

# Neutrino energy reconstruction from semi-inclusive samples



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## Neutrino energy reconstruction from semi-inclusive samples

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# Introduction

A proton and a muon are detected in coincidence (no pions). Then, assuming the shape of the neutrino flux is given, we provide estimates of the mean neutrino energy ( $\equiv$ reconstructed neutrino energy) and its 1-sigma error.

We consider that the detected proton comes from a CCQE interaction (with a neutron). This neutron may be an 'independent particle' or it may belong to a SRC pair. The struck outgoing proton can undergo elastic and inelastic final-state interactions.

Therefore, the final state consists in a muon and at least a proton.

## Our goals are:

1. to look for events in which the neutrino energy can be reconstructed with high precision (and understand why that is so),
2. to identify whether the probability of such events is high,
3. to study the dependence of the outcomes with the cross section model (i.e. with the description of the initial state and the final-state interactions).

We define **an event** as a complete set of **muon and proton kinematics**:

$$k_\mu, \theta_\mu, \phi_\mu, p_N, \theta_N, \text{ and } \phi_N.$$

For each event, we compute the **mean energy** and its **1-sigma error** as follows:  
(Van Orden and Donnelly, PRC 100, 044620 (2019))

**Mean neutrino energy**  
( $\equiv$ reconstructed energy):

$$\langle \varepsilon_\nu \rangle = \frac{\int d\varepsilon_\nu \varepsilon_\nu \phi(\varepsilon_\nu) \frac{d^6 \sigma(\varepsilon_\nu)}{d\Omega_\mu dk_\mu d\Omega_N dp_N}}{\int d\varepsilon_\nu \phi(\varepsilon_\nu) \frac{d^6 \sigma(\varepsilon_\nu)}{d\Omega_\mu dk_\mu d\Omega_N dp_N}},$$

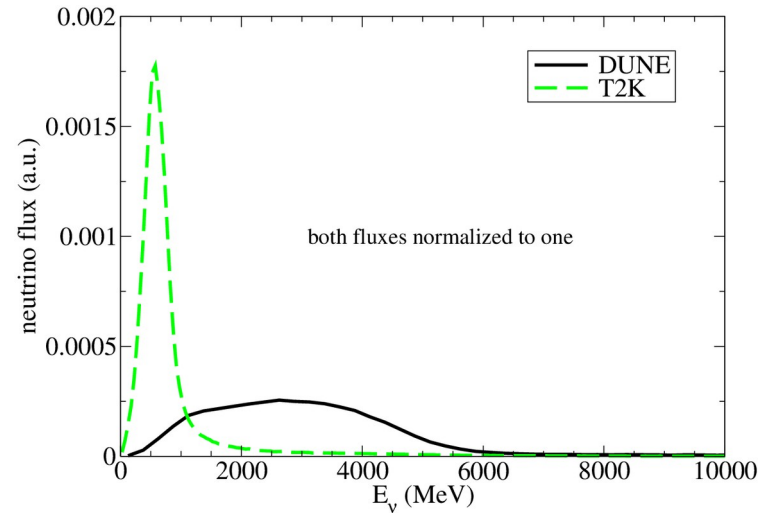
**Intrinsic error:**

$$\Delta \varepsilon_\nu = \sqrt{\langle \varepsilon_\nu^2 \rangle - \langle \varepsilon_\nu \rangle^2}$$

with

$$\langle \varepsilon_\nu^2 \rangle = \frac{\int d\varepsilon_\nu \varepsilon_\nu^2 \phi(\varepsilon_\nu) \frac{d^6 \sigma(\varepsilon_\nu)}{d\Omega_\mu dk_\mu d\Omega_N dp_N}}{\int d\varepsilon_\nu \phi(\varepsilon_\nu) \frac{d^6 \sigma(\varepsilon_\nu)}{d\Omega_\mu dk_\mu d\Omega_N dp_N}}$$

For the moment, we focus on **oxygen 16**



and DUNE and T2K fluxes.

# The models

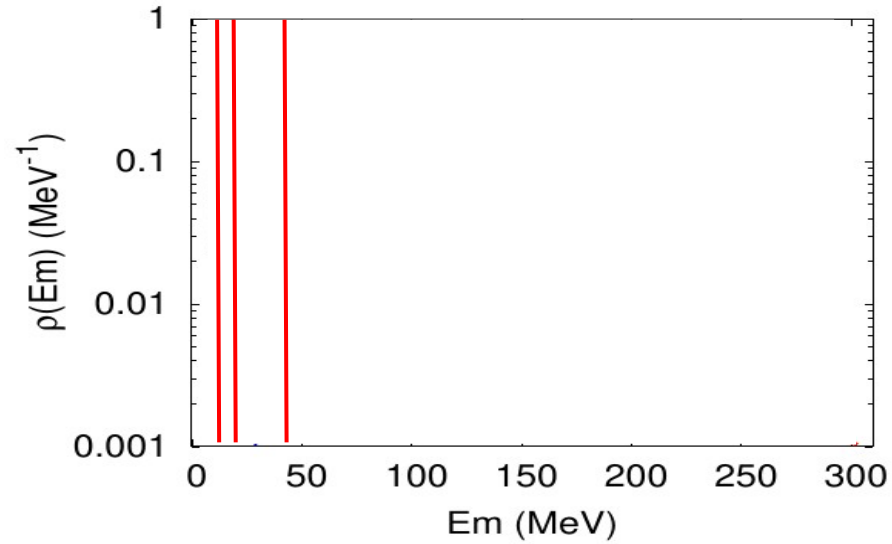
The starting point is the six differential cross section:

$$\frac{d^6\sigma}{dk_l d\Omega_l dp_N d\Omega_N} = K \rho_\kappa(E_m) \ell_{\mu\nu} H_\kappa^{\mu\nu}$$

The hadronic current is:

$$J_{had}^\mu = \int d^3\mathbf{p} \bar{\Psi}_f(\mathbf{p}_f, \mathbf{q} + \mathbf{p}) \mathcal{O}_{\text{one-body}}^\mu \Psi_i(\mathbf{p})$$

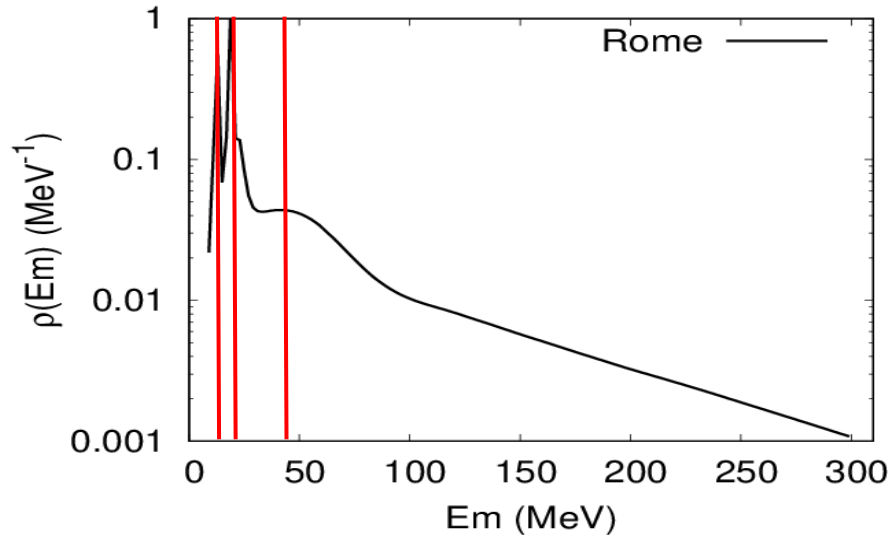
## Missing energy distribution in a **pure shell model**:



$$\rho_{\kappa}(E_m) = \delta(E_m - E_m^{\kappa})$$

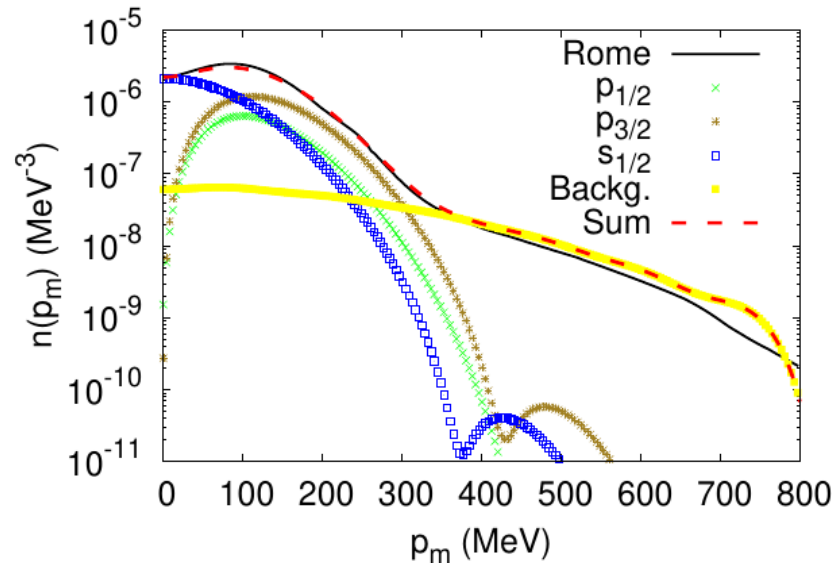
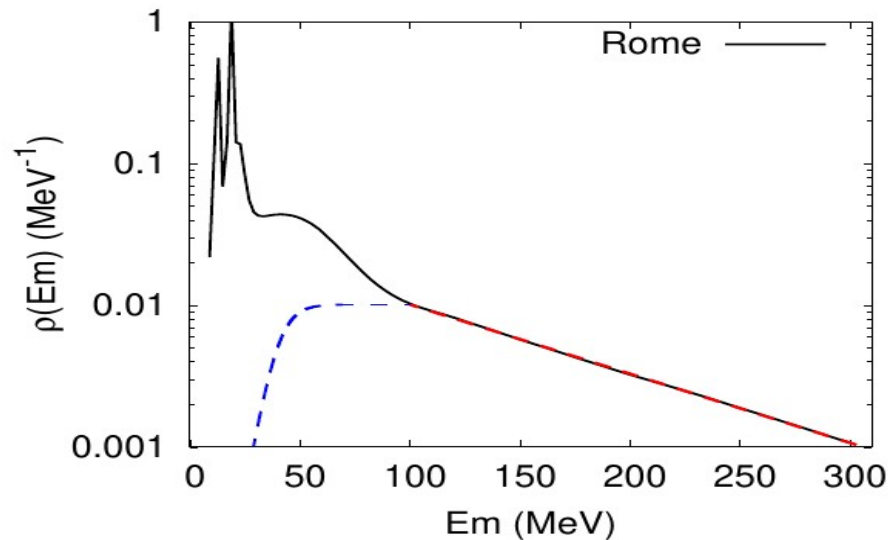


**Missing energy distribution** from the Rome spectral function (O. Benhar et al. NPA 579, 493 (1994); PRD 72, 053005 (2005)):



$$\rho(E_m) = \int d^3 \mathbf{p}_m S(E_m, p_m)$$

**Missing energy and momentum distributions** from the Rome spectral function (O. Benhar et al. NPA 579, 493 (1994); PRD 72, 053005 (2005)) and the shell model used in this work:



$E_m$ (MeV)	Shells	$^{16}\text{O}$
0 – 16.5	$p_{1/2}$	1.51
16.5 – 25	$p_{3/2}$	3.47
25 – 100	$s_{1/2}$ + backg.	2.22
	$s_{1/2}$	1.62
	backg.	0.60
100 – 300	backg.	0.80

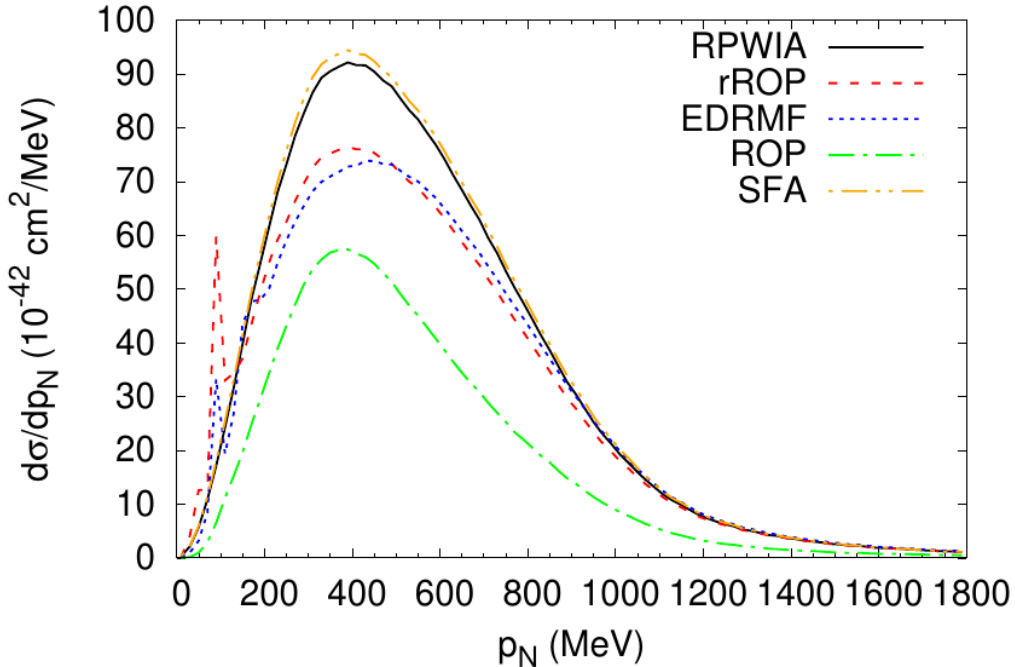
$$n(p_m) = \int dE_m S(E_m, p_m)$$

$$\int d^3\mathbf{p}_m n(p_m) = 8$$

TABLE: Correspondence between missing energy regions and shells in oxygen. The last column are the occupation numbers.

# The models

Single-differential cross section for the **T2K flux**.



**RPWIA** and **SFA**: no FSI, no Pauli blocking (PB). They are easy to implement, the SFA is already in some MC generators, therefore they are shown as reference.

**rROP** and **EDRMF**: FSI and PB included.

They provide an estimate of the signal definition: **at least a proton** in the final state. (Other hadrons can populate the final state due to FSI and correlations in the initial state).

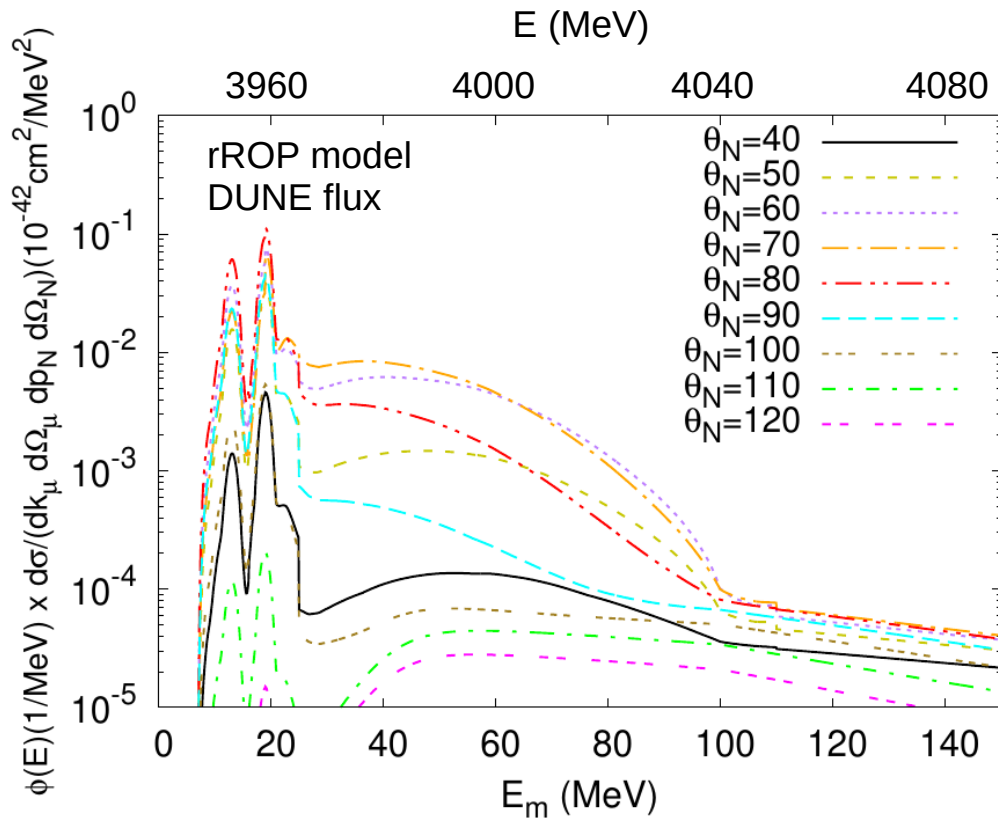
**ROP**: The struck nucleon does not suffer inelastic FSI, but does elastic FSI.

It provides a closer estimate of the signal definition: “one proton and no other hadrons in the final state”.

To deeper understand the definition of the reconstructed energy and its error, let's take a look to the 6-differential cross section as a function of  $E_m$ .

$$\left\langle \frac{d^6\sigma}{dk_l d\Omega_l dp_N d\Omega_N} \right\rangle = \int d\varepsilon_\nu \phi(\varepsilon_\nu) \frac{d^6\sigma(\varepsilon_\nu)}{dk_l d\Omega_l dp_N d\Omega_N}$$

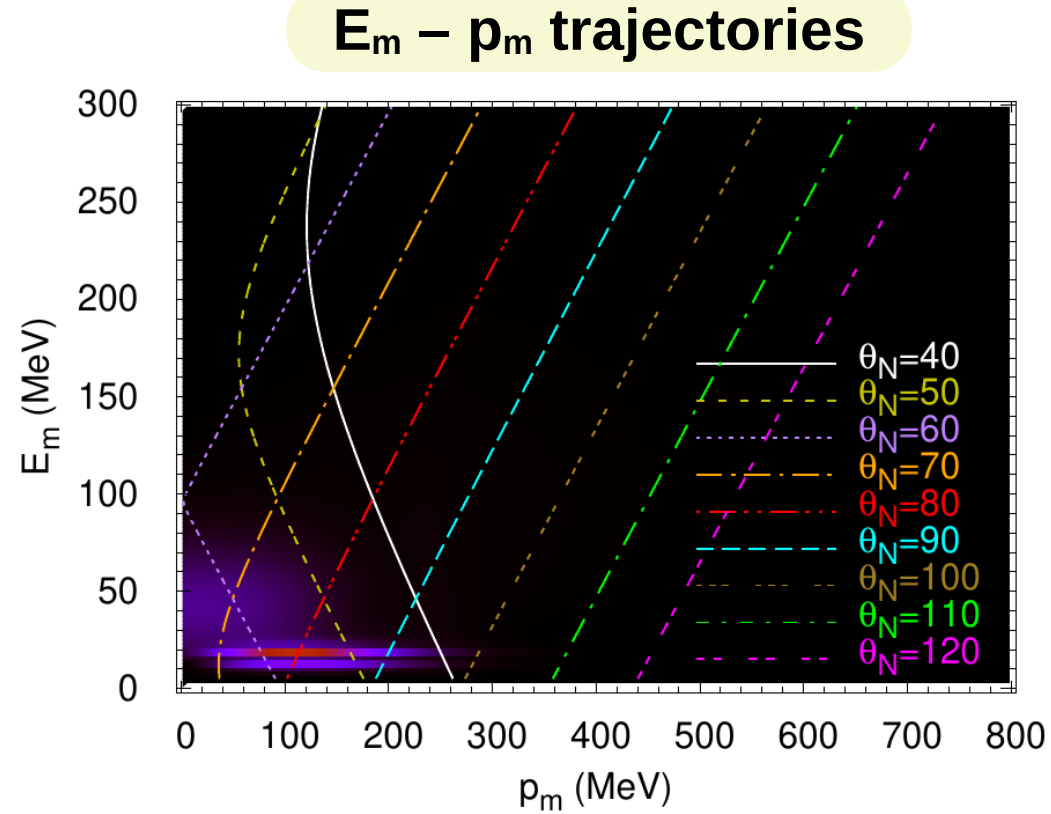
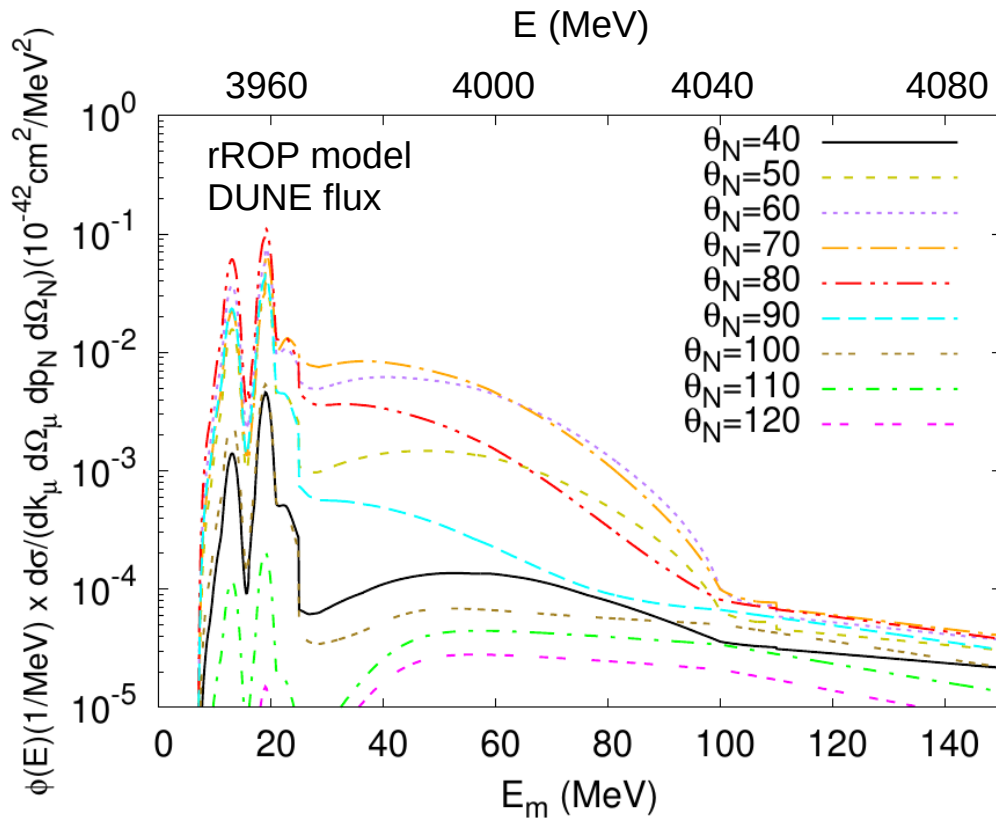
$$E_m = E - E_l - T_N - T_B ,$$



1. The reconstructed neutrino energy is the average value from the distributions.

2. The one-sigma error will be small when the strength concentrates in a small  $E_m$  region.

Kinematic: ( $E_i = 3800$  MeV,  $\theta_i = 7$  deg,  $T_N = 140$  MeV,  $\varphi_N = 180$  deg)



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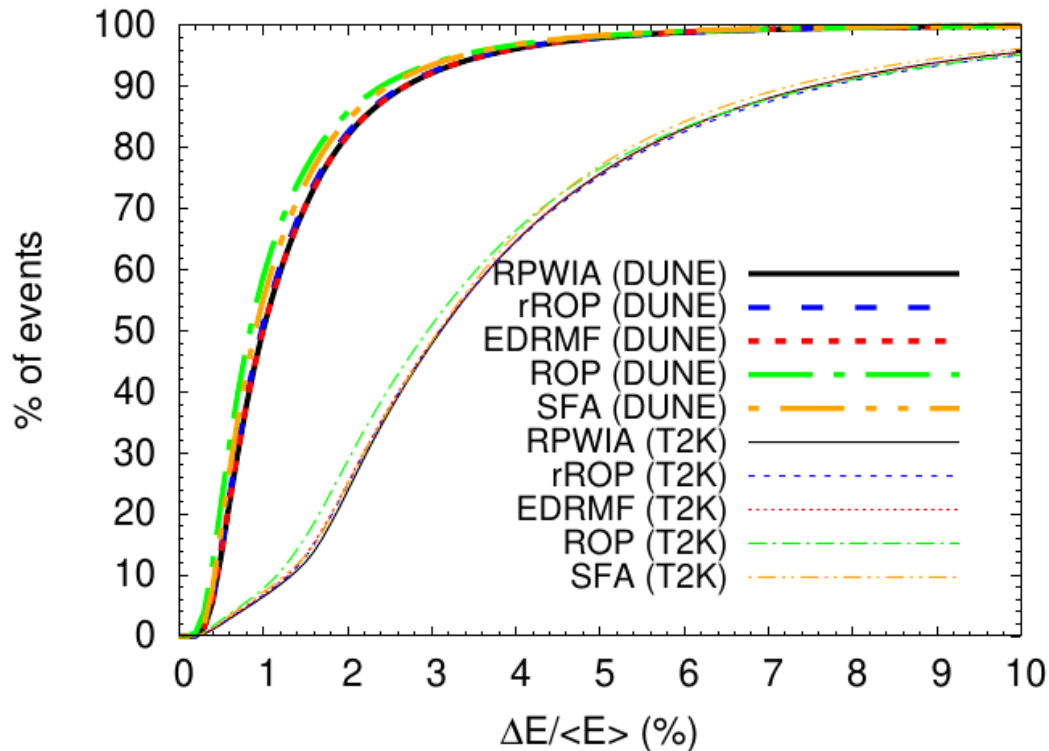
We have populated the phase-space with a few million of events.

$$\left\langle \frac{d^6\sigma}{dk_l d\Omega_l dp_N d\Omega_N} \right\rangle = \int d\varepsilon_\nu \phi(\varepsilon_\nu) \frac{d^6\sigma(\varepsilon_\nu)}{dk_l d\Omega_l dp_N d\Omega_N}$$

Each event has a reconstructed <energy> and its error.

We do the same for different nuclear models.

## Cumulative distribution as a function of the error in the reconstructed neutrino energy.



### Results:

DUNE: for ~50% of the events the error is lower than 1%.

T2K: for ~50% of the events the error is lower than 3%.

We get essentially the same with all models. (Models differ from each other, mainly, in the treatment of FSI .)

### Messages:

1. Very small errors for a lot of events.
2. The error estimates are essentially independent on the modeling of FSI.

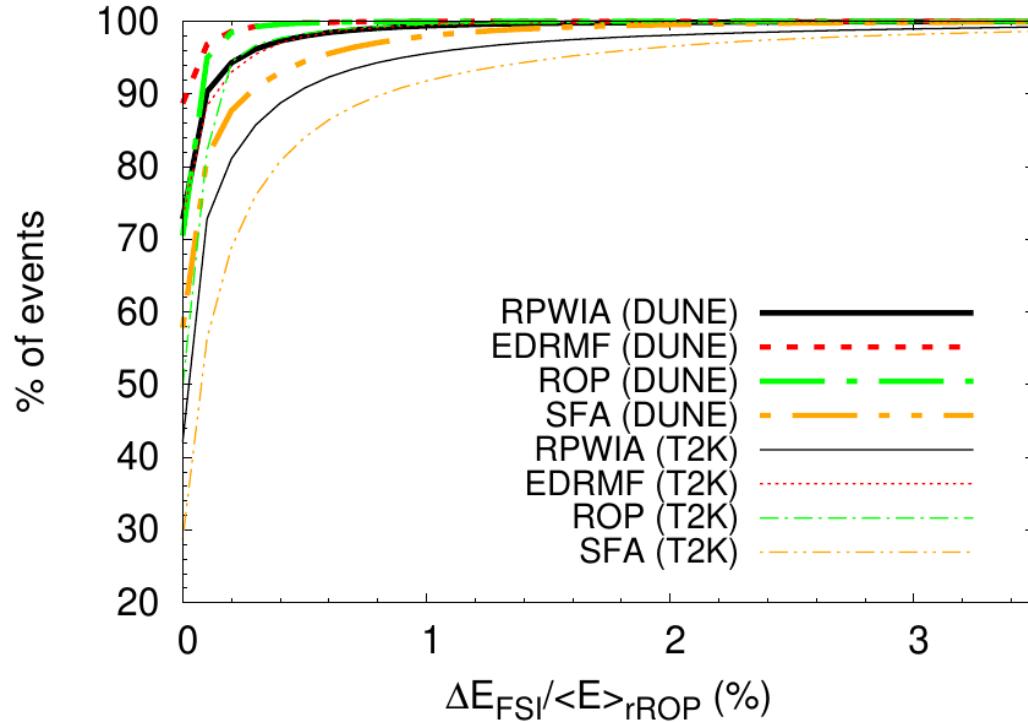
This is good news!



For a given event ( $\mathbf{k}_\mu$  and  $\mathbf{p}_N$ ),  
**does the reconstructed energy depend on FSI?**  
**How much?**

**Cumulative distributions** as a function of the FSI error, that is estimated as :

$$\Delta E_{\text{FSI},i} = \frac{1}{2} |\langle E \rangle_{\text{rROP}} - \langle E \rangle_i| ,$$



**Results:**

For ~98% (~90%) of the DUNE (T2K) events, the neutrino energy is reconstructed to the same value  $\pm 1\%$ .

**Message:**

The reconstructed neutrino energy shows a small dependence on FSI.

This is good news again!



**Where in the phase space is most of the strength? And where is the error small?**

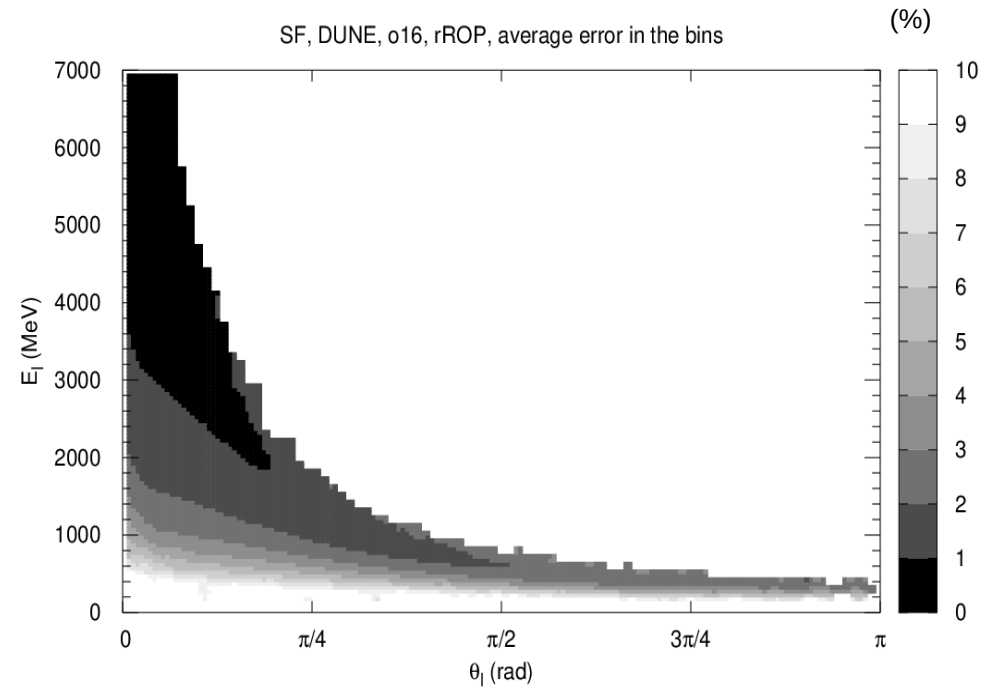
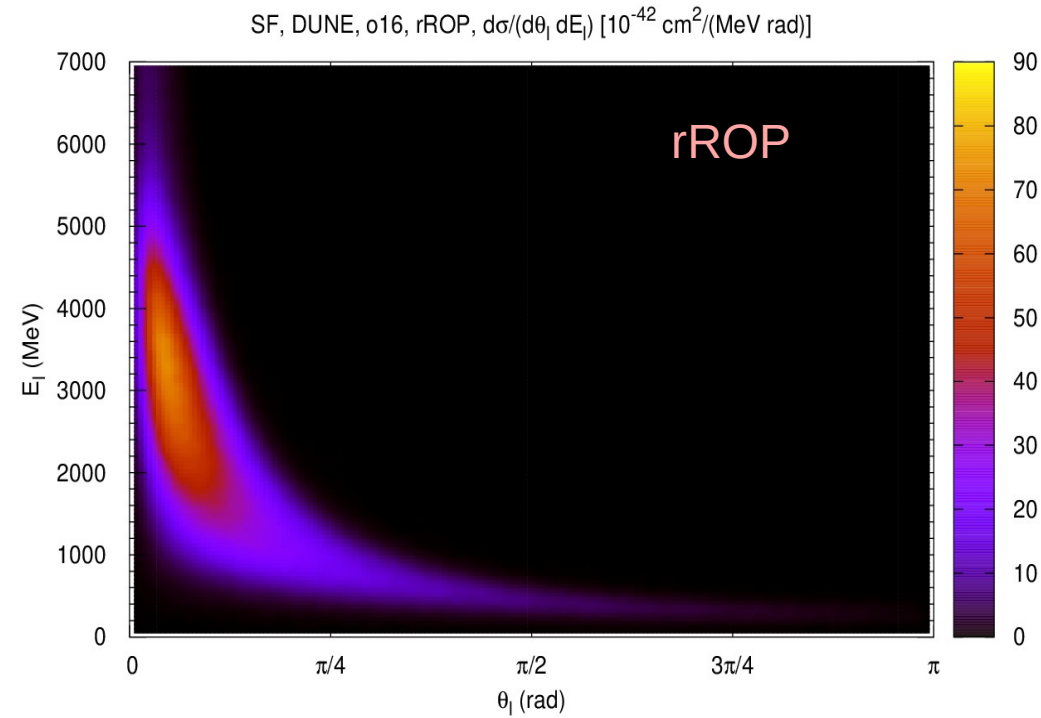
**In the next figures :**

**Left panels:** Flux folded double differential cross sections as a function of muon and proton laboratory variables.

**Right panels:** Average  $\langle \text{energy} \rangle$  error (in %) for the events in the bins.

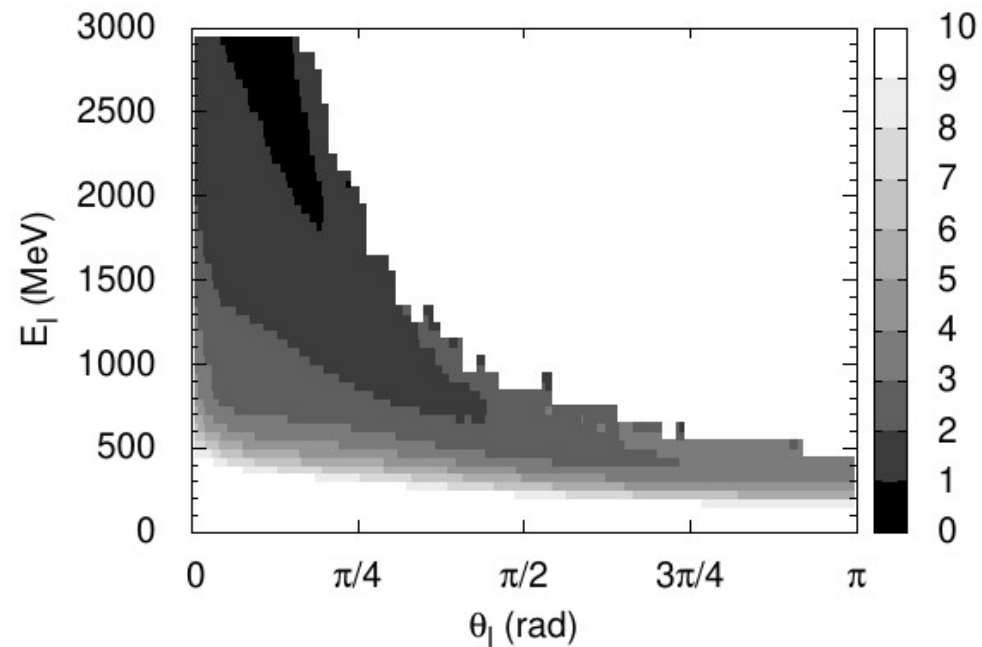
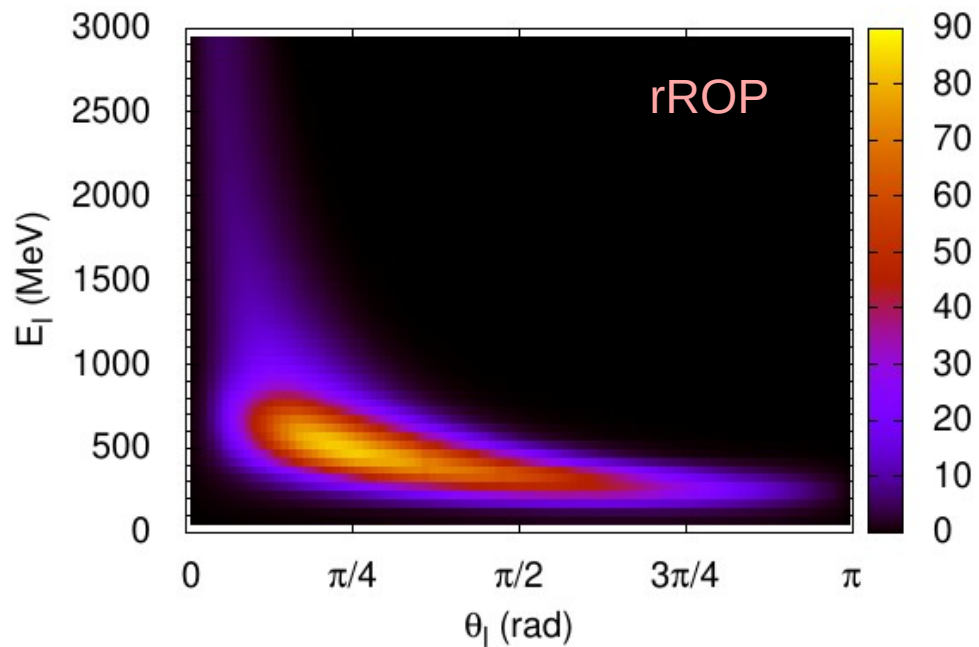
# DUNE flux

## $E_I$ vs $\theta_I$



# T2K flux

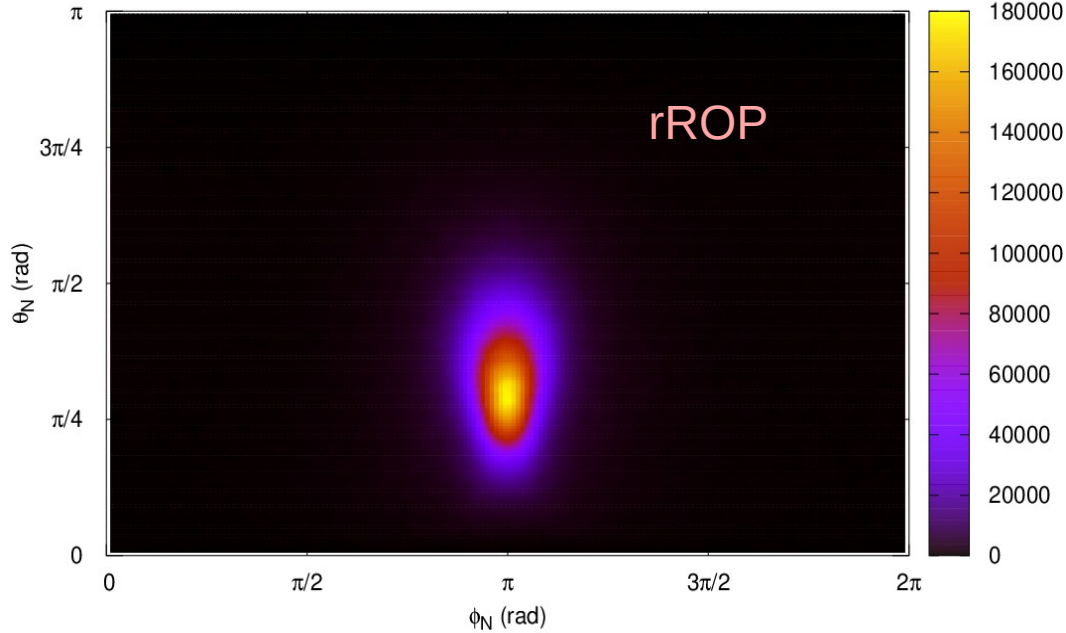
## $E_l$ vs $\theta_l$



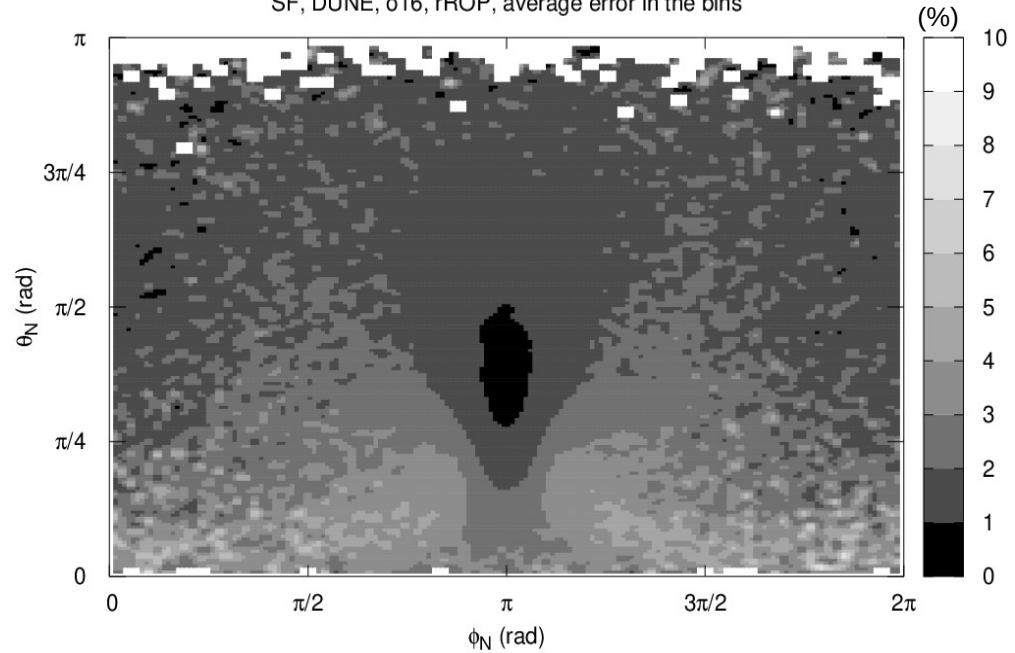
# DUNE flux

## $\theta_N$ vs $\phi_N$

SF, DUNE, o16, rROP,  $d\sigma/(d\theta_N d\phi_N)$  [ $10^{-42}$  cm<sup>2</sup>/(rad<sup>2</sup>)]

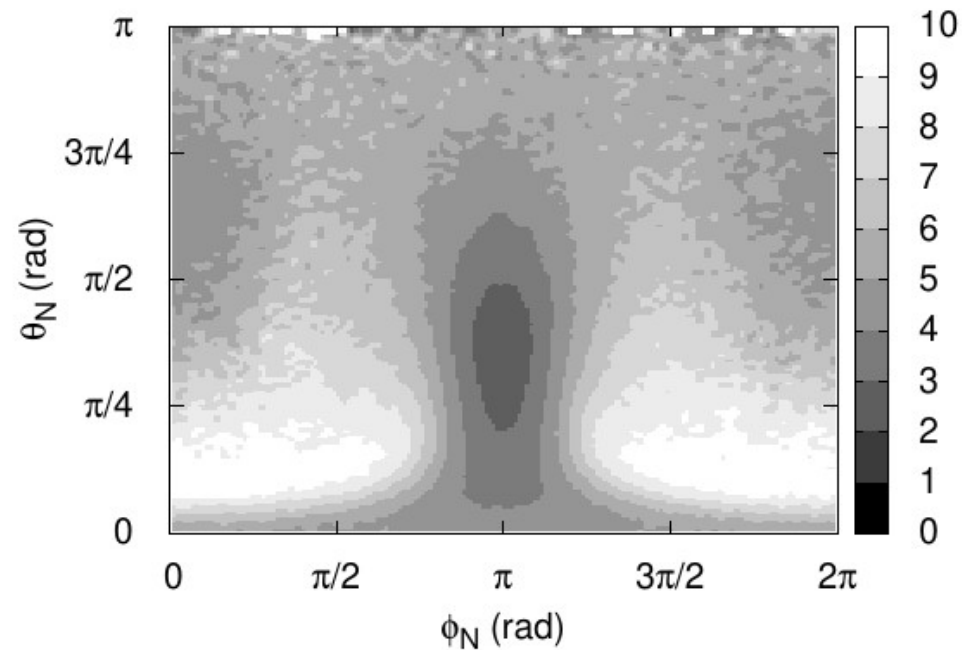
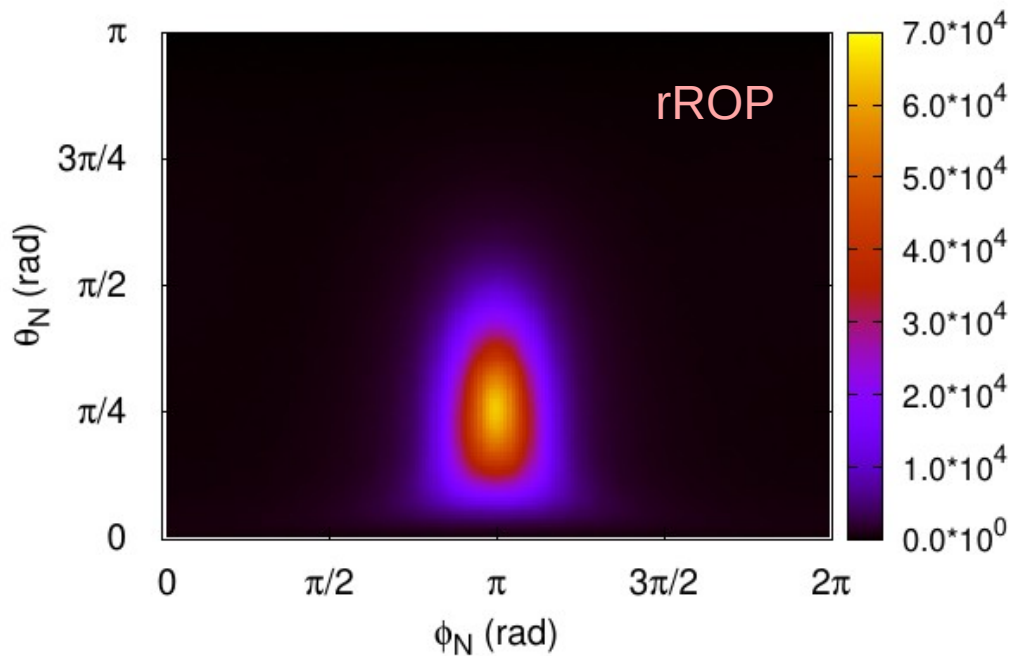


SF, DUNE, o16, rROP, average error in the bins



# T2K flux

$\theta_N$  vs  $\phi_N$







For a given event ( $\mathbf{k}_\mu$  and  $\mathbf{p}_N$ ),

**does the reconstructed energy depend on  
the description of the initial state?**

**How much?**

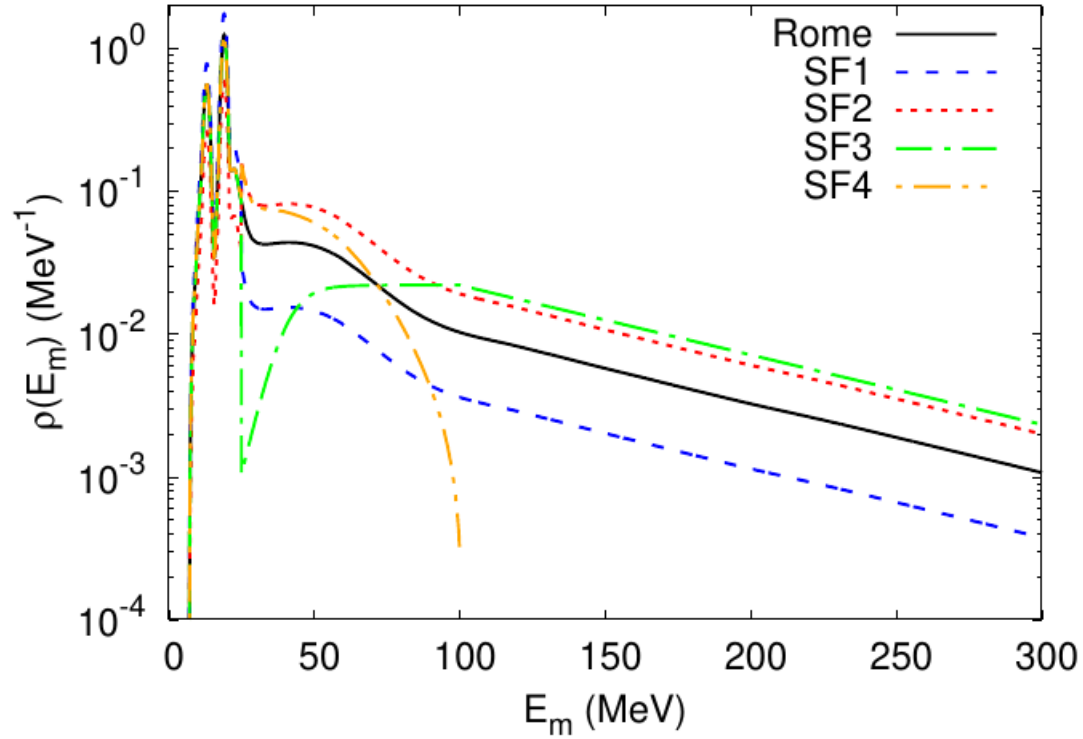
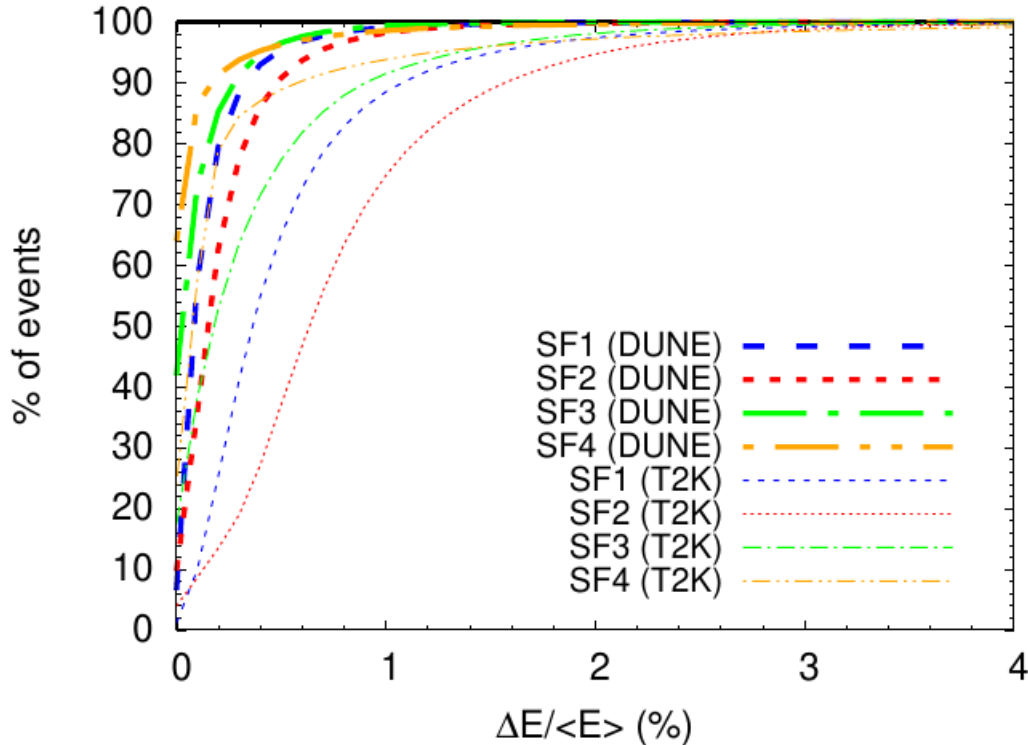


FIG. 8: Different missing energy profiles employed to analyze the impact of the description of the initial state in the reconstructed neutrino energy and its error.

**Cumulative distributions** as a function of the *initial-state error*, that is estimated as:

$$\Delta E_{\text{model},i} = \frac{1}{2} |\langle E \rangle_{\text{rROP}} - \langle E \rangle_{\text{SF}i}| ,$$



### Results:

For the DUNE (T2K) flux, more than 95% (~80%) of the events reconstruct to the same neutrino energy  $\pm 1\%$ , for all studied missing energy profiles.

### Message:

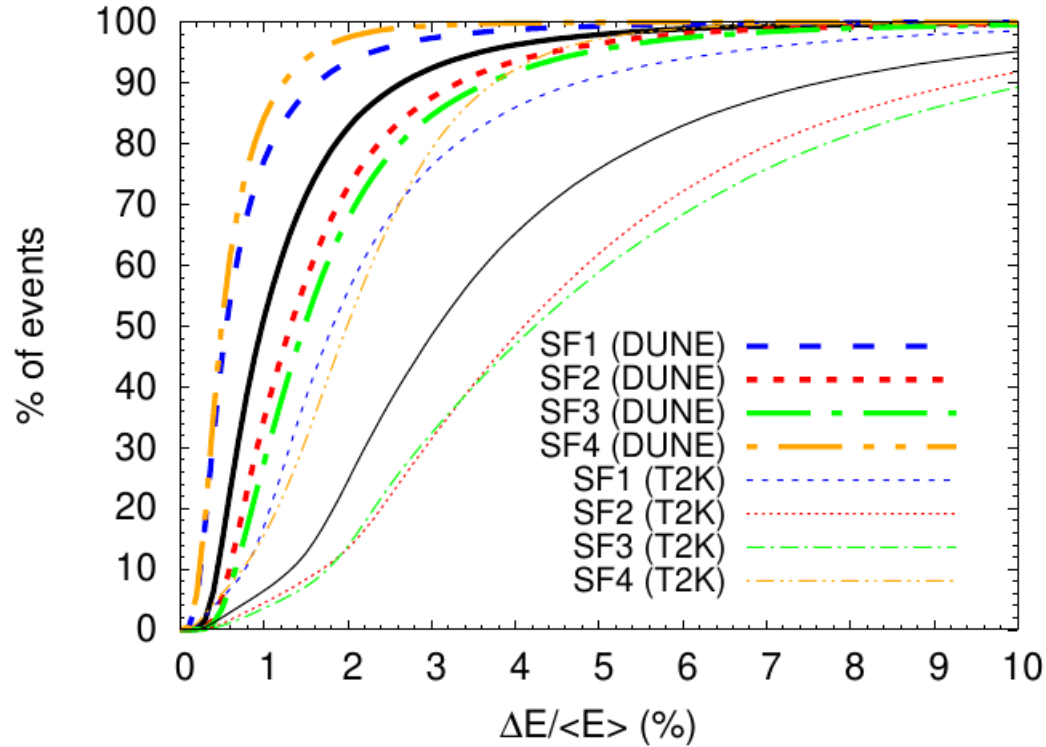
The reconstructed neutrino energy shows a very small dependence on the description of the initial state.

This is good news again!

**does the error depend on the description  
of the initial state?**

**how much?**

## Cumulative distribution as a function of the error in the reconstructed neutrino energy for the ‘test missing energy profiles’.



### Result:

The different models show large differences in the estimated errors.

### Message:

The estimated error is quite sensitive to the description of the initial state.

This is something to take into account because using a simple model can lead to underestimation of the errors.

# Summary and Conclusions:

We have populated the whole phase-space with millions of events, according to some cross section model. For each event we compute the average (reconstructed) neutrino energy and its 1-sigma error.

Our models account for the initial state ( $E_m$ - $p_m$  distribution) in a realistic way. FSI are considered in a fully relativistic and quantum mechanical way.

These are some numbers (for oxygen 16):

- + For ~50% of the events the error in the reconstructed neutrino energy is <1% for the DUNE flux and <3% for the T2K flux.
- + It is observed that for the “good events” (small error + large cross section), the strength comes mainly from the  $p$  shells.

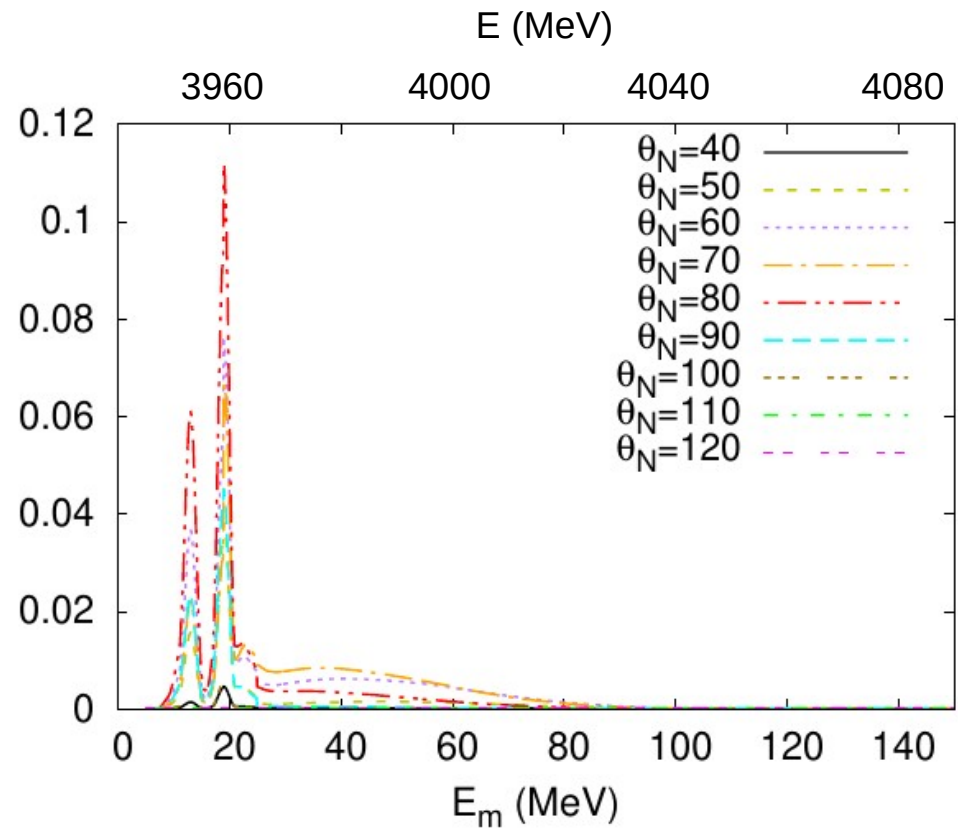
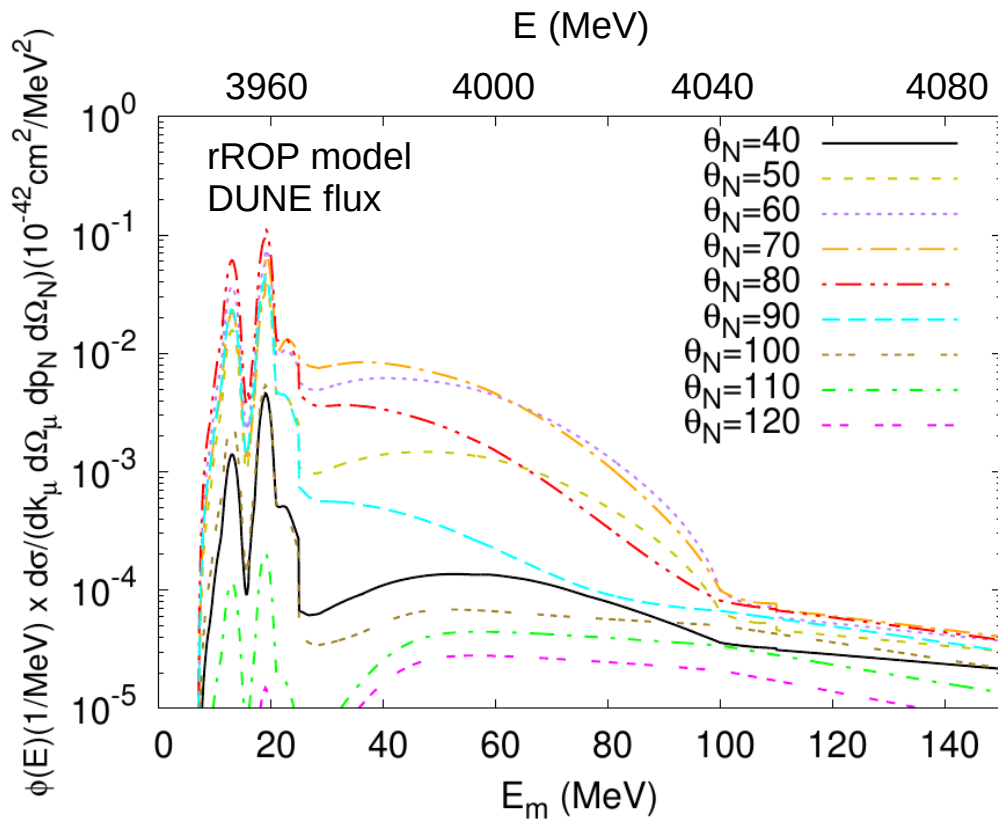
## **IMPORTANT:**

- + Both the reconstructed neutrino energy and its error are nearly independent on the final-state interactions.
- + The reconstructed neutrino energy depends only slightly on the description of the initial state while the error depends on it quite a lot.

**Thanks for  
the attention**



# Backup slides



Kinematic: ( $E_i = 3800$  MeV,  $\theta_i = 7$  deg,  $T_N = 140$  MeV,  $\phi_N = 180$  deg)

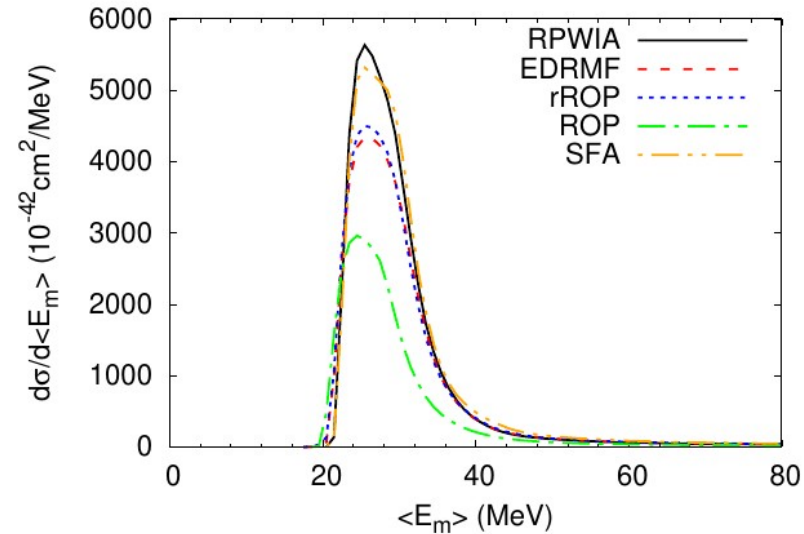
$\theta_N$ (deg)	$\langle E \rangle$ (MeV)	$\Delta E / \langle E \rangle$ (%)	c.s. ( $10^{-42} \text{cm}^2 / \text{MeV}^2$ )	$p$ -shell weight ( $r$ )
40	3985	1.3	0.028	0.63
50	3971	0.69	0.23	0.67
60	3974	0.60	0.56	0.49
70	3978	0.58	0.51	0.31
80	3964	0.47	0.60	0.77
90	3963	0.55	0.22	0.88
100	3976	1.1	0.030	0.76
110	4025	1.3	0.0053	0.18
120	4040	0.91	0.0027	0.023

TABLE II: For the kinematic  $E_l = 3800$  MeV,  $\theta_l = 7$  deg,  $T_N = 140$  MeV, and  $\phi_N = 180$  deg and different  $\theta_N$  (first column), we show the mean neutrino energy  $\langle E \rangle$ , its relative one-sigma error  $\Delta E / \langle E \rangle$ , the cross section (c.s.), and the weight of the  $p$  shells [ $r$ , defined in eq.(14)]. We have used the DUNE flux and the rROP model.

**Let's show that the majority of events with an error in the averaged neutrino energy lower than 1% are those in which the p shells dominate.**

Cross section as a function of the average missing energy.

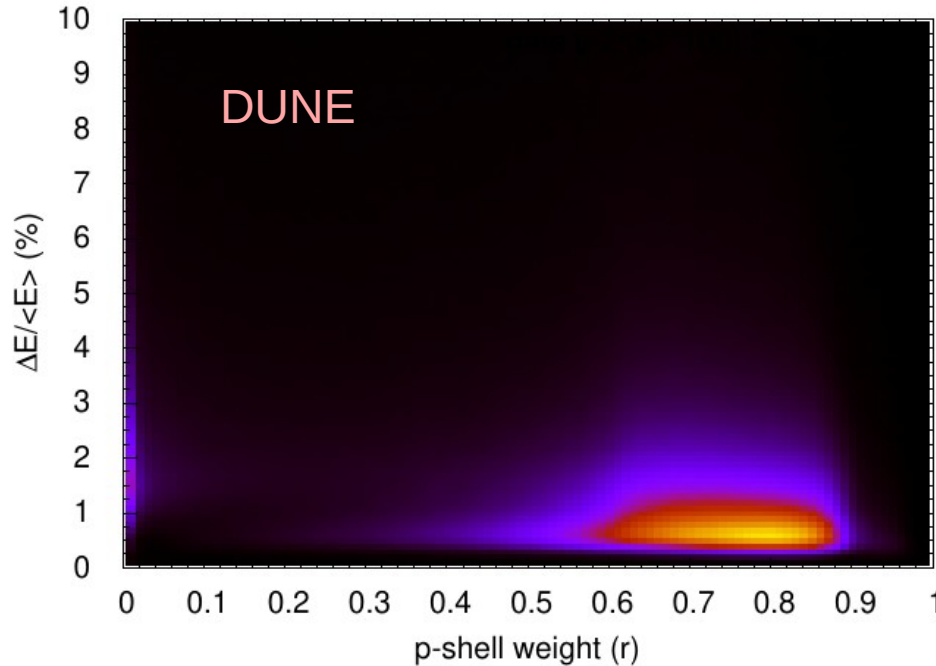
$$\langle E_m \rangle = \langle E \rangle - E_l - T_N .$$



**The majority of events with an error in the averaged neutrino energy lower than 1% are those in which the p shells dominate.**

# Energy error versus the ratio "p-shells only cross section / full cross section".

The meaning of this ratio is the following: if it is close to 1 then the p shells dominate, if it is close to 0 then the p-shells do not contribute.

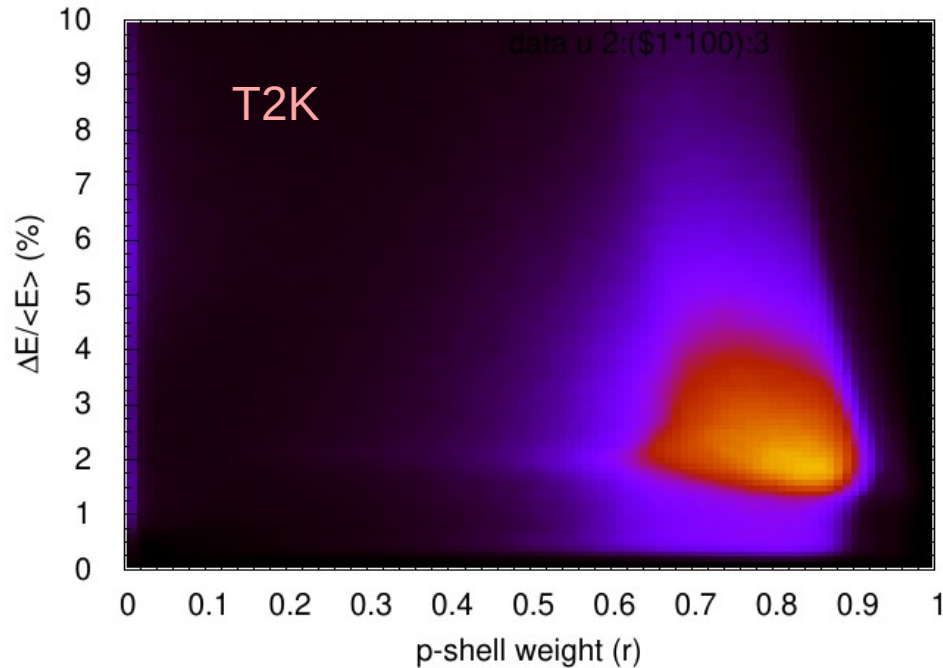


## Three messages:

- 1 Dominance of the  $E_m < 30$  MeV region (where the p-shells live).
- 2 The probability of finding events with error  $< 1\%$  and  $r < 0.6$  is not significant  $\implies$  For events with error  $< 1\%$ , the proton was likely to arise from the p shells.
- 3 All the models report the same conclusions  $\implies$  independence on the modeling of FSI.

# Energy error versus the ratio "p-shells only cross section / full cross section".

The meaning of this ratio is the following: if it is close to 1 then the p shells dominate, if it is close to 0 then the p-shells do not contribute.



## Three messages:

- 1** Dominance of the  $E_m < 30$  MeV region (where the p-shells live).
- 2** The probability of finding events with error  $< 3\%$  and  $r < 0.6$  is not significant  $\implies$  For events with error  $< 3\%$ , the proton was likely to arise from the p shells.
- 3** All the models report the same conclusions  $\implies$  independence on the modeling of FSI.





**Where in the phase space is most of the strength? And where is the error small?**

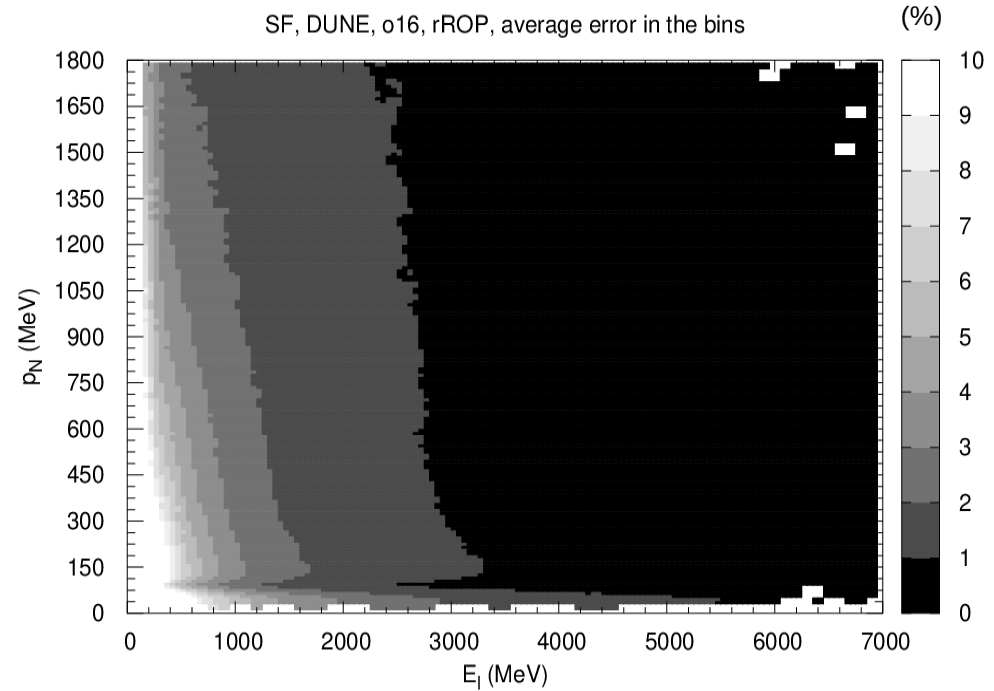
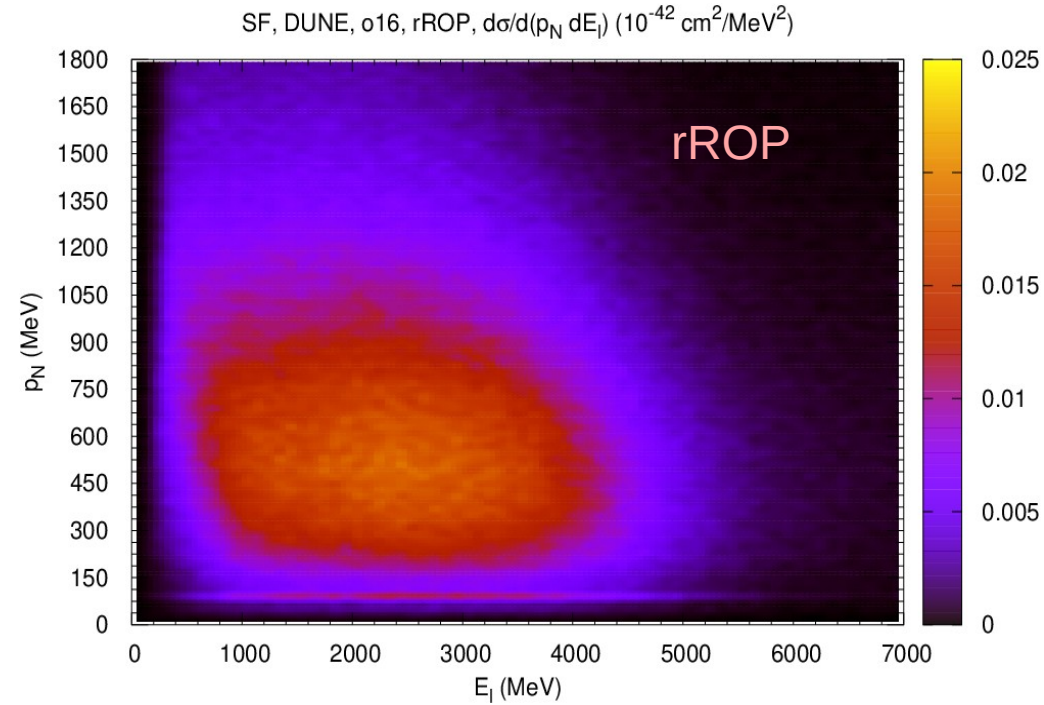
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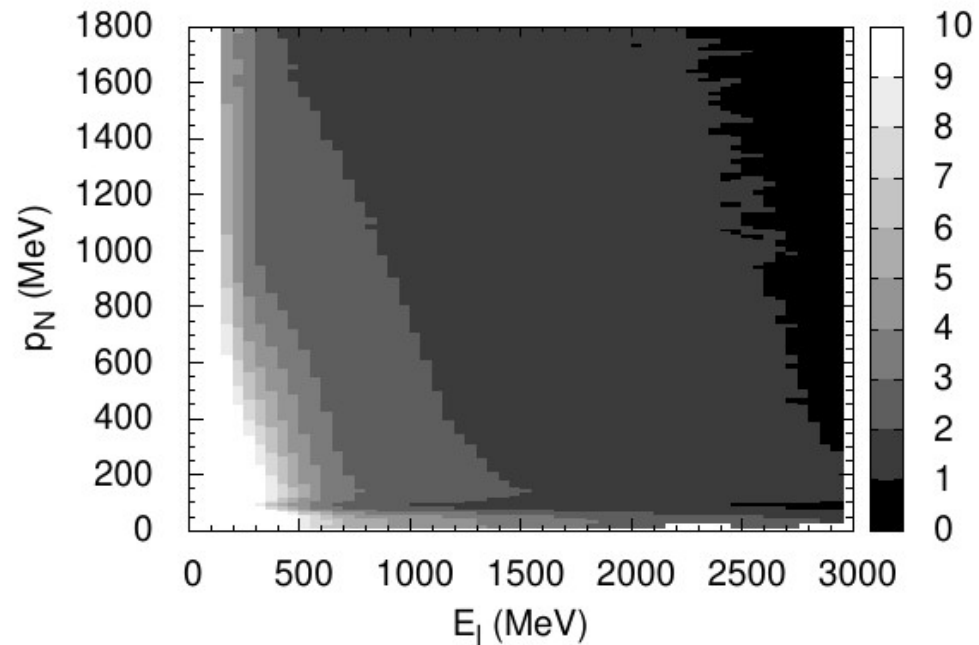
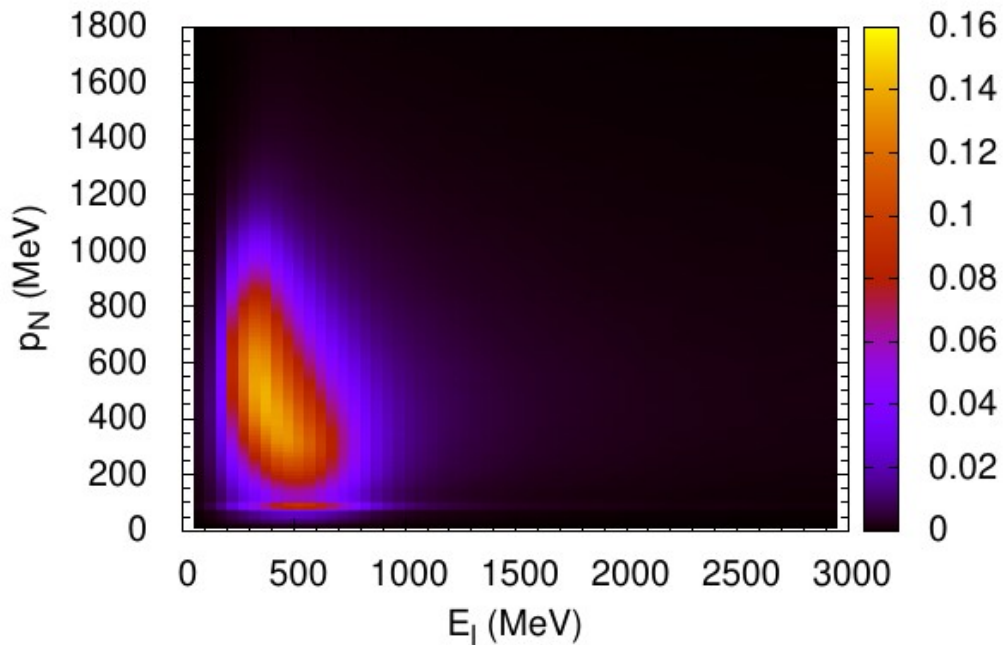
# DUNE flux

## $p_N$ vs $E_I$



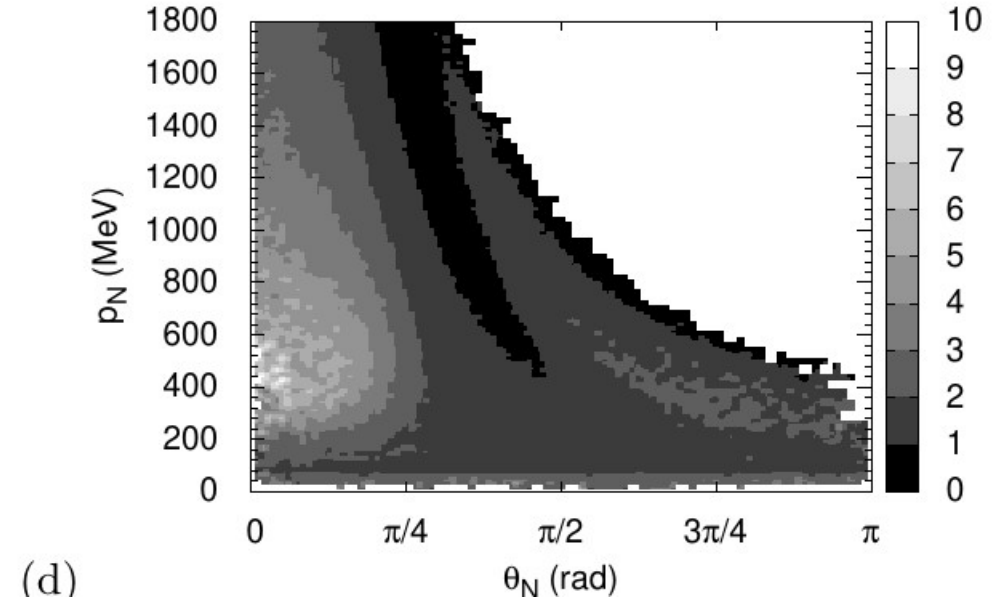
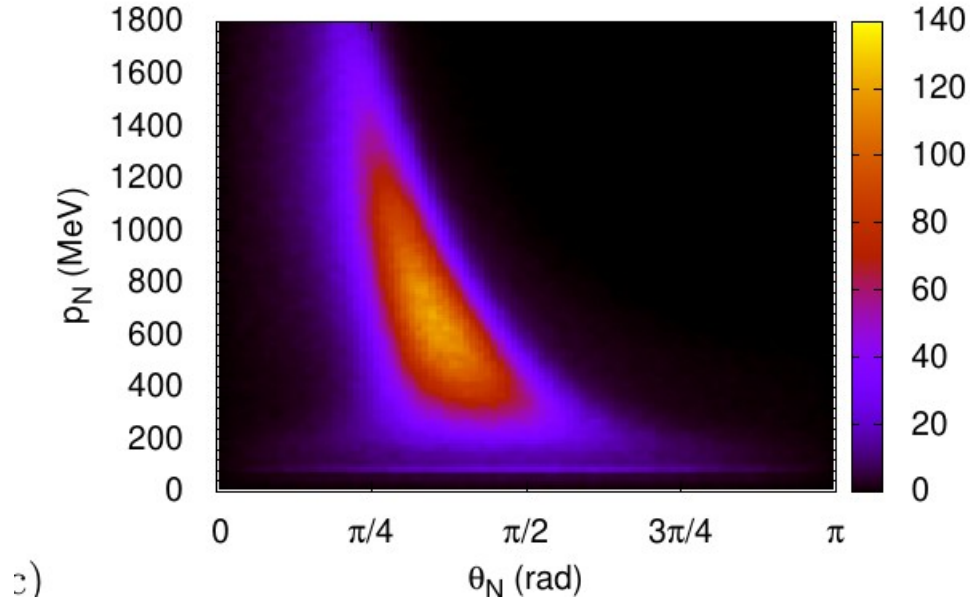
# T2K flux

$p_N$  vs  $E_l$



# DUNE flux

$p_N$  vs  $\theta_N$



# T2K flux

$p_N$  vs  $\theta_N$

