Angular distributions in Monte Carlo event generation of weak single-pion production

Kajetan Niewczas

The physics problem

$$\nu + N \to l^- + N' + \pi$$

- **Single-pion production** (SPP) is an essential dynamics for accelerator-based experiments
- There many measurements sensitive to **pion angular distributions** ($\cos\theta_\pi$)

$$\nu + N \to l^- + (\Delta \to N' + \pi)$$

- **NuWro** models the **Δ-resonance** excitation
  $\to$ it decays according to the ANL/BNL angular fits

$$\frac{d^2\sigma_\Delta}{dQ^2 dW} \to \frac{d^4\sigma_\pi}{dQ^2 dW} \times \frac{df_\Delta(Q^2)}{d\Omega_\pi^*}$$

- The nonresonant background is extrapolated from the DIS formalism into the lower regions of W, Q^2

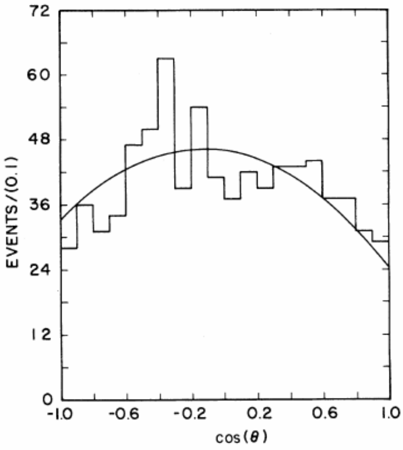


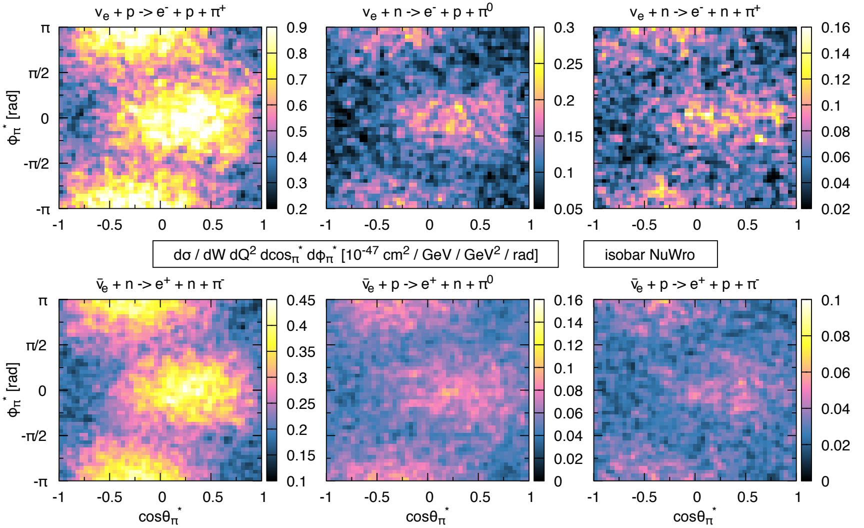
FIG. 15. Distribution of events in the pion polar angle $\cos\theta$ for the final state $\mu^- p \pi^+$, with $M(p\pi^+) < 1.4$ GeV. The curve is the area-normalized prediction of the Adler model.

Radecky et al. [ANL Collaboration], Phys.Rev. D 25 (1982) 1161

Pion angular distributions

- **Default NuWro**
- Free nucleon
- Fixed kinematics:

$$E = 1\ \text{GeV}$$
$$Q^2 = 0.1\ \text{GeV}^2$$
$$W = 1230\ \text{MeV}$$



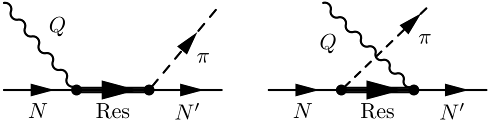
(total number of 10^7 **events** over the **whole phase space**)

Ghent low energy model of SPP

- The model of Ref. [**R. González-Jiménez et al., Phys.Rev. D 95 (2017) 113007**]
- The **low-energy** part based on the **Valencia model**

Resonances

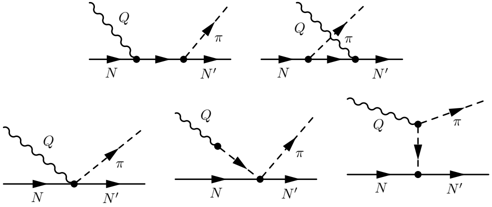
$P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$



based on [PRD 76 033005, PRD 87 113009, PRD 93 014016]

+

ChPT background



- **Bottleneck** for the implementation is the code **execution time**
- Adding a **nuclear model** will further increase the complexity of the implementation

Implementation

- Working in the **Adler frame**, generating an event requires the value of

$$\frac{d^4\sigma}{dQ^2 dW d\Omega_\pi^*} = \frac{\mathcal{F}^2}{(2\pi)^4}\frac{k_\pi^*}{k_l^2}\left[A + B\cos(\phi_\pi^*) + C\cos(2\phi_\pi^*) + D\sin(\phi_\pi^*) + E\sin(2\phi_\pi^*)\right]$$

$\rightarrow$ that is **time consuming** and the MC sampling has an **efficiency** of $10 - 15\,\%$

- Sampling Q^2, W from **precomputed arrays** allows to build the **muon kinematics**
- Then, $\cos\theta_\pi^*$ is given by the A function that is mostly **parabolic** (fit using 3-7 points)
- Finally, for other variables fixed, ϕ_π^* is given by an **analytical expression**

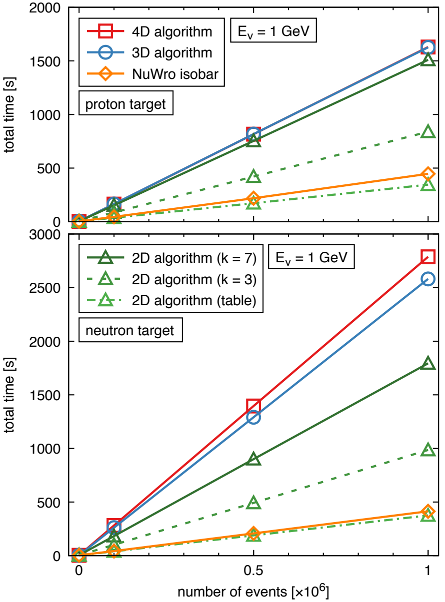
$$\frac{d^2\sigma}{dQ^2 dW} \xrightarrow[\text{numerical}]{\text{fix } Q^2,\, W} \frac{d^3\sigma}{dQ^2 dW d\cos\theta_\pi^*} \xrightarrow[\text{numerical / analytical}]{\text{fix } \cos\theta_\pi^*} \frac{d^4\sigma}{dQ^2 dW d\Omega_\pi^*} \xrightarrow[\text{analytical}]{\text{fix } \phi_\pi^*} \text{event...}$$

Performance

We propose:

- **4D algorithm**: sampling $(Q^2, W, \cos\theta^*_\pi, \phi^*_\pi)$ together
  (1 cross section calculation per accepted event)
- **3D algorithm**: sampling $(Q^2, W, \cos\theta^*_\pi)$ together
  $+\ \phi^*_\pi$ analytical
  (2 cross section calculation per accepted event)
- **2D algorithm**: sampling (Q^2, W) from tables
  $+ \cos\theta^*_\pi$ from k points or from tables
  $+\ \phi^*_\pi$ analytical
  ($k+1$ cross section calculation per accepted event)

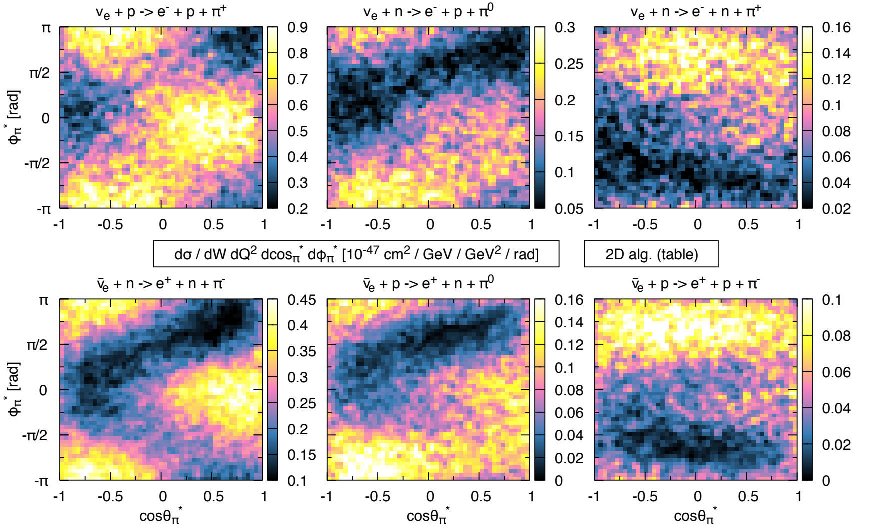
→ $\nu - n$ scattering requires one more code evaluation because it has two channels $(p+\pi^0, n+\pi^+)$



Pion angular distributions

- **Ghent LEM**
- Free nucleon
- Fixed kinematics:

$$E = 1\ \text{GeV}$$
$$Q^2 = 0.1\ \text{GeV}^2$$
$$W = 1230\ \text{MeV}$$



(total number of 10^7 **events** over the **whole phase space**)

Summary

- We have implemented the **Ghent low energy model** into **NuWro**
  - → so far, only off the nucleon
- We have investigated various **methods of optimization** in SPP
  - → different trade-offs between efficiency, precision and reliance on precomputed assets
- Our framework is based on kinematics, and therefore, **model-independent**
- The work is exhaustively presented in Ref. [**Phys.Rev. D 103 (2021) 053003**]

Angular distributions in Monte Carlo event generation of weak single-pion production

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Backup slides

Performance

$$S_N = N \cdot \tau \cdot (1+\alpha) + (\frac{N}{\epsilon} - N) \cdot \tau = N \cdot \tau \cdot (\frac{1}{\epsilon} + \alpha)$$

N - accepted events τ - trial event cost [arb. unit] ϵ - efficiency α - additional cost per accepted event [arb. unit]

model	$\sigma[\mathrm{cm}^2]$	$s_{1M}[\mathrm{cm}^2]$	τ	ϵ	α	S_{1M}
4D alg.	5.1724e-39	7.8e-42	8.01e-07	0.12	-	6.9
3D alg.	5.1661e-39	7.7e-42	8.02e-07	0.13	1.0	6.9
2D alg. $(k=7)$	5.1586e-39	7.5e-42	4.04e-08	0.16	143.9	6.1
2D alg. $(k=3)$	5.1623e-39	7.5e-42	4.04e-08	0.16	72.0	3.2
2D alg. (table)	5.1613e-39	7.5e-42	4.03e-08	0.16	18.6	1.0

(a) $E = 1.0$ GeV neutrinos off proton target.

model	$\sigma[\mathrm{cm}^2]$	$s_{1M}[\mathrm{cm}^2]$	τ	ϵ	α	S_{1M}
4D alg.	2.5105e-39	2.7e-42	1.83e-06	0.15	-	12.1
3D alg.	2.5095e-39	2.7e-42	1.83e-06	0.18	0.5	11.2
2D alg. $(k=7)$	2.5126e-39	2.6e-42	4.11e-08	0.21	169.4	7.2
2D alg. $(k=3)$	2.5124e-39	2.6e-42	4.10e-08	0.21	85.1	3.7
2D alg. (table)	2.5116e-39	2.6e-42	4.08e-08	0.21	22.0	1.1

(b) $E = 1.0$ GeV neutrinos off neutron target.

model	$\sigma[\mathrm{cm}^2]$	$s_{1M}[\mathrm{cm}^2]$	τ	ϵ	α	S_{1M}
4D alg.	6.8637e-39	11.2e-42	8.04e-07	0.08	-	9.9
3D alg.	6.8634e-39	10.8e-42	8.01e-07	0.10	1.0	8.8
2D alg. $(k=7)$	6.8327e-39	10.5e-42	3.98e-08	0.12	149.1	6.3
2D alg. $(k=3)$	6.8510e-39	10.5e-42	4.08e-08	0.12	72.6	3.3
2D alg. (table)	6.8450e-39	10.5e-42	4.04e-08	0.12	19.0	1.1

(c) $E = 2.5$ GeV neutrinos off proton target.

model	$\sigma[\mathrm{cm}^2]$	$s_{1M}[\mathrm{cm}^2]$	τ	ϵ	α	S_{1M}
4D alg.	4.5860e-39	4.7e-42	1.84e-06	0.14	-	13.5
3D alg.	4.5851e-39	4.4e-42	1.83e-06	0.18	0.5	11.4
2D alg. $(k=7)$	4.5762e-39	4.2e-42	4.19e-08	0.20	169.6	7.3
2D alg. $(k=3)$	4.5805e-39	4.2e-42	4.13e-08	0.20	86.0	3.8
2D alg. (table)	4.5809e-39	4.2e-42	4.12e-08	0.20	22.3	1.1

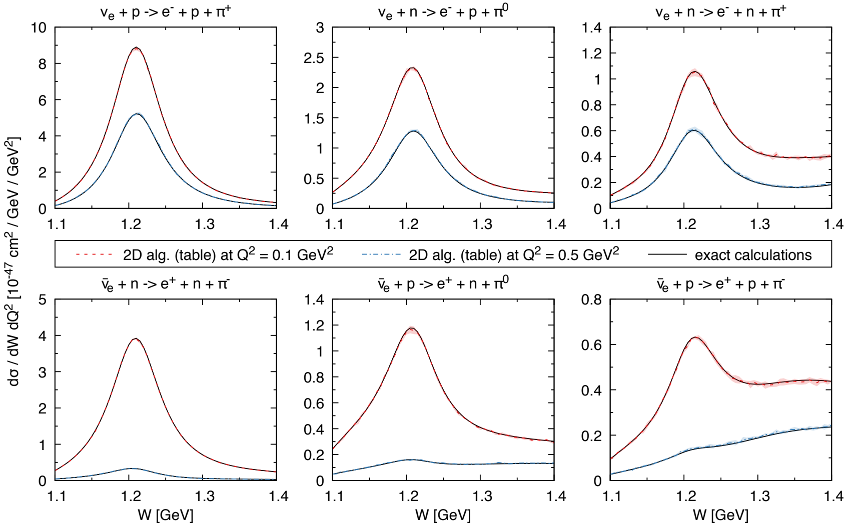
(d) $E = 2.5$ GeV neutrinos off neutron target.

(the values of τ are normalized to obtain $S_{1M} = 1.0$ for the "2D alg. (table)" model)

Pion angular distributions

- **Ghent LEM**
- Free nucleon
- Fixed kinematics:

$$E = 1\ \text{GeV}$$
$$Q^2 = 0.1\ \text{GeV}^2 \vee 0.5\ \text{GeV}^2$$

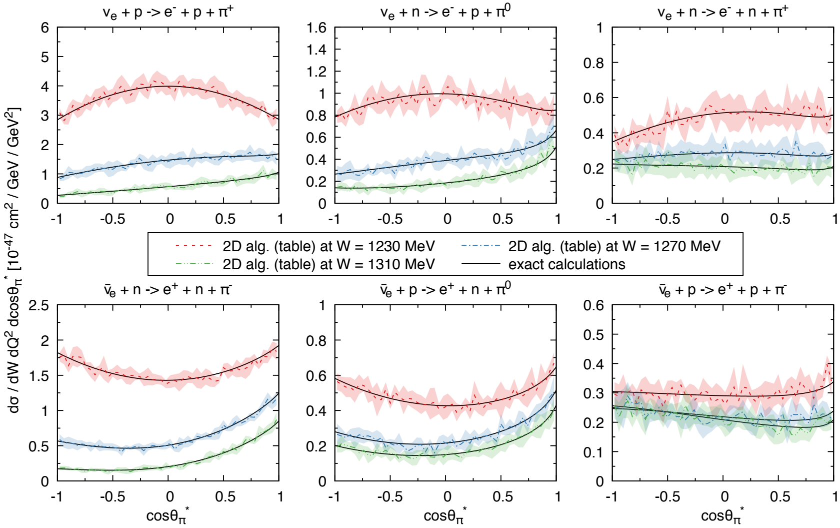


(total number of 10^7 **events** over the **whole phase space**)

Pion angular distributions

- **Ghent LEM**
- Free nucleon
- Fixed kinematics:

$$E = 1\ \text{GeV}$$
$$Q^2 = 0.1\ \text{GeV}^2$$
$$W = 1230\ \text{MeV} \vee 1270\ \text{MeV} \vee 1310\ \text{MeV}$$

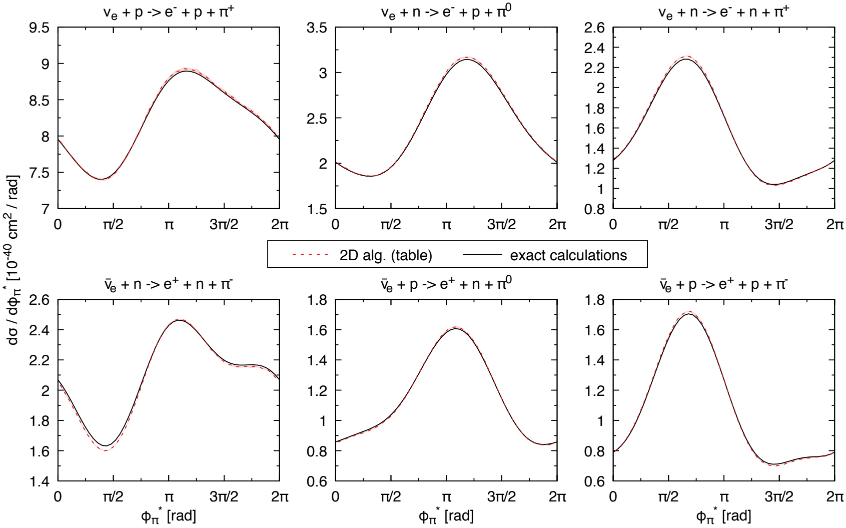


(total number of 10^7 **events** over the **whole phase space**)

Pion angular distributions

- **Ghent LEM**
- Free nucleon
- Fixed kinematics:

  $E = 1$ GeV

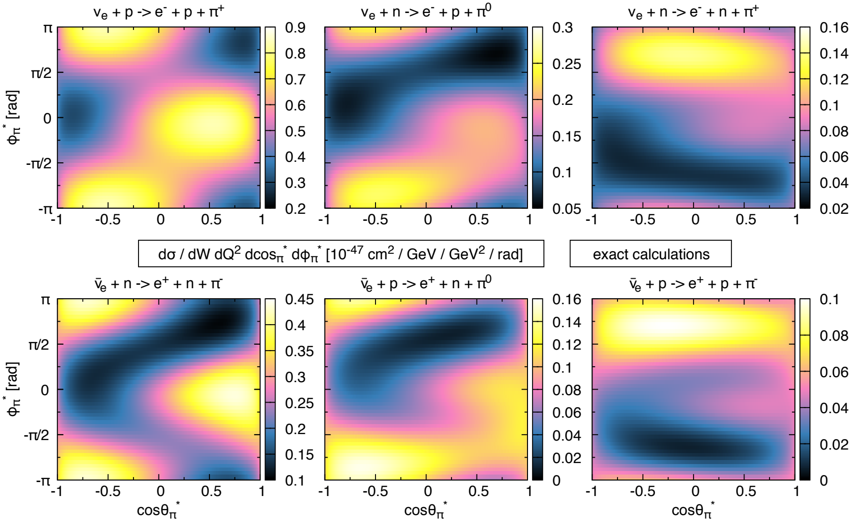


(total number of 10^7 **events** over the **whole phase space**)

Pion angular distributions

- **Ghent LEM**
- Free nucleon
- Fixed kinematics:

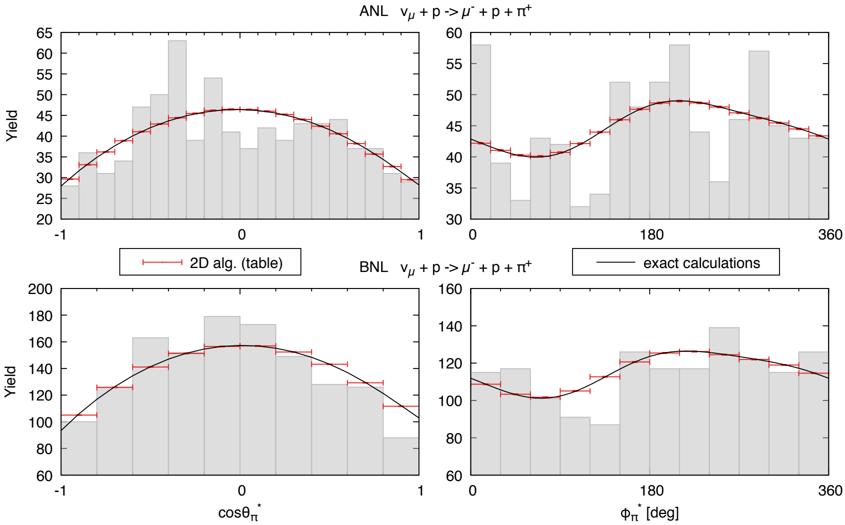
$$E = 1\ \mathrm{GeV}$$
$$Q^2 = 0.1\ \mathrm{GeV}^2$$
$$W = 1230\ \mathrm{MeV}$$



(numerical results on a **dense grid** with **selected kinematics**)

Pion angular distributions

- **Ghent LEM**
- Free proton
- ANL / BNL data



(total number of 10^7 **events** over the **whole phase space**)