Extracting the Nucleon Axial Form Factor from Lattice QCD using \( \chi \)PT

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**Nucleon Axial Form Factor,** \[ G_A(q^2) \]

- fundamental property of the nucleon
- Information on the spins distribution
- \[ A^i_\mu(x) = \bar{q}(x) \gamma_\mu \gamma_5 \frac{i}{2} q(x) \]
- \[ \langle N(p')|A^i_\mu|N(p)\rangle = \bar{u} \left\{ \gamma_\mu G_A(q^2) + \frac{q_\mu}{2m_N} G_P(q^2) \right\} \gamma_5 \frac{i}{2} u(p) \]

\[ G_A(q^2) = g_A \left[ 1 + \frac{1}{6} \langle r_A^2 \rangle q^2 + \mathcal{O}(q^4) \right] \]

- \( g_A \) and \( G_A \) dependence in \( q^2 \) are necessary in \( \nu \) oscillations experiments.

- \( G_A(q^2) \) enters \( \sigma_{\nu N} \). Input for MC and theoretical models.
Measurements

- \( G_A(q^2) = g_A \left[ 1 + \frac{1}{6} \langle r_A^2 \rangle q^2 + \mathcal{O}(q^4) \right] \)

- \( \beta \) decay: \( g_A = 1.2723(23) \)
  
  (C. Patrignani et al. (PDG), Chin. Phys. C 40 (2016))

- \( G_A(q^2) \) more challenging
  
  - Neutrino-induced charged-current QE scattering on deuteron:
    - Low statistics, unknown \( E_\nu \)
  
  - Old experiments: low accuracy
  
  - New experiments in nuclei \( \rightarrow \) problem: nuclear physics
  
  - Also experiments in: \( \pi \) electro-production and weak capture in muonic hydrogen

Figure: Fermilab
Lattice

- several studies on $G_A(q^2)$
  - $\rightarrow$ some technical difficulties
- Experimental and lattice $q^2$ parametrisation:
  - dipole ansatz
    $\implies$ different results
  - $z$-expansion

**Chiral Perturbation Theory (χPT)**
- QCD based parametrisation of $q^2$ and $M_\pi$ dependencies
- χPT is useful to extrapolate Lattice to the physical limit
- Fits to lattice can determine the low energy constants (LECs) values:
  - Lattice-χPT synergy: $M_\pi$ is "variable" in both theories
Chiral Perturbation Theory (B$_\chi$PT)

- $\chi$PT is the effective field theory for QCD at low energy
  - Invariance under chiral symmetry: $G \equiv SU(n_f)_L \otimes SU(n_f)_R$
  - quarks, gluons $\rightarrow$ hadronic degrees of freedom
  - Successful description of meson properties

Baryon $\chi$PT

- Problem: $m_B \not\rightarrow 0 \Rightarrow$ Power counting breaking (PCB)
  - $\chi$ limit
  - $\Rightarrow$ additional finite renormalisation: extended on mass-shell (EOMS) renormalisation
    - PCB terms absorbed by LECs
    - Covariance and analytic properties of loops preserved
$G_A$ calculation in (BχPT)

- Yao, Alvarez-Ruso, Vicente Vacas, PRD 96 (2017)
- Up to leading one-loop $\mathcal{O}(p^3)$
- Explicit $\Delta(1232)$
  - $\delta = m_\Delta - m_N \sim \mathcal{O}(p)$
- EOMS renormalisation
- Correct power counting and analytical properties
  $\Rightarrow$ appropriate for chiral extrapolations
- $G_A$ fit to combined lattice data
Combined fit

- **\( G_A \) fit to combined lattice data**
  - 2017 fit data = PNDME\(^1\) + ”Cyprus”\(^2\) + ”Mainz”\(^3\)
  - 2021 fit data = 2017 (updating ”Cyprus”\(^4\)) + RQCD\(^5\) + PACS\(^6\)
  - lattice spacing dependence corrected
  - \( \chi^2 \)PT range:
    - \( Q^2 < 0.36 \text{ GeV}^2, M_\pi < 400 \text{ MeV} \) (\( M_\pi L \geq 3.5 \))
  - LECs \( g, d_{16} \) and \( d_{22} \) are determined by the fit
  - \( \Delta \) LECs (\( h_A, g_1 \)) fixed to large-\( N_c \)
  - Good description: improved by \( \Delta \)

5. G. S. Bali et al., JHEP 126 (2020)
Axial charge, $g_A$

- $G_A(q^2) = g_A \left[ 1 + \frac{1}{6} \langle r_A^2 \rangle q^2 + \mathcal{O}(q^4) \right]$

<table>
<thead>
<tr>
<th></th>
<th>Fit-2017</th>
<th>Fit-2021</th>
<th>Experimental*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A$</td>
<td>1.2444(48)</td>
<td>1.2295(37)</td>
<td>1.2723(23)</td>
</tr>
</tbody>
</table>

Our fits: statistical errors alone
*(C. Patrignani et al. (PDG), Chin. Phys. C 40 (2016))
Axial radius, $\langle r_A^2 \rangle$

$$G_A(q^2) = g_A \left[ 1 + \frac{1}{6} \langle r_A^2 \rangle q^2 + \mathcal{O}(q^4) \right]$$

<table>
<thead>
<tr>
<th>Source</th>
<th>$\langle r_A^2 \rangle$</th>
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<tbody>
<tr>
<td>Fit-2017</td>
<td>0.2732(32)</td>
</tr>
<tr>
<td>Fit-2021</td>
<td>0.2937(22)</td>
</tr>
<tr>
<td>$\nu D$ and electroprod. dipole (Bodek\textsuperscript{7})</td>
<td>0.454(13)</td>
</tr>
<tr>
<td>$\nu D$ z-exp. (Meyer\textsuperscript{8})</td>
<td>0.46(22)</td>
</tr>
</tbody>
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\textsuperscript{8} A. S. Meyer et al., Phys. Rev. D 93, 113015 (2016)
\textsuperscript{9} R. J. Hill et al. Rept. Prog. Phys. 81 (2018)

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\textsuperscript{9} R. J. Hill et al. Rept. Prog. Phys. 81 (2018)
A more precise description of $G_A(q^2)$ and a determination of the chiral series truncation error $\Rightarrow \mathcal{O}(p^4)$ calculation

We have calculated the new diagrams $\longrightarrow$ new $\mathcal{O}(p^2)$ LECs appear

- $L_{\pi N}^{(2)} = \sum_i c_i \bar{\Psi} O_i \Psi \Rightarrow c_1, c_2, c_3, c_4$
- $L_{\pi \Delta}^{(2)} = a_1 \bar{\Psi}_\mu \langle \chi_+ \rangle \Psi^{\mu} + \text{h. c.}$
- $L_{\pi N \Delta}^{(2)} = \sum_i \bar{\Psi}_\alpha b_i O^{\alpha}_i \Psi_N + \text{h. c.} \Rightarrow b_1, b_2, b_4, b_5$

+...

+ w. f. renormalisation

+ mass insertions
$g_A \mathcal{O}(4) \Delta$-less

- $g_A \mathcal{O}(4) \Delta$-less
  - $g_A$ lattice dataset*
  - The $\mathcal{O}(3)$ and $\mathcal{O}(4) \Delta$-less LECs ($d_{16}$ and the $c_i$) extracted from $\pi N \to \pi N$ and $\pi N \to \pi \pi N$ (Siemens et al. PRC 96 (2017)) do not describe the $g_A$ in the lattice.

* $g_A$ dataset: "Mainz" (PRD 100 (2019)), RQCD$^5$, PNDME (PRD 98 (2018)), CallLat (Nature 558 (2018))

**Figure:** Gray line: our fit of the $g_A \mathcal{O}(3)$ to the lattice data (correcting $a$). Red line: $g_A \mathcal{O}(3)$ prediction of PRC 96 (2017) (their $d_{16}$, but our $g$). Blue line: $g_A \mathcal{O}(4)$ prediction of PRC 96 (2017) (their $d_{16}$ and $c_i$, but our $g$). The bands show the errors (only statistical errors are considered for our fit). The points represent the lattice data.
$g_A \mathcal{O}(4) \Delta$-full

- $g_A \mathcal{O}(4) \Delta$-full fit:
  - $g_A = 1.199 \pm 0.012$
  - $d_{16}$
    - key source of uncertainty in $\chi$PT extrap. of nuclear observables
    - $\pi N \rightarrow \pi \pi N \Rightarrow$
      - $d_{16} = 2.98 \pm 1.00 \text{ GeV}^{-2}$
    - $g_A$ is more sensitive to $d_{16}$ than $\pi N \rightarrow \pi \pi N$
    - our fit $\Rightarrow d_{16} = 0.24 \pm 0.18 \text{ GeV}^{-2}$ (stat. error only)

† Siemens et al. PRC 96 (2017) (value converted to standard EOMS)

Figure: Red curve: $g_A \mathcal{O}(3) \Delta$-full fit; blue curve: $g_A \mathcal{O}(4) \Delta$-full fit. The points are the lattice data, except for the filled black square, which is the experimental determination.
Conclusions and Outlook

- $G_A(q^2)$ essential in $\nu$ oscillations.

- $\chi$PT and Lattice complement one another

- The extraction of $\langle r_A^2 \rangle$ is challenging both from experiment and lattice

- Our combined fit $\mathcal{O}(p^3)$ $\Delta$-full successfully describes the lattice data
  
  - $g_A = 1.2295(37)$, $\langle r_A^2 \rangle = 0.2937(22)$ (fm$^2$)
  
  - No ad hoc parametrisation

- $\mathcal{O}(p^4)$ $\Delta$-full fit $\Rightarrow d_{16} = 0.24 \pm 0.18$ GeV$^{-2}$

Thanks for watching!

Any questions?
Nucleon Axial Form Factor: Extra

- Dipole ansatz: $G_A(q^2) = g_A \left( 1 - \frac{q^2}{M_A^2} \right)^{-2}$
- $z$-exp.: $G_A(q^2) = \sum_k a_k z^k(q^2)$, with $z(q^2, t_{\text{cut}}, t_0)$