Neutrino-Nucleon Form Factors from Lattice QCD

Aaron S. Meyer (asmeyer.physics@gmail.com)

UC Berkeley/LBNL

March 16, 2021

New Directions in Neutrino Nucleus Scattering
Outline

▶ Introduction
  - Motivation
  - Interaction Modes
  - $z$ expansion
▶ (High-Level) LQCD Results
  - Fermilab Lattice $g_A$
  - Callat $g_A$
  - Quick survey of $g_A$, $r^2_A$
▶ Future Prospects
  - QE
  - RES
  - NN
▶ Conclusions
Introduction
Challenges with Oscillation Experiments

Deep Underground Neutrino Experiment – Flagship $1b experiment at LBNF

Measure osc. prob. over first two oscillation peaks

\[ E_\nu \text{ btw. } [0.5, 10] \text{ GeV} \]

Osc. prob. is fn of \( L/E_\nu \)

\[ \Rightarrow \text{ must classify events by } E_\nu \]

Neutrinos from secondary beam

\[ \Rightarrow E_\nu \text{ not known event by event} \]

\[ \Rightarrow E_\nu \text{ inferred from distribution} \]

Many requisite inputs difficult/impractical to measure from expt

\[ \Rightarrow \text{ Need ambitious theory support to supplement the experimental effort!} \]
Liquid Argon target
Nuclear xsec $\implies$ nuclear modeling

Nuclear amplitudes constructed from one/few nucleon response

Event characteristics change in nuclear medium

In general: $E_\nu \neq E_{\text{vis}}$.

$\implies$ Must infer $E_\nu$ statistically from Monte Carlo
Neutrino-Nucleon Interaction Modes

Event rate/\(E_\nu\) bin \(\sim \int_{\text{bin}} dE_\nu\) (Flux) \(\times\) (Xsec)

DUNE appx. 1:1:1 events for QE:RES:DIS

\[\Rightarrow\] all interaction topologies are important

Focus on QE & RES, but LQCD comp. strategies exist for DIS (& SIS) too
**$z$ Expansion Fit to QE $F_A$**

- Dipole has strict $Q^2$ shape, inconsistent w/ QCD
- Dipole FF ansatz significantly underestimates FF uncertainty

- Nucl. xsec uncertainty from FF same size as data-MC tensions
- Source of tensions unclear btw. nucleon/nuclear

**Model-independent parameterization**

Order of mag. increase in $\delta \sigma$

---

**Graphs and Data References**

- Dipole has strict $Q^2$ shape, inconsistent w/ QCD
- Dipole FF ansatz significantly underestimates FF uncertainty
- Nucl. xsec uncertainty from FF same size as data-MC tensions
- Source of tensions unclear btw. nucleon/nuclear

---

Aaron S. Meyer  
Section: Introduction 7/24
Correlated differences between data & MC in leptonic, hadronic models

Balancing act to reconcile two variables

Insufficient model to describe interactions?
Moving Forward

Room for improvement, but what is needed?

Ideal: Modern high stats $\nu$-$D_2$ scattering bubble chamber expt

Some community push, safety concerns

$\Rightarrow$ LQCD as an alternative/complement to expt

- No nuclear effects
- Realistic uncertainty estimates
- Systematically improvable
- Computers are (relatively) inexpensive
How can Lattice Help?

Lattice is well suited to compute matrix elements:

\[ \mathcal{M}_{\nu\mu n \rightarrow \mu p} = \langle \mu | (V - A)_{\mu} | \nu \rangle \langle p | (V - A)_{\mu} | n \rangle \]
Lattice QCD
Lattice QCD: Formalism 1/2

Numerical eval of path integral
Quark, gluon DOFs —

\[ \langle O \rangle = \frac{1}{Z} \int D\psi D\bar{\psi} DU \exp(-S) O_\psi [U] \]

Few inputs —

Computational:
\[ am(u,d), \text{bare} \]
\[ am_s, \text{bare} \]
\[ \beta = \frac{6}{g_{\text{bare}}^2} \]

Scale setting:
\[ \text{e.g. } \frac{M_\pi}{M_\Omega}, \frac{M_K}{M_\Omega}, M_\Omega \]
one per computational input

Results — first principles predictions of QCD

\[ M_{\text{hadron}}, \langle F|O|I \rangle \]

Euclidean time \[ \Rightarrow \]
\[ C(t) \sim e^{-M \cdot t} \]

“Complete” error budget \[ \Rightarrow \] extrapolation in \( a, L, (M_\pi) \)
Lattice QCD: Formalism 2/2

Correlation functions computed in euclidean time:

2-point function
\[\langle O_1(t)O_2(0) \rangle = \sum_n \langle 0|O_1|n \rangle \langle n|O_2|0 \rangle e^{-E_n t}\]

3-point function
\[\langle O_1(t)O_2(\tau)O_3(0) \rangle = \sum_{mn} \langle 0|O_1|n \rangle \langle n|O_2|m \rangle \langle m|O_3|0 \rangle e^{-E_n(t-\tau) - E_m \tau}\]

Large \(t\): excited states decay away, signal-to-noise degrades

Computations performed on “gauge ensembles” with fixed physics:
\[\{a, L, M_\pi, \ldots\}\]

Extrapolate in ensemble parameters to arrive at physical point:

- \(a \rightarrow 0\) (continuum limit)
- \(L \rightarrow \infty\) (infinite volume limit)
- \(M_\pi \rightarrow M_\pi^{\text{phys}}\) (chiral limit)

Word of caution:

Many collaborations will compare experiment to unextrapolated results...
These values will have uncontrolled systematics.
Make sure you know what you are looking at!
Fermilab Lattice — $M_N$

Nucleon mass — $C_{2pt}(t) \sim \sum_n z_n z_n^\dagger e^{-E_n t}$, $z_n = \langle 0 | O | n \rangle$

Demonstration of method:
Baryons w/ “Highly-Improved Staggered Quarks” (HISQ)
Additional $SU(4)$ “taste” symmetry
Baryon octet $\rightarrow$ baryon 572-plet

Complicated group thy, cheaper computation (1-comp spinors)

3 ensembles: all $M_{\pi}^{\text{phys}}$; various $a$, $L$

Analysis credit: Yin Lin

[Phys.Rev.D 103 (2021)]

Data generation in progress

Best fit: $\chi^2$/dof = 0.79

$M_{N, \text{phy}} \approx 940$ MeV
Fermilab Lattice — $g_A$, $g_V$

Axial charge —

$$C_{3pt}(t, \tau) \sim \sum_{mn} z_n z_m^\dagger \langle n|A_\mu|m\rangle e^{-E_n(t-\tau)-E_m\tau}$$

$$\langle N|A_\mu|N\rangle \sim g_A$$

Single ensemble: $M_\pi \approx 305$ MeV

World-first 3-point fn w/ HISQ

Interpreting results requires

$\text{SU}(2)_{\text{flavor}} \times \text{SU}(4)_{\text{taste}} \subset \text{SU}(8)$ CG coefs/Wigner-Eckart

Analysis credit: Yin Lin

$\sim g_A/Z_A$

$\sim g_V/Z_V$

[2010.10455[hep-lat]](in press)
Callat — $g_A$

Left: $O(1\%)$ on $g_A$ using Feynman-Hellman inspired technique:

$$\frac{dE_\lambda}{d\lambda} = \langle \psi_\lambda | \frac{dH_\lambda}{d\lambda} | \psi_\lambda \rangle \quad \text{w/ source term} \quad \lambda \int d^4 x \ A_\mu(x)$$

2-pt

3-pt

Right: Work in progress —

One ensemble: $a \approx 0.09 \text{ fm}$, $M_\pi \approx 310 \text{ MeV}$

5-state fit, detailed analysis of excited state contamination

Compare w/ traditional three-point method

Analysis credit: Jinchen He
Survey — $g_A$


Historically low $g_A$ attributed to $N$ excitations ($N\pi$, RES) led to apparent violations of PCAC relation

Now:

- Agreement w/ PDG seen consistently
- Details about excitations still unclear

Survey — $r_A^2$

Avoid $M_A$: only makes sense in dipole 

$$r_A^2 = -(6/g_A)dF_A/dQ^2\bigg|_{Q^2=0}$$

$$\implies r_{A,dipole}^2 = (1 \text{ GeV}^2/m_A^2) \times 0.466\text{fm}^2$$

Most collaborations have adopted $z$ expansion parameterization

Uncertainties will continue to decrease w/ time

Opinion: Still too early by few years — wait for resolution of excited state issues

See refs. in: [2103.05599[hep-lat]]

[18/ 24] Lattice QCD
Prospects
Timeline

Very rough sketch of my interpretation of timeline for LQCD computations

\[ \langle N | J_\mu (Q^2) | N \rangle \]
\[ \langle N\pi | J_\mu | N \rangle \]
\[ \langle NN | J_\mu | NN \rangle \]
Forward-Looking: QE FFs

High-precision meas. \( \langle N | \mathcal{J}(\bar{q}, \nu) | N \rangle \sim F_A(Q^2), F_V(Q^2) \),

Realistically: \( Q^2 \sim 0 - 1 \text{ GeV}^2 \)

Reduce uncertainties of axial/pseudoscalar FFs
  Fill in where difficult/impractical to get expt data

Resolve tensions btw vector FF parameterizations
  Discrepancy of proton mag FF \( \implies \) uncertainty floor for axial FF

\[ \begin{align*}
\sigma_{\nu,n \rightarrow \mu^- p}(E_\nu) \\
\sigma(E_\nu) \text{ [10}^{-39} \text{ cm}^2] \\
\end{align*} \]

(black solid) z expansion: [Phys.Rev.D 102 (2020)]

Aaron S. Meyer  Section: Prospects
Forward-Looking: Resonant Transitions

Constrain amplitudes/FFs that are inaccessible to expt

\[ \langle N\pi|\mathcal{J}_\mu(\vec{q}, W)|N\rangle \ [\text{res.}+\text{nonres.}] \sim C_5 A(Q^2) \]

Theoretically & computationally challenging:

\[ \Rightarrow \text{ hard cutoff at } N\pi\pi \text{ threshold (for now)} \]
\[ \Rightarrow \text{ raise } M_\pi \text{ to circumvent, } \Delta \rightarrow \text{ stable} \]

Dense spectrum of states

\[ \Rightarrow \text{ In practice requires } N\pi, (N\pi\pi) \text{ operators} \]

\[ [1912.04917[hep-lat]] \]
\[ [\text{github.com/lehner/Wick}] \]

\[ M_\pi L = 4, \ M_\pi = 135 \text{ MeV} \]
Forward-Looking: $NN \ g_A$

$$\langle NN| J_\mu | NN \rangle \sim F_{V}^{D_2}, F_{A}^{D_2}$$

**Direct comparisons** to $D_2$ scattering, nucl. model

$\Rightarrow$ Test nucl. corrections to QE assumption on $D_2$

Signal to noise exponentially degrades $\sim e^{-(2M_N - 3M_\pi)t}$

$\Rightarrow$ heavy $M_\pi$ required (for now)

Nuclear models w/ large $M_\pi$ could offer direct comparisons in near future

$M_\pi \approx 713$ MeV

[Phys.Rev.C 103 (2021)]
Closing Remarks

Nuclear xsecs necessary for $\nu$ oscillation experiments, but nucleon amplitude uncertainties still leave something to be desired —

- Nuclear corrections make extraction of nucleon amplitudes difficult
- Nucleon amplitudes from $D_2$ have large uncertainties
- New $D_2$ data not forthcoming (yet)

In absence of modern $D_2$ expt, LQCD can fill missing pieces to puzzle —

- Improved stats on QE form factors
- Resolve tensions in vector form factor parameterizations
- Compute amplitudes that are difficult/impractical to measure in expt
- Match directly to (small) nuclear target data/models

Lots of work to be done, but path forward is clear!