Neutrino-Nucleon Form Factors from Lattice QCD

Aaron S. Meyer (asmeyer.physics@gmail.com)

UC Berkeley/LBNL

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New Directions in Neutrino Nucleus Scattering

Outline

- Introduction
  - Motivation
  - Interaction Modes
  - z expansion
- (High-Level) LQCD Results
  - Fermilab Lattice g_A
  - Callat g_A
  - Quick survey of g_A, r_A^2
- Future Prospects
  - QE
  - RES
  - NN
- Conclusions

Introduction

Challenges with Oscillation Experiments

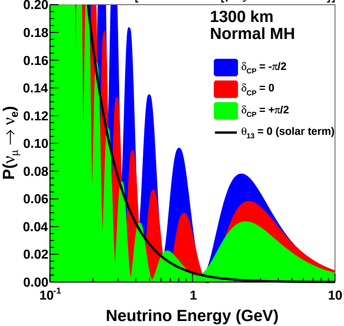
Deep Underground Neutrino Experiment – Flagship \$1b experiment at LBNF

Measure osc. prob. over first two oscillation peaks
$\implies$ E_ν btw. $[0.5, 10]$ GeV

Osc. prob. is fn of L/E_ν
$\implies$ must classify events by E_ν

Neutrinos from secondary beam
$\implies$ E_ν not known event by event
$\implies$ E_ν inferred from distribution

[1512.06148[physics.ins-det]]



Many requisite inputs difficult/impractical to measure from expt
$\implies$ Need ambitious theory support to supplement the experimental effort!

Final State Interactions

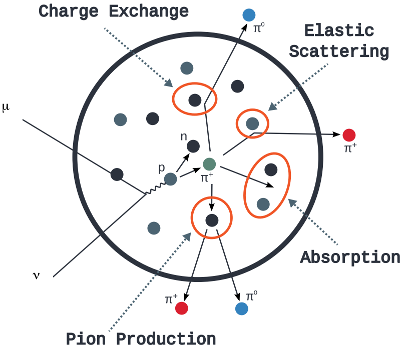
Liquid Argon target
  Nuclear xsec $\Longrightarrow$ nuclear modeling

Nuclear amplitudes constructed from
  one/few nucleon response

Event characteristics change in nuclear medium

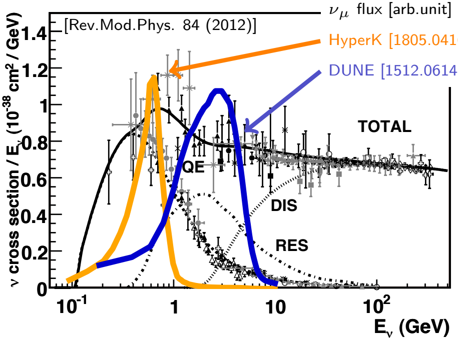
In general: $E_\nu \neq E_{\text{vis.}}$

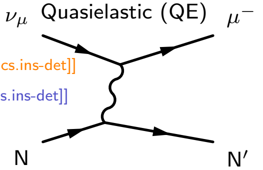
$\Longrightarrow$ Must infer E_ν statistically from Monte Carlo

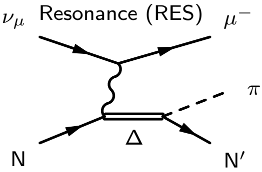


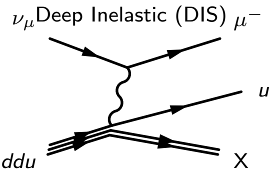
[image:T.Golan]

Neutrino-Nucleon Interaction Modes









Event rate/E_ν bin $\sim \int_{\text{bin}} dE_\nu$ (Flux) × (Xsec)

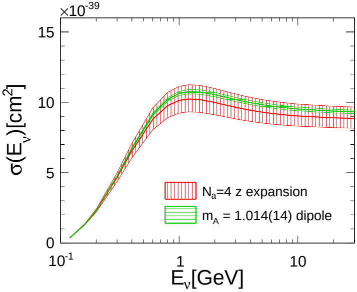
DUNE appx. 1:1:1 events for QE:RES:DIS

$\Longrightarrow$ all interaction topologies are important

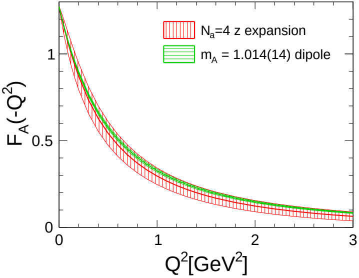
Focus on QE & RES, but LQCD comp. strategies exist for DIS (& SIS) too

z Expansion Fit to QE F_A

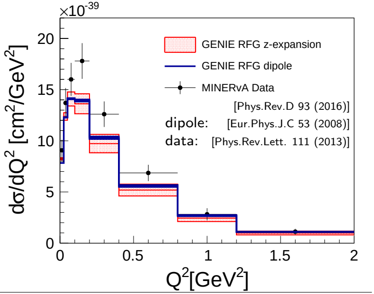
- Dipole has strict Q^2 shape, inconsistent w/ QCD
- Dipole FF ansatz significantly underestimates FF uncertainty



- Nucl. xsec uncertainty from FF same size as data-MC tensions
- Source of tensions unclear btw. nucleon/nuclear

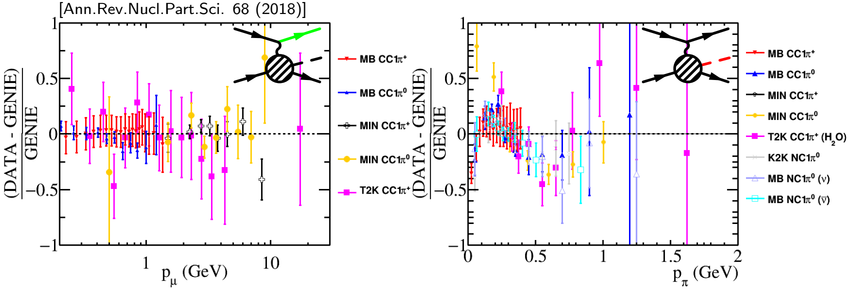


- Model-independent parameterization Order of mag. increase in $\delta\sigma$



[Phys.Rev.D 93 (2016)]
dipole: [Eur.Phys.J.C 53 (2008)]
data: [Phys.Rev.Lett. 111 (2013)]

Leptonic vs Hadronic



Correlated differences between data & MC in leptonic, hadronic models

Balancing act to reconcile two variables

Insufficient model to describe interactions?

Moving Forward

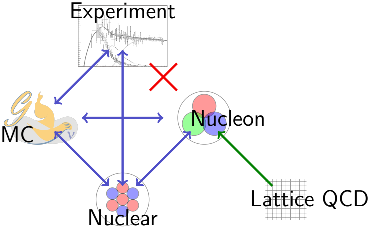
Room for improvement, but what is needed?

Ideal: Modern high stats ν-D_2 scattering bubble chamber expt

Some community push, safety concerns

$\Longrightarrow$ LQCD as a alternative/complement to expt

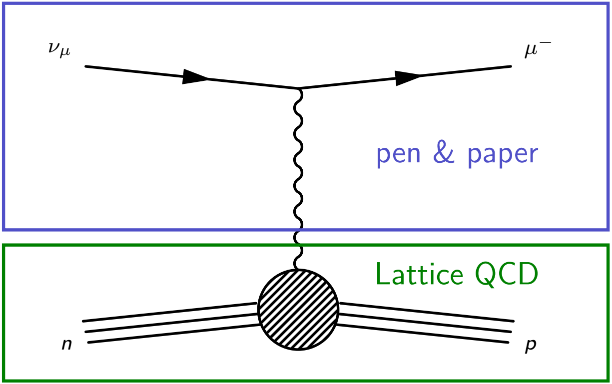
- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



How can Lattice Help?

Lattice is well suited to compute matrix elements:

$$\mathcal{M}_{\nu_\mu n\to\mu p} = \langle\mu|\,(V-A)_\mu\,|\nu\rangle\,\langle p|\,(V-A)_\mu\,|n\rangle$$



Lattice QCD

Lattice QCD: Formalism 1/2

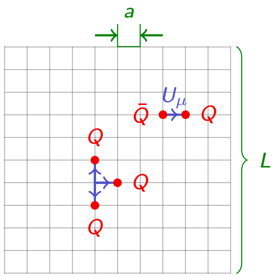
Numerical eval of path integral
Quark, gluon DOFs —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\, \mathcal{D}U \, \exp(-S)\, \mathcal{O}_{\psi}\, [U]$$

Few inputs —

Computational: $am_{(u,d),\text{bare}}$
$am_{s,\text{bare}}$
$\beta = 6/g^2_{\text{bare}}$

Scale setting: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
one per computational input



Results — first principles predictions of QCD

$$M_{\text{hadron}},\ \langle \text{F}|\mathcal{O}|\text{I}\rangle$$

Euclidean time $\implies$ $C(t) \sim e^{-M\cdot t}$

“Complete” error budget $\implies$ extrapolation in a, L, (M_π)

Lattice QCD: Formalism 2/2

Correlation functions computed in euclidean time:

2-point function

$\langle \mathcal{O}_1(t)\mathcal{O}_2(0)\rangle = \sum_n \langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle e^{-E_n t}$

3-point function

$\langle \mathcal{O}_1(t)\mathcal{O}_2(\tau)\mathcal{O}_3(0)\rangle = \sum_{mn} \langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|m\rangle\langle m|\mathcal{O}_3|0\rangle e^{-E_n(t-\tau)-E_m\tau}$

Large t: excited states decay away, signal-to-noise degrades

Computations performed on "gauge ensembles" with fixed physics:

$\{a,\ L,\ M_\pi,\ \ldots\}$

Extrapolate in ensemble parameters to arrive at physical point

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\text{phys}}$ (chiral limit)

Word of caution:

Many collaborations will compare experiment to unextrapolated results...

These values will have uncontrolled systematics.

Make sure you know what you are looking at!

Fermilab Lattice — M_N

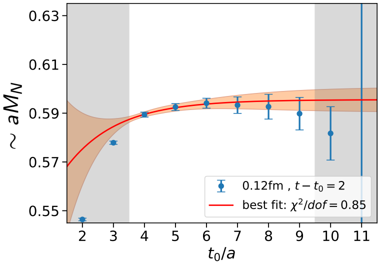
Nucleon mass — $C_{2pt}(t) \sim \sum_n z_n z_n^\dagger e^{-E_n t}, \quad z_n = \langle 0|\mathcal{O}|n\rangle$

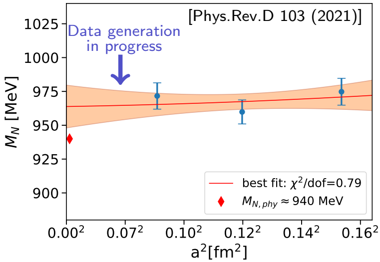
Demonstration of method:
- Baryons w/ “Highly-Improved Staggered Quarks” (HISQ)
- Additional SU(4) “taste” symmetry
- Baryon octet → baryon 572-plet

Complicated group thy, cheaper computation (1-comp spinors)

3 ensembles: all M_π^{phys}; various a, L

Analysis credit: Yin Lin





Fermilab Lattice — g_A, g_V

Axial charge —

$C_{3pt}(t,\tau) \sim \sum_{mn} z_n z_m^\dagger \langle n|\mathcal{A}_\mu|m\rangle e^{-E_n(t-\tau)-E_m\tau}$

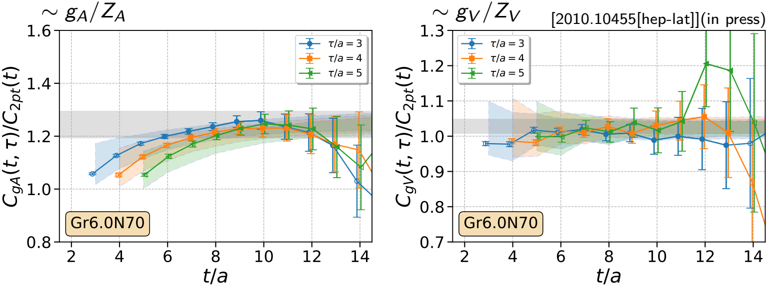
$\langle N|\mathcal{A}_\mu|N\rangle \sim g_A$

Single ensemble: $M_\pi \approx 305$ MeV

World-first 3-point fn w/ HISQ

Interpreting results requires
$SU(2)_{\text{flavor}} \times SU(4)_{\text{taste}} \subset SU(8)$ CG coefs/Wigner-Eckart

Analysis credit: Yin Lin



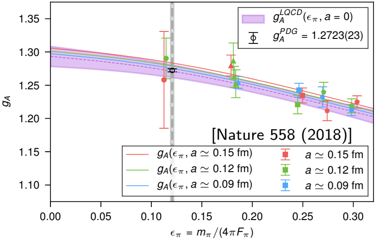
Callat — g_A

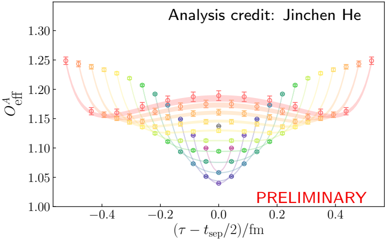
Left: $O(1\%)$ on g_A using Feynman-Hellman inspired technique:

$$\underbrace{\frac{dE_\lambda}{d\lambda}}_{\text{2-pt}} = \underbrace{\langle\psi_\lambda|\frac{dH_\lambda}{d\lambda}|\psi_\lambda\rangle}_{3-pt} \quad \text{w/ source term} \quad \lambda\int d^4x\, \mathcal{A}_\mu(x)$$

Right: Work in progress —

- One ensemble: $a \approx 0.09$ fm, $M_\pi \approx 310$ MeV
- 5-state fit, detailed analysis of excited state contamination
- Compare w/ traditional three-point method





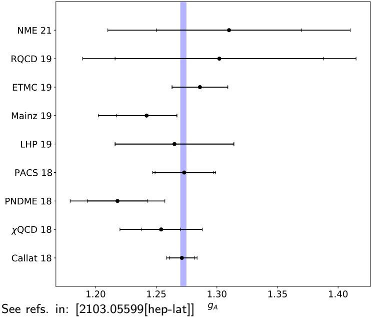
Survey — g_A

State of field circa 2019 summarized in white paper,
written for nonpractitioners: [Eur.Phys.J.A 55 (2019)]

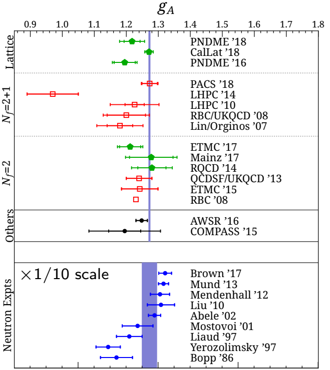
Historically low g_A attributed to N excitations ($N\pi$, RES)
led to apparent violations of PCAC relation

Now:

- Agreement w/ PDG seen consistently
- Details about excitations still unclear



See refs. in: [2103.05599[hep-lat]]



[Eur.Phys.J.A 55 (2019)]

Survey — r_A^2

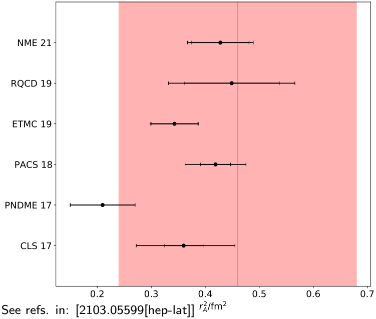
Avoid M_A: only makes sense in dipole $\implies r_A^2 = -(6/g_A) dF_A/dQ^2\big|_{Q^2=0}$

$$\implies r_{A,\text{dipole}}^2 = (1\ \text{GeV}^2/m_A^2) \times 0.466\text{fm}^2$$

Most collaborations have adopted z expansion parameterization

Uncertainties will continue to decrease w/ time

Opinion: Still too early by few years — wait for resolution of excited state issues



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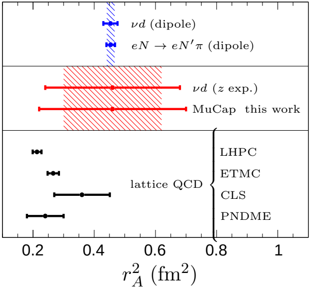
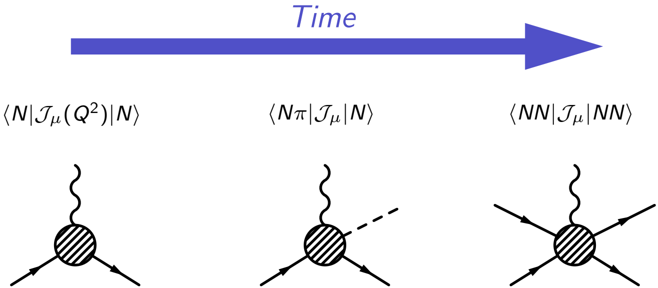


[Eur.Phys.J.A 55 (2019)]

Prospects

Timeline

Very rough sketch of my interpretation of timeline for LQCD computations



$\langle N|\mathcal{J}_\mu(Q^2)|N\rangle$ $\langle N\pi|\mathcal{J}_\mu|N\rangle$ $\langle NN|\mathcal{J}_\mu|NN\rangle$

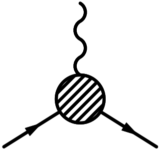
Forward-Looking: QE FFs

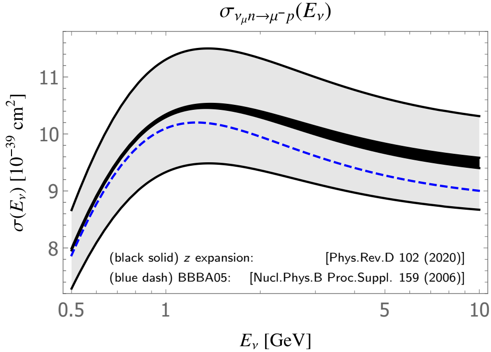
High-precision meas. $\langle N|\mathcal{J}(\vec{q},\nu)|N\rangle \sim F_A(Q^2)$, $F_V(Q^2)$,

Realistically: $Q^2 \sim 0-1\ \mathrm{GeV}^2$

Reduce uncertainties of axial/pseudoscalar FFs
  Fill in where difficult/impractical to get expt data

Resolve tensions btw vector FF parameterizations
  Discrepancy of proton mag FF $\Longrightarrow$ uncertainty floor for axial FF





Forward-Looking: Resonant Transitions

Constrain amplitudes/FFs that are inaccessible to expt

$\langle N\pi|\mathcal{J}_\mu(\vec{q}, W)|N\rangle$ [res.+nonres.] $\sim C_{5A}(Q^2)$

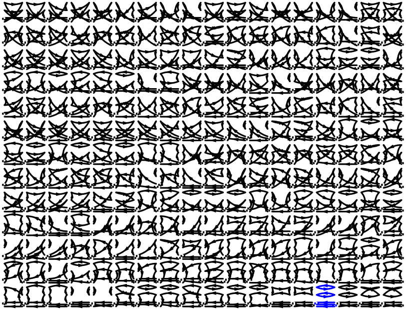
Theoretically & computationally challenging:

$\implies$ hard cutoff at $N\pi\pi$ threshold (for now)

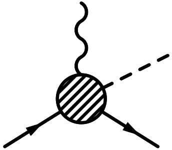
$\implies$ raise M_π to circumvent, $\Delta \to$ stable

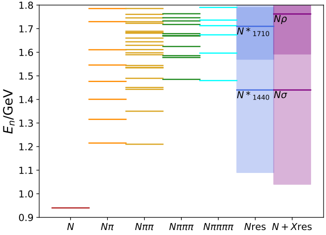
Dense spectrum of states

$\implies$ In practice requires $N\pi$, $(N\pi\pi)$ operators



[1912.04917[hep-lat]]
[github.com/lehner/Wick]





$M_\pi L = 4,\ M_\pi = 135$ MeV

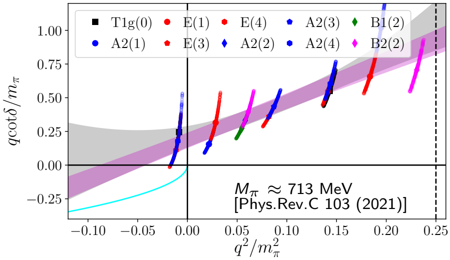
Forward-Looking: NN g_A

$\langle NN|\mathcal{J}_\mu|NN\rangle \sim F_V^{D_2}, F_A^{D_2}$

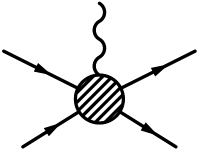
Direct comparisons to D_2 scattering, nucl. model
$\implies$ Test nucl. corrections to QE assumption on D_2

Signal to noise exponentially degrades $\sim e^{-(2M_N-3M_\pi)t}$
$\implies$ heavy M_π required (for now)

Nuclear models w/ large M_π could offer direct comparisons in near future



$M_\pi \approx 713$ MeV
[Phys.Rev.C 103 (2021)]



Closing Remarks

Nuclear xsecs necessary for ν oscillation experiments,
but nucleon amplitude uncertainties still leave something to be desired —

- Nuclear corrections make extraction of nucleon amplitudes difficult
- Nucleon amplitudes from D_2 have large uncertainties
- New D_2 data not forthcoming (yet)

In absence of modern D_2 expt, LQCD can fill missing pieces to puzzle —

- Improved stats on QE form factors
- Resolve tensions in vector form factor parameterizations
- Compute amplitudes that are difficult/impractical to measure in expt
- Match directly to (small) nuclear target data/models

Lots of work to be done, but path forward is clear!