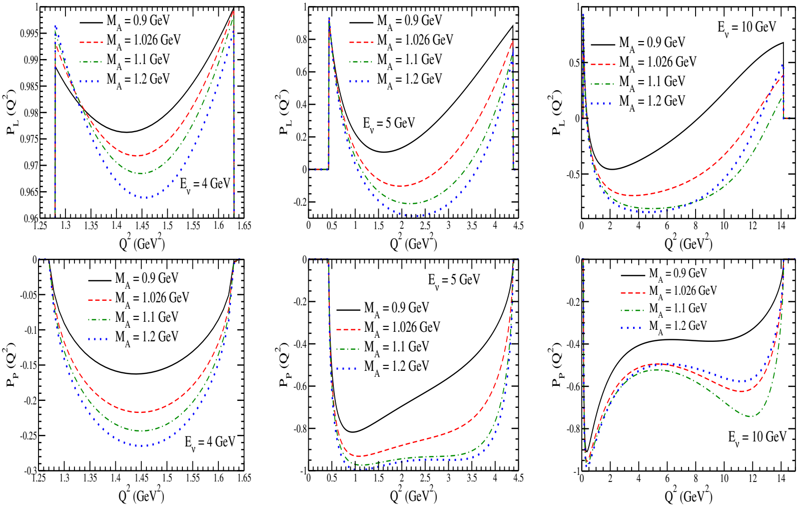
A. Fatima, MSA, SKS, Front. in Phys. 7, 13 (2019).

Polarization components vs E_e for the process $e^- + p \longrightarrow \nu_e + \Lambda$



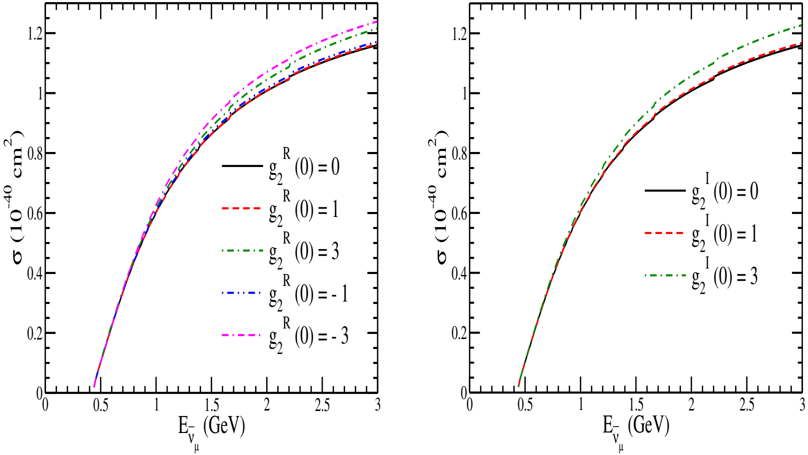
A. Fatima, MSA, SKS, Eur. Phys. J. A 54, 95 (2018).

Polarization components vs Q^2 for the process $\bar{\nu}_\tau + p \longrightarrow \tau^+ + \Lambda$



A. Fatima, MSA, SKS, To be communicated.

σ vs $E_{\bar{\nu}_\mu}$ for the process $\bar{\nu}_\mu + p \longrightarrow \mu^+ + \Sigma^-$



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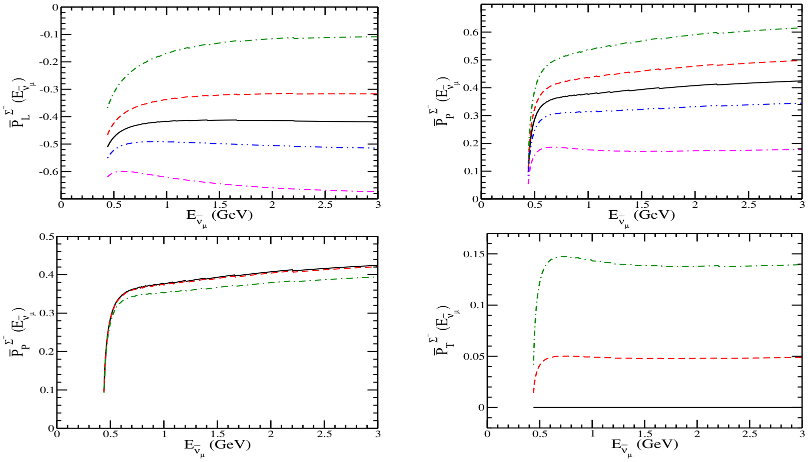
Polarization components vs $E_{\bar{\nu}_\mu}$ for the process $\bar{\nu}_\mu + p \longrightarrow \mu^+ + \Sigma^-$



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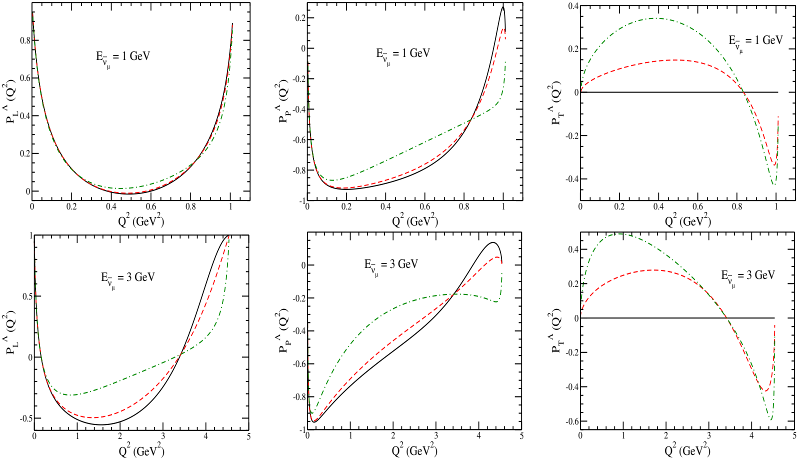
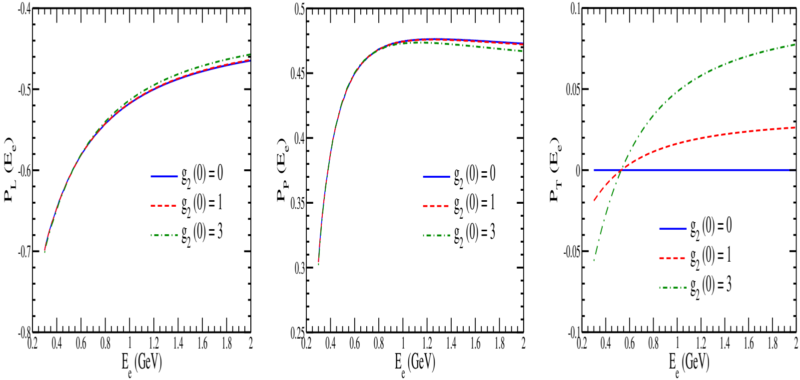


A. Fatima, MSA, SKS, Front. in Phys. 7, 13 (2019).

Polarization components vs E_e for the process $e^- + p \longrightarrow \nu_e + \Sigma^0$



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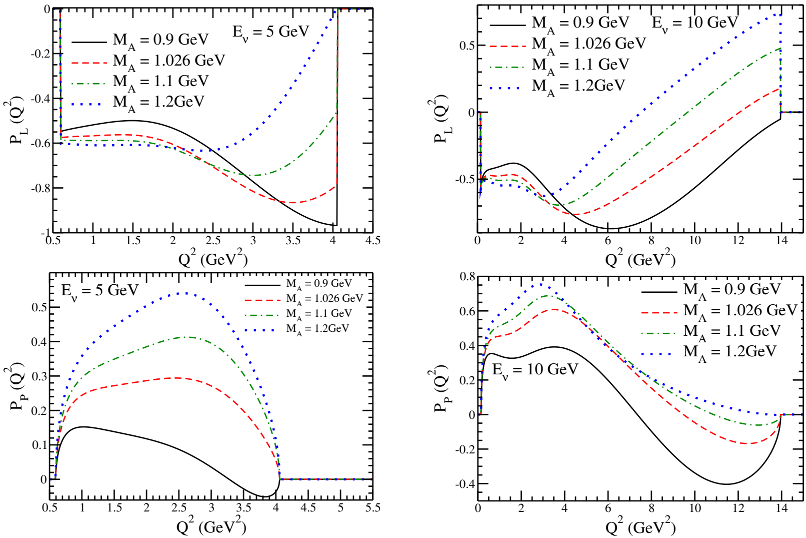
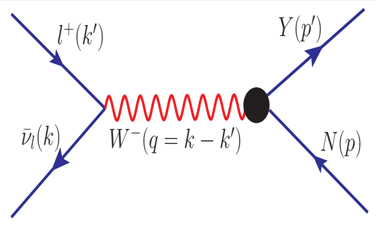
Polarization components vs Q^2 for the process $\bar{\nu}_\tau + p \longrightarrow \tau^+ + \Sigma^0$



$|\Delta S| = 1$ processes

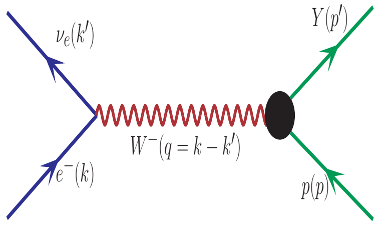
Antineutrino induced single hyperon production

$$\bar{\nu}_l(k) + p(p) \rightarrow l^+(k') + \Lambda(p')$$
$$\bar{\nu}_l(k) + p(p) \rightarrow l^+(k') + \Sigma^0(p')$$
$$\bar{\nu}_l(k) + n(p) \rightarrow l^+(k') + \Sigma^-(p')$$



Electron induced single hyperon production

$$e^-(k) + p(p) \rightarrow \nu_e(k') + \Lambda(p')$$
$$e^-(k) + p(p) \rightarrow \nu_e(k') + \Sigma^0(p')$$
$$e^-(k) + n(p) \rightarrow \nu_e(k') + \Sigma^-(p')$$



Differential cross section

$$\bar{\nu}_l(k) + N(p) \longrightarrow l^+(k') + Y(p'),$$

$$d\sigma = \frac{1}{4M_N E_\nu}(2\pi)^4\delta^4(k+p-k'-p')\frac{d^3k'}{(2\pi)^3 2E_{k'}}\frac{d^3p'}{(2\pi)^3 2E_{p'}}\overline{\sum}\sum|\mathcal{M}|^2$$

$$e^-(k) + N(p) \longrightarrow \nu_e(k') + Y(p'),$$

$$d\sigma = \frac{1}{4M_N E_e}(2\pi)^4\delta^4(k+p-k'-p')\frac{d^3k'}{(2\pi)^3 2E_{k'}}\frac{d^3p'}{(2\pi)^3 2E_{p'}}\overline{\sum}\sum|\mathcal{M}|^2$$

- $q = p' - p = k - k'$ **is the four momentum transfer**
- $\mathcal{M}$ **is the transition matrix element**

G-parity

- **This is a symmetry of strong interactions, defined as**

$$G = Ce^{i\pi I_y}$$

- **Under G-parity, V^μ and A^μ transforms as**

$$GV^\mu G^{-1} = V^\mu \qquad GA^\mu G^{-1} = -A^\mu$$

- $f_1(Q^2)$**,** $f_2(Q^2)$**,** $g_1(Q^2)$ **and** $g_3(Q^3)$ **transform the same way as above and are terms as first class currents.**

- $f_3(Q^2)$ **and** $g_2(Q^2)$ **transform in opposite way and are termed as second class currents.**

Symmetry properties

- **T-invariance ⇒ form factors are real**
- **CVC ⇒** $f_3(Q^2) = 0$
- **G-invariance ⇒** $f_3(Q^2) = 0$ **and** $g_2(Q^2) = 0$
- **PCAC ⇒ relates** $g_3(Q^2)$ **with** $g_1(Q^2)$ **through GT relation**

Vector form factors

The assumption of SU(3) symmetry and CVC leads to the determination of $f_1(Q^2)$ and $f_2(Q^2)$ in terms of EM form factors of the nucleon $f_1^N(Q^2)$ and $f_2^N(Q^2)$.

FF	$p \to n$	$p \to \Lambda$	$p \to \Sigma^0$
$f_1(Q^2)$	$f_1^p(Q^2) - f_1^n(Q^2)$	$-\sqrt{\frac{3}{2}} f_1^p(Q^2)$	$-\frac{1}{\sqrt{2}}(f_1^p(Q^2) + 2f_1^n(Q^2))$
$f_2(Q^2)$	$f_2^p(Q^2) - f_2^n(Q^2)$	$-\sqrt{\frac{3}{2}} f_2^p(Q^2)$	$-\frac{1}{\sqrt{2}}(f_2^p(Q^2) + 2f_2^n(Q^2))$

The EM form factors of the nucleon $f_1^N(Q^2)$ and $f_2^N(Q^2)$ are expressed in terms of the electric and magnetic Sach's form factors

$$f_1^{p,n}(Q^2) = \frac{1}{1+\frac{Q^2}{4M^2}}\left[G_E^{p,n}(Q^2) + \frac{Q^2}{4M^2} G_M^{p,n}(Q^2)\right]$$

$$f_2^{p,n}(Q^2) = \frac{1}{1+\frac{Q^2}{4M^2}}\left[G_M^{p,n}(Q^2) - G_E^{p,n}(Q^2)\right]$$

Axial vector form factors

$g_1(Q^2)$ and $g_2(Q^2)$ are given in terms of two functions $F^A(Q^2)$ and $D^A(Q^2)$.

FF	$p \rightarrow n$	$p \rightarrow \Lambda$	$p \rightarrow \Sigma^0$
$g_1(Q^2)$	$g_A(Q^2)$	$-\frac{1}{\sqrt{6}}\frac{D^A(Q^2)+3F^A(Q^2)}{D^A(Q^2)+F^A(Q^2)}g_A(Q^2)$	$\frac{1}{\sqrt{2}}\frac{D^A(Q^2)-F^A(Q^2)}{D^A(Q^2)+F^A(Q^2)}g_A(Q^2)$
$g_2(Q^2)$	$g_2(Q^2)$	$-\frac{1}{\sqrt{6}}\frac{D^A(Q^2)+3F^A(Q^2)}{D^A(Q^2)+F^A(Q^2)}g_2(Q^2)$	$\frac{1}{\sqrt{2}}\frac{D^A(Q^2)-F^A(Q^2)}{D^A(Q^2)+F^A(Q^2)}g_2(Q^2)$

F and D are determined from the semileptonic decays and are taken as 0.463 and 0.804 respectively.

Axial vector form factor (Contd.)

$$g_A(Q^2) = \frac{g_A(0)}{\left(1+\frac{Q^2}{M_A^2}\right)^2}, \qquad g_A(0) = D(0) + F(0) = 1.267$$

$$g_2^{pn}(Q^2) = \frac{g_2(0)}{\left(1+\frac{Q^2}{M_2^2}\right)^2}, \qquad M_2 = 1.026 \text{ GeV}.$$

Axial vector form factor (Contd.)

$$g_A(Q^2) = \frac{g_A(0)}{\left(1+\frac{Q^2}{M_A^2}\right)^2}, \qquad g_A(0) = D(0)+F(0) = 1.267$$

$$g_2^{pn}(Q^2) = \frac{g_2(0)}{\left(1+\frac{Q^2}{M_2^2}\right)^2}, \qquad M_2 = 1.026 \text{ GeV}.$$

$g_3(Q^2)$ **is determined in terms of** $g_1(Q^2)$ **assuming PCAC along with the GT relation.**

$$g_3^{np}(Q^2) = \frac{2M^2 g_A(Q^2)}{m_\pi^2 + Q^2}$$

$$g_3^{NY}(Q^2) = \frac{(M+M_Y)^2 g_1^{NY}(Q^2)}{2(m_K^2 + Q^2)}$$

Differential and total scattering cross sections

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 a^2}{8\pi M^2 E^2} \mathcal{L}_{\mu\nu} \mathcal{J}^{\mu\nu}$$

- $a = \sin\theta_C$ **for** $\Delta S = 1$ **processes,**
- $a = \cos\theta_C$ **for** $\Delta S = 0$ **processes.**

$$\mathcal{L}_{\mu\nu} = b \mathrm{Tr}\left[\gamma_\mu (1 \pm \gamma_5) \Lambda(k') \gamma_\nu (1 \pm \gamma_5) \Lambda(k)\right]$$

- $b = 1$ **for** $\nu/\bar{\nu}$ **induced processes,** $b = \frac{1}{2}$ **for** e^- **induced processes.**

$$\mathcal{J}_{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left[\Lambda(p') \Gamma_\mu \Lambda(p) \tilde{\Gamma}_\nu\right]$$

$$\Gamma_\mu = V_\mu - A_\mu, \qquad \tilde{\Gamma}_\nu = \gamma^0 \gamma_\nu^\dagger \gamma^0, \qquad \Lambda(P) = \not{P} + M_P$$

Helicity projection operator

- **projects spin states of the particle**
- **must commute with $\not{p}$**

Helicity projection operator

- projects spin states of the particle
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★ a four-vector for spin polarization ξ^μ is required

ξ^μ satisfies the following relations:

- $\boldsymbol{\xi}^2 = -1$
- $\boldsymbol{\xi} \cdot p = 0$

$$\Lambda(\boldsymbol{\xi}) = \frac{1}{2}(1 - \gamma_5 \not{\xi})(\not{p} + M_p)$$

In the covariant density matrix formalism

- **the spin density matrix**

$$\rho_f = \mathcal{L}^{\alpha\beta}\Lambda(p')J_\alpha\Lambda(p)\tilde{J}_\beta\Lambda(p')$$

- **the polarization vector ξ_τ; average of the spin density matrix**

$$\xi^\tau = \frac{\mathrm{Tr}[\gamma^\tau\gamma_5\ \rho_f]}{\mathrm{Tr}[\rho_f]}$$

$$\xi^\tau = \left(g^{\tau\sigma} - \frac{p'^\tau p'^\sigma}{m_Y^2}\right)\frac{\mathcal{L}^{\alpha\beta}\mathrm{Tr}[\gamma_\sigma\gamma_5\Lambda(p')J_\alpha\Lambda(p)\tilde{J}_\beta]}{\mathcal{L}^{\alpha\beta}\mathrm{Tr}[\Lambda(p')J_\alpha\Lambda(p)\tilde{J}_\beta]}$$

Case I: When T-invariance is assumed

The vector part of the hyperon polarization ξ^τ is given by

$$\vec{\xi} = A(Q^2)\ \vec{k} + B(Q^2)\ \vec{p}'$$

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$$\vec{\xi} = \xi_P \hat{e}_P + \xi_L \hat{e}_L$$

the independent unit vectors are taken along:

- longitudinal direction, $\vec{e}_L = \frac{\vec{p}'}{|\vec{p}'|}$
- transverse direction, $\vec{e}_T = \frac{\vec{p}' \times \vec{k}}{|\vec{p}' \times \vec{k}|}$
- perpendicular direction, $\vec{e}_P = \vec{e}_L \times \vec{e}_T$

$$\xi_L = \vec{\xi} \cdot \vec{e}_L; \qquad \xi_P = \vec{\xi} \cdot \vec{e}_P; \qquad \xi_T = \vec{\xi} \cdot \vec{e}_T$$

Polarization of the final hyperon

Longitudinal component: $P_L(Q^2) = \frac{M_Y}{E_Y}\vec{\xi}\cdot\vec{e}_L$

$$P_L(Q^2) = \frac{M_Y}{E_Y}\frac{A(Q^2)\vec{k}.\vec{p}' + B(Q^2)|\vec{p}'|^2}{|\vec{p}'|}$$

Perpendicular component: $P_P(Q^2) = \vec{\xi}\cdot\vec{e}_P$

$$P_P(Q^2) = \frac{A(Q^2)[(\vec{k}.\vec{p}')^2 - |\vec{k}|^2|\vec{p}'|^2]}{|\vec{p}'|\ |\vec{p}'\times\vec{k}|}$$

Case II: When T-violation is assumed

The vector part of the hyperon polarization ξ^τ is given by

$$\vec{\xi} = A(Q^2)\ \vec{k} + B(Q^2)\ \vec{p}' + C(Q^2)\ M_p(\vec{k} \times \vec{p}')$$

Case II: When T-violation is assumed

The vector part of the hyperon polarization ξ^τ is given by

$$\vec{\xi} = A(Q^2)\ \vec{k} + B(Q^2)\ \vec{p}\,' + C(Q^2)\ M_p(\vec{k} \times \vec{p}\,')$$

$$\vec{\xi} = \xi_P \hat{e}_P + \xi_L \hat{e}_L + \xi_T \hat{e}_T$$

Transverse component: $P_T(Q^2) = \vec{\xi} \cdot \vec{e}_T$

$$P_T(Q^2) = \frac{C(Q^2) M_P[(\vec{k}.\vec{p}\,')^2 - |\vec{k}|^2 |\vec{p}\,'|^2]}{|\vec{p}\,' \times \vec{k}|}$$

Expression of $A(Q^2)$, $B(Q^2)$ and $C(Q^2)$

$$
\begin{aligned}
A(Q^2) &= f_1^2 A_1(Q^2) + f_2^2 A_2(Q^2) + g_1^2 A_3(Q^2) + g_2^2 A_4(Q^2) + f_1 f_2 A_5(Q^2) \\
&+ f_1 g_1 A_6(Q^2) + f_2 g_1 A_7(Q^2)
\end{aligned}
$$

$$
\begin{aligned}
B(Q^2) &= f_1^2 B_1(Q^2) + f_2^2 B_2(Q^2) + g_1^2 B_3(Q^2) + g_2^2 B_4(Q^2) + f_1 f_2 B_5(Q^2) \\
&+ f_1 g_1 B_6(Q^2) + f_2 g_1 B_7(Q^2)
\end{aligned}
$$

$$C(Q^2) = Im(g_1 g_2 C_1(Q^2) + f_1 g_2 C_2(Q^2) + f_2 g_2 C_3(Q^2))$$

Expression of $A(Q^2)$, $B(Q^2)$ and $C(Q^2)$

$$
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&+ f_1 g_1 A_6(Q^2) + f_2 g_1 A_7(Q^2)
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$$

$$
\begin{aligned}
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&+ f_1 g_1 B_6(Q^2) + f_2 g_1 B_7(Q^2)
\end{aligned}
$$

$$C(Q^2) = Im(g_1 g_2 C_1(Q^2) + f_1 g_2 C_2(Q^2) + f_2 g_2 C_3(Q^2))$$

If T-invariance is assumed

$$C(Q^2) = 0, \quad P_T(Q^2) = 0$$