Extracting the Nucleon Axial Form Factor from Lattice QCD using χPT

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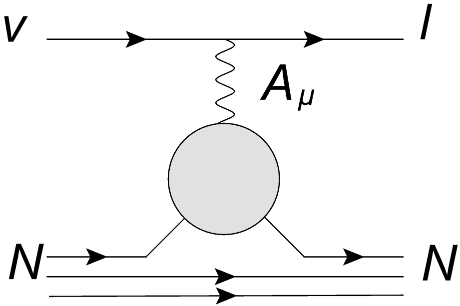
Nucleon Axial Form Factor

- **Nucleon Axial Form Factor**, $\boxed{G_A(q^2)}$
  - fundamental property of the nucleon
  - Information on the spins distribution
  - $A^i_\mu(x) = \bar{q}(x)\gamma_\mu\gamma_5\frac{\tau^i}{2}q(x)$
  - $\langle N(p')|A^i_\mu|N(p)\rangle =$
    $\bar{u}\left\{\gamma_\mu G_A(q^2) + \frac{q_\mu}{2m_N}G_P(q^2)\right\}\gamma_5\frac{\tau^i}{2}u(p)$
- $\boxed{G_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]}$
- g_A and G_A dependence in q^2 are necessary in **ν oscillations experiments**.
  - $G_A(q^2)$ enters $\sigma_{\nu N}$. Input for MC and theoretical models.



Measurements

- $G_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$
- β decay: $g_A = 1.2723(23)$

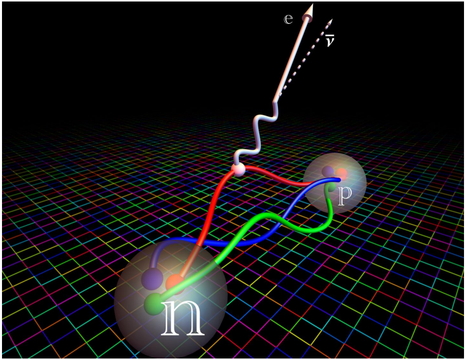
  (C. Patrignani et al. (PDG), Chin. Phys. C 40 (2016))
- $G_A(q^2)$ more challenging
  - Neutrino-induced charged-current QE scattering on deuteron:
    - Low statistics, unknown E_ν
  - Old experiments: low accuracy
  - New experiments in nuclei $\rightarrow$ problem: nuclear physics
  - Also experiments in: π electro-production and weak capture in muonic hydrogen

Figure: Fermilab

Lattice

- **Lattice**
  - several studies on $G_A(q^2)$
    - $\rightarrow$ some technical difficulties
  - Experimental and lattice q^2 parametrisation:
    - dipole ansatz

      $\Longrightarrow$ different results
    - z-expansion
- **Chiral Perturbation Theory** (χPT)
  - QCD based parametrisation of q^2 and M_π dependencies
  - χPT is useful to extrapolate Lattice to the physical limit
  - Fits to lattice can determine the low energy constants (LECs) values:
    - Lattice-χPT synergy: M_π is "variable" in both theories

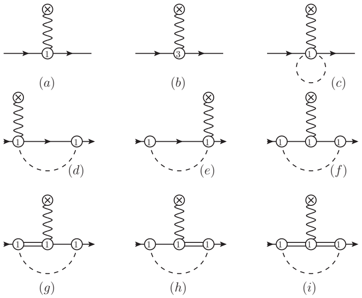


Chiral Perturbation Theory (BχPT)

- χPT is the effective field theory for QCD at low energy
  - Invariance under chiral symmetry: $G \equiv SU(n_f)_L \otimes SU(n_f)_R$
  - quarks, gluons $\longrightarrow$ hadronic degrees of freedom
  - Successful description of meson properties
- Baryon χPT
  - Problem: $\underbrace{m_B \nrightarrow 0}_{\chi\text{limit}} \Rightarrow$ Power counting breaking (PCB)
  - $\Rightarrow$ additional finite renormalisation: extended on mass-shell (EOMS) renormalisation
    - PCB terms absorbed by LECs
    - Covariance and analytic properties of loops preserved

G_A calculation in (BχPT)

- Yao, Alvarez-Ruso, Vicente Vacas, PRD 96 (2017)
- Up to leading one-loop $\mathcal{O}(p^3)$
- Explicit $\Delta(1232)$
  - $\delta = m_\Delta - m_N \sim \mathcal{O}(p)$
- EOMS renormalisation
- Correct power counting and analytical properties
  $\Rightarrow$ appropriate for chiral extrapolations
- G_A fit to combined lattice data

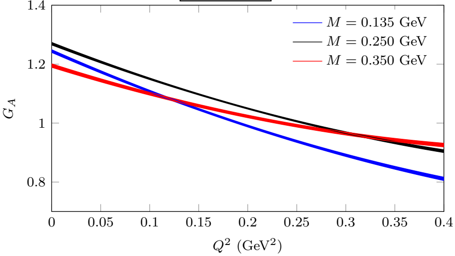


Combined fit

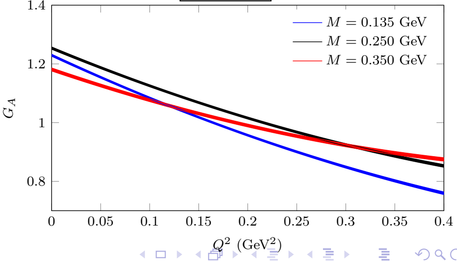
- **G_A fit to combined lattice data**
  - 2017 fit data = PNDME[1] +"Cyprus"[2] + "Mainz"[3]
  - 2021 fit data = 2017 (updating "Cyprus"[4]) + RQCD[5] + PACS[6]
    - lattice spacing dependence corrected
  - χPT range:
    - $Q^2 < 0.36$ GeV^2, $M_\pi < 400$ MeV ($M_\pi L \geq 3.5$)
  - LECs g, d_{16} and d_{22} are determined by the fit
  - Δ LECs (h_A, g_1) fixed to large-N_c
  - **Good description: improved by Δ**

1. R. Gupta et al., Phys. Rev. D 96 (2017)
2. C. Alexandrou et al., Phys. Rev. D 96 (2017)
3. S. Capitani et al., Int.J.Mod.Phys.A 34 (2019)
4. Phys. Rev. D 103 (2021)
5. G. S. Bali et al., JHEP 126 (2020)
6. Phys. Rev. D 102 (2020)

Fit 2017



Fit 2021



Axial charge, g_A

- $G_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$
-

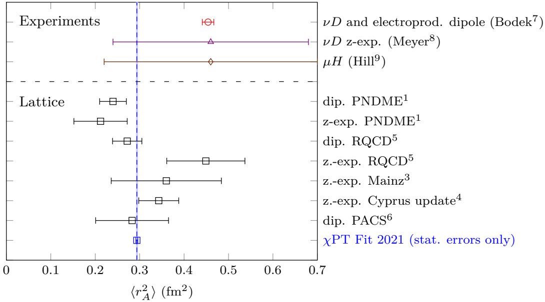
	Fit-2017	Fit-2021	Experimental*
g_A	1.2444(48)	1.2295(37)	1.2723(23)

- Our fits: statistical errors alone

*(C. Patrignani et al. (PDG), Chin. Phys. C 40 (2016))

$$G_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$$

Source	$\langle r_A^2 \rangle$
Fit-2017	0.2732(32)
Fit-2021	0.2937(22)
νD and electroprod. dipole (Bodek[7])	0.454(13)
νD z-exp. (Meyer[8])	0.46(22)



7 A. Bodek et al., Eur. Phys. J. C 53, 349 (2008)

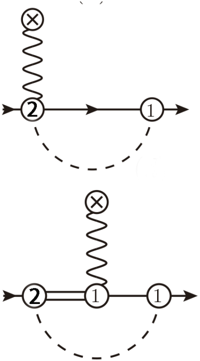
8 A. S. Meyer et al., Phys. Rev. D 93, 113015 (2016)

9 R. J. Hill et al. Rept. Prog. Phys. 81 (2018)

G_A $\mathcal{O}(4)$

G_A $\mathcal{O}(p^4)$

- A more precise description of $G_A(q^2)$ and a determination of the chiral series truncation error $\Longrightarrow$ $\boxed{\mathcal{O}(p^4)\text{ calculation}}$
- We have calculated the new diagrams
  $\longrightarrow$ new $\mathcal{O}(p^2)$ LECs appear
  - $\mathcal{L}^{(2)}_{\pi N} = \sum_i c_i \bar{\Psi} O_i \Psi \Longrightarrow c_1, c_2, c_3, c_4$
    $\mathcal{L}^{(2)}_{\pi \Delta} = a_1 \bar{\Psi}^i_\mu \langle \chi_+ \rangle \Psi^{j\mu} + \text{h. c.}$
    $\mathcal{L}^{(2)}_{\pi N\Delta} = \sum_i \bar{\Psi}^j_\alpha b_i O^{\alpha j} \Psi_N + \text{h. c.} \Longrightarrow b_1, b_2, b_4, b_5$



$+ \dots$

$+$ w. f. renormalisation

$+$mass insertions

- g_A $\mathcal{O}(4)$ Δ-less
  - g_A lattice dataset*
  - The $\mathcal{O}(3)$ and $\mathcal{O}(4)$ Δ-less LECs (d_{16} and the c_i) extracted from $\pi N \to \pi N$ and $\pi N \to \pi\pi N$ (Siemens et al. PRC 96 (2017)) do not describe the g_A in the lattice.

*g_A dataset: "Mainz" (PRD 100 (2019)), RQCD[5], PNDME (PRD 98 (2018)), CallLat (Nature 558 (2018))

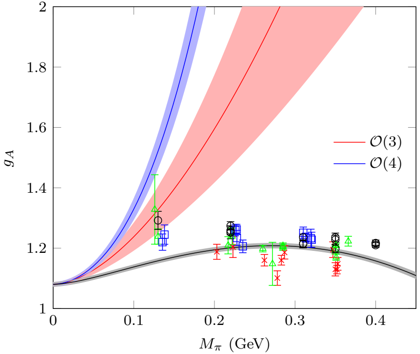


Figure: Gray line: our fit of the g_A $\mathcal{O}(3)$ to the lattice data (correcting a). Red line: g_A $\mathcal{O}(3)$ prediction of PRC 96 (2017) (their d_{16}, but our g). Blue line: g_A $\mathcal{O}(4)$ prediction of PRC 96 (2017) (their d_{16} and c_i, but our g). The bands show the errors (only statistical errors are considered for our fit). The points represent the lattice data.

g_A $\mathscr{O}(4)$ Δ-full

- g_A $\mathscr{O}(4)$ Δ-full fit:
  - $g_A = 1.199 \pm 0.012$
  - d_{16}
    - key source of uncertainty in χPT extrap. of nuclear observables
    - $\pi N \to \pi\pi N \Rightarrow d_{16} = 2.98 \pm 1.00$ GeV$^{-2\dagger}$
    - g_A is more sensitive to d_{16} than $\pi N \to \pi\pi N$
    - our fit $\Rightarrow$ $d_{16} = 0.24 \pm 0.18$ GeV^{-2} (stat. error only)

† Siemens et al. PRC 96 (2017) (value converted to standard EOMS)

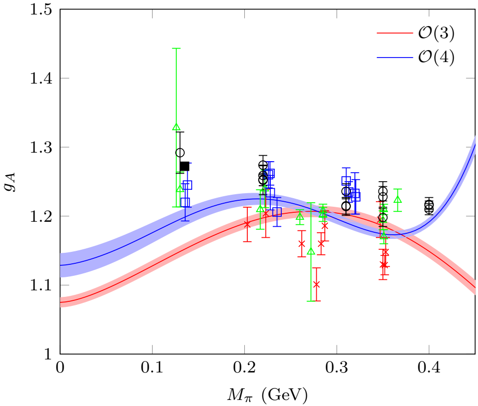


Figure: Red curve: g_A $\mathscr{O}(3)$ Δ-full fit; blue curve: g_A $\mathscr{O}(4)$ Δ-full fit. The points are the lattice data, except for the filled black square, which is the experimental determination.

Conclusions and Outlook

- $G_A(q^2)$ essential in ν oscillations.
- χPT and Lattice complement one another
- The extraction of $\langle r_A^2 \rangle$ is challenging both from experiment and lattice
- Our combined fit $\mathcal{O}(p^3)$ Δ-full successfully describes the lattice data
  - $g_A = 1.2295(37)$, $\langle r_A^2 \rangle = 0.2937(22)$ (fm^2)
  - No ad hoc parametrisation
- $\mathcal{O}(p^4)$ Δ-full fit $\Rightarrow$ $d_{16} = 0.24 \pm 0.18$ GeV^{-2}

Thanks for watching!

Any questions?

Nucleon Axial Form Factor: Extra

- Dipole ansatz: $G_A(q^2) = g_A(1 - \frac{q^2}{M_A^2})^{-2}$
- z-exp.: $G_A(q^2) = \sum_k a_k z^k(q^2)$, with $z(q^2, t_{\text{cut}}, t_0)$