

# New directions in Neutrino-Nucleus Scattering

## NUSTEC Workshop

16 March, 2021

# Radiative corrections



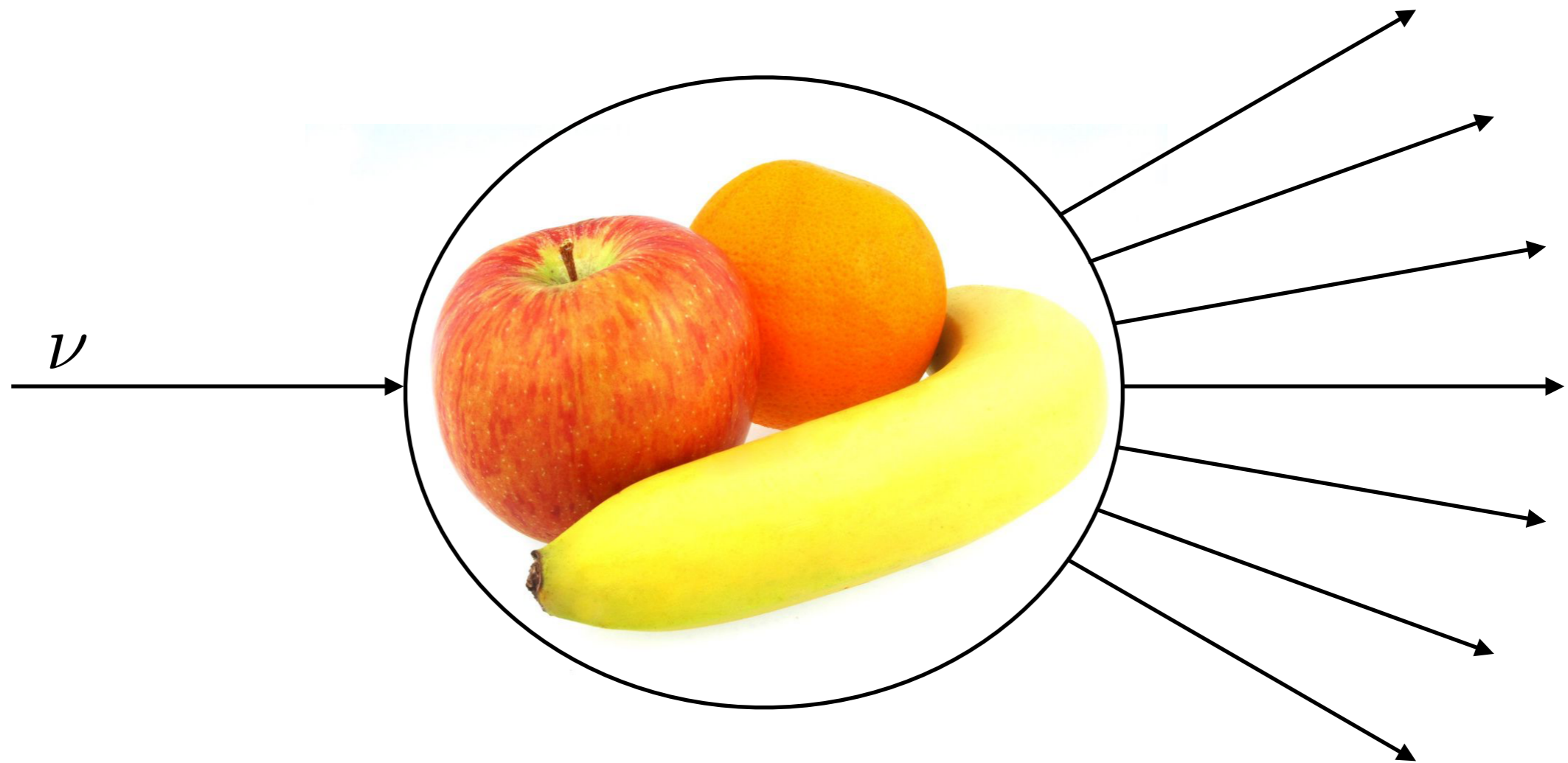
Oleksandr Tomalak

URA Visiting Scholar at Fermilab

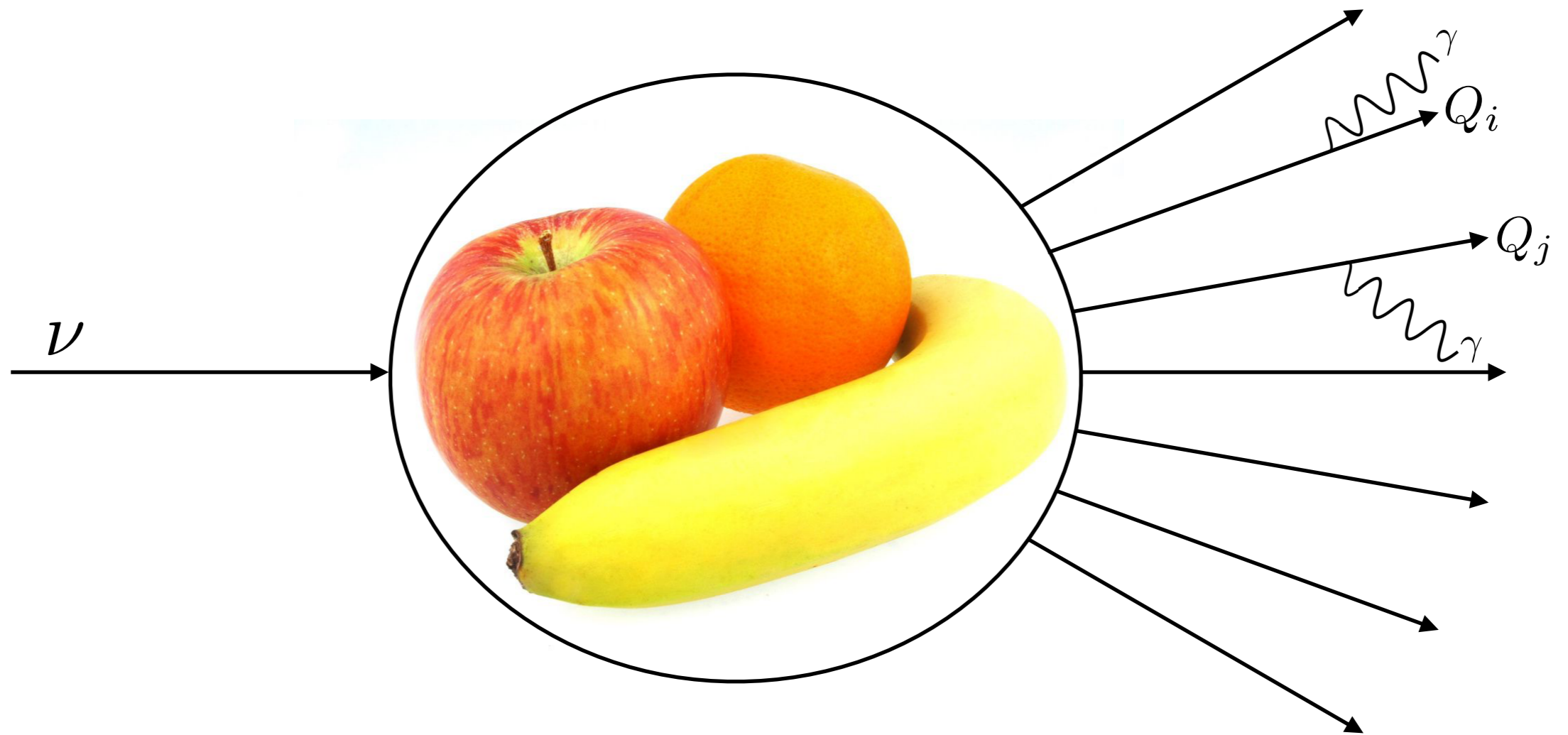
# Outline

- 1) microscopic EFT for neutrino physics
- 2) coherent elastic **neutrino-nucleus** scattering (CE $\nu$ NS)
- 3) charged-current scattering on **nucleons**

# Neutrino interactions

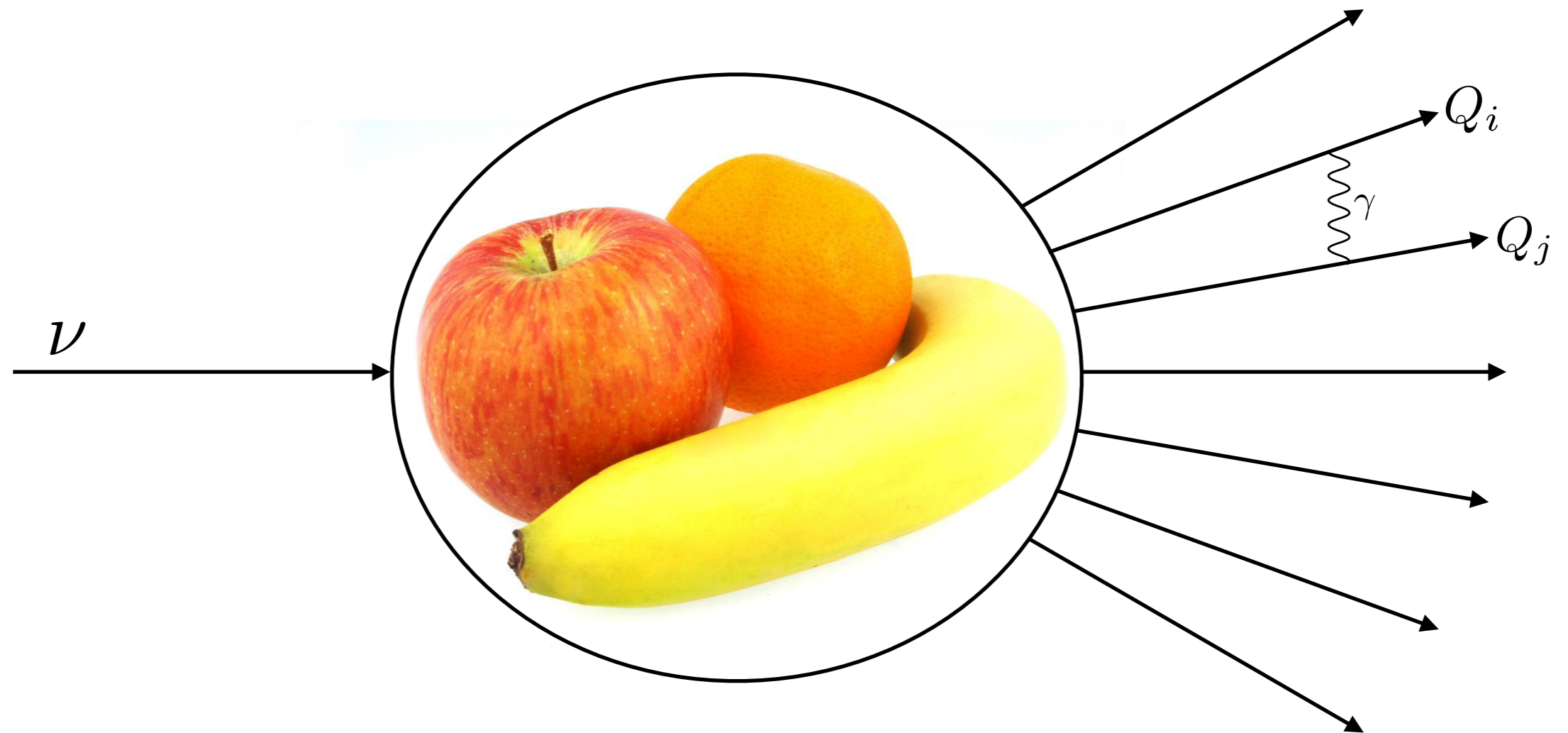


# QED corrections



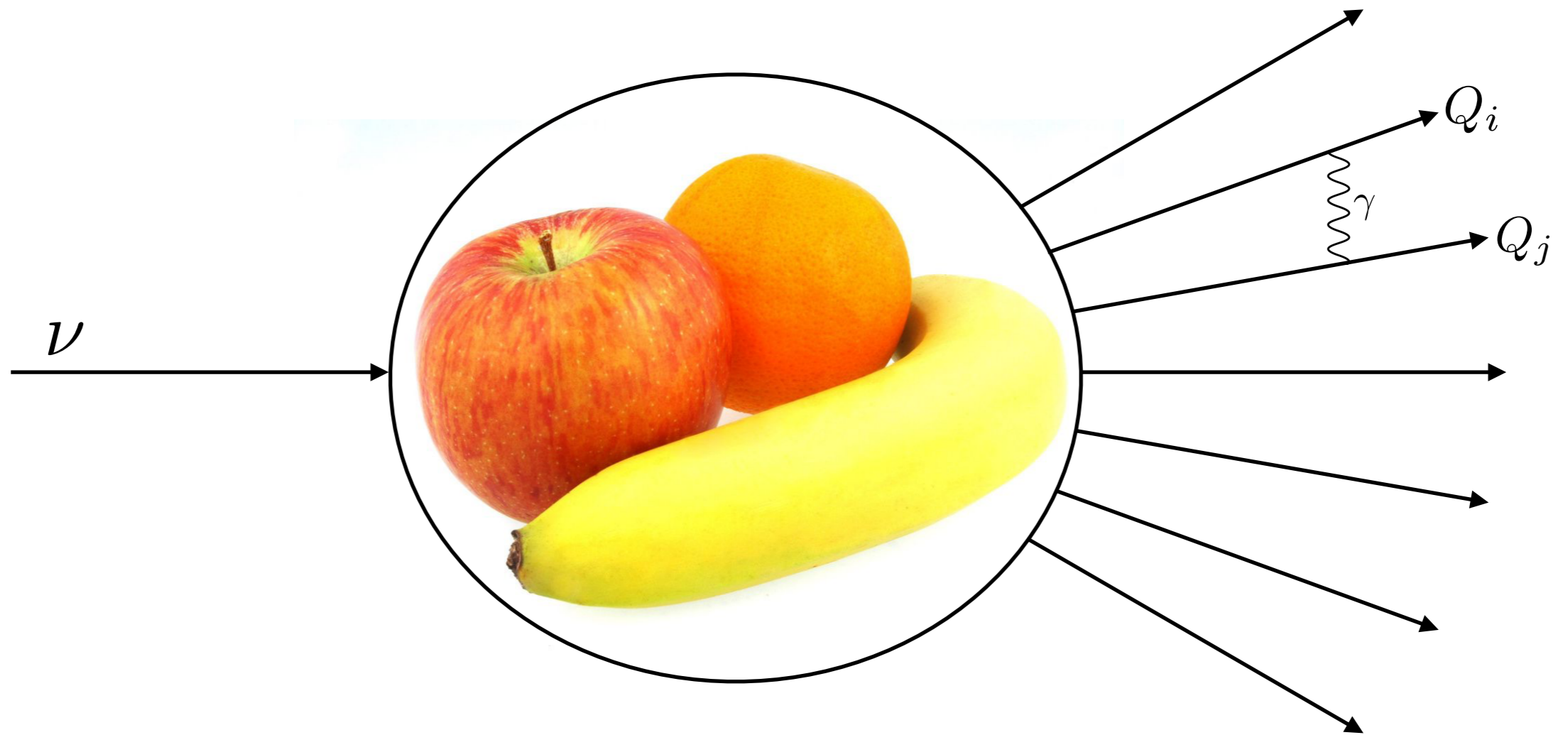
- all charged particles couple to real and virtual photons

# QED corrections



- all charged particles couple to real and virtual photons

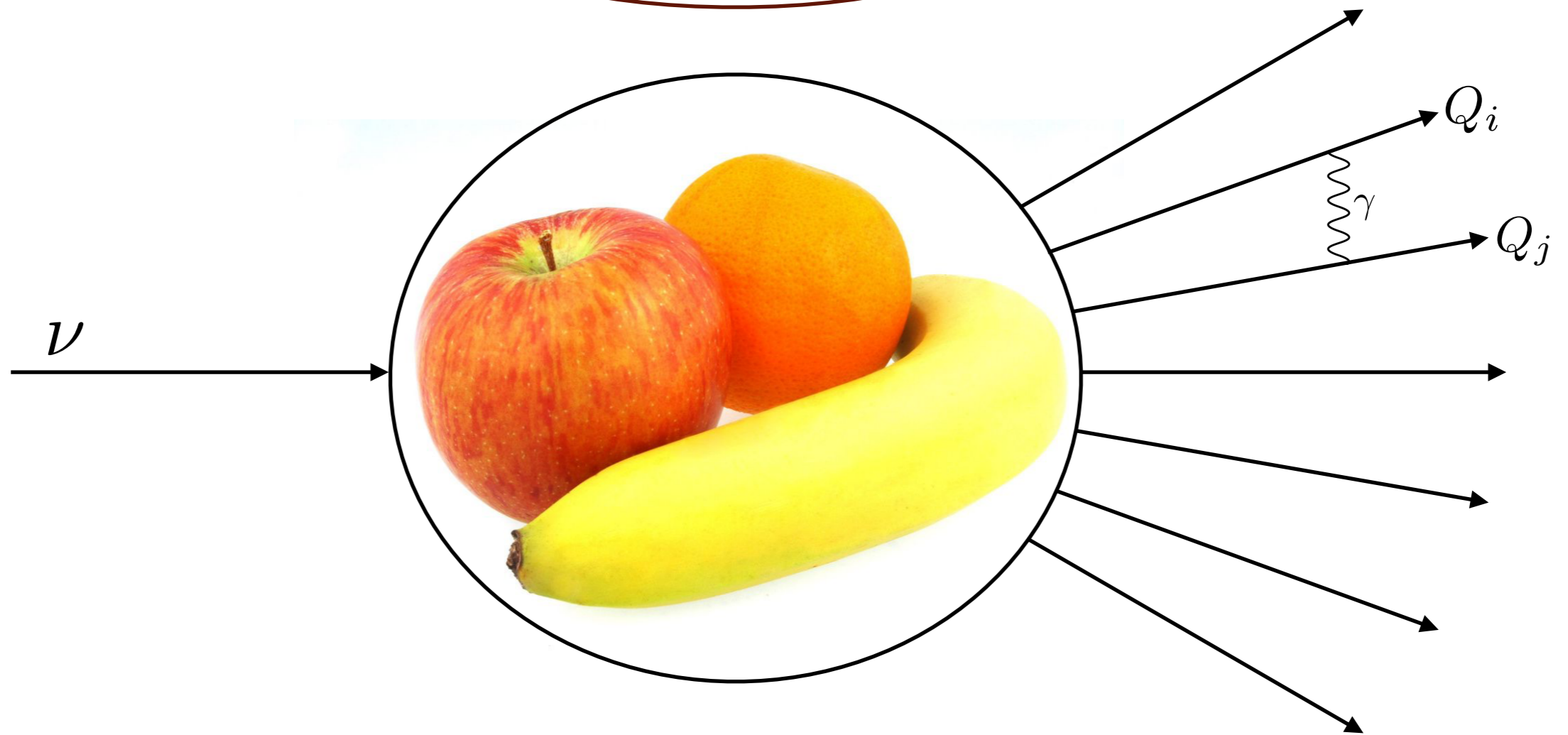
# QED corrections



-  $\frac{\alpha}{\pi} \sim 0.2\%$  suppression by electromagnetic coupling constant

# QED corrections

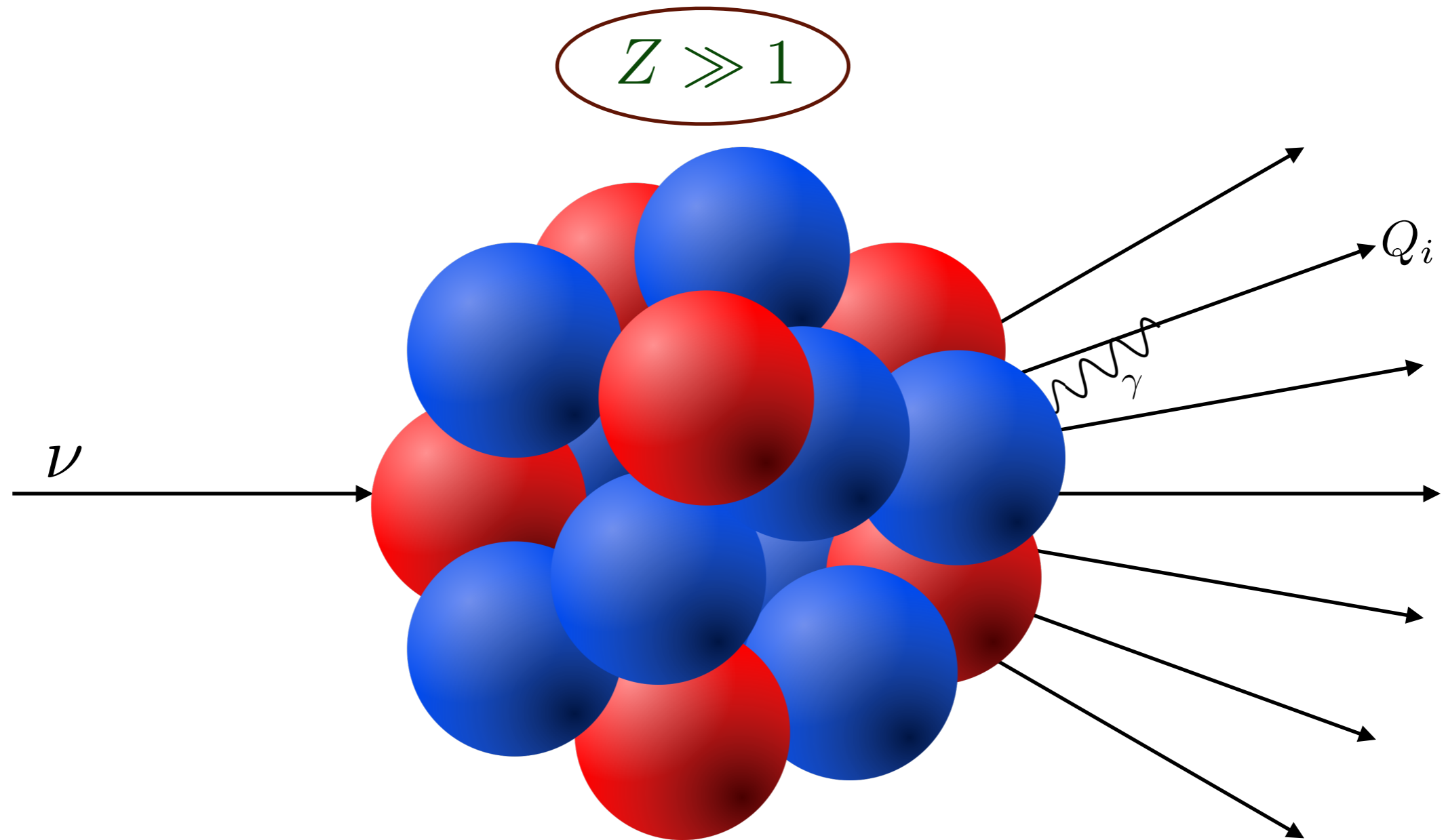
$$m_e \ll m_\mu \ll E_\nu$$



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

- scale separation introduces large flavor-dependent QED logarithms

# QED corrections



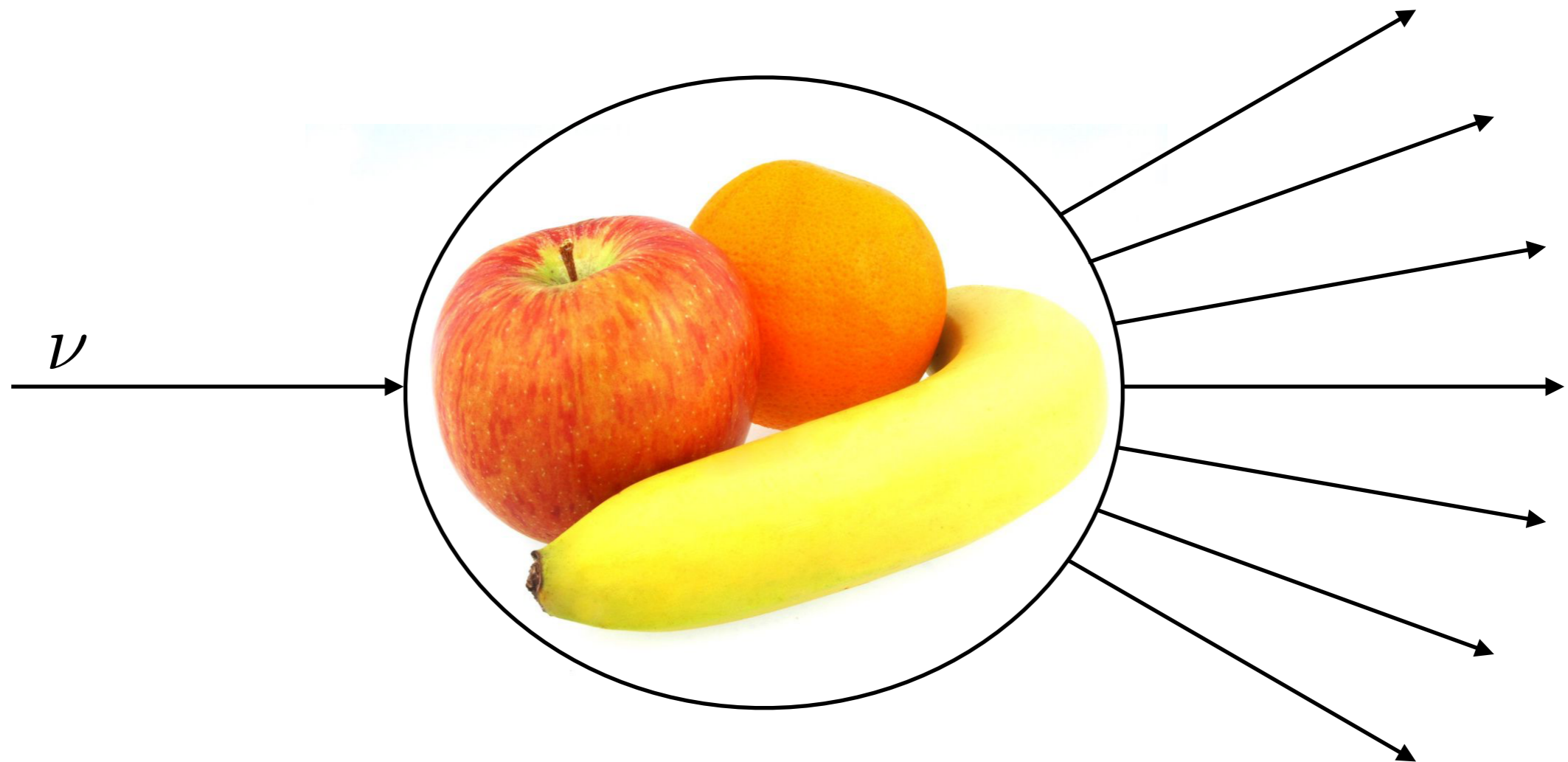
$\frac{\alpha}{\pi} \sim 0.2\%$  multiplied by target nucleus charge  $Z \lesssim 10 - 20$   
talk by Ryan Plestid

- Coulomb corrections are enhanced by nucleus charge factor



# QED corrections

neutral-current interactions

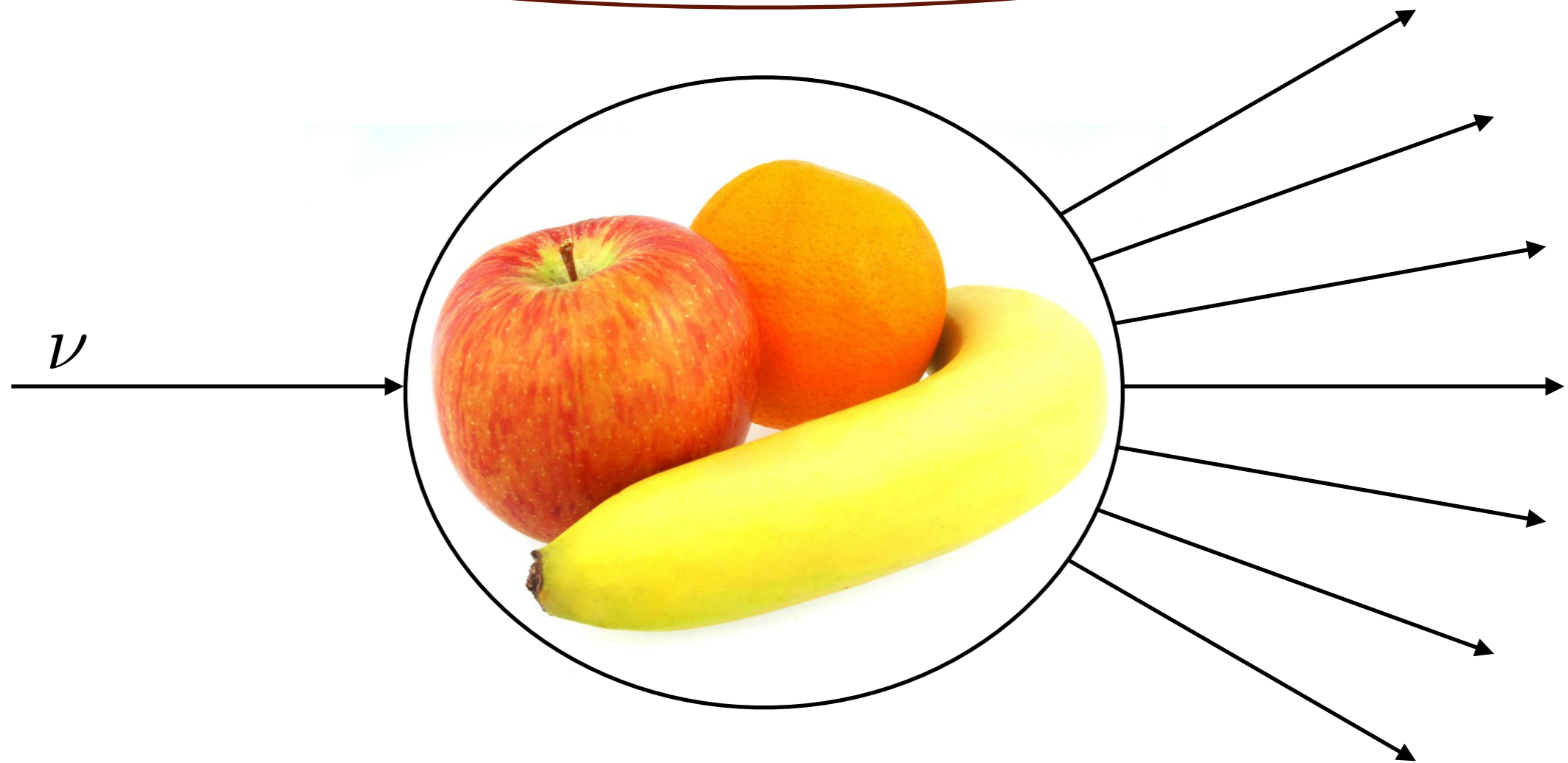


$X \frac{\alpha}{\pi} \sim 0.2\%$  multiplied by kinematic-dependent factors

- kinematic dependence and factor  $X$  can enhance QED corrections

# Electroweak corrections

$$m_e, m_\mu, M, E_\nu \ll M_W, M_Z, m_t, m_H$$



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \frac{1}{\sin^2 \theta_W}, \ln \frac{M_Z}{M}, \ln \frac{M_t}{M}, \dots$$

- electroweak corrections can be included in low-energy interactions

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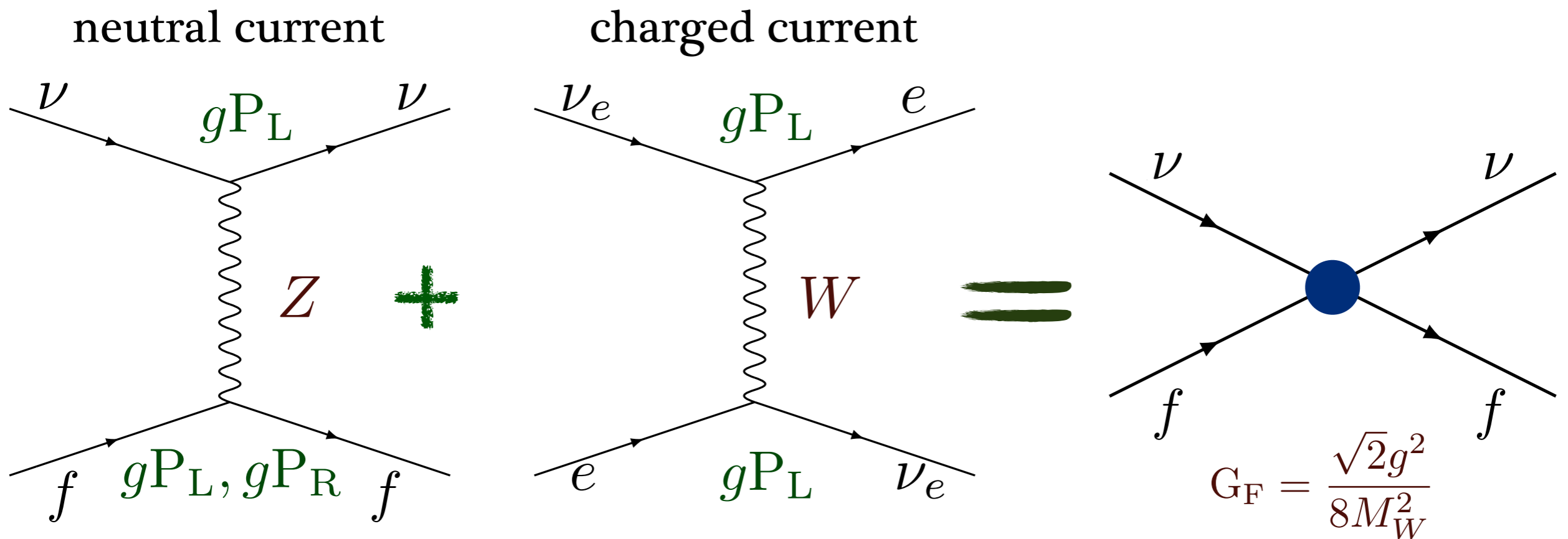
# Microscopic EFT for neutrino physics

O. T. and Richard J Hill, *Phys Lett B* 805 (2020) 135466

# Neutrino scattering in EFT. Matching

- tree-level matching to low-energy EFT

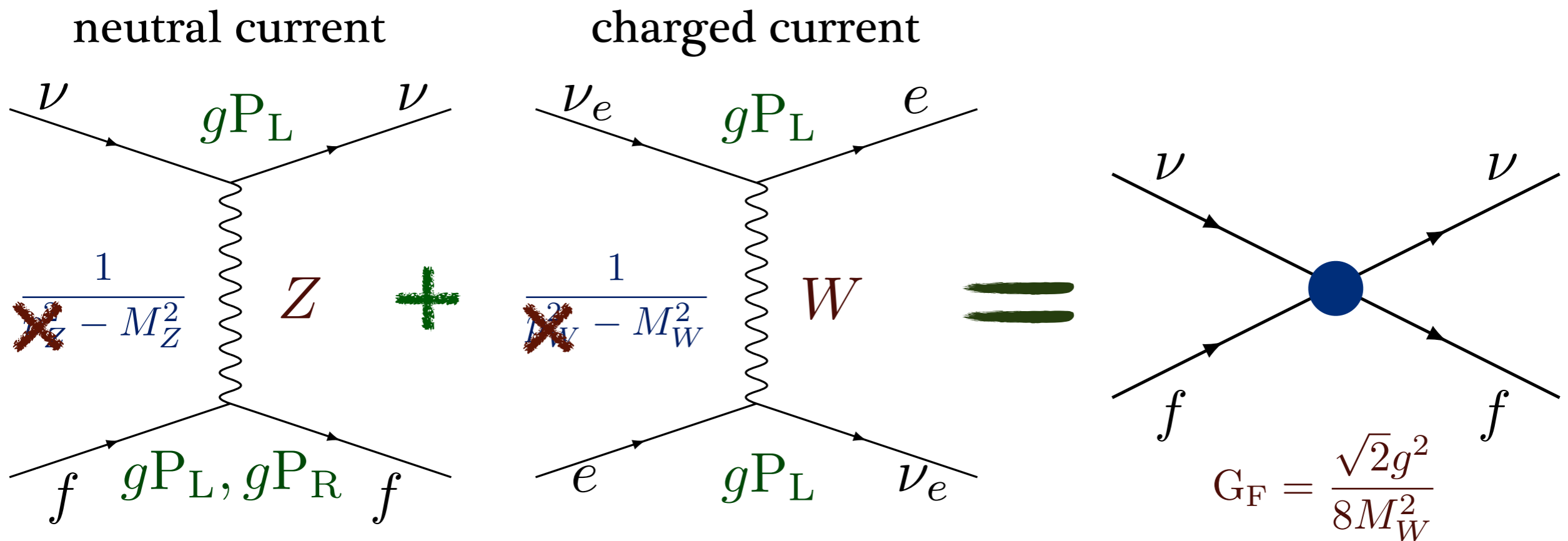
$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \bar{f} \gamma^\mu \left( c_L^{\nu_\ell f} P_L + c_R^{\nu_\ell f} P_R \right) f$$



# Neutrino scattering in EFT. Matching

- tree-level matching to low-energy EFT

$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \bar{f} \gamma^\mu \left( c_L^{\nu_\ell f} P_L + c_R^{\nu_\ell f} P_R \right) f$$



- masses of W and Z are large: integrate out W and Z at tree level

# Neutrino scattering in EFT. Matching

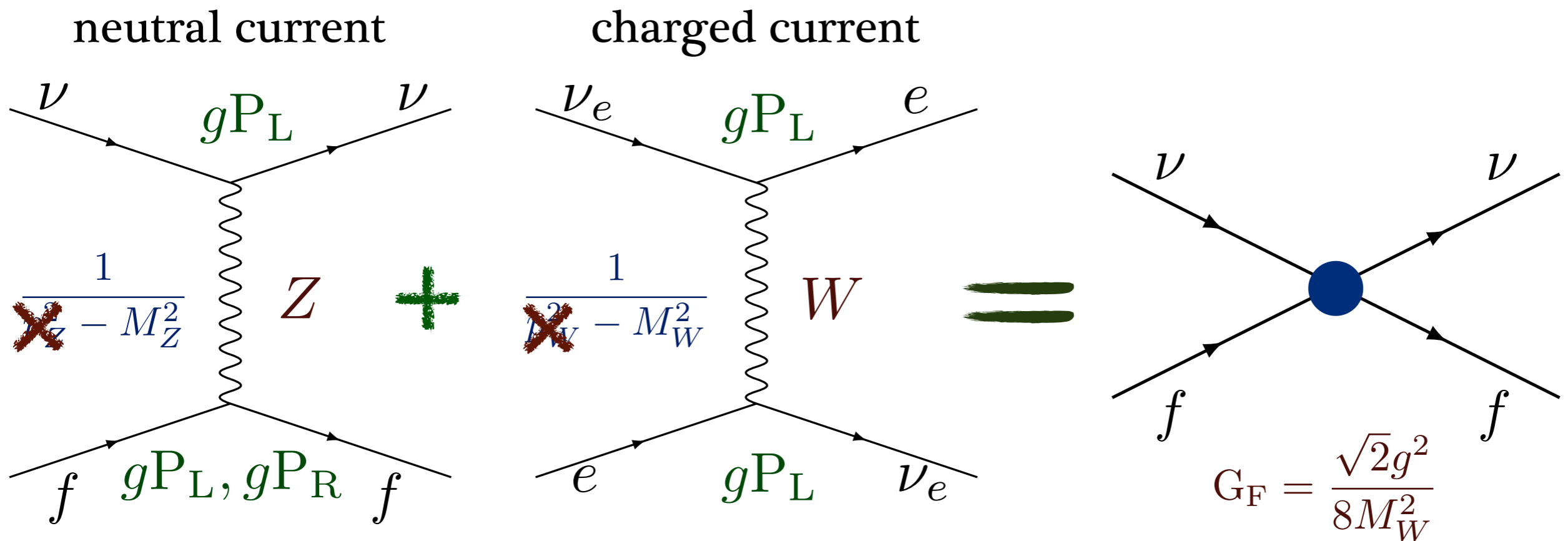
- tree-level matching to low-energy EFT

$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \bar{f} \gamma^\mu \left( c_L^{\nu_\ell f} P_L + c_R^{\nu_\ell f} P_R \right) f$$

couplings to electron

$$c_R = 2\sqrt{2}G_F \sin^2 \theta_W \quad c_L = 2\sqrt{2}G_F (\sin^2 \theta_W - 0.5 + \delta_{\nu, \nu_e})$$

Weinberg (1967), 't Hooft (1971)



- masses of W and Z are large: integrate out W and Z at tree level

# Neutrino scattering in EFT. Matching

- matching to low-energy EFT

$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \bar{f} \gamma^\mu \left( c_L^{\nu_\ell f} P_L + c_R^{\nu_\ell f} P_R \right) f$$

- consider only leading in  $G_F$  terms: loop corrections in  $\alpha$ ,  $\alpha_s$
- gauge-invariant matching of amplitudes, renormalized in  $\overline{\text{MS}}$  scheme

$$\mathcal{M}^{\text{SM}} = \mathcal{M}^{\text{EFT}}$$

- $G_F$ : combination of parameters is precisely measured

$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$$

MULAN (2012)

- matching at order  $\alpha\alpha_s$ : left- and right-handed couplings
- muon lifetime measurement improves precision

# Running to low scales

$M_Z$  - integrate out top, Z, W, h

% running effects

$m_b$

$m_\tau$  - integrate out GeV particles

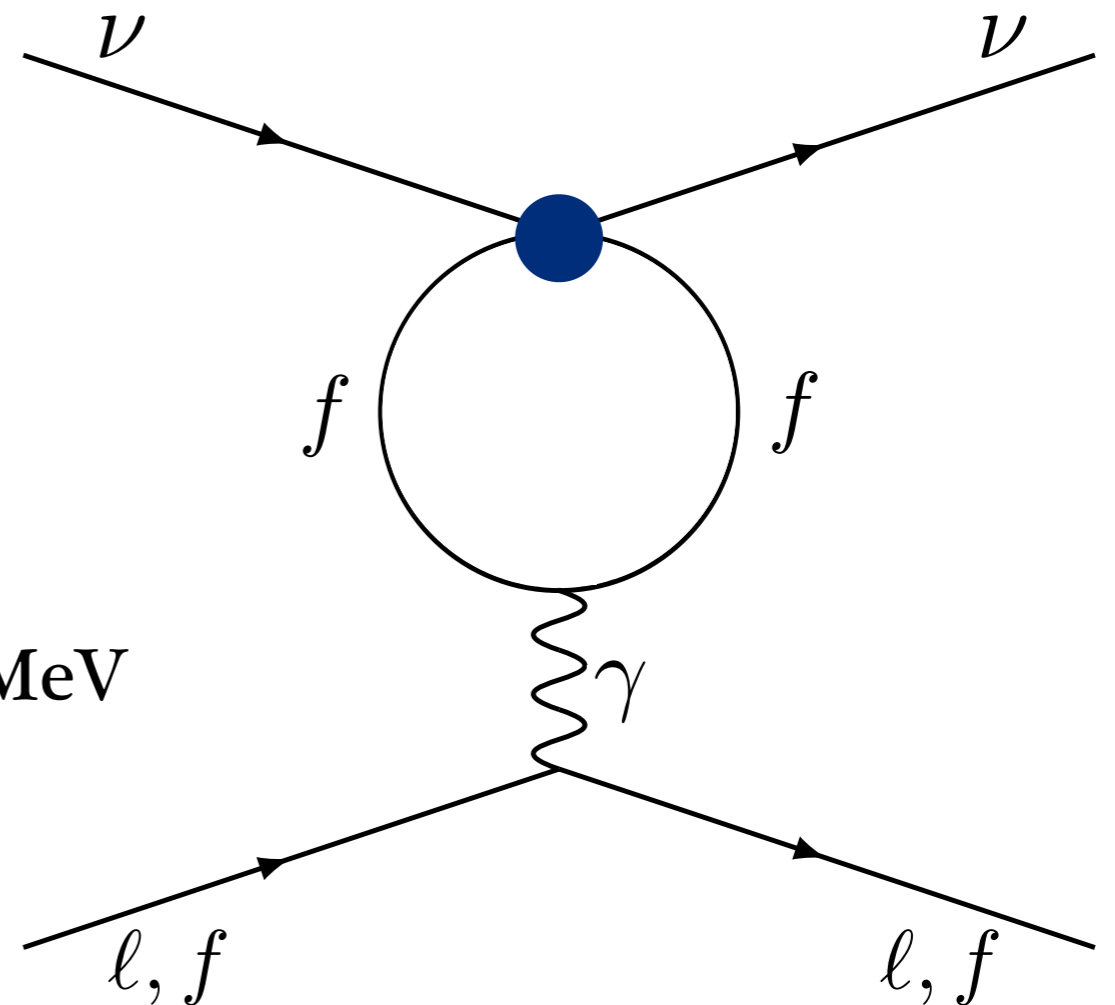
$m_c$

-  $\alpha_s$  becomes too strong

- hadronic physics down to 140 MeV

$m_\pi$

- theory with leptons





# Running to low scales

$M_Z$  - integrate out top, Z, W, h

$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \bar{f} \gamma^\mu \left( c_L^{\nu_\ell f} P_L + c_R^{\nu_\ell f} P_R \right) f$$

$m_b$

$m_\tau$  - integrate out GeV particles

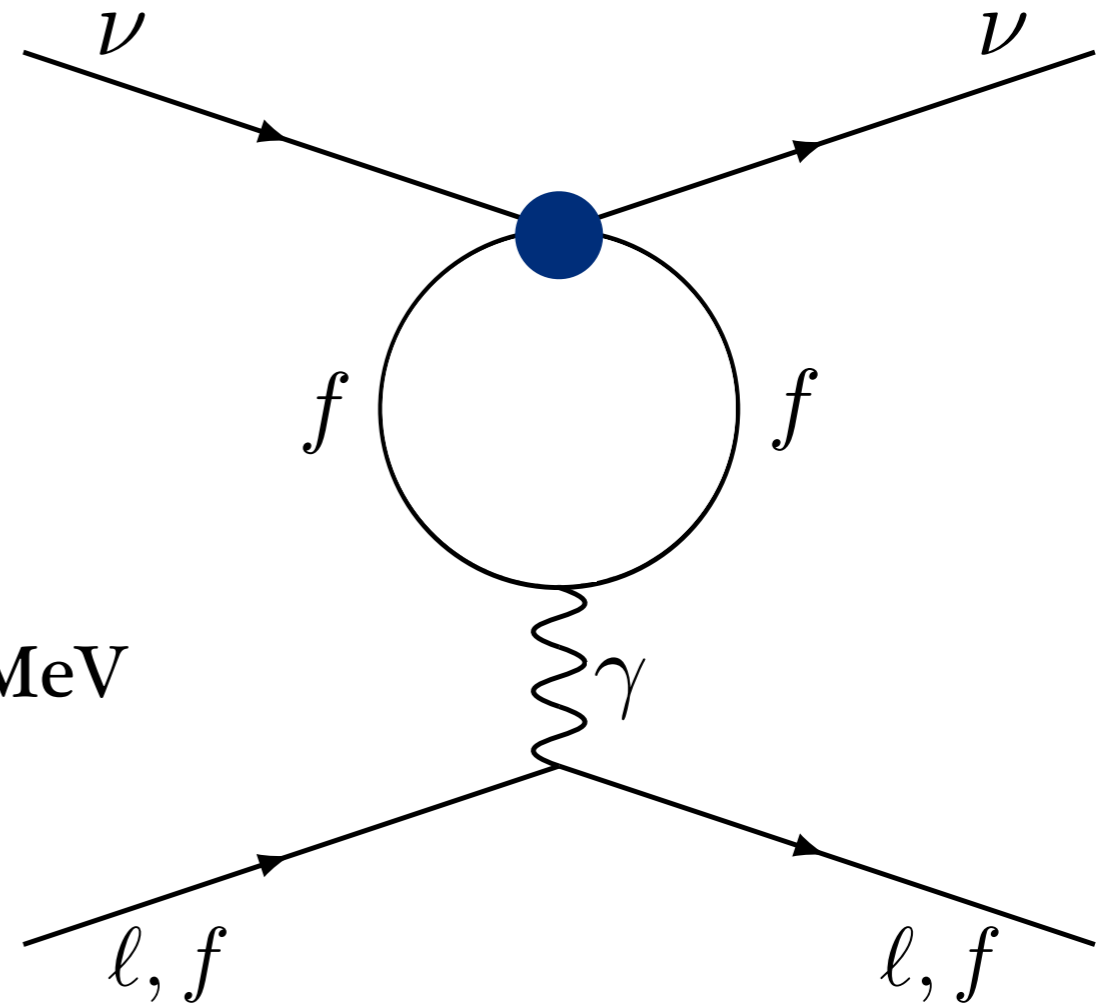
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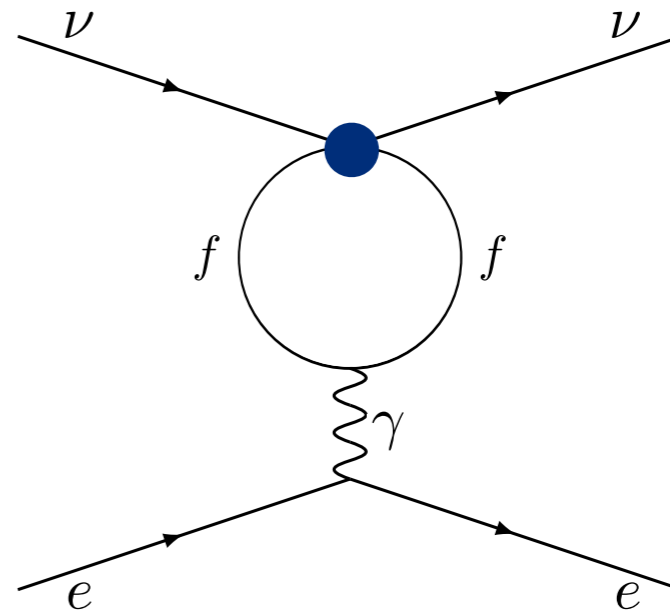
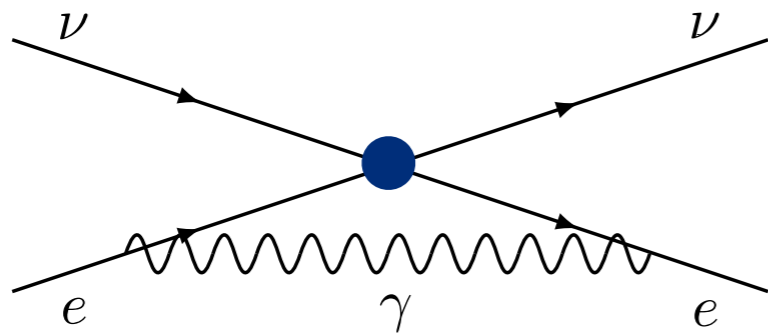
- hadronic physics down to 140 MeV

$m_\pi$

- theory with leptons



- precise mapping from electroweak to hadronic scales



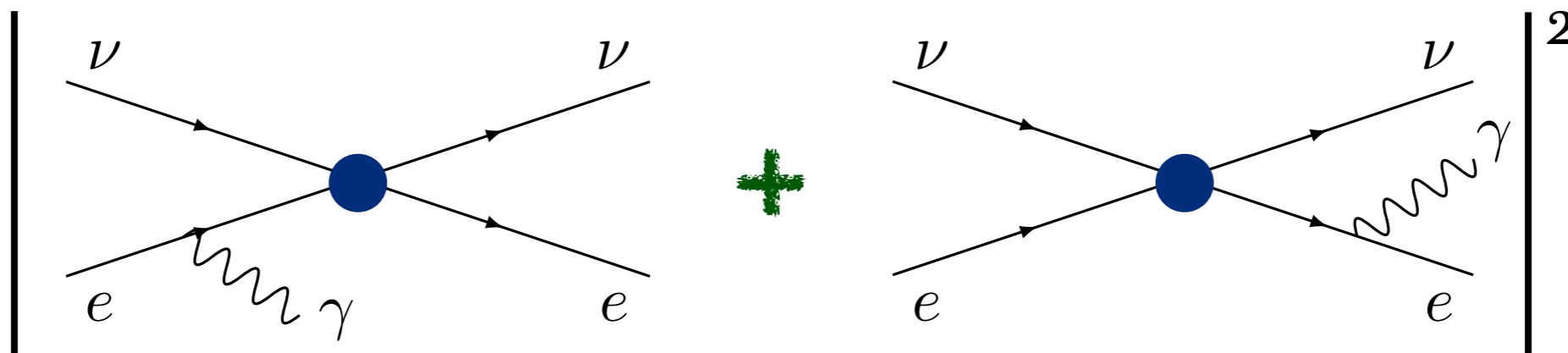
# Neutrino-electron scattering

O. T. and Richard J Hill, Phys. Rev. D 101 (2020) 3, 033006

poster at Neutrino 2020:

<https://youtu.be/mrW4aYjP57w>

known at permille level





# Coherent elastic neutrino-nucleus scattering

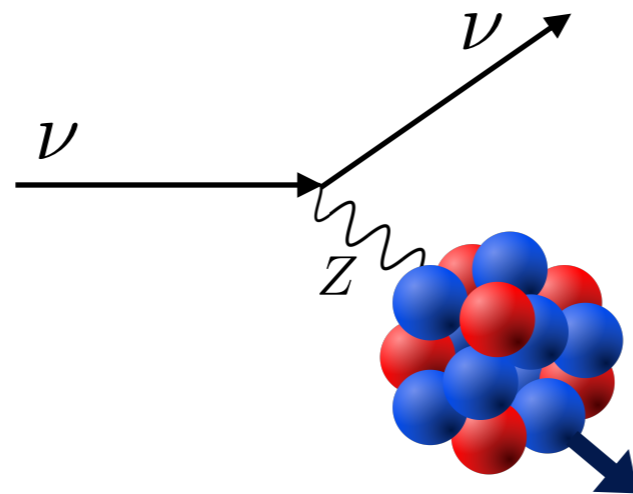
O.T., Pedro Machado, Vishvas Pandey and Ryan Plestid, JHEP 2102, 097 (2021)

talks by Yiyu Zhang, Erin Conley, Diego Aristizabal,  
Dr. Matthew Heath, Dr. Sonia Bacca

# Coherent elastic neutrino-nucleus scattering

- at low neutrino energies (<50 MeV) nuclear state is unchanged  
nucleus recoils as a whole

Stodolsky (1966), Freedman (1974), Kopeliovich and Frankfurt (1974)



recoil nucleus energy  $T$

- large cross section scales as squared number of neutrons  $N^2$

$$\frac{d\sigma}{dT} \approx \frac{G_F^2 M_A}{4\pi} \left(1 - \frac{M_A T}{2E_\nu^2}\right) (N - (1 - 4\sin^2 \theta_W) Z)^2$$

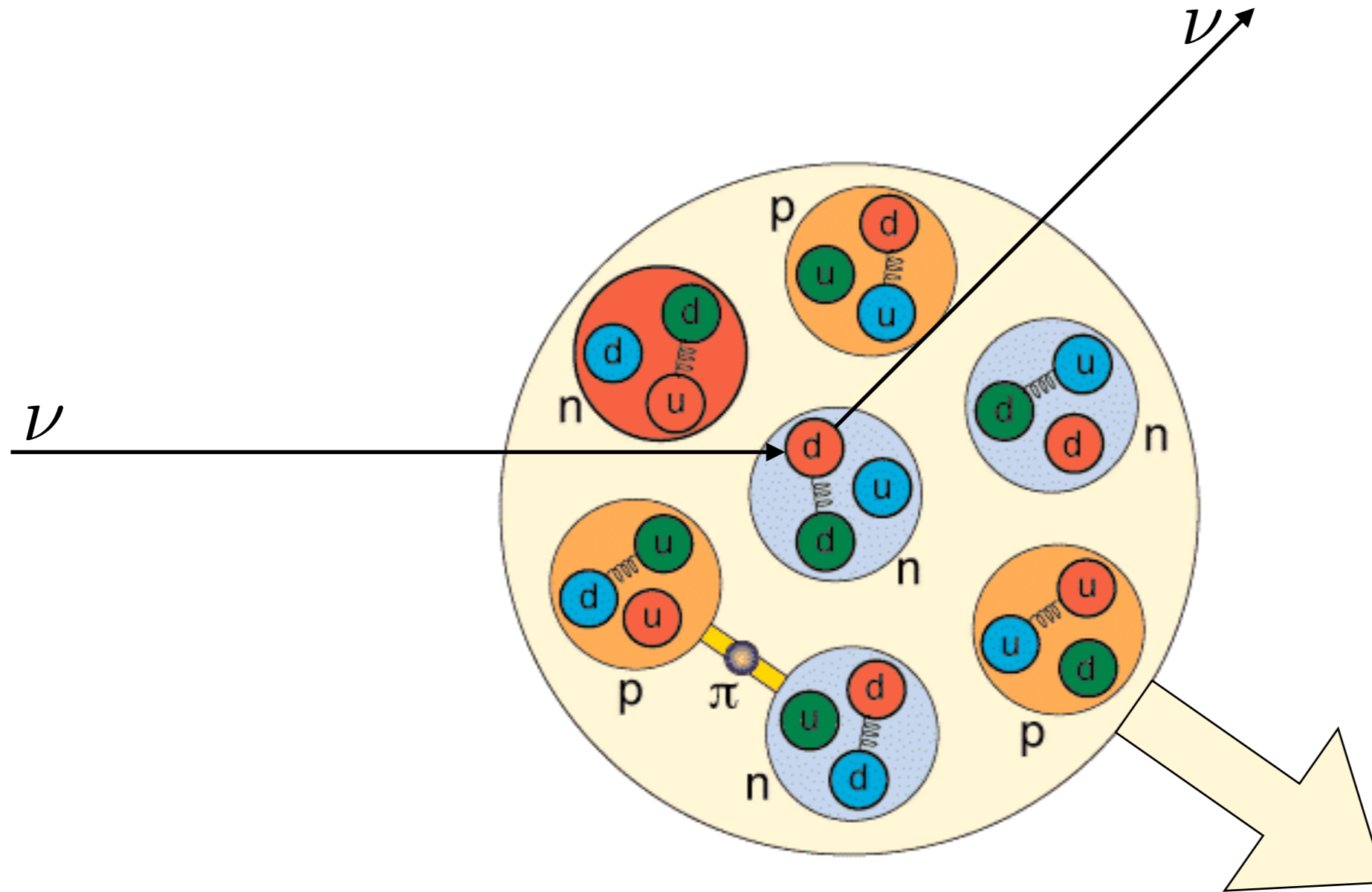
- first detection in 2017 at SNS, measured on CsI and Ar

COHERENT, Science 357 (2017) 6356, 1123-1126

- rapidly developing field nowadays

- CEvNS enters precision era with  $\pi$ DAR sources

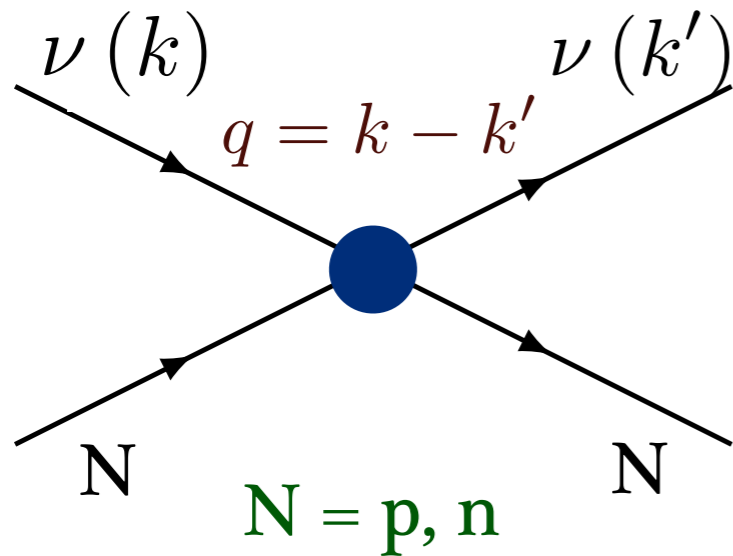
# From quarks to nuclei



[fafnir.phyast.pitt.edu](http://fafnir.phyast.pitt.edu)

- scattering on quarks in nucleons in nucleus

# From quarks to nucleons



momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- neutral-current nucleon matrix elements

$$P_{L,R} = \frac{1 \mp \gamma_5}{2}$$

$$\mathcal{M} \sim \bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \langle N | \sum_q \bar{q} \gamma^\mu (c_L^{\nu\ell q} P_L + c_R^{\nu\ell q} P_R) q | N \rangle$$

$$\mathcal{M} \sim G_E(Q^2), G_M(Q^2), F_A(Q^2), F_P(Q^2)$$

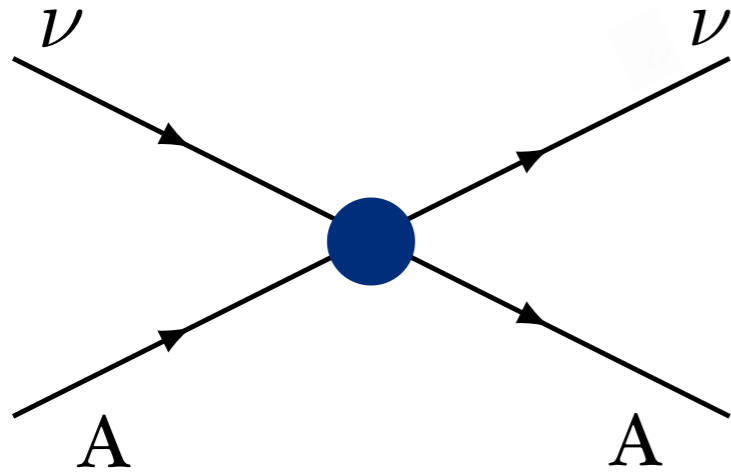
form factors:

electric and magnetic

axial and pseudoscalar

- form factors describe matrix elements of quark currents
- $\pi$ DAR sources: only normalizations and charge radii

# From nucleons to nuclei



- tree-level cross section

$$\frac{d\sigma}{dT} = \frac{G_F^2 M_A}{4\pi} \left( 1 - \frac{T}{E_\nu} - \frac{M_A T}{2E_\nu^2} \right) F_W^2(Q^2)$$

spin-0 nuclei

- sum over nucleons with point-nucleon form factors  $f_p, f_n$

$$F_W = \left( \frac{c_L^{\nu\ell u} + c_R^{\nu\ell u}}{\sqrt{2}G_F} G_E^{n,u} + \frac{c_L^{\nu\ell d} + c_R^{\nu\ell d}}{\sqrt{2}G_F} G_E^{n,d} \right) f_n + (n \leftrightarrow p)$$

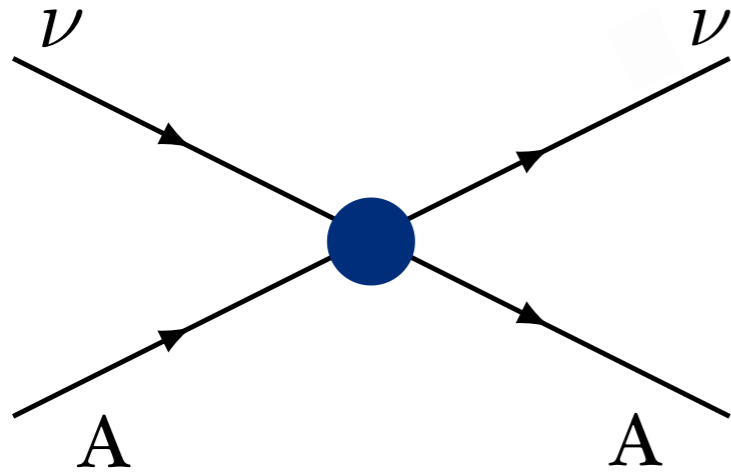
- flavor-independent form factor above GeV scale

-  $Q^2/M^2$  corrections and spin-dependent terms are known

Hoferichter et al. (2020)

- point-nucleon form factors: distribution of nucleons in nuclei
- $\pi$ DAR sources: factorization starting from quark level

# CEvNS cross section on spin-0 nuclei

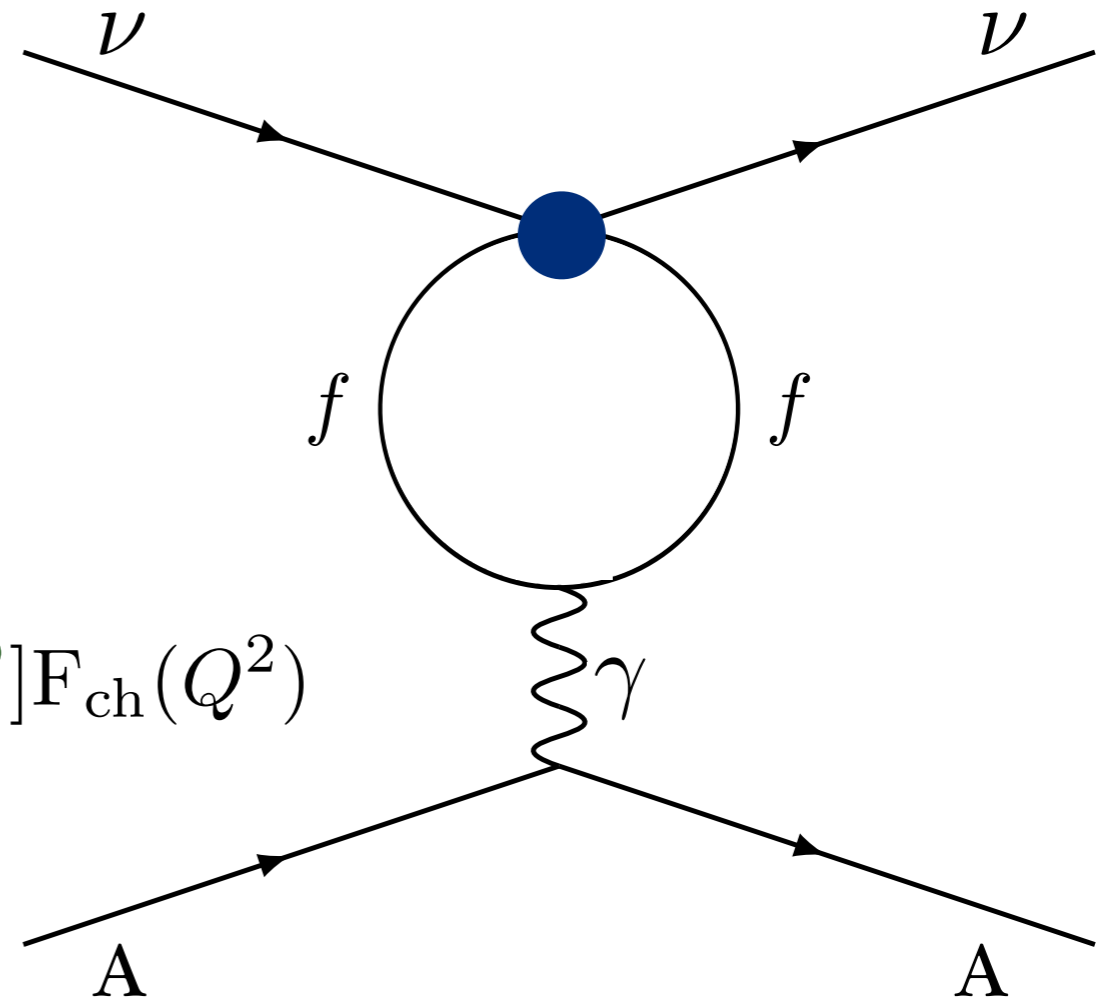


- tree-level cross section

$$\frac{d\sigma}{dT} = \frac{G_F^2 M_A}{4\pi} \left( 1 - \frac{T}{E_\nu} - \frac{M_A T}{2E_\nu^2} \right) F_W^2(Q^2)$$

- effect of radiative corrections

$$F_W(Q^2) \rightarrow F_W(Q^2) + \frac{\alpha}{\pi} [\delta^{\nu e} + \delta^{\text{QCD}}] F_{\text{ch}}(Q^2)$$



- radiative corrections enter with the nucleus charge form factor



# Virtual QED corrections. Fermion loop

- all charged fermions contribute to elastic scattering at one loop

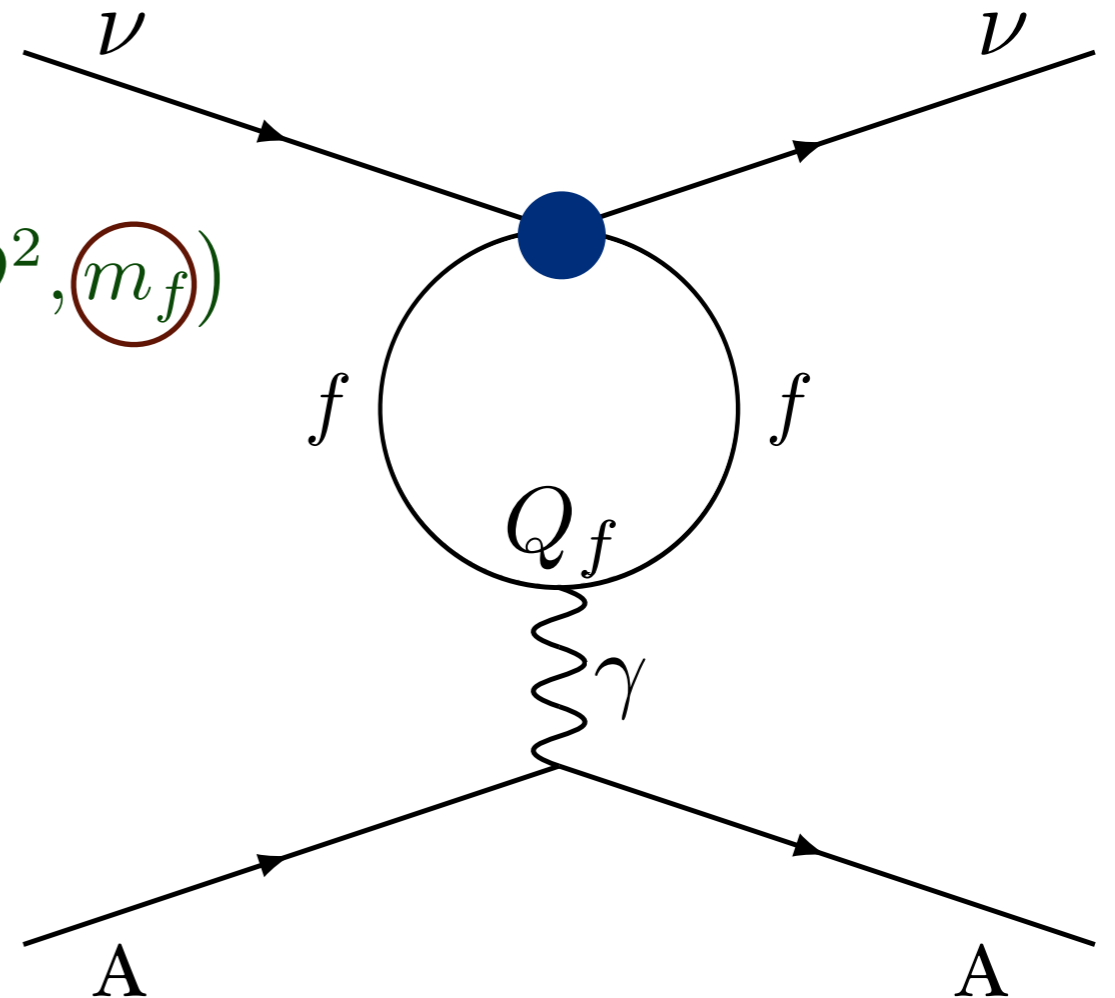
$$\mathcal{L}_{\text{eff}} = -\bar{\nu}_\ell \gamma_\mu P_L \nu_\ell \cdot \bar{f} \gamma^\mu \left( c_L^{\nu_\ell f} P_L + c_R^{\nu_\ell f} P_R \right) f$$

- lepton loops

$$\delta^{\nu_\ell} = - \sum_f \frac{c_L^{\nu_\ell f} + c_R^{\nu_\ell f}}{\sqrt{2} G_F} Q_f \Pi(Q^2, m_f)$$

- origin of flavor dependence

$$c_L^{\nu_e \mu} = c_L^{\nu_\mu e} \neq c_L^{\nu_\mu \mu} = c_L^{\nu_e e}$$



- lepton mass breaks "flavor universality"

# Light-quark contribution

- description in terms of quarks is invalid at CEvNS kinematics

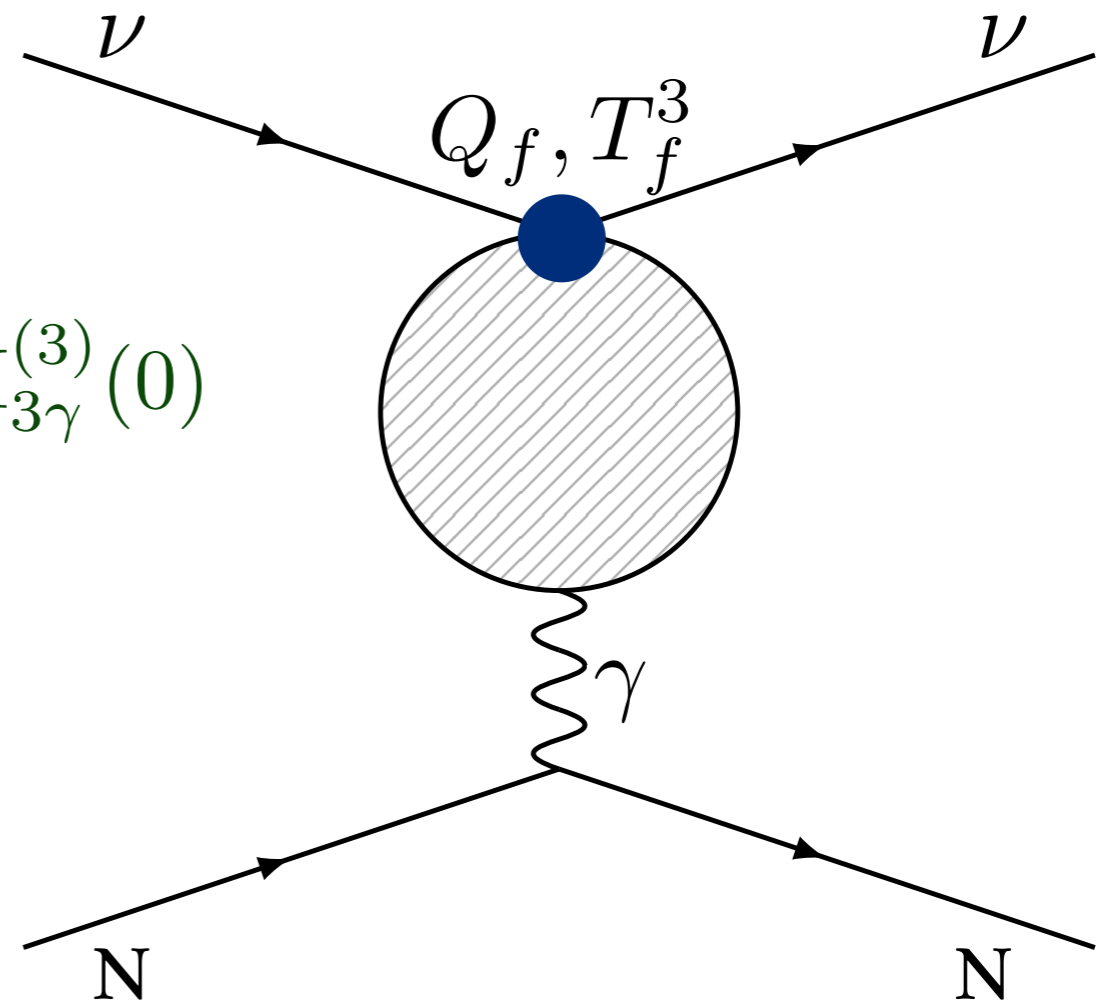
$$Q^2 \ll \Lambda_{\text{QCD}}^2$$

- light quarks

$$\delta^{\text{QCD}} = 4\Pi_{\gamma\gamma}^{(3)}(0) \sin^2 \theta_W - 2\Pi_{3\gamma}^{(3)}(0)$$

- chiral symmetry approximation

- flavor independent

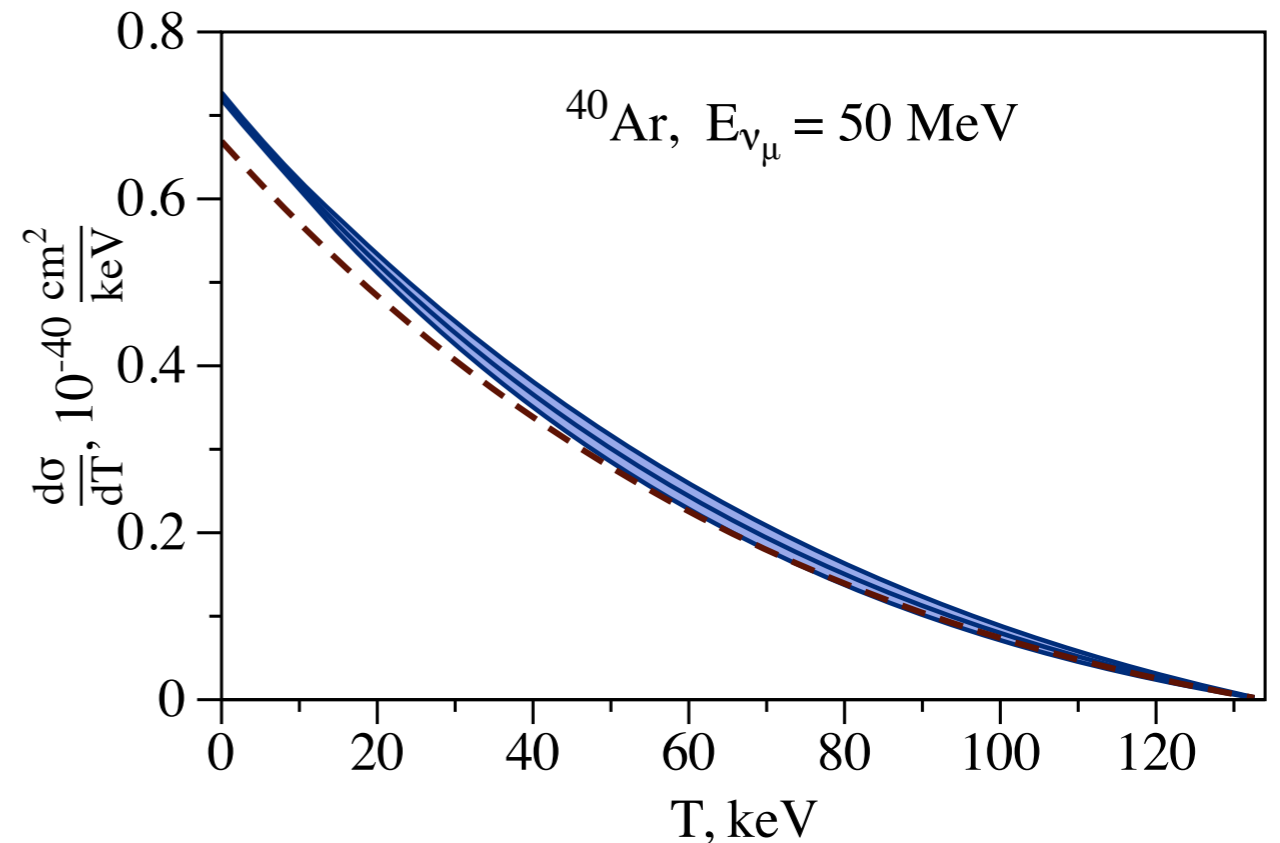
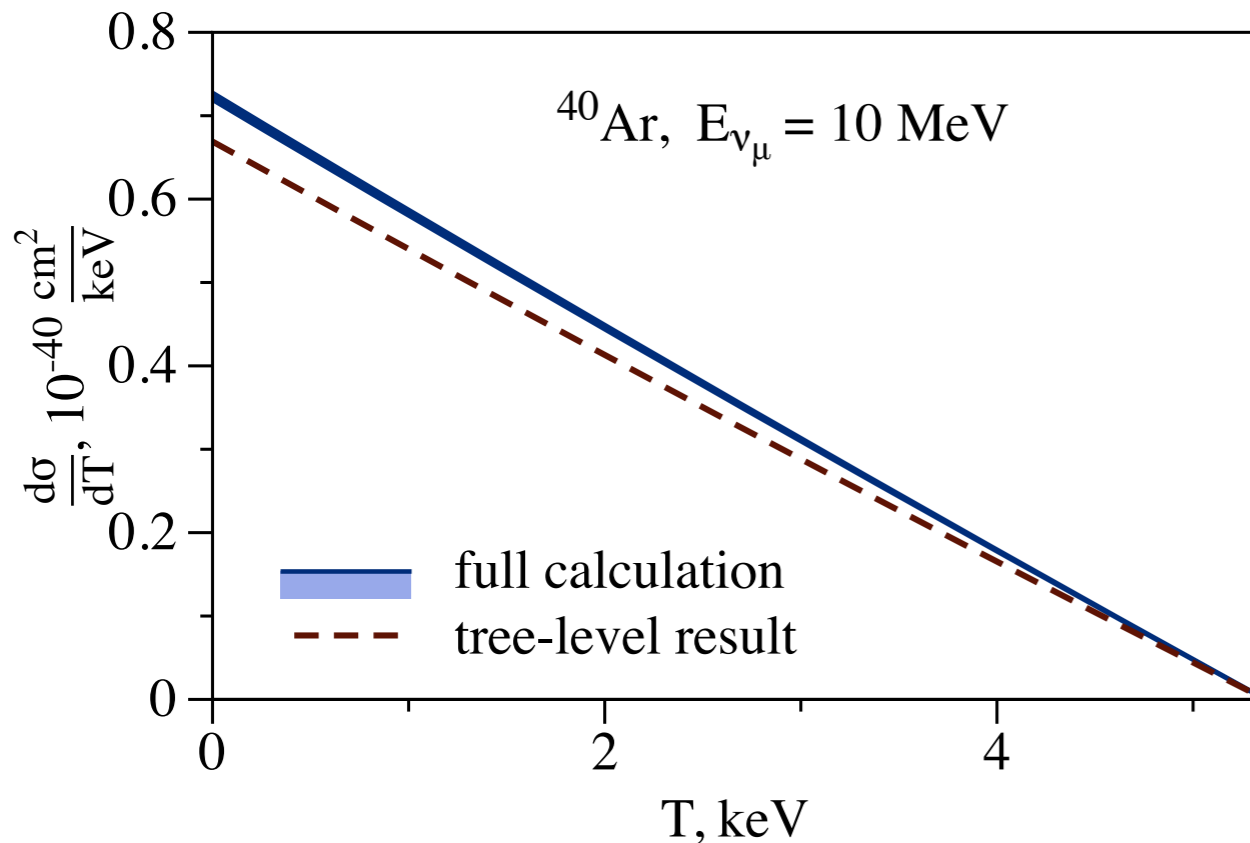


- non-perturbative light-quark contribution: error at low energy

# Total and differential cross section

- recoil nucleus energy spectrum: one-loop vs tree level

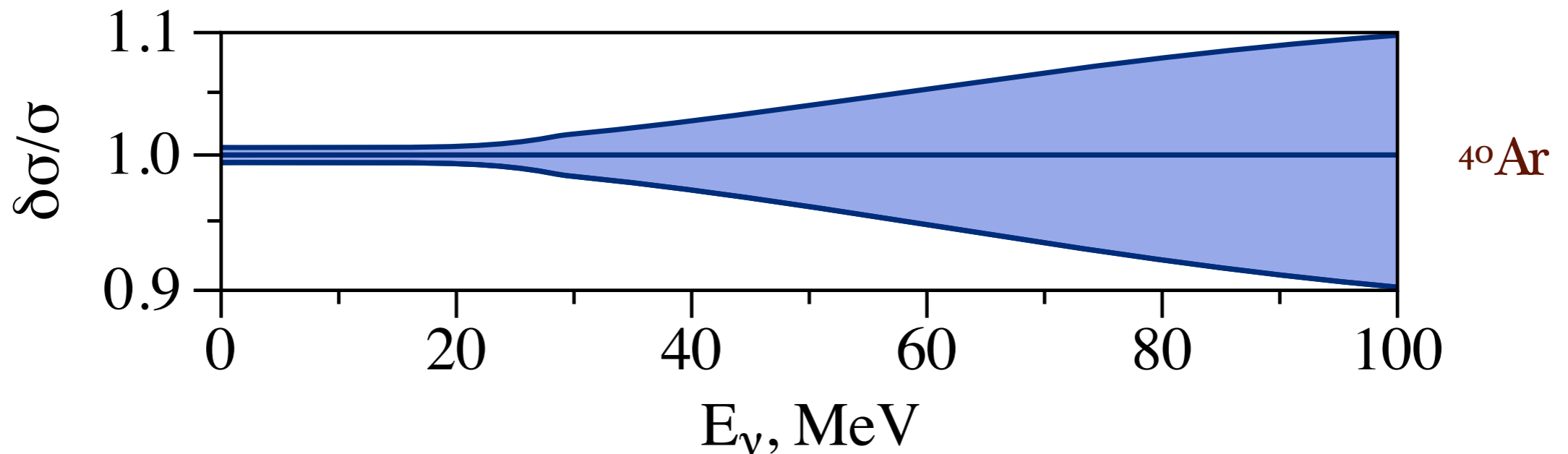
nuclear models for point-nucleon form factors:  
Yang et al. (2019), Payne et al. (2019), Hoferichter et al. (2020), Van Dessel et al. (2020)



- % effect of radiative corrections on cross sections

# Total cross section errors

- relative cross section error



- sources of uncertainty (%)

$E_\nu$ , MeV	Nuclear	Nucleon	Hadronic	Quark	Perturbative	Total
50	4	0.06	0.56	0.13	0.08	4.05
30	1.5	0.014	0.56	0.13	0.03	1.65
10	0.04	0.001	0.56	0.13	0.004	0.58

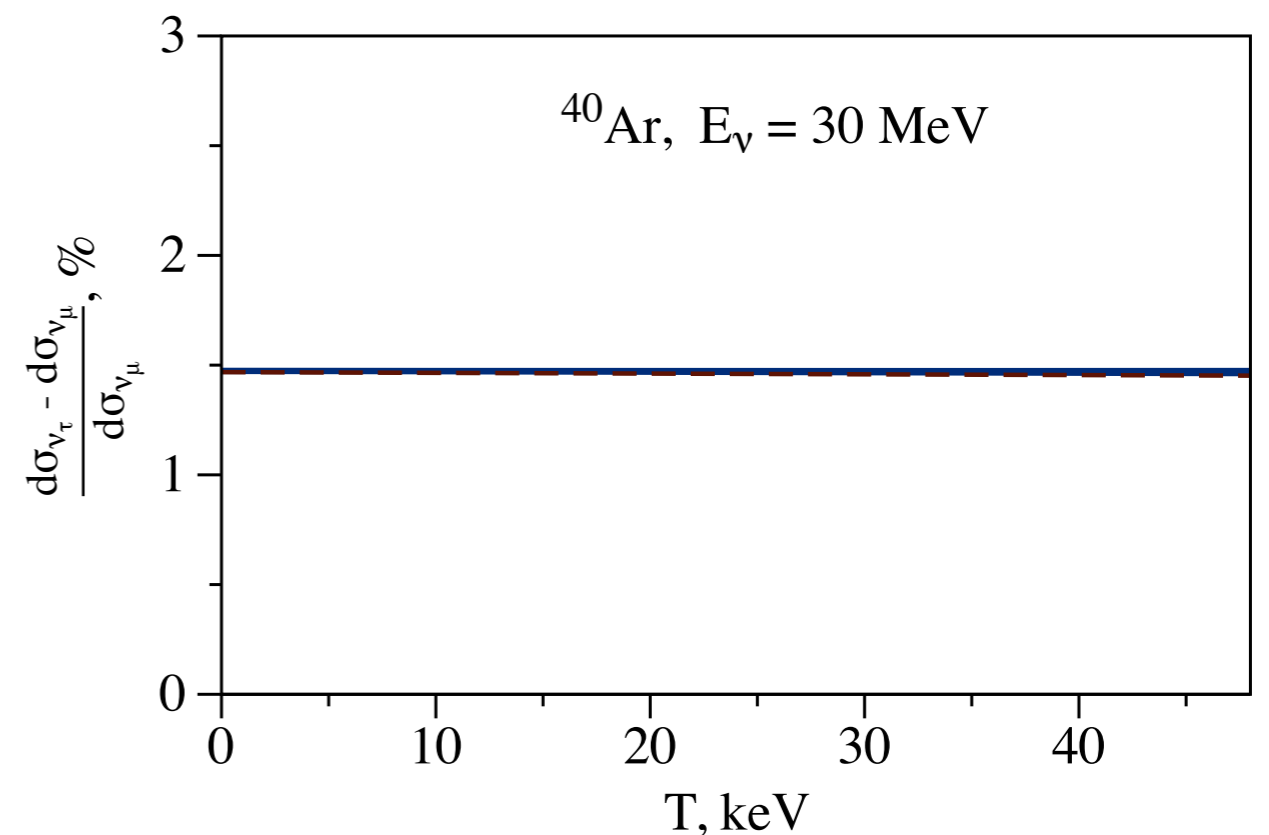
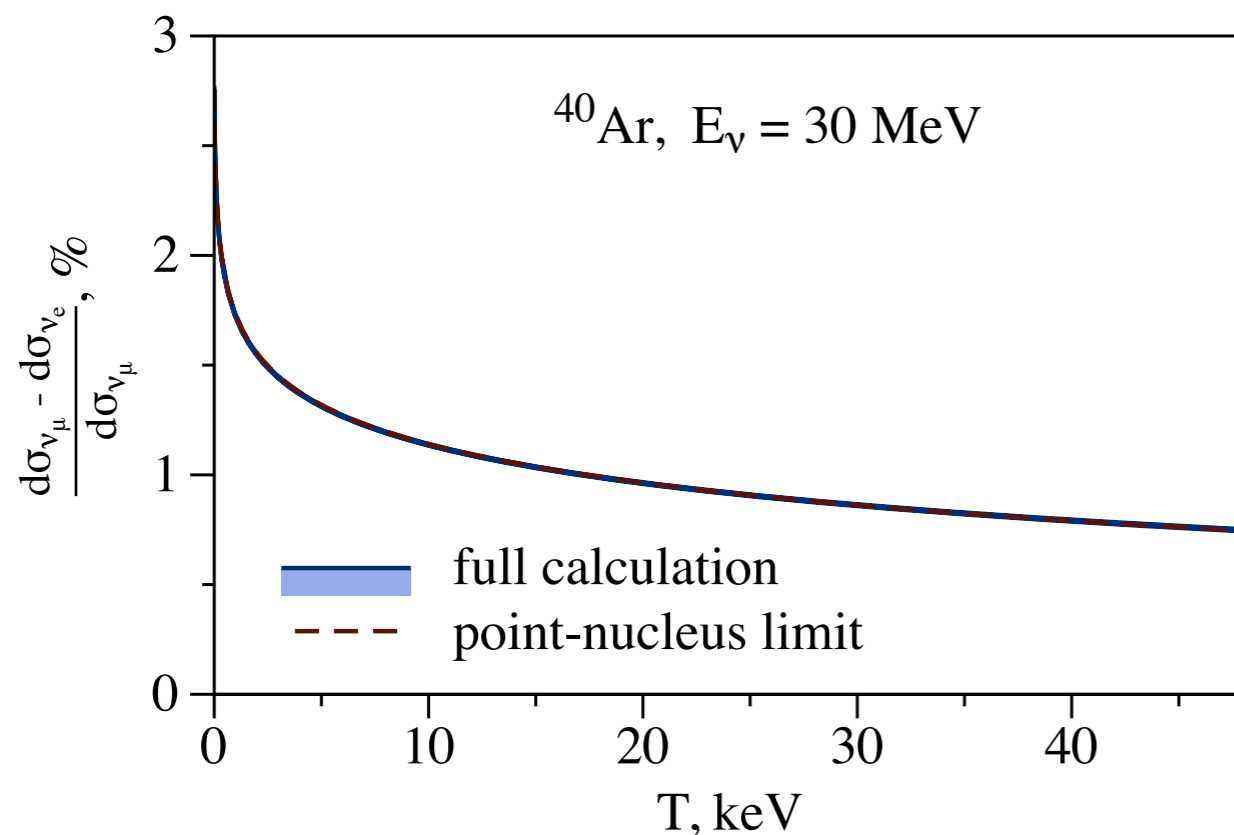
- hadronic error 0.6% at low energy, nuclear error at higher energy

# Flavor difference

- well described by point-nucleus limit

$$\lim_{R_p, R_n \rightarrow 0} \frac{d\sigma_{\nu\ell} - d\sigma_{\nu\ell'}}{d\sigma_{\nu\ell}} = 4 \frac{\alpha_0}{\pi} \frac{Z}{Q_W} [\Pi(Q^2, m_\ell) - \Pi(Q^2, m_{\ell'})]$$

- kinematic dependence: full result vs point-nucleus limit



- factor 3-6 change in precisely predicted electron-muon asymmetry

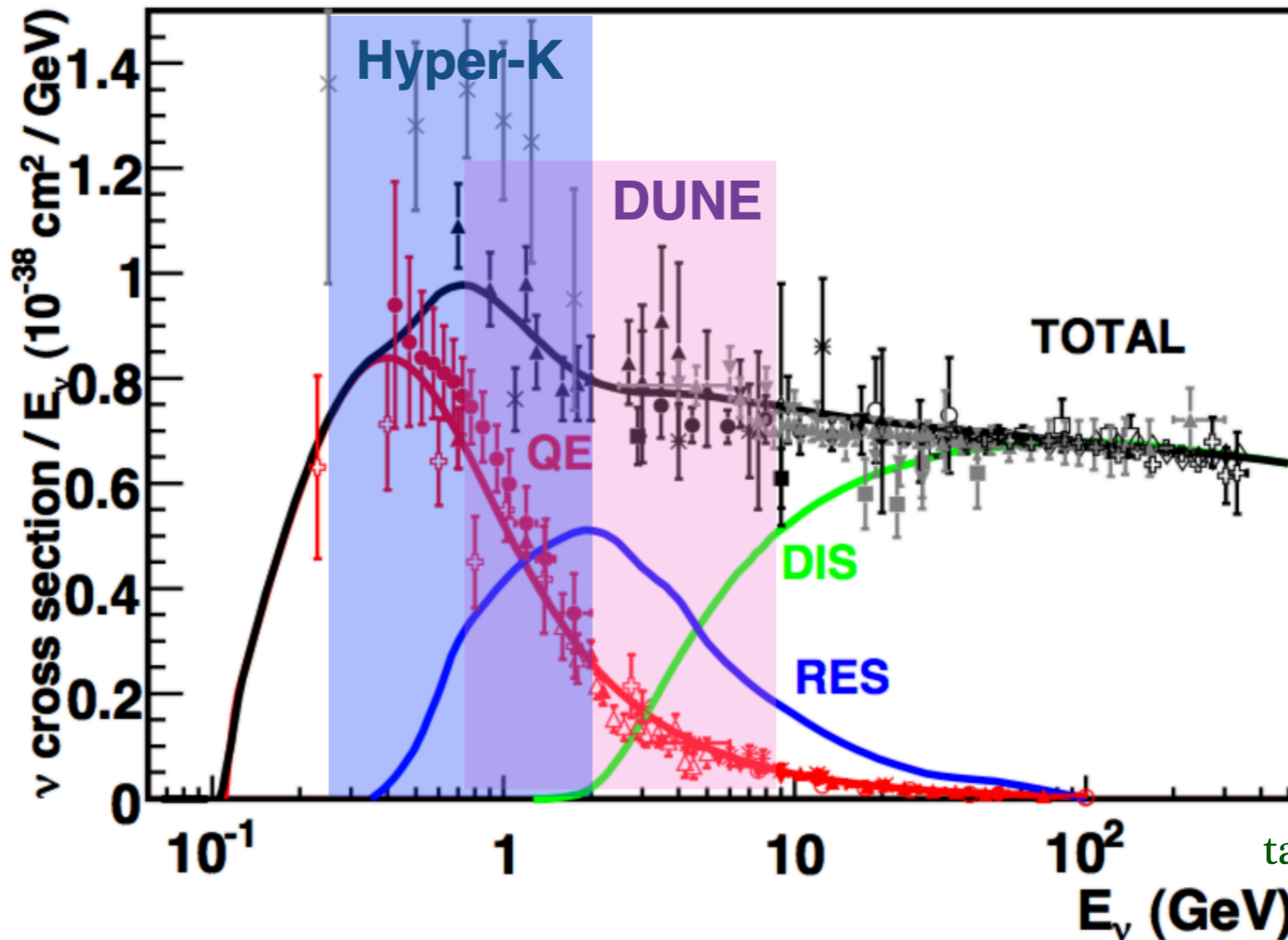


# Radiative corrections in CCQE on free nucleons

Qing Chen, Richard J. Hill, Kevin S. McFarland and O. T. (arXiv: to appear)

# CCQE. Why should we care?

- neutrino-nucleus cross sections and future accelerator-based fluxes

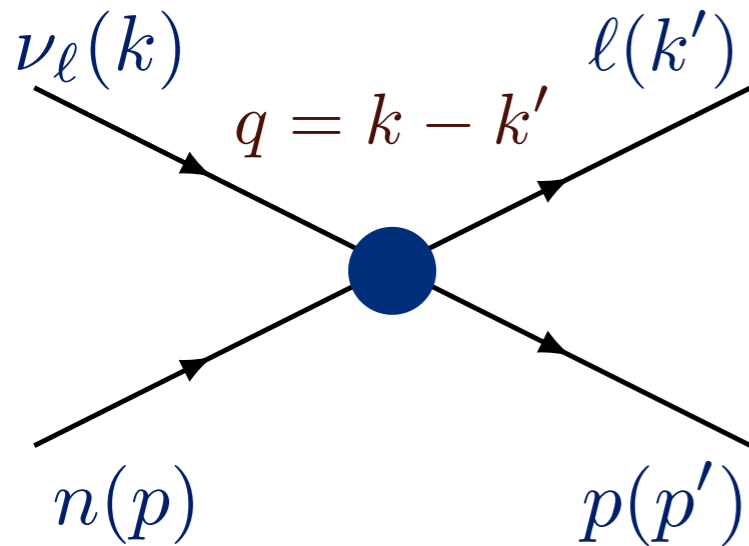


Formaggio  
and Zeller  
(2013)

Noemi Rocco  
talk at Neutrino 2020

- basic process: bulk of events at Hyper-K and DUNE
- best channel for reconstruction of neutrino energy

# CCQE scattering on free nucleon



neutrino energy

$$E_\nu$$

momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu(Q^2) = \langle p | \bar{u} (\gamma^\mu - \gamma^\mu \gamma_5) d | n \rangle$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P^V(Q^2) + \gamma^\mu \gamma_5 F_A(Q^2) + \frac{q^\mu}{M} \gamma_5 F_P(Q^2)$$

form factors: isovector Dirac and Pauli

axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

talks by Aaron Meyer, Fernando Alvarado,  
Beata Kowal, Dr. Rafi Alam

tree-level amplitude

$$T = \frac{G_F V_{ud}}{\sqrt{2}} (\bar{\ell}(k') \gamma_\mu (1 - \gamma_5) \nu_\ell(k)) (\bar{p}(p') \Gamma^\mu(Q^2) n(p))$$



# Radiative corrections in CCQE

- large kinematic logarithms enhance radiative corrections

$$\frac{\alpha}{\pi} \sim 0.2 \% \quad \text{multiplied by} \quad \ln \frac{E_\nu}{m_e} \sim 6 - 10$$

- CCQE with electron flavor is subject to large corrections

- phase-space restrictions enhance radiative corrections

$$\frac{\alpha}{\pi} \sim 0.2 \% \quad \text{multiplied by} \quad \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

$$E_\gamma < \Delta E \quad \text{soft photons} \quad 2 \ln \frac{E_\nu}{m_e} \ln \frac{\Delta E}{m_e} \sim 35 - 60$$

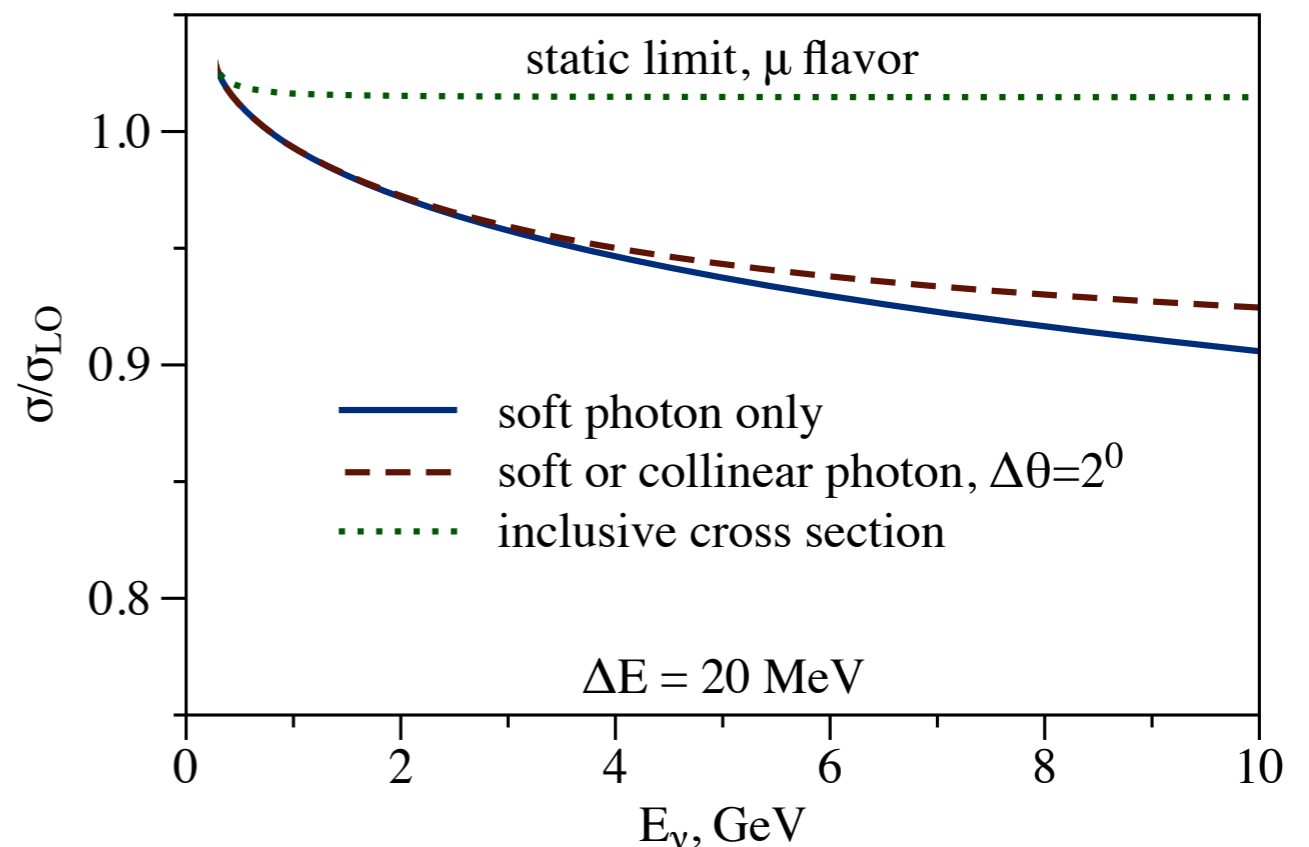
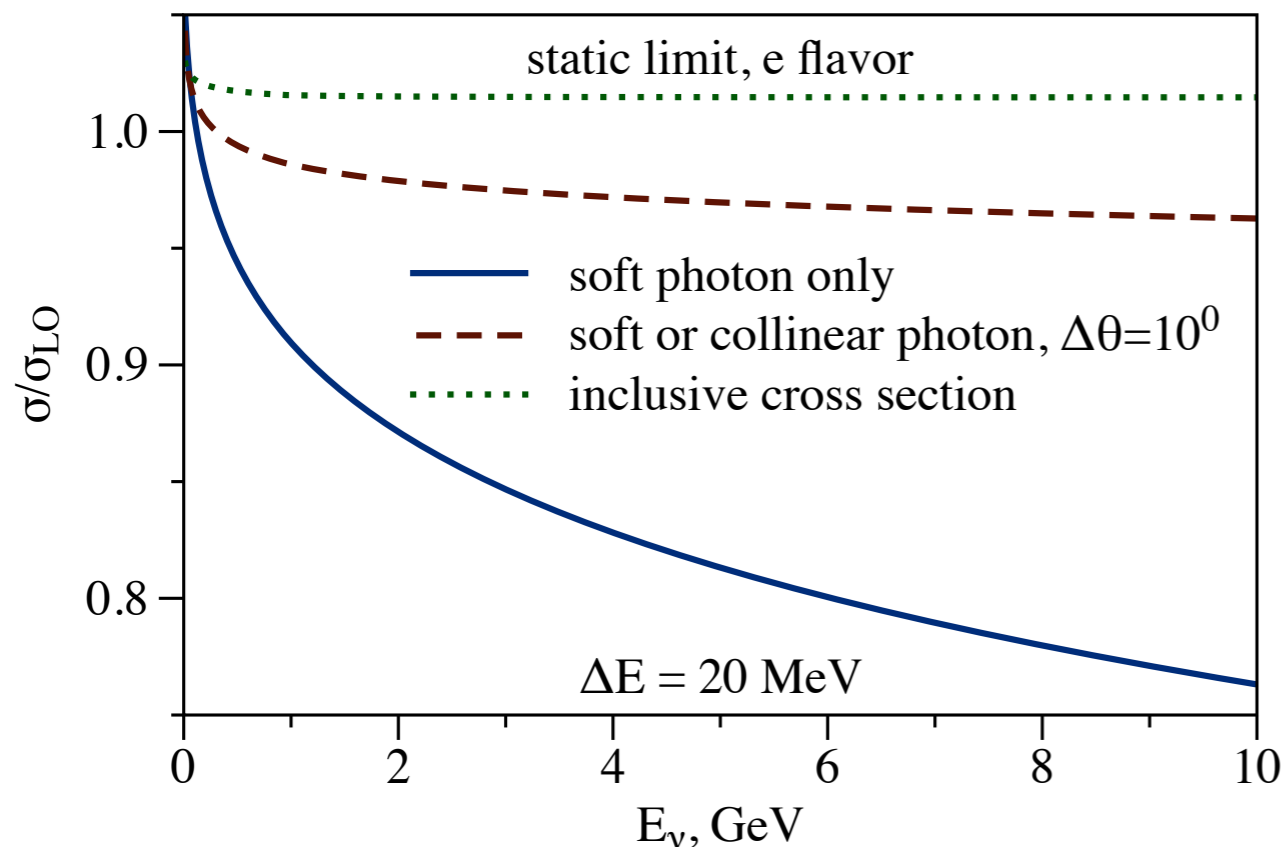
smaller collinear logarithms

- crucial dependence on detector details

- radiative corrections crucial for %-level oscillation program

# Static nucleon limit

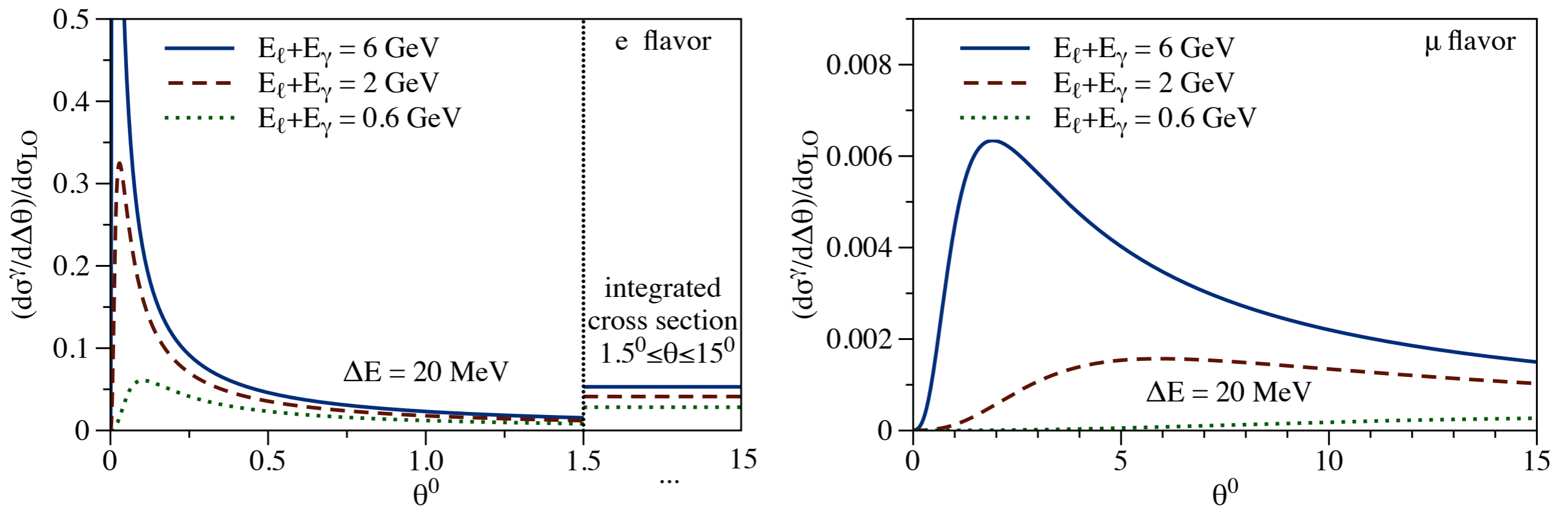
- formal limit of infinitely heavy nucleus  $m_\ell \ll E_\ell \ll M$
- provides correct soft and collinear logarithms
- soft-photon energy  $< 20$  MeV, jet size:  $10^\circ$  for electron and  $2^\circ$  for muon



- flavor-dependent effect, same for  $\nu_\ell n \rightarrow \ell^- p$  vs  $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- collinear observable: cancellation of virtual vs real logs
- inclusive observables (+ $\gamma$ ): few % level, flavor independent

# Electron vs muon jets

- factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy  $> 20$  MeV, fixed energy in the cone



- flavor-dependent effect, same for  $\nu_\ell n \rightarrow \ell^- p$  vs  $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

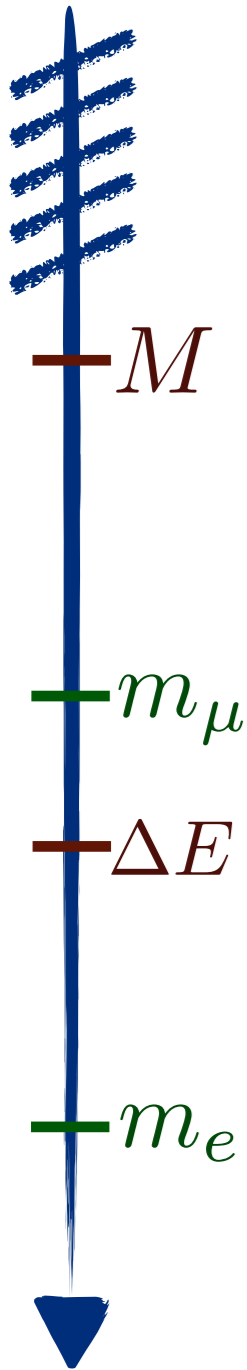
# Factorization approach

- cross section is given by factorization formula

$$d\sigma \sim S \left( \frac{\Delta E}{\mu} \right) J \left( \frac{m_\ell}{\mu} \right) H \left( \frac{M}{\mu} \right)$$

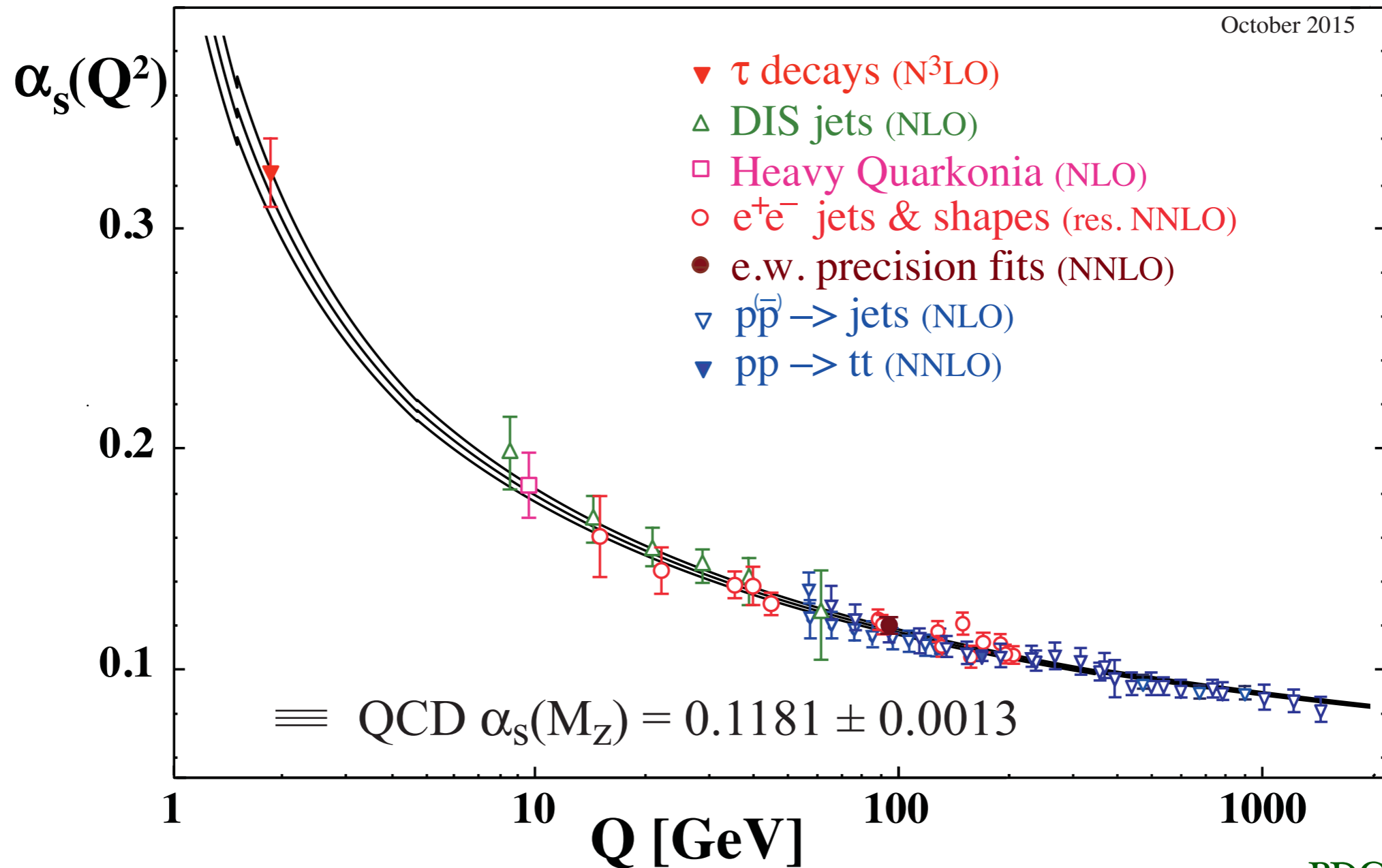
- determine hard function at hard scale by matching experiment or model to the theory with heavy nucleon

- soft and collinear functions are evaluated perturbatively



# Interaction with nucleons

- QCD running coupling

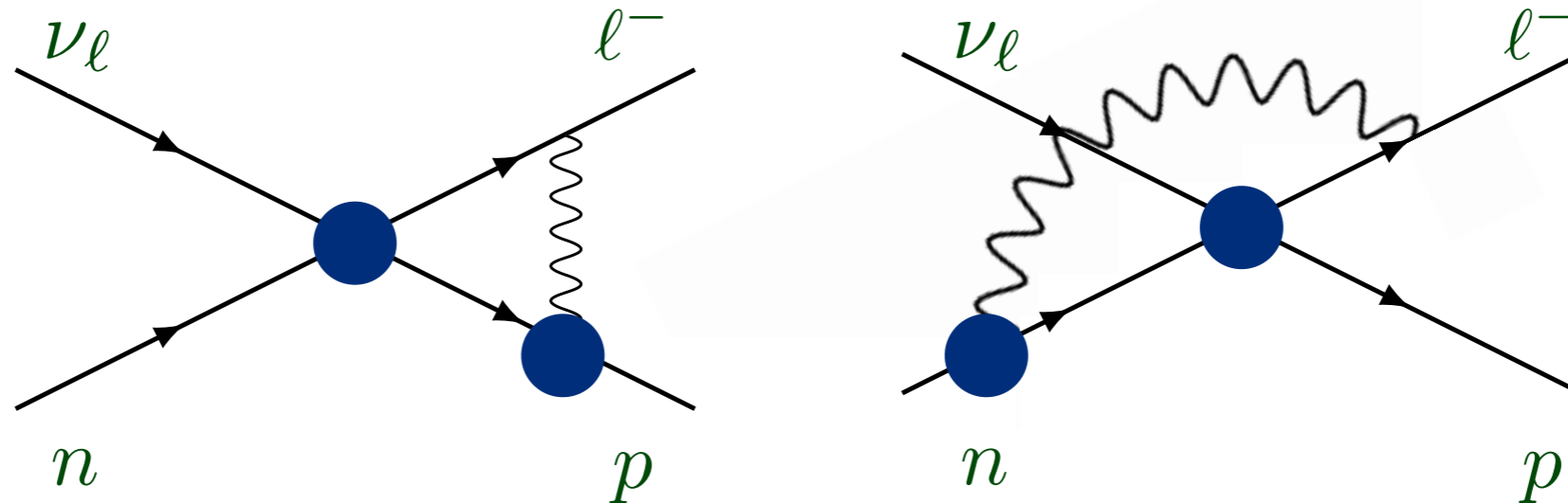


PDG 2015

- hadrons are correct degrees of freedom at GeV energy

# Hadronic model at GeV scale

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- exchange of photon between the charged lepton and nucleons
- assume **onshell form** for each interaction with dipole form factors  
discussed for CCQE: Graczyk (2013)
- add **self energy** for charged particles
- best determination of hard function by matching to low-energy EFT

- gauge-dependent vertex and gauge-dependent form factors

# Factorization approach

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$$d\sigma \sim S \left( \frac{\Delta E}{\mu} \right) J \left( \frac{m_\ell}{\mu} \right) H \left( \frac{M}{\mu} \right)$$

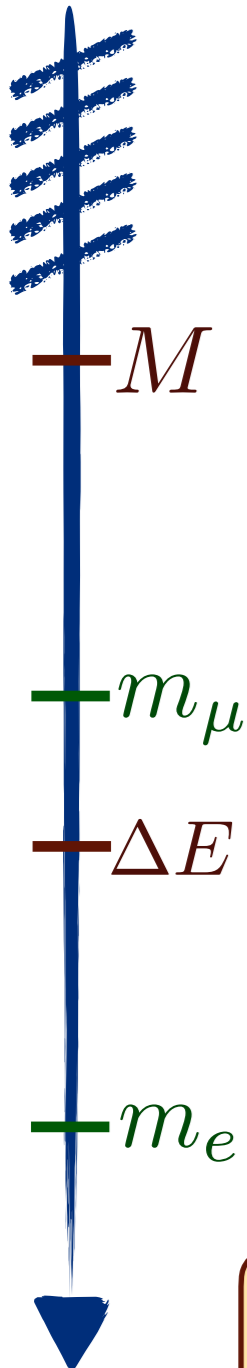
- determine hard function at hard scale by matching experiment or model to the theory with heavy nucleon

- RGE evolution of the hard function to scales  $\Delta E, m_\ell$

- soft and collinear functions are evaluated perturbatively

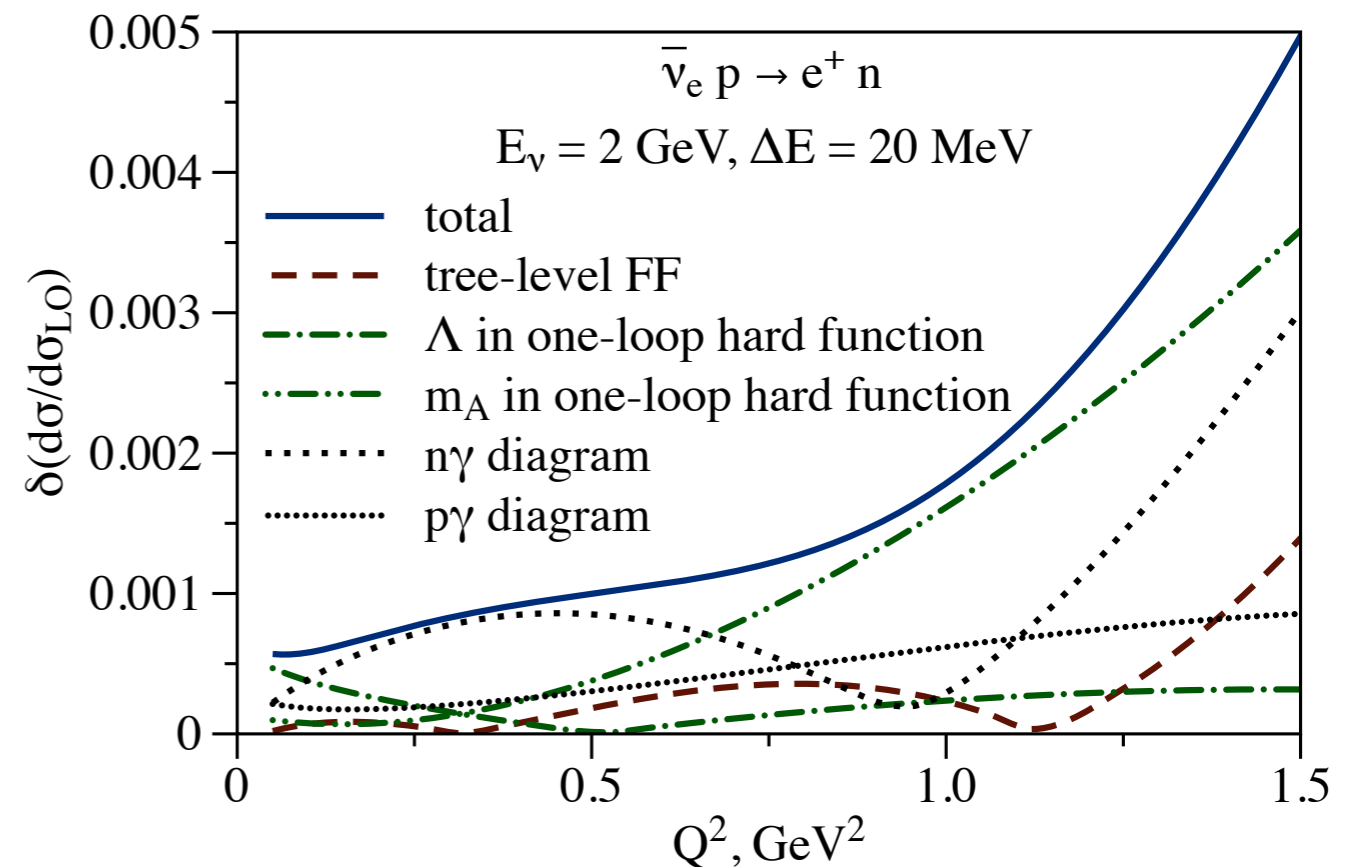
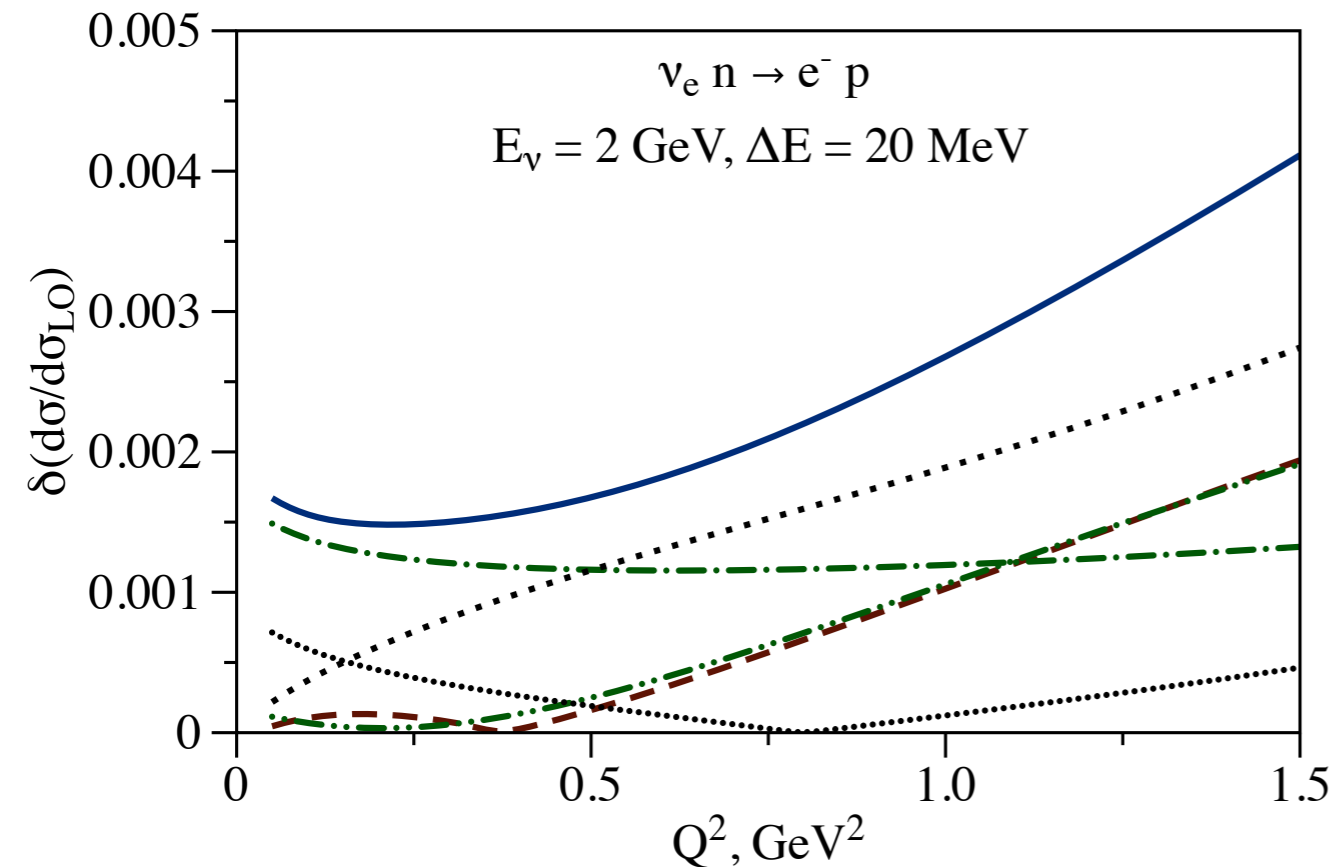
- calculate cross section at low energies accounting for **all large logs**  
ep scattering with soft radiation only: Hill (2016)

- soft and collinear functions obtained **analytically**
- **hard** function describes physics at GeV energies



# Exclusive case: assignment of errors

- uncertainties from hard function



- nucleon form factors
- add perturbative errors by variation of scale

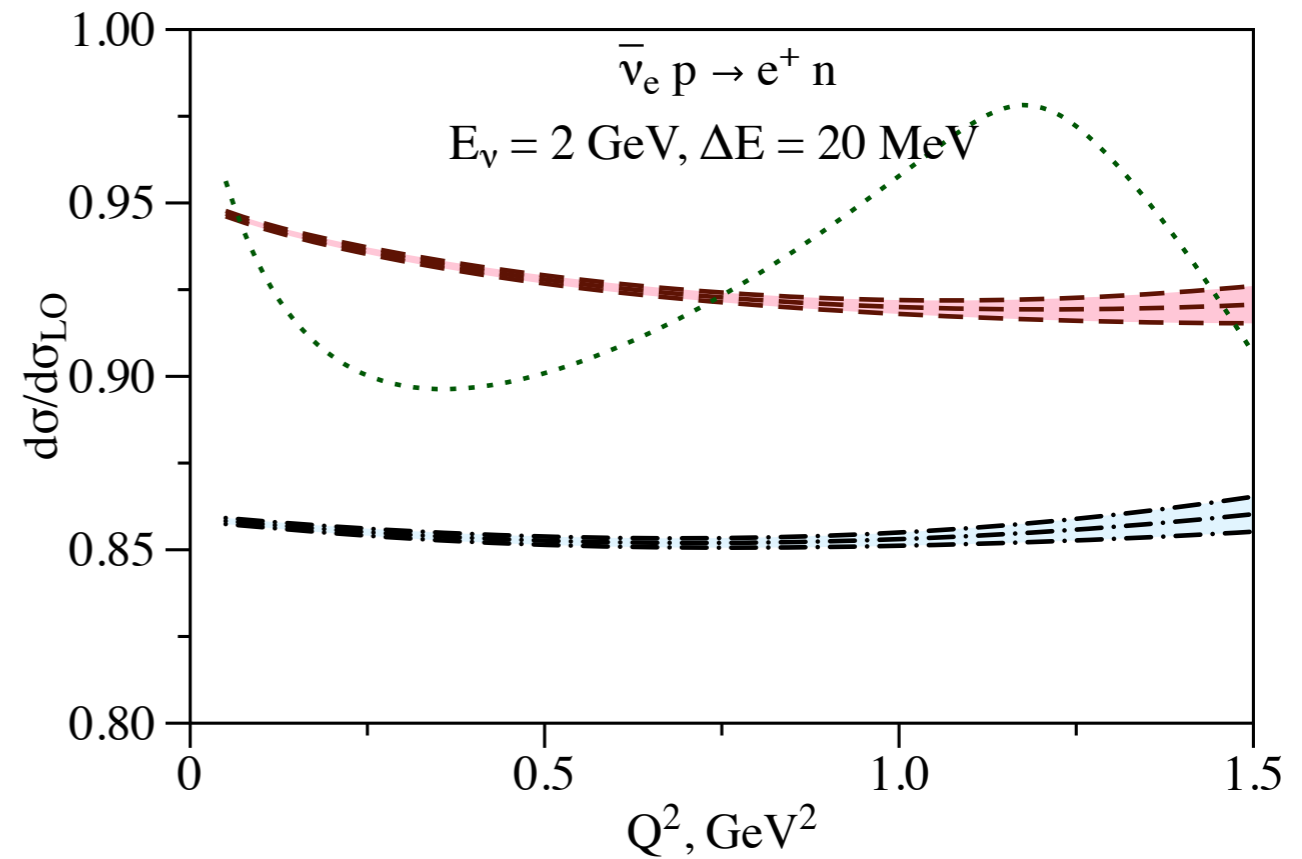
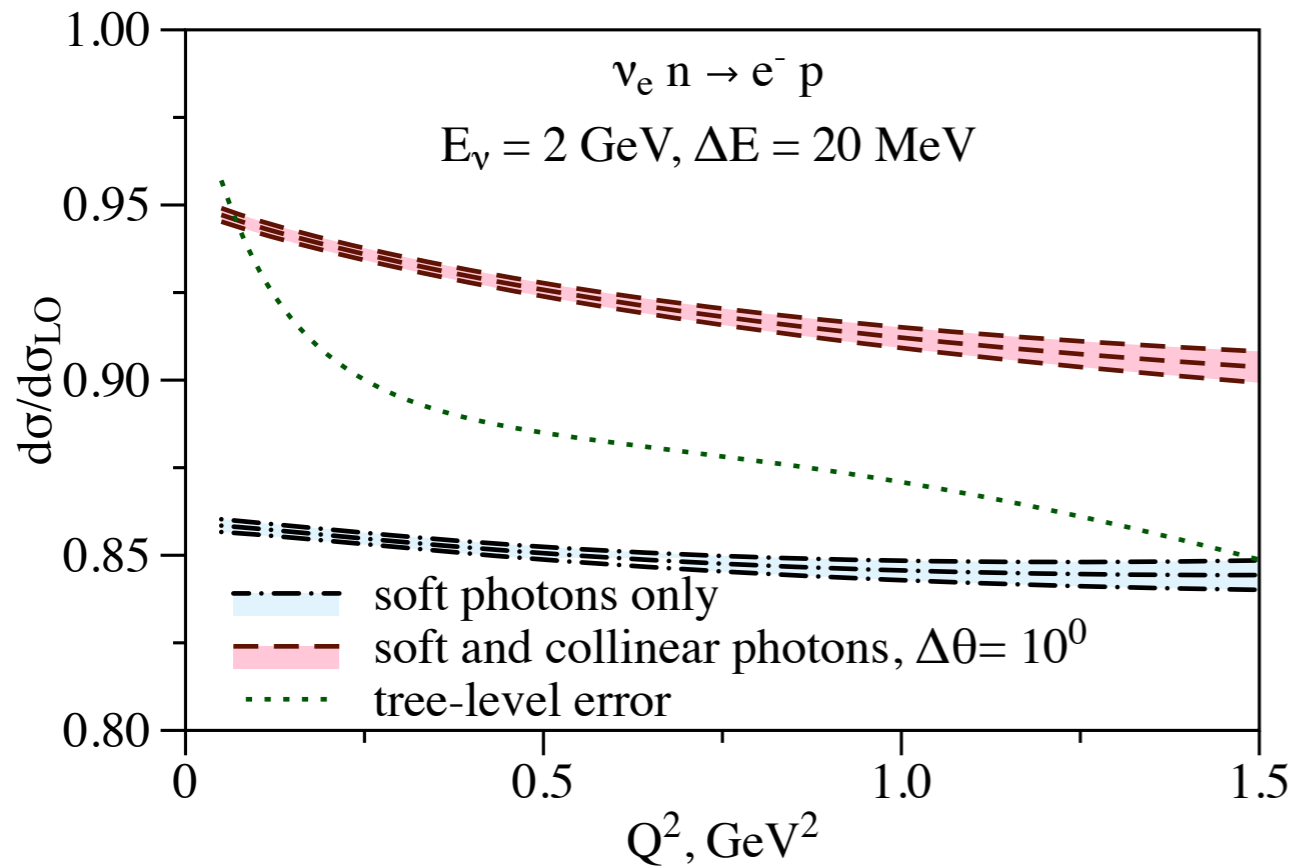
Kaushik Borah, Gabriel Lee, Richard J. Hill and O. T. (2020)

Meyer, Betancourt, Gran and Hill (2016)

- uncertainty of per mille level for the ratio to LO result

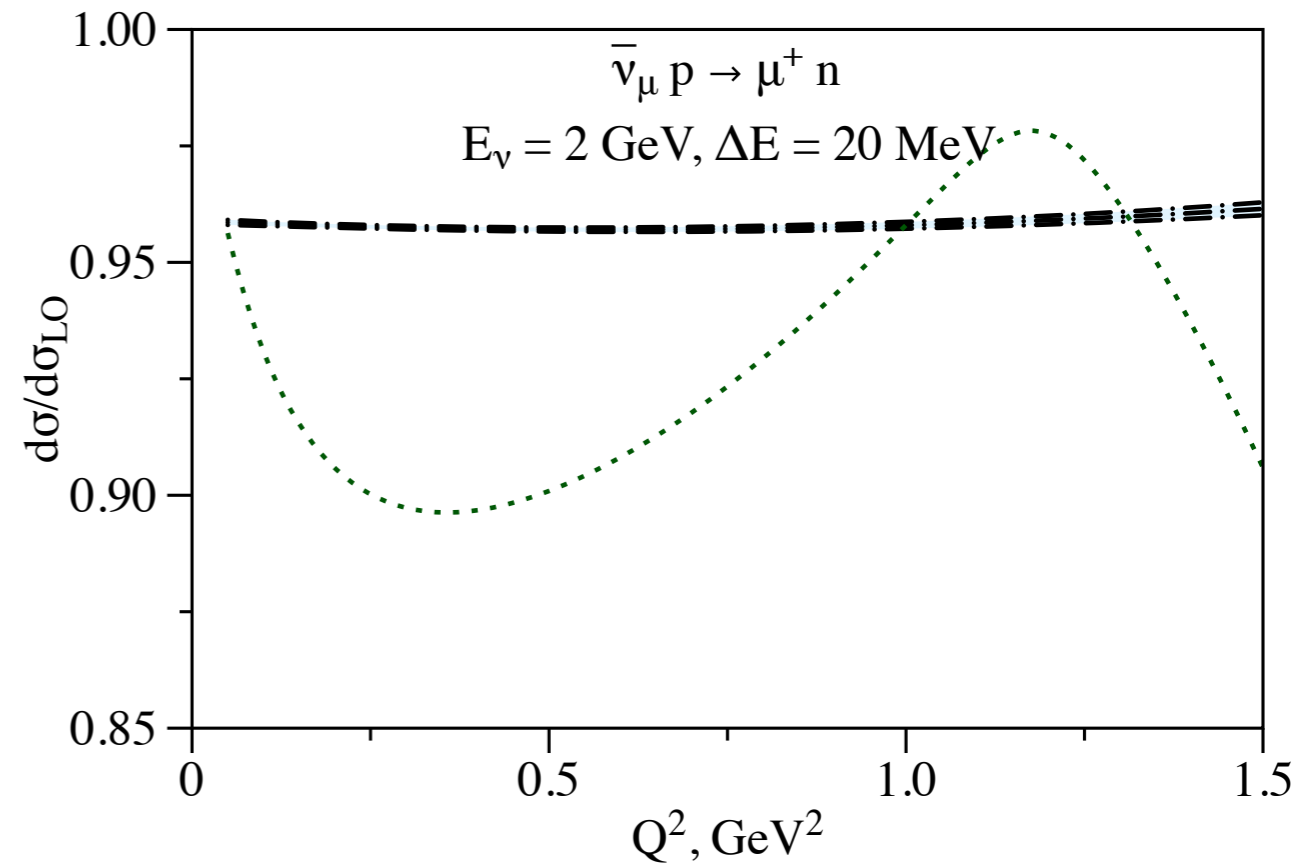
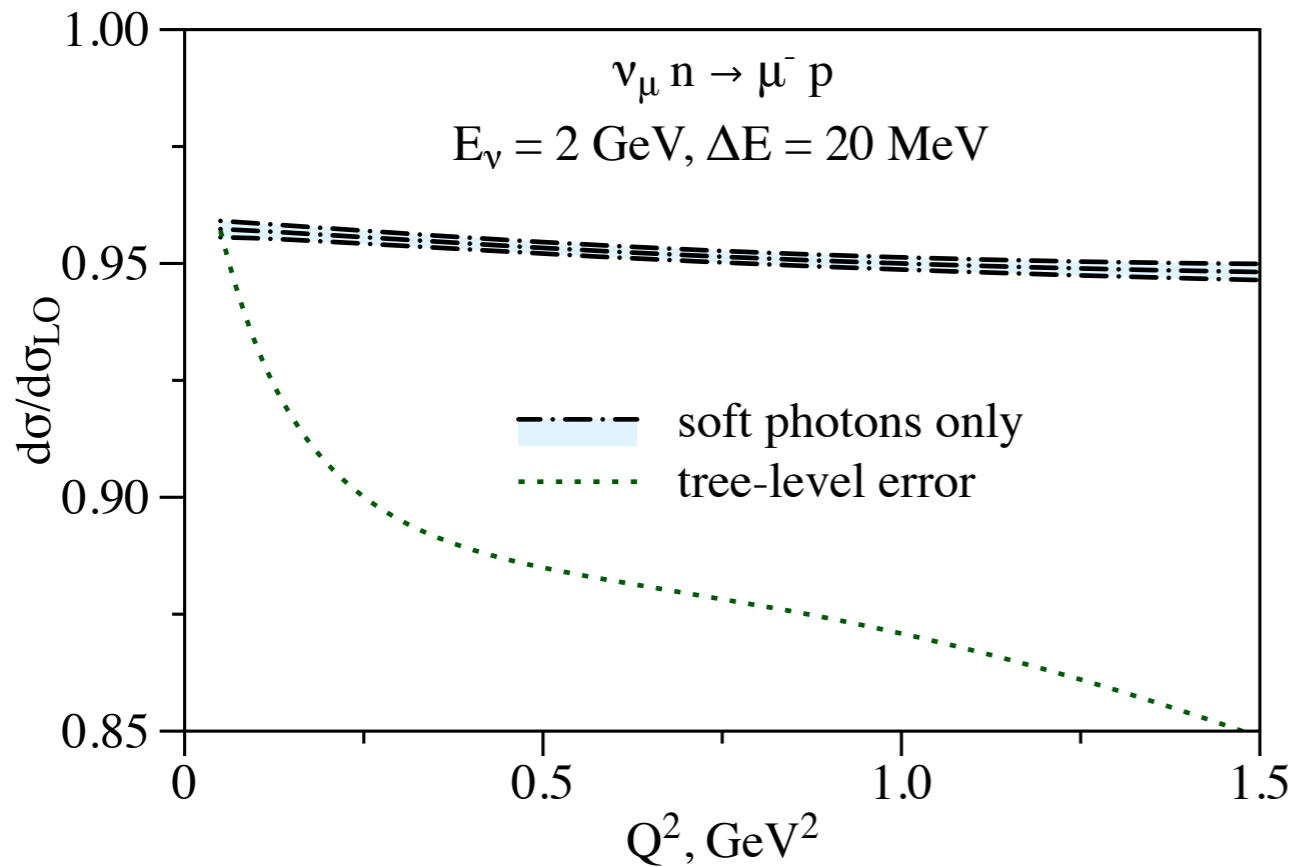


# Cross section. Electron flavor



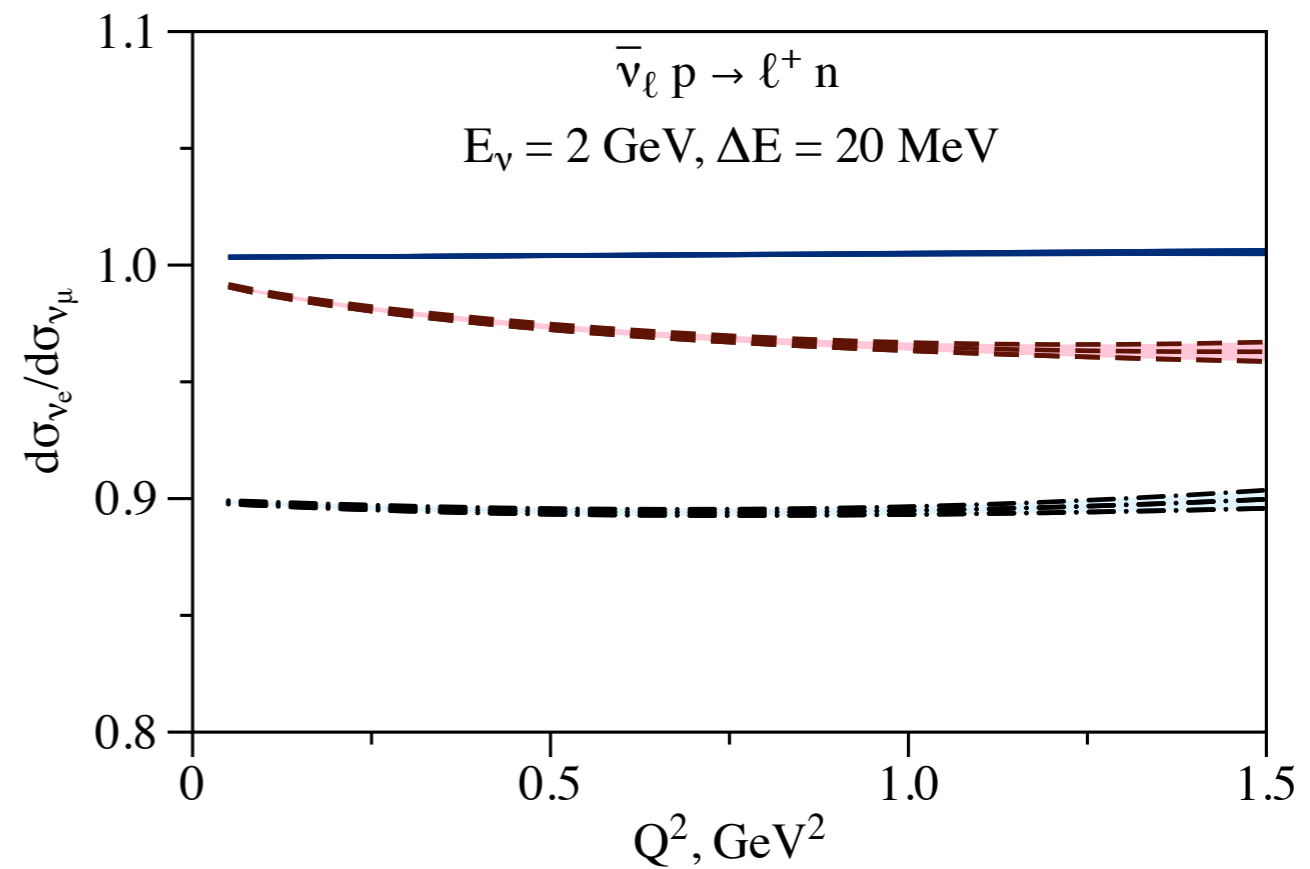
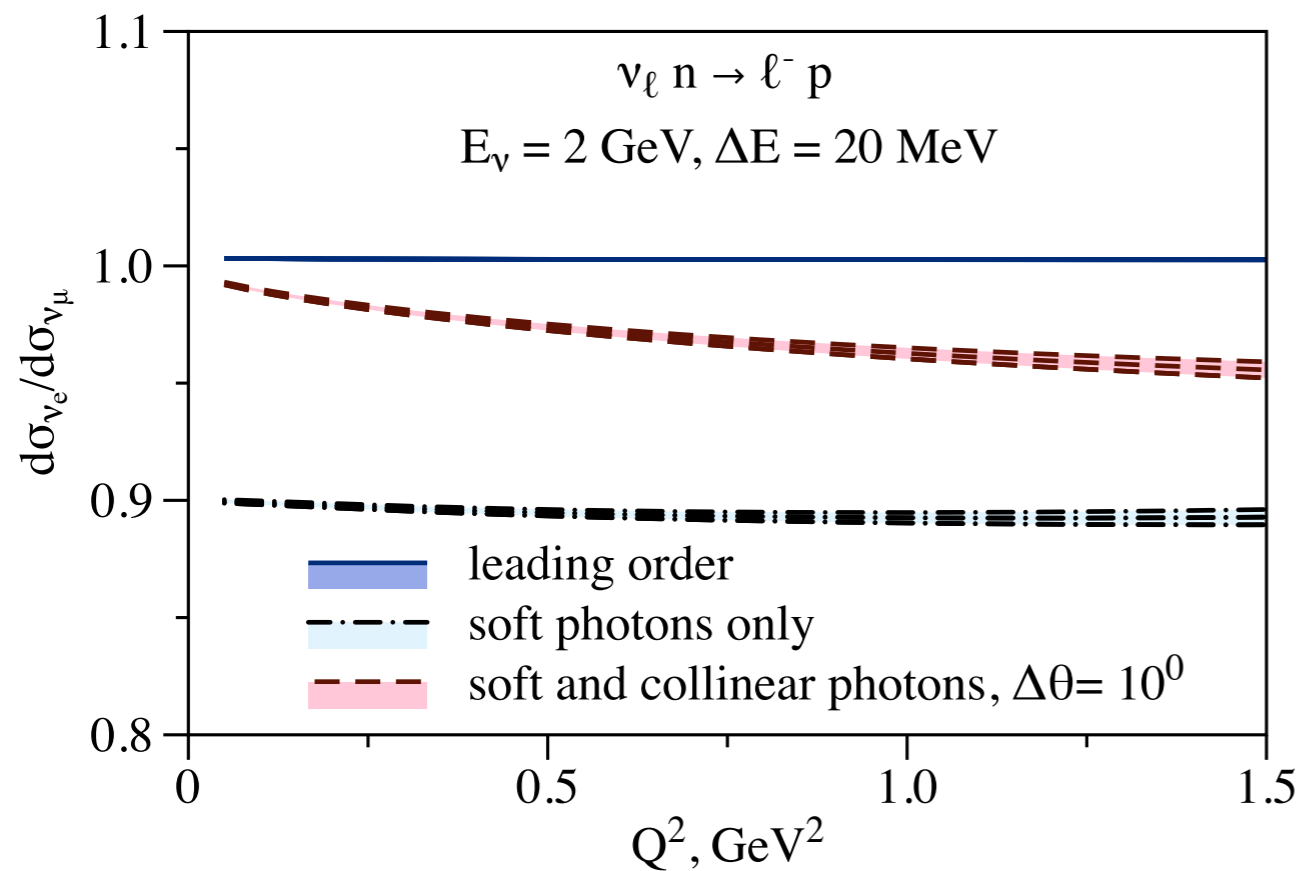
- cancellation of error from hard function for ratio to LO

# Cross section. Muon flavor



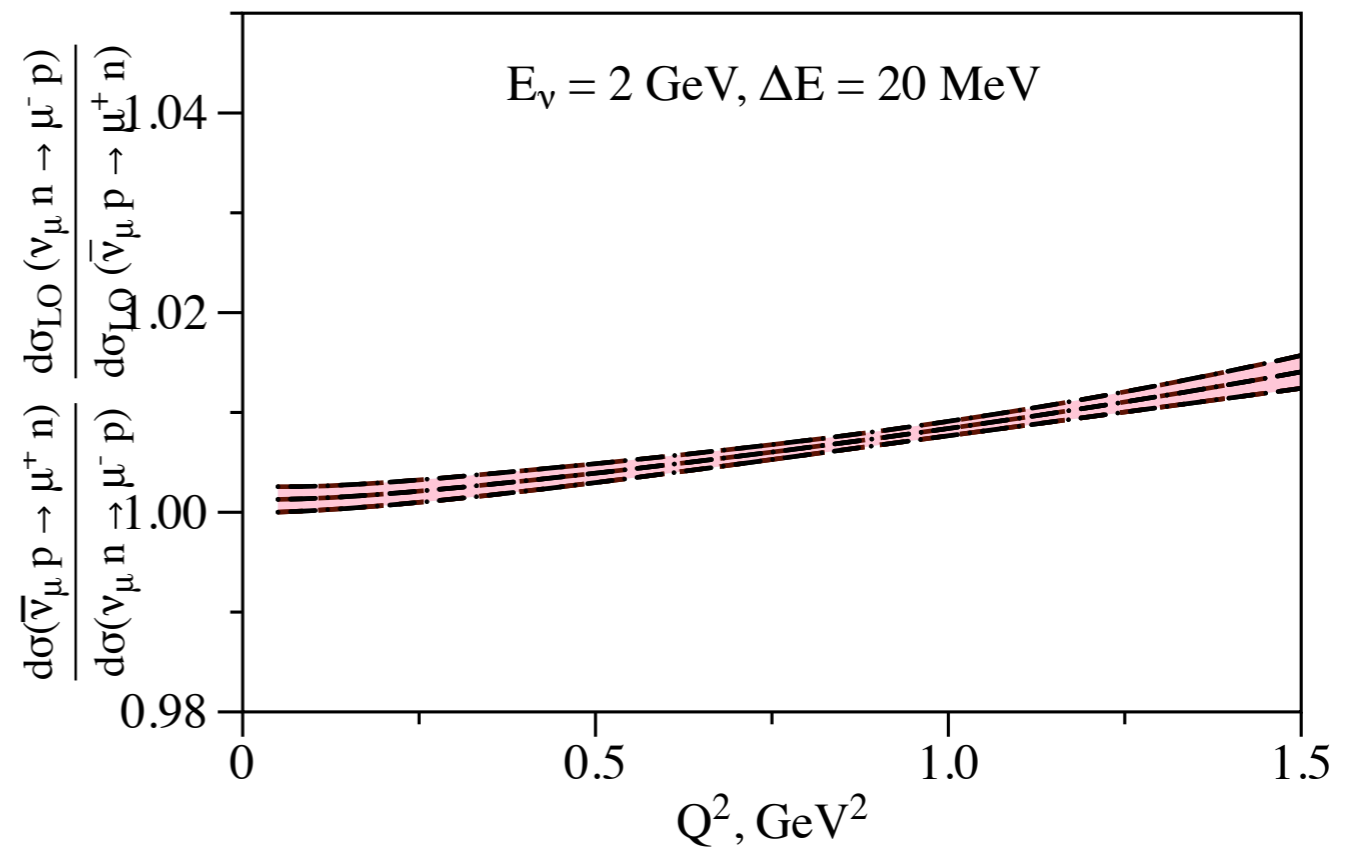
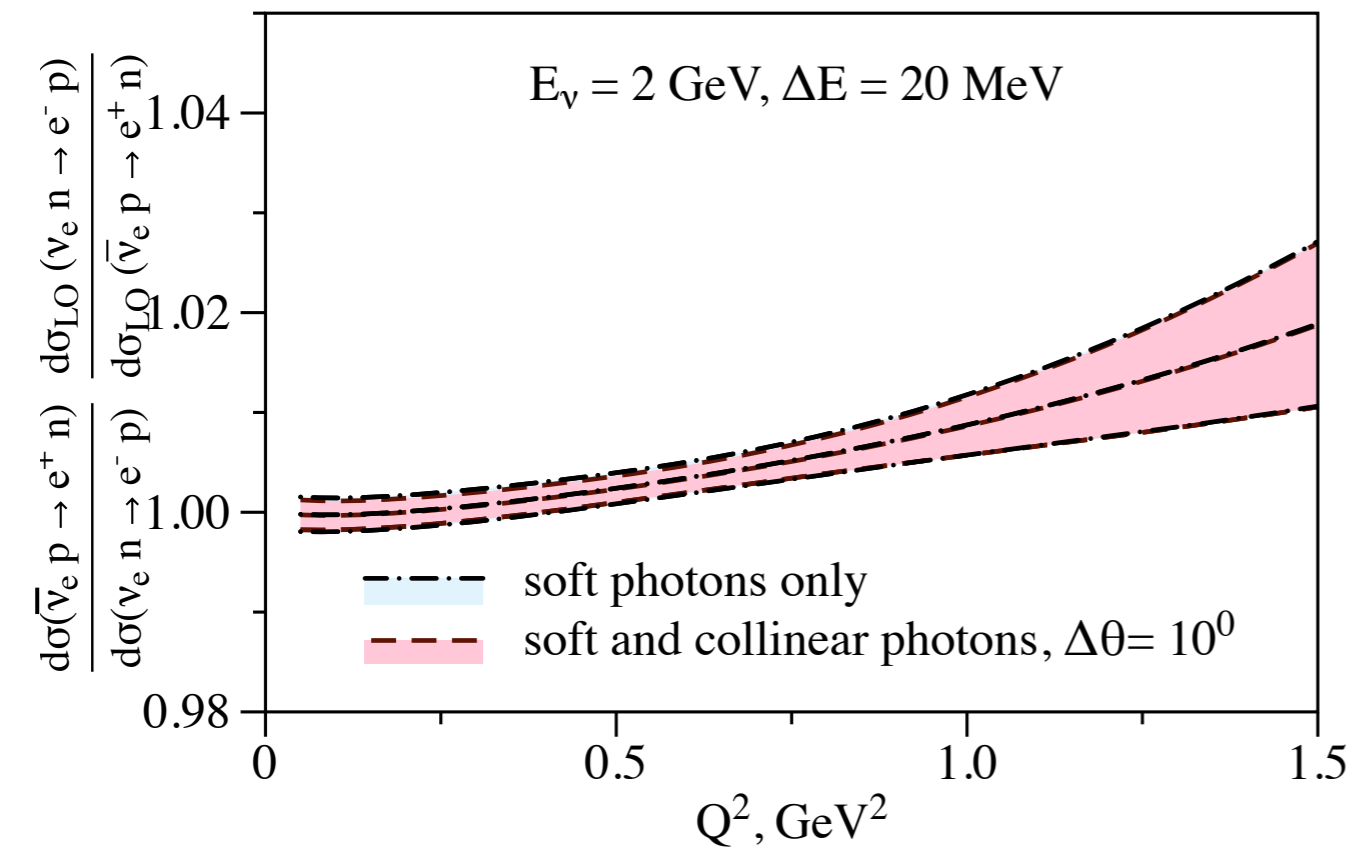
- cancellation of error from hard function for ratio to LO

# Electron/muon ratio



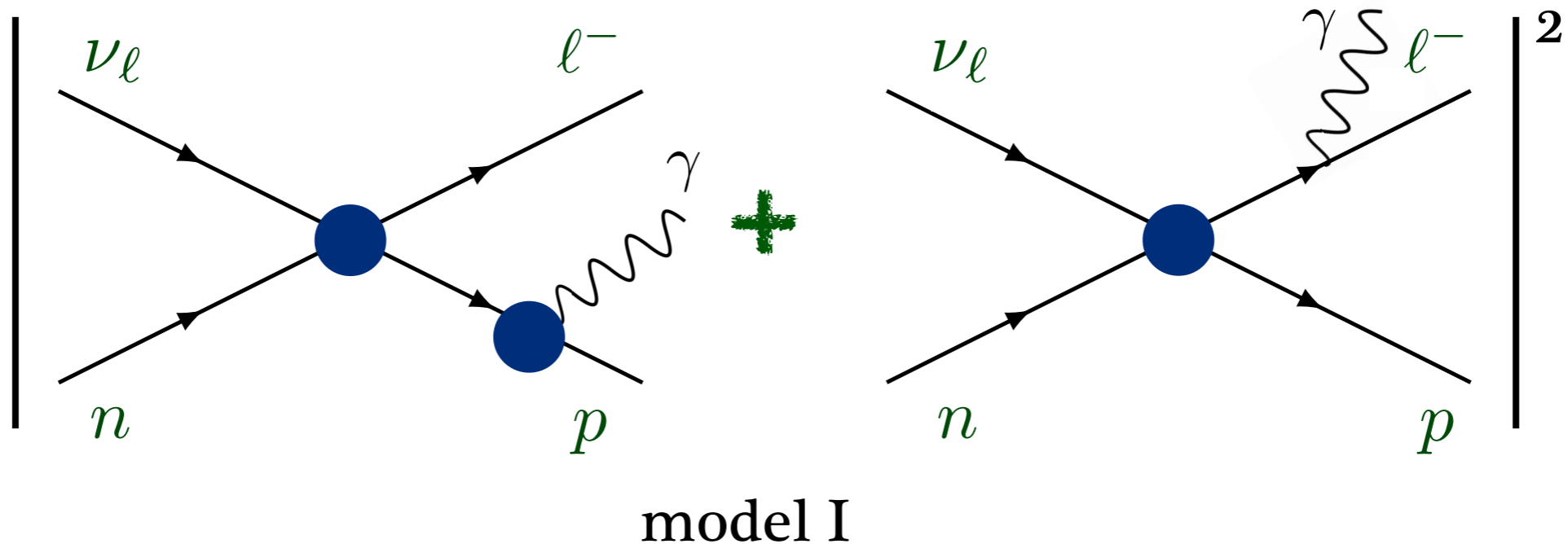
- small error: hard function does not depend on mass

# Neutrino/antineutrino double ratio



- jet function is the same for  $\nu_\ell n \rightarrow \ell^- p$  vs  $\bar{\nu}_\ell p \rightarrow \ell^+ n$

# Radiation of hard noncollinear photons



- tree-level EW vertex with nucleon-leg kinematics for both diagrams
- reproduces SCET for collinear radiation
- gauge-invariant bremsstrahlung

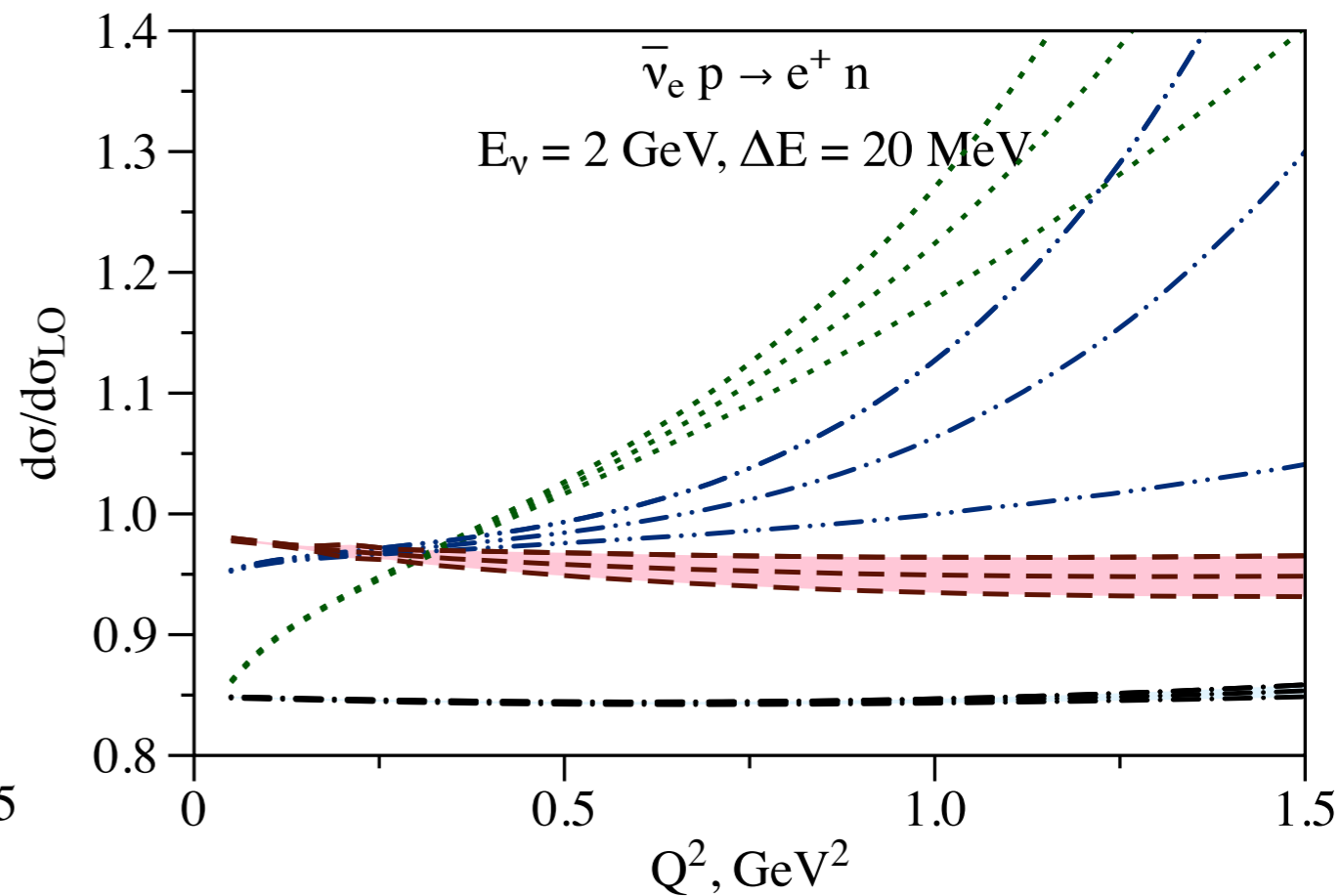
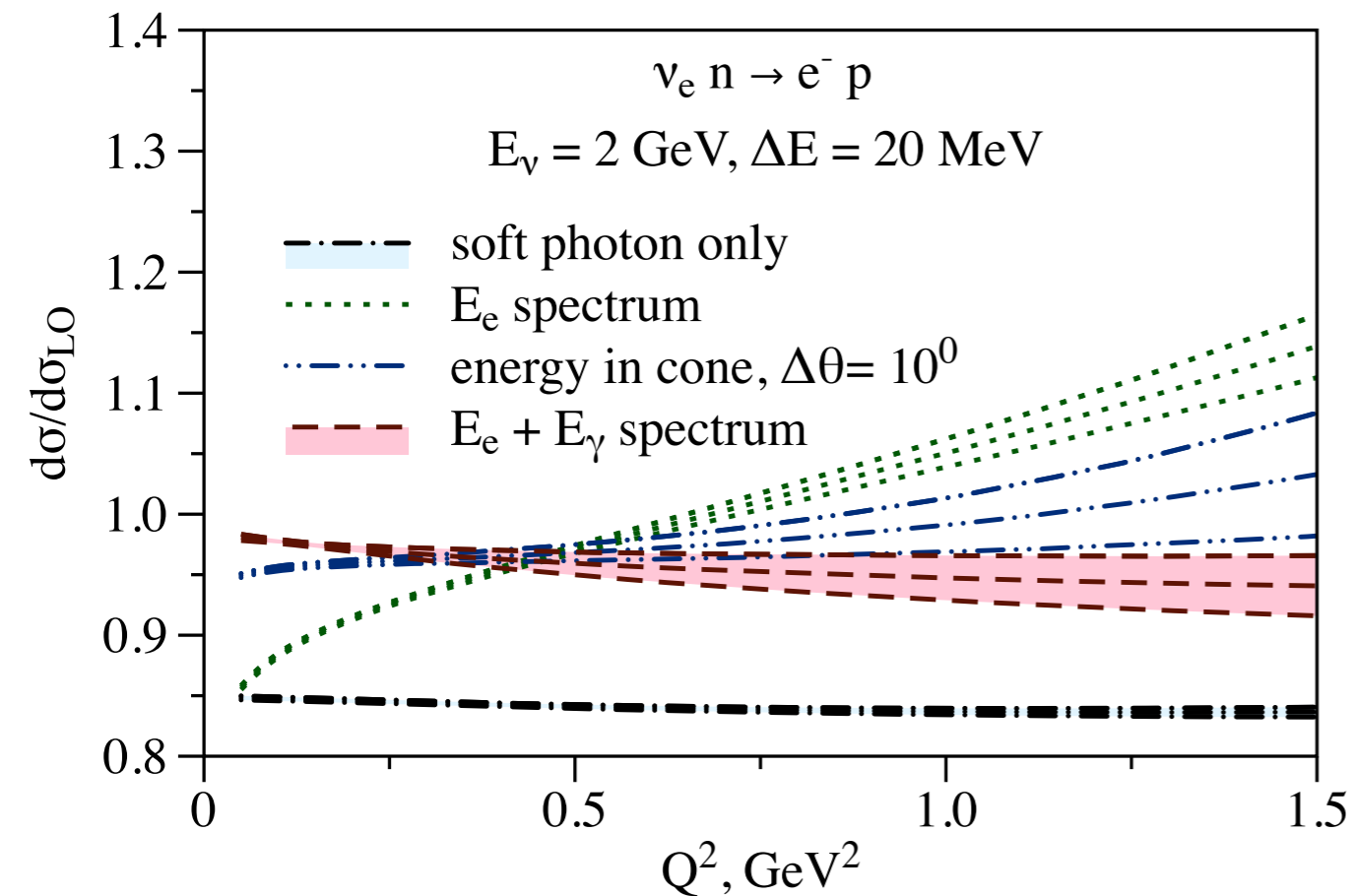
model II

- all EW vertex arguments as for local field theory

- uncertainty is given as a difference between two models

# Cross section. Electron flavor

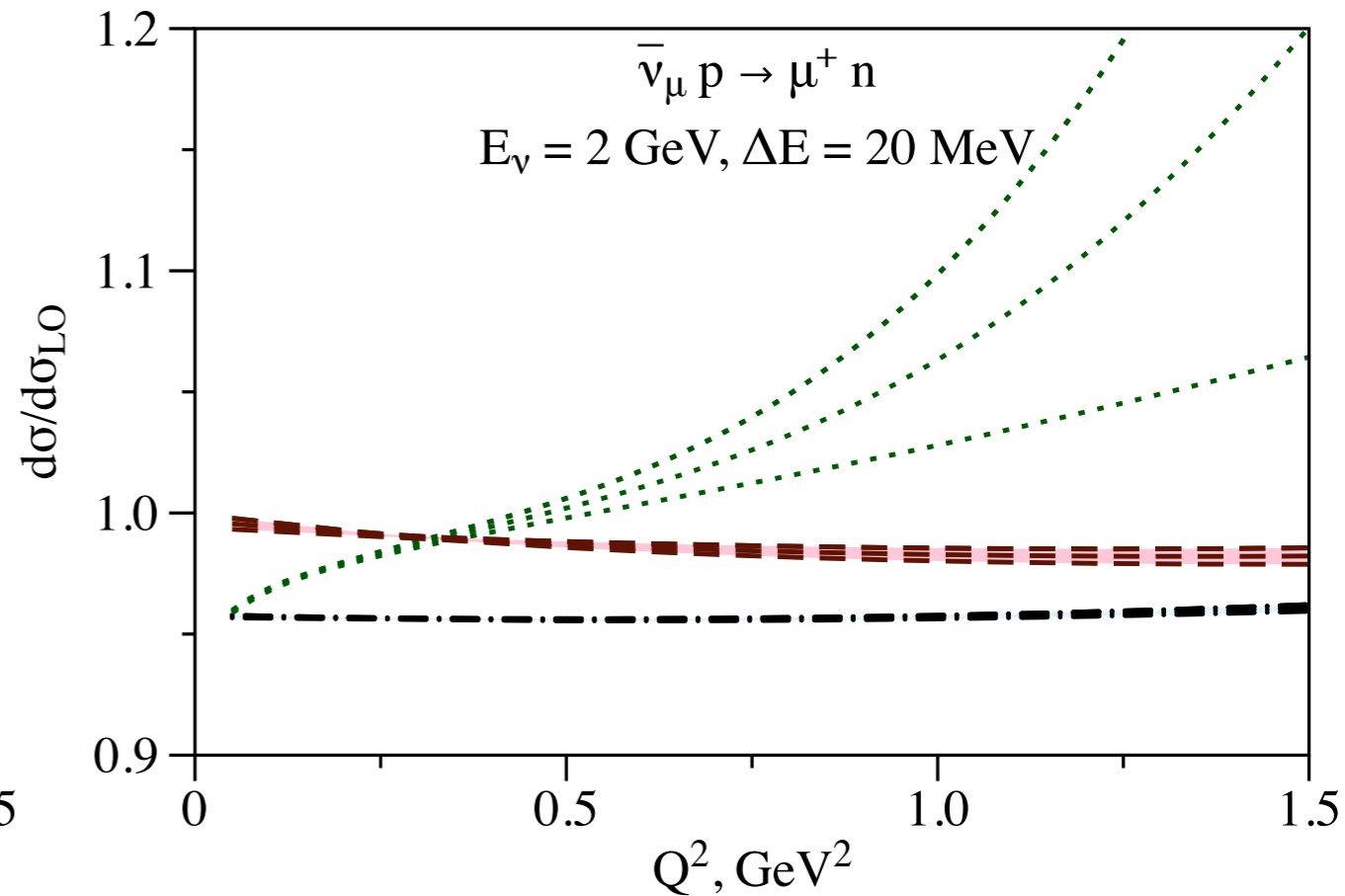
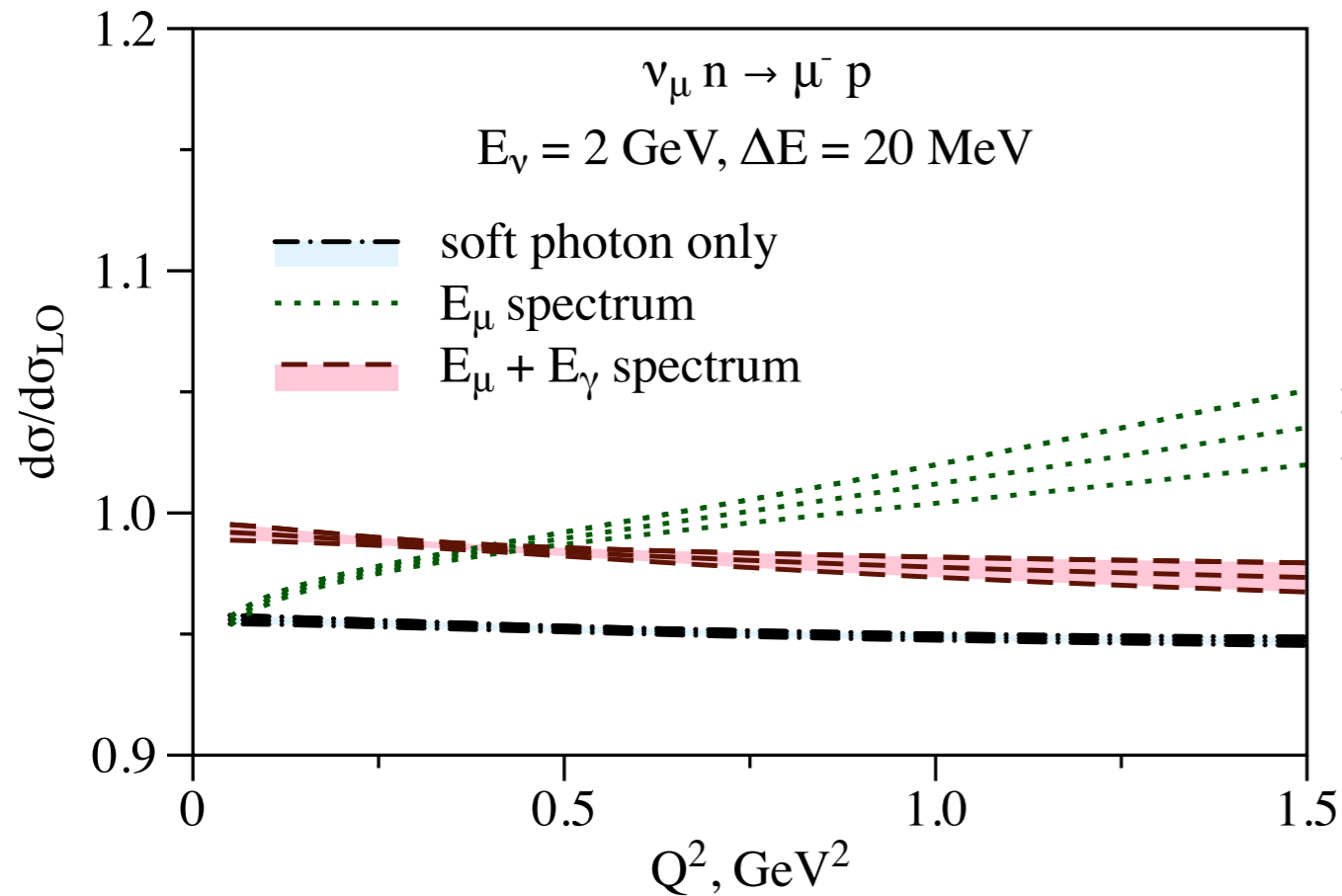
- momentum transfer is defined as  $Q^2 = 2M(E_\nu - E_\ell - E_\gamma)$   
 $Q^2 = 2M(E_\nu - E_\ell)$  or from the energy in the cone



- preliminary uncertainty on plots with inclusive observables

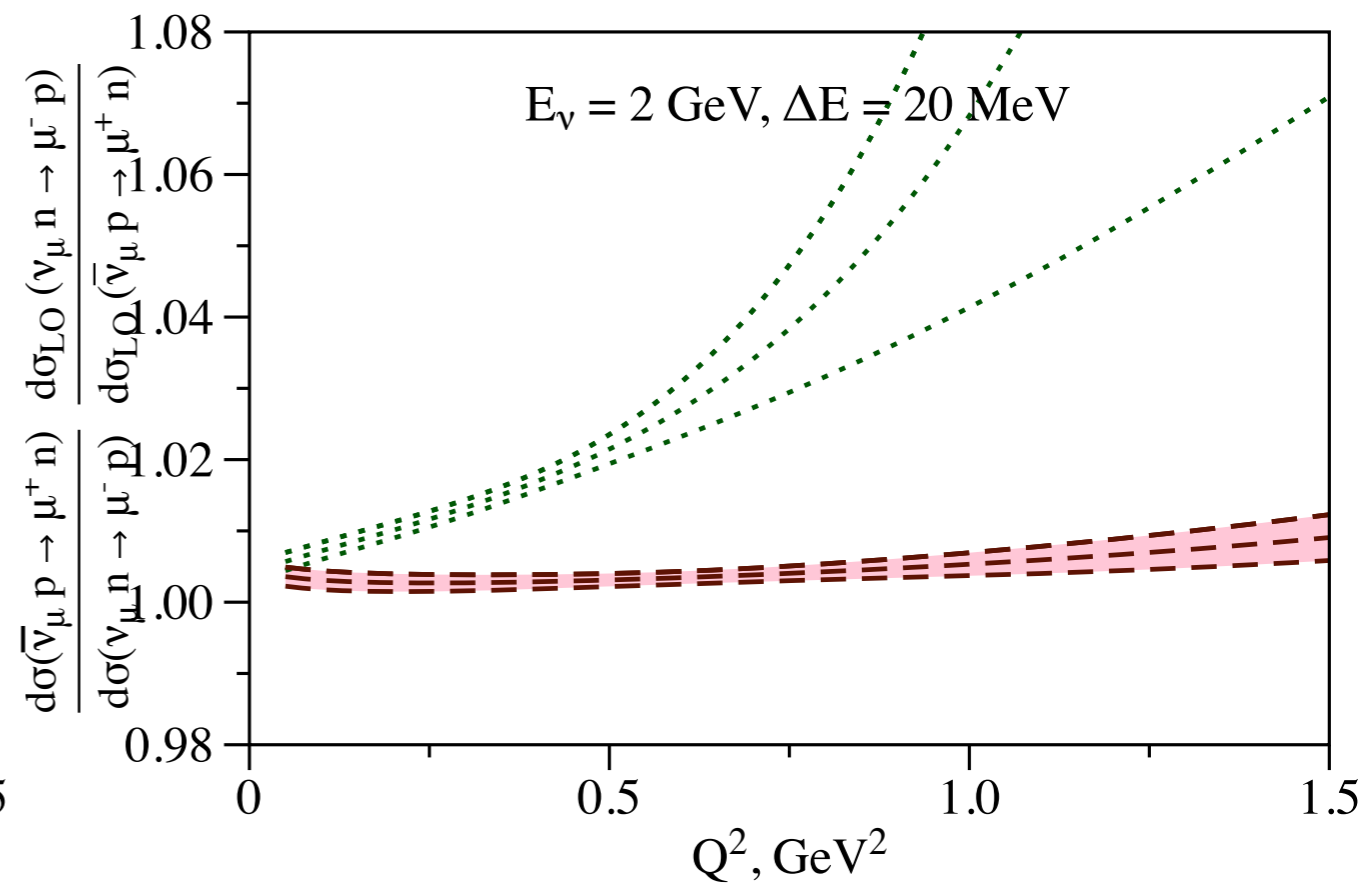
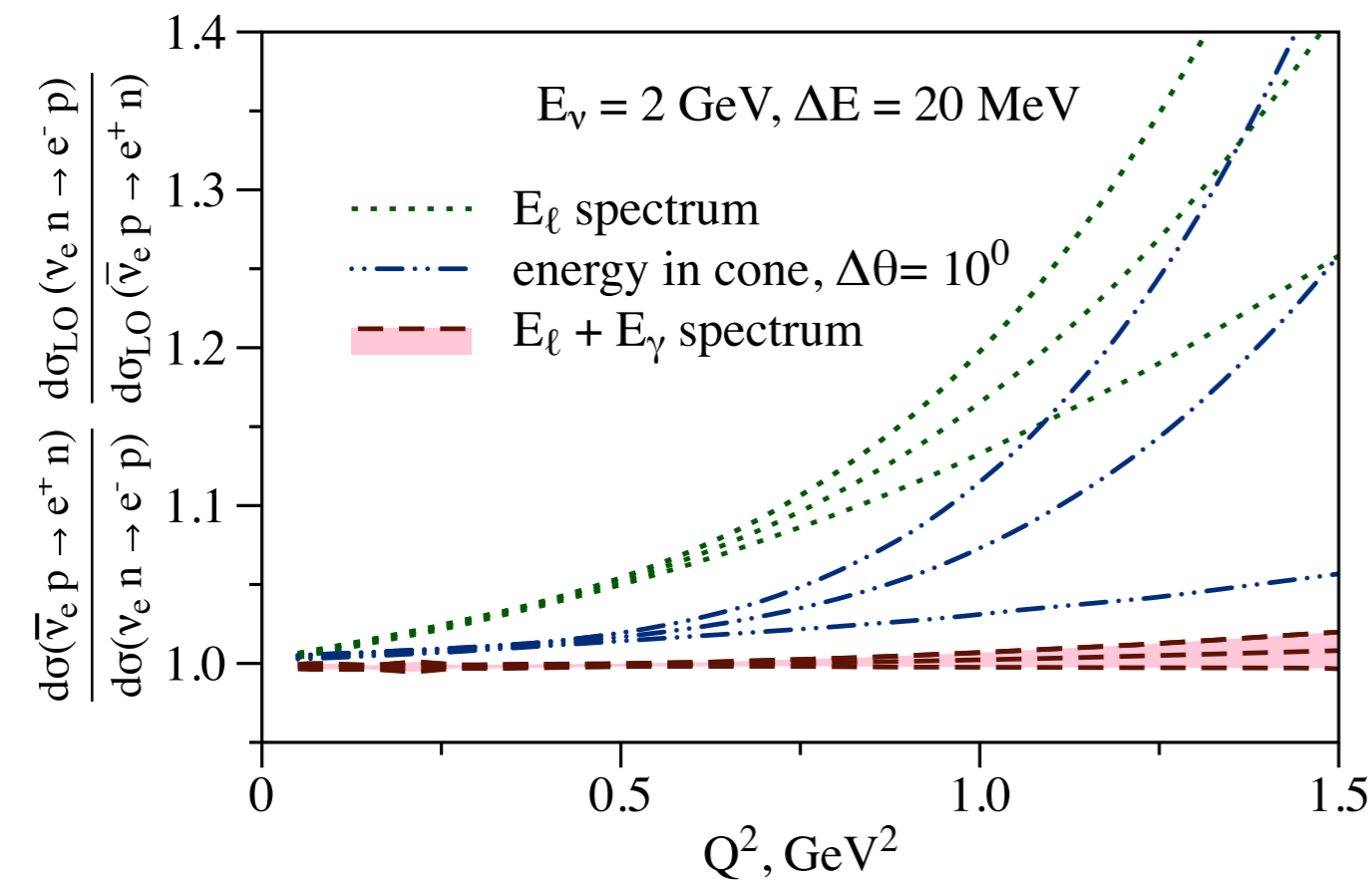
- dependence on reconstruction of kinematics

# Cross section. Muon flavor



- dependence on reconstruction of kinematics

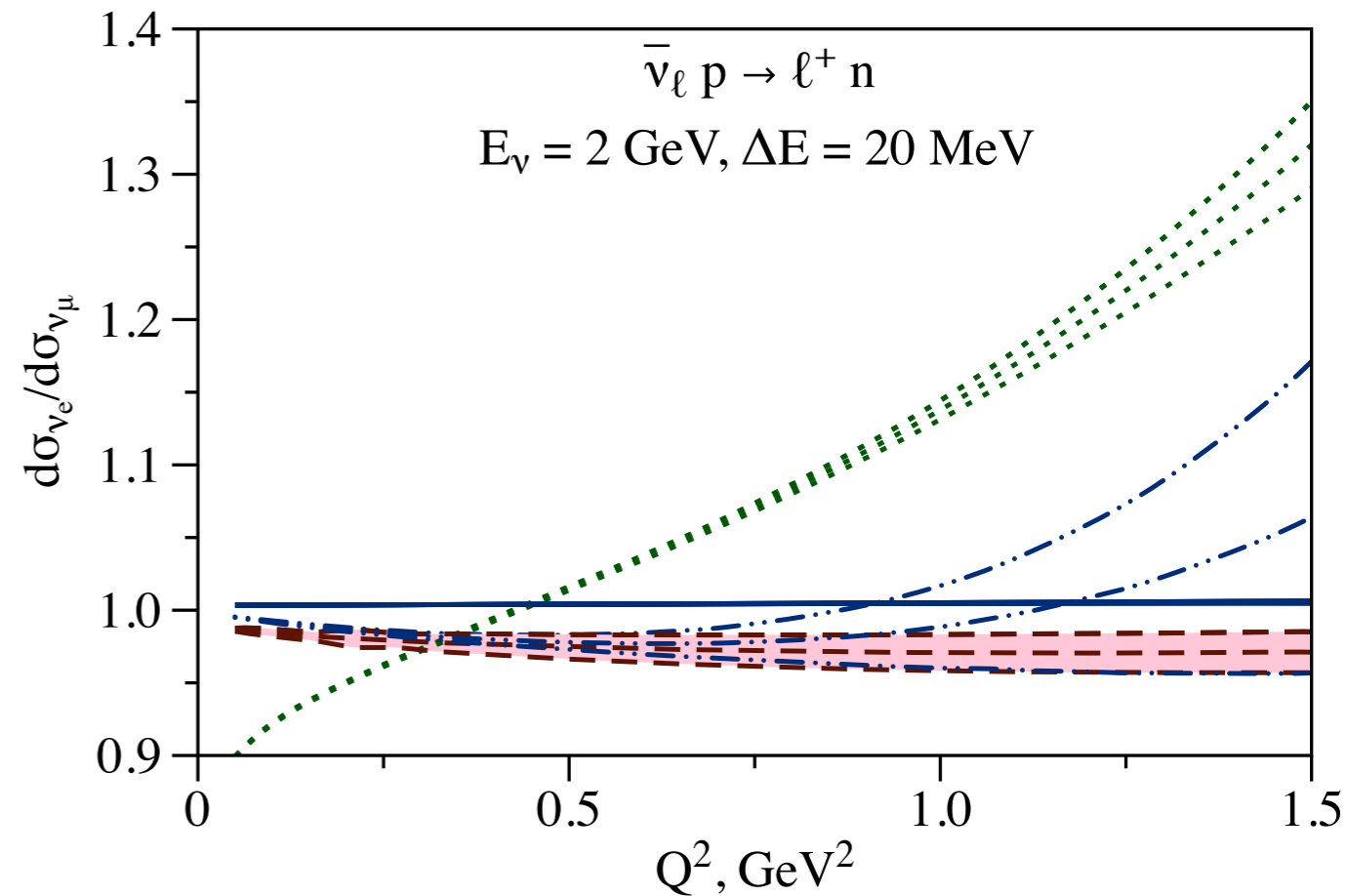
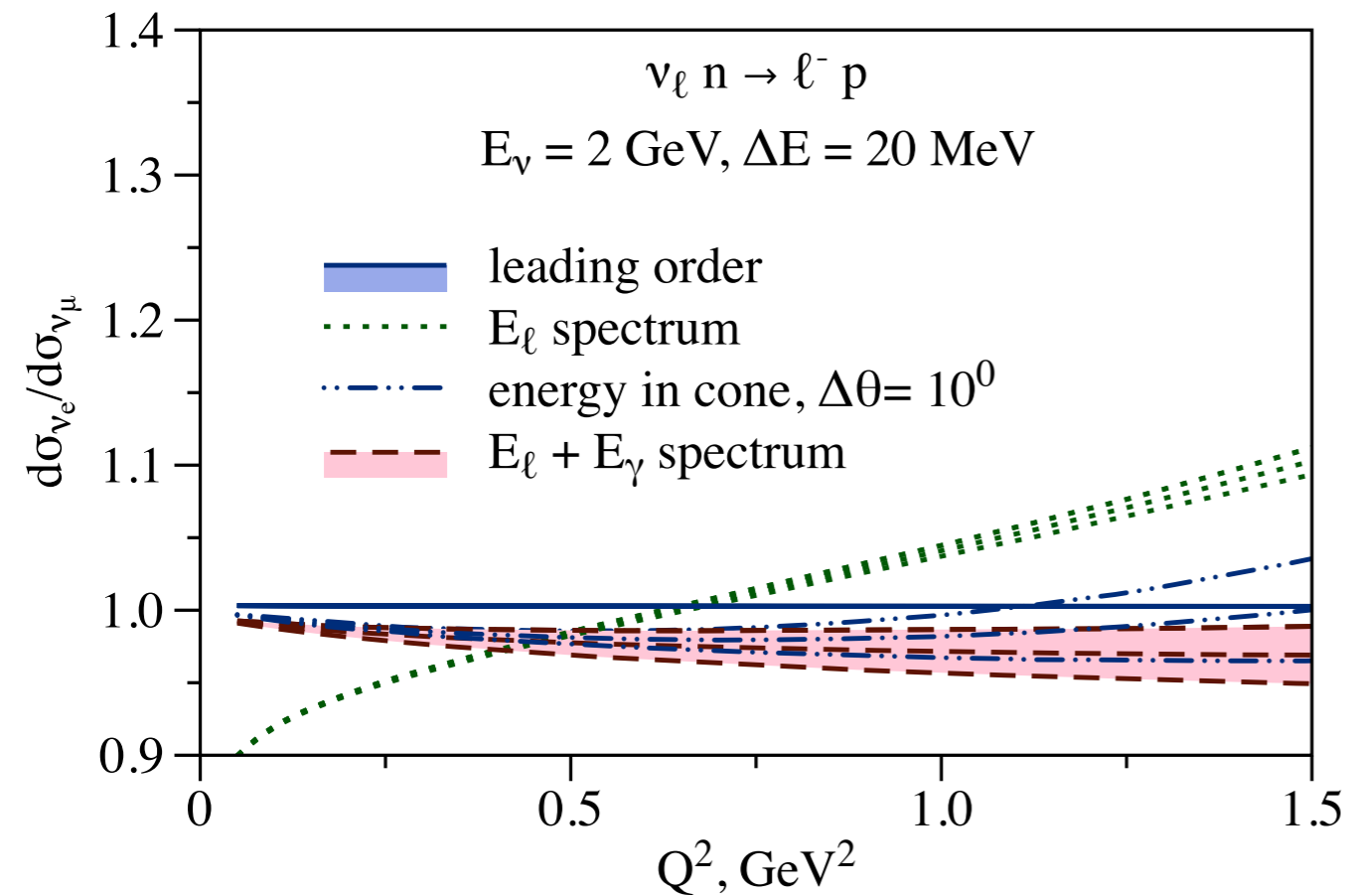
# Neutrino/antineutrino double ratio



- deviations from 1 for  $E_\ell$  spectrum

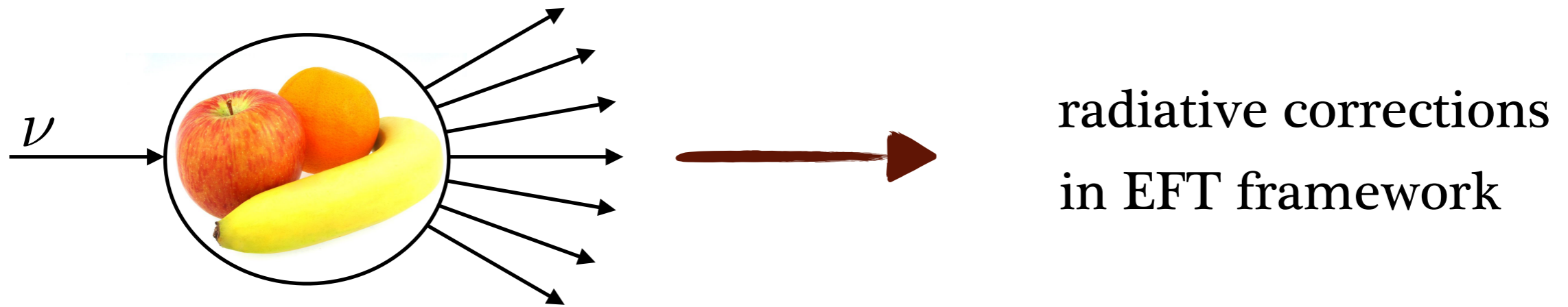


# Electron/muon ratio



- energy in cone observable is close to  $E_\ell + E_\gamma$  spectrum

# Conclusions



- precision four-Fermi effective theory: basis for computations with sub-percent accuracy in neutrino interactions
- total and differential  $\nu e$ , CE $\nu$ NS and CCQE cross sections evaluated from theory with first rigorous error analysis
- corrections to CCQE significantly depend on experimental details

Thanks for your attention !!!



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