Short Time Approximation

New Directions in Neutrino-Nucleus Scattering - NuSTEC

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Computational Resources awarded by the DOE ALCC and INCITE programs

Microscopic (or *ab initio*) Description of Nuclei

Goal:

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions

Requirements:

- Accurate understanding of the interactions/correlations between nucleons in paris, triplets, ... (two- and three-nucleon forces)
- Accurate understanding of the electroweak interactions of leptons with nucleons, correlated nucleon-pairs, … (one- and two-body electroweak currents)
- Computational methods to solve the many-body nuclear problem of strongly interacting particles

Strategy

Validate the Nuclear Model against available data for strong and electroweak observables

- Energy Spectra, Electromagnetic Form Factors, Electromagnetic Moments, ...
- Electromagnetic and Beta decay rates, ...
- Muon Capture Rates, ...
- **Electron-Nucleus Scattering Cross Sections**, ...

Use attained information to make (accurate) predictions for BSM searches and precision tests

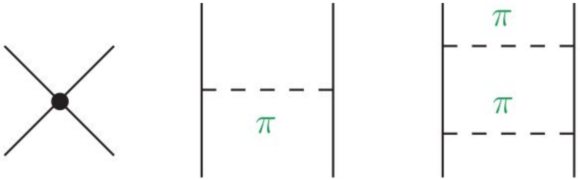
- EDMs, Hadronic PV, ...
- BSM searches with beta decay, ...
- Neutrinoless double beta decay, ...
- **Neutrino-Nucleus Scattering Cross Sections**, ...
- ...

Many-body Nuclear Interactions

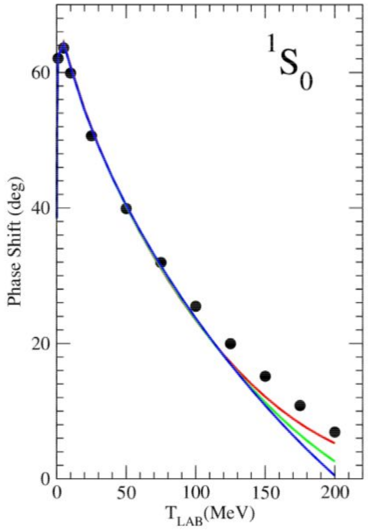
Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

v_{ij} and V_{ijk} are two- and three-nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics



Contact term: short-range
Two-pion range: intermediate-range $r \propto (2\, m_\pi)^{-1}$
One-pion range: long-range $r \propto m_\pi^{-1}$



SP et al. PRC80(2009)034004

In Quantum Monte Carlo methods we use:

AV18+UIX; AV18+IL7 Wiringa, Schiavilla, Pieper *et al.*
chiral πN N2LO+N2LO Gerzelis, Tews, Lynn *et al.*
chiral πNΔ N3LO+N2LO Piarulli *et al.* Norfolk Models

Quantum Monte Carlo Methods

Minimize the expectation value of the nuclear Hamiltonian: H = T +v_{ij} + V_{ijk}

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using the trial wave function:

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

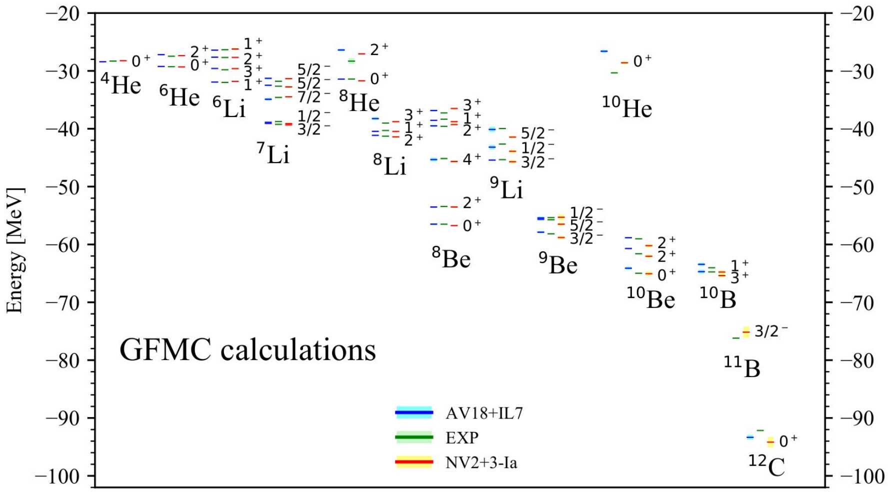
Further improve the trial wave function by eliminating spurious contaminations via a Green's Function Monte Carlo (GFMC) propagation in imaginary time

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

$$\Psi(\tau \to \infty) = a_0 \psi_0$$

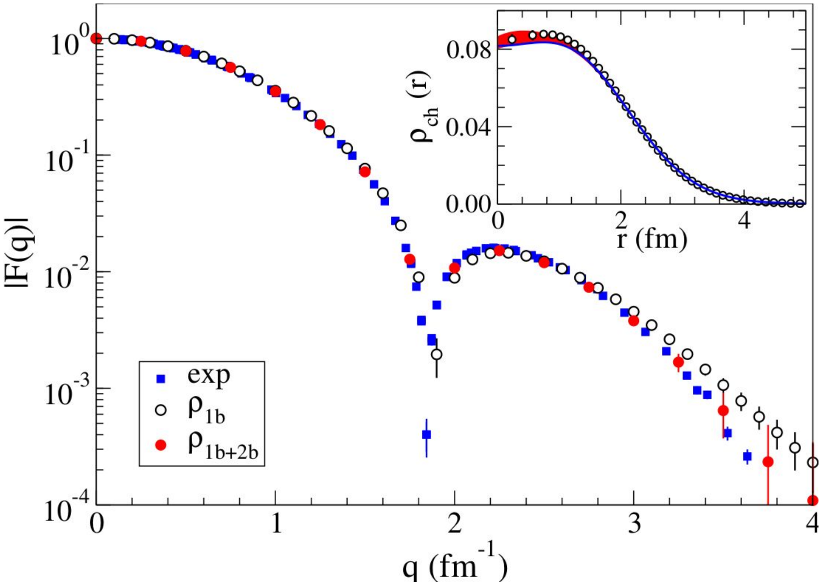
AV18+UIX; AV18+IL7 Wiringa, Schiavilla, Pieper *et al.*
chiral πN N2LO+N2LO Gerzelis, Tews, Lynn *et al.*
chiral πNΔ N3LO+N2LO Piarulli *et al.* Norfolk Models

Energies and Shapes of Nuclei

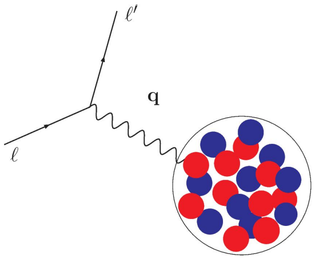


Spectra of light nuclei
Piarulli *et al.* PRL120(2018)052503

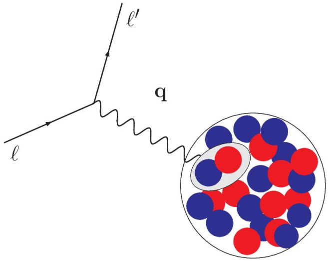
Charge form factor ^{12}C
Lovato *et al.* PRL111(2013)092501



Many-body Nuclear Electroweak Currents



one-body



two-body

- One-body currents: non-relativistic reduction of covariant nucleons' currents
- Two-body currents are a manifestation of two-nucleon correlations
- Electromagnetic two-body currents are required to satisfy current conservation

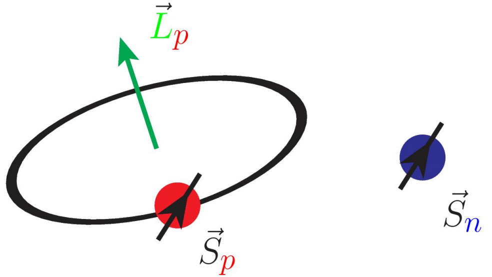
$$\mathbf{q} \cdot \mathbf{j} = [\,H,\, \rho\,] = [t_i + v_{ij} + V_{ijk},\, \rho\,]$$

Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots$$

Nuclear (Vector) Current Operator

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$



Magnetic Moment: Single Particle Picture

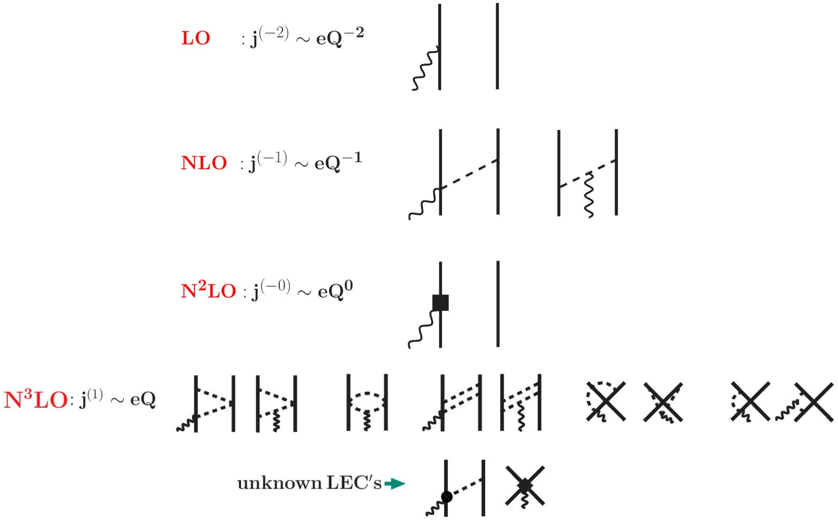
Many-body Currents: Available Models

- **Meson Exchange Currents** (MEC)

Constrain the MEC current operators by imposing that the current conservation relation is satisfied with the given two-body potential

- **Chiral Effective Field Theory Currents**

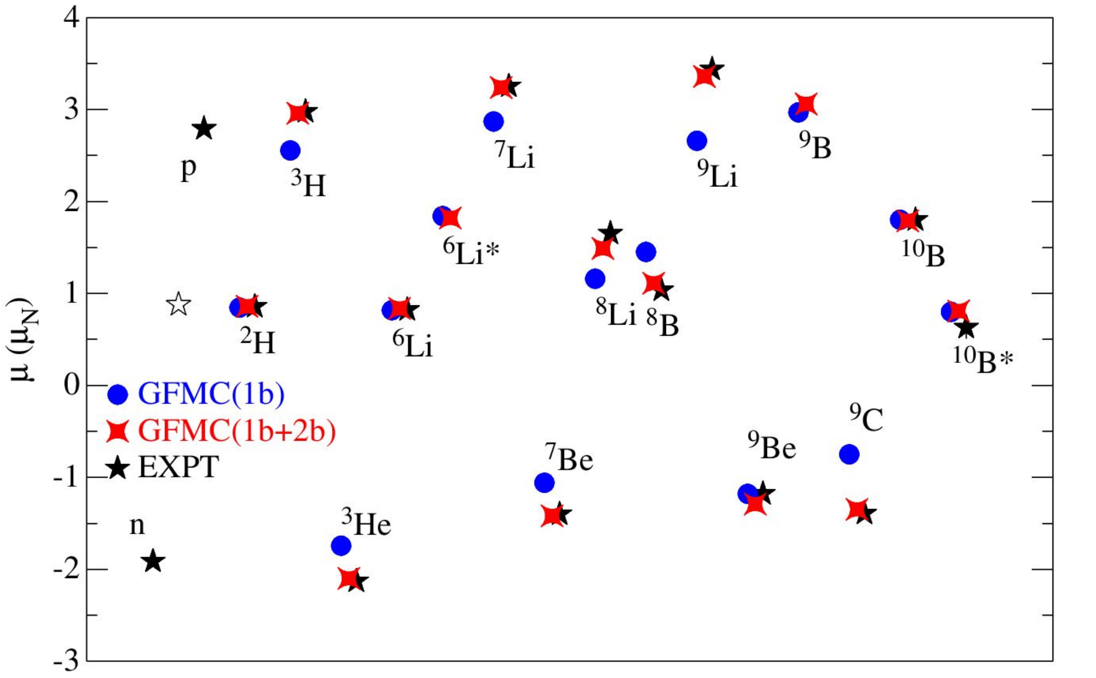
Are constructed consistently with the two-body chiral potential; Unknown parameters, or Low Energy Constants (LECs), need to be determined by either fits to experimental data or by QCD calculations, as well as nucleonic form factors



Electromagnetic Current Operator

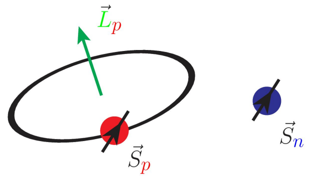
SP *et al.* PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001, PRC87(2013)014006
Park *et al.* NPA596(1996)515, Phillips (2005)
Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008

Magnetic Moments of Light Nuclei



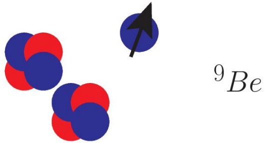
SP *et al*. PRC87(2013)035503

Single particle picture

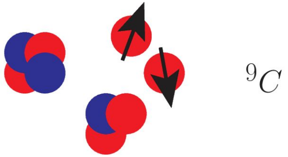


$$\mu_N(1\mathrm{b}) = \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

Small two-body current effects

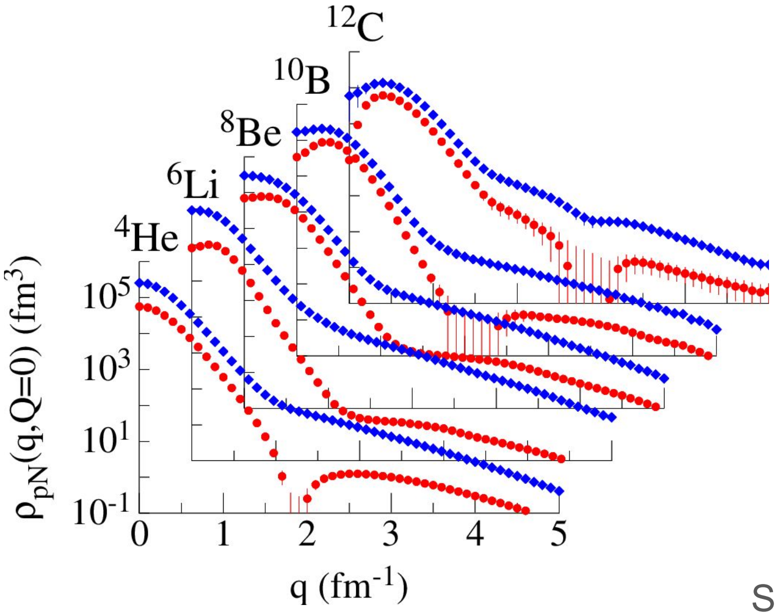


Large two-body current effects

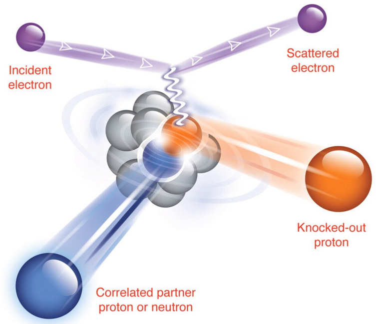


Electron-Nucleus Scattering

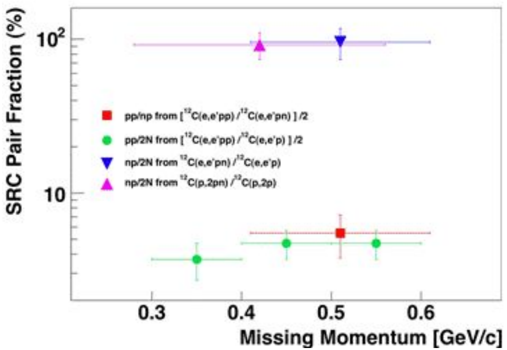
Two-body momentum distribution
Wiringa *et al.* PRC89(2014)024305



pp-pairs; np-pairs



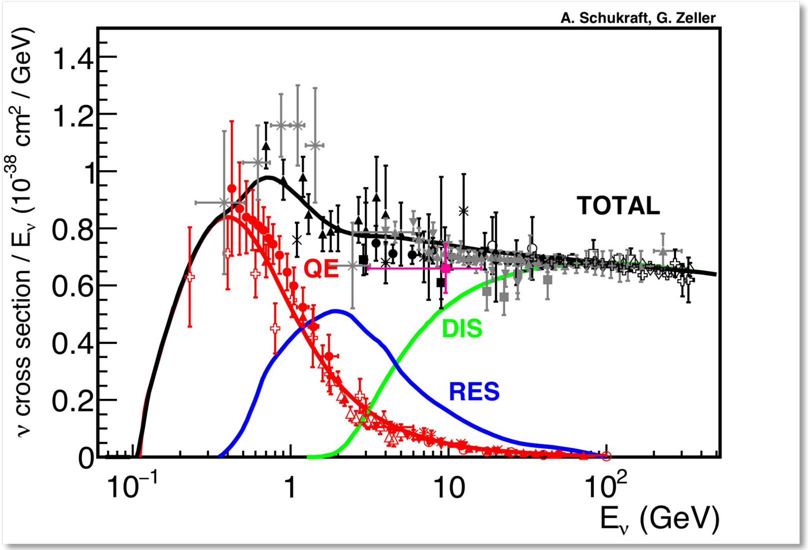
Nuclear properties are strongly affected by two-body correlations and currents in a wide range of energy and momentum transfer



Subedi et al. Science320(2008)1475

Neutrino-Nucleus Interactions

High Energy (on Nuclear Physics Scale):
Neutrino-Nucleus Cross Section



Formaggio and Zeller

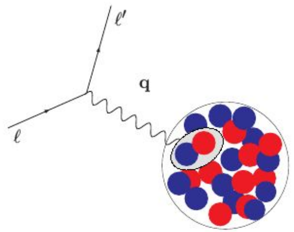
Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_\alpha(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) |\langle f|O_\alpha(\mathbf{q})|0\rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$
Transverse response induced by the current operator $O_T = \boldsymbol{j}$
5 Responses in neutrino-nucleus scattering

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$

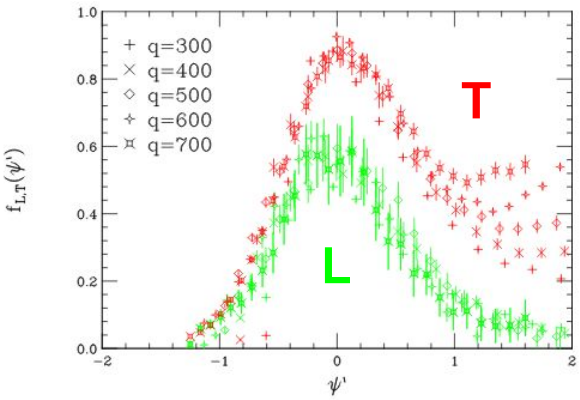


For a recent review see Rocco *Front.inPhys.*8 (2020)116

Lepton-Nucleus scattering: Data

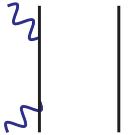
Transverse Sum Rule

$$S_T(q) \propto \langle 0|\mathbf{j}^\dagger\ \mathbf{j}|0\rangle \propto \langle 0|\mathbf{j}_{1b}{}^\dagger\ \mathbf{j}_{1b}|0\rangle + \langle 0|\mathbf{j}_{1b}{}^\dagger\ \mathbf{j}_{2b}|0\rangle + \ldots$$



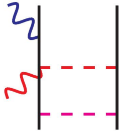
^{4}He Electromagnetic Data
Carlson *et al.* PRC65(2002)024002

Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term



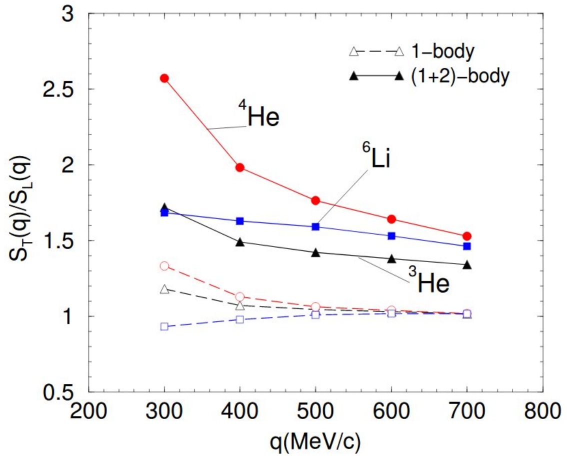
$\langle \mathbf{j}_{1b}^\dagger\ \ \mathbf{j}_{1b}\rangle\ > 0$

Leading one-body term



$\langle \mathbf{j}_{1b}^\dagger\ \mathbf{j}_{2b}\ v_\pi\rangle \propto \langle v_\pi^2\rangle\ > 0$

Interference term

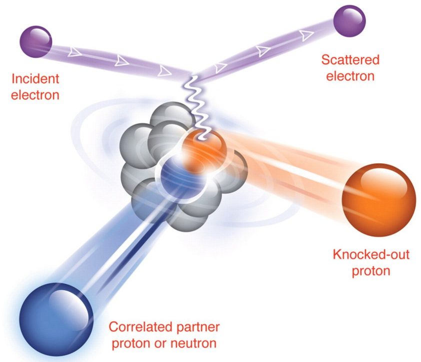


Transverse/Longitudinal Sum Rule
Carlson *et al.* PRC65(2002)024002

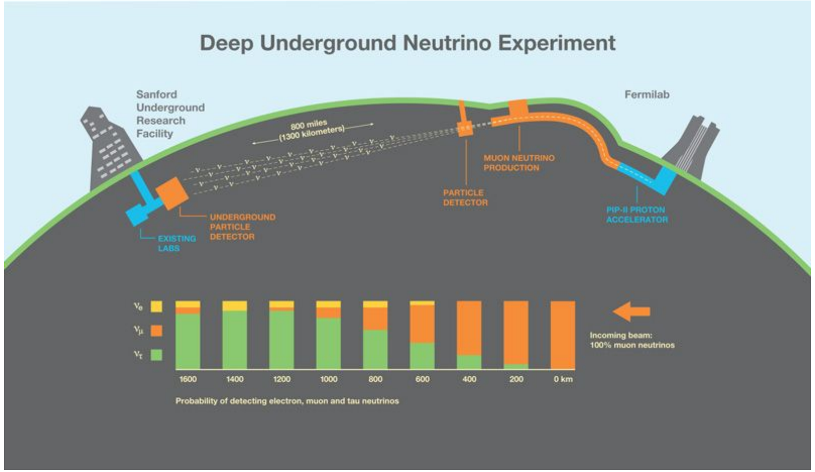
Beyond Inclusive: Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from A > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



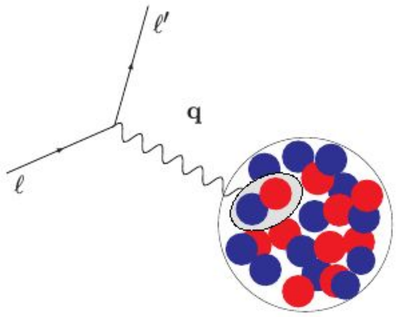
Subedi et al. Science320(2008)1475



Stanford Lab article

e4u collaboration

Short-Time-Approximation



Short-Time-Approximation:

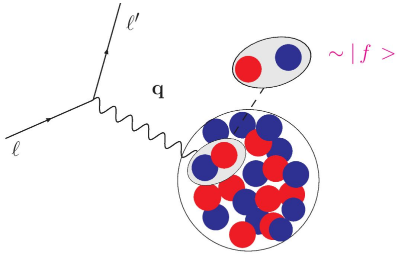
- Based on Factorization
- Ratain two-body physics
- Correctly accounts for interference

$$R(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_0)t} \langle 0|O^\dagger e^{-iHt} O|0\rangle$$

$$O_i^\dagger e^{-iHt} O_i + O_i^\dagger e^{-iHt} O_j + O_i^\dagger e^{-iHt} O_{ij} + O_{ij}^\dagger e^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i<j} v_{ij}$$

Short-Time-Approximation



Short-Time-Approximation:

- Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides “more” exclusive information in terms of nucleon-pair kinematics via the Response Densities

Response Functions

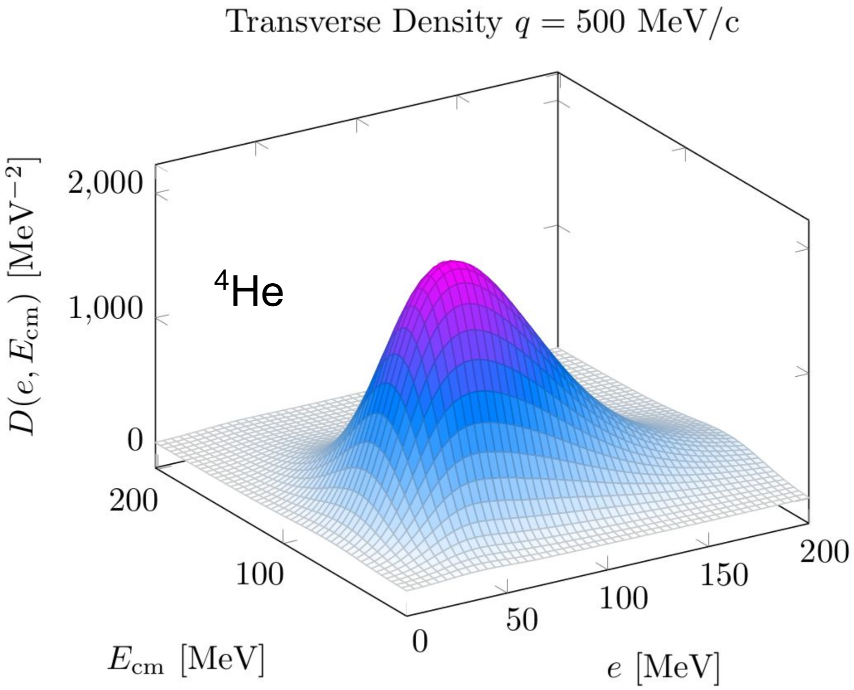
$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) |\langle f|O_{\alpha}(\mathbf{q})|0\rangle|^2$$

Response Densities

$$R(q,\omega) \sim \int \delta\left(\omega + E_0 - E_f\right)\, dP'\, dp' \mathcal{D}(p', P'; q)$$

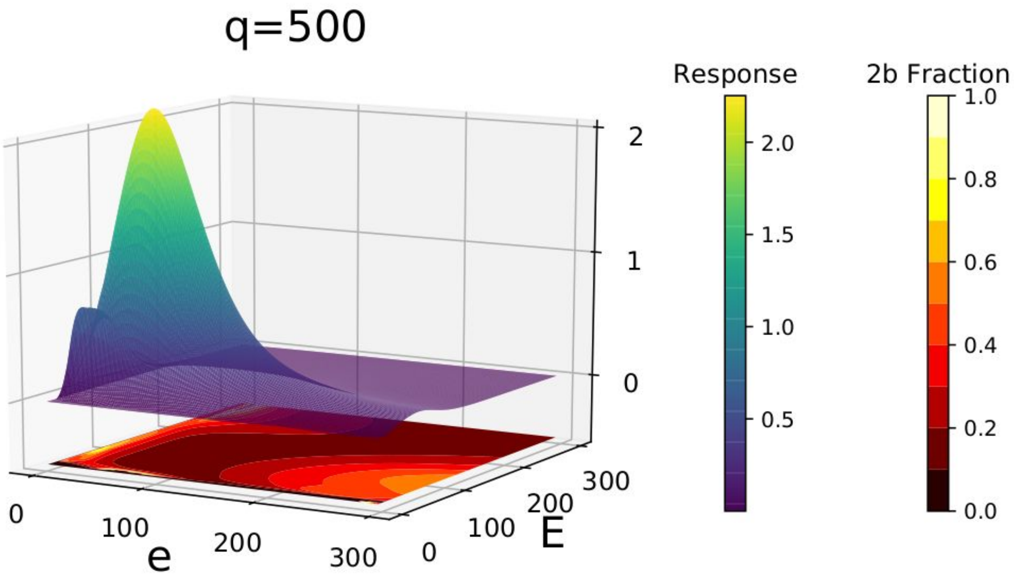
P’ and *p’* are the CM and relative momenta of the struck nucleon pair

Transverse Response Density: e-^{4}He scattering

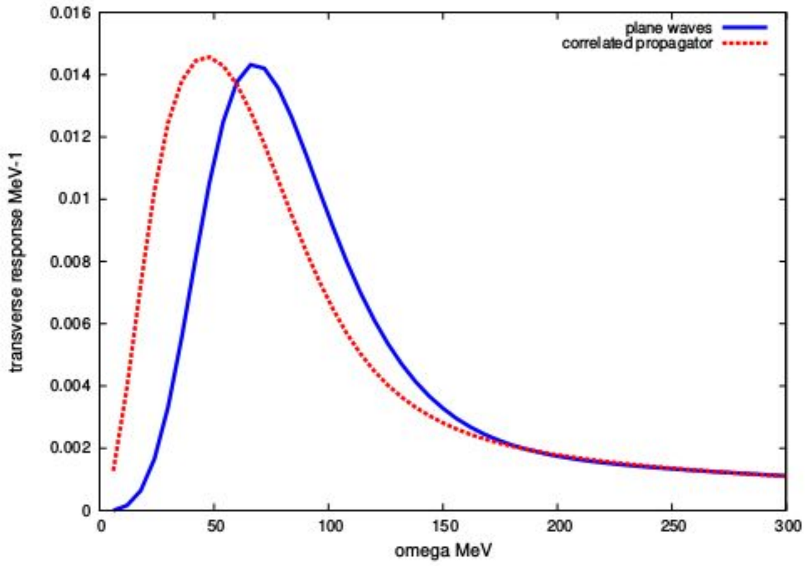


SP *et al.* PRC101(2020)044612

Transverse Response Density: two-body physics

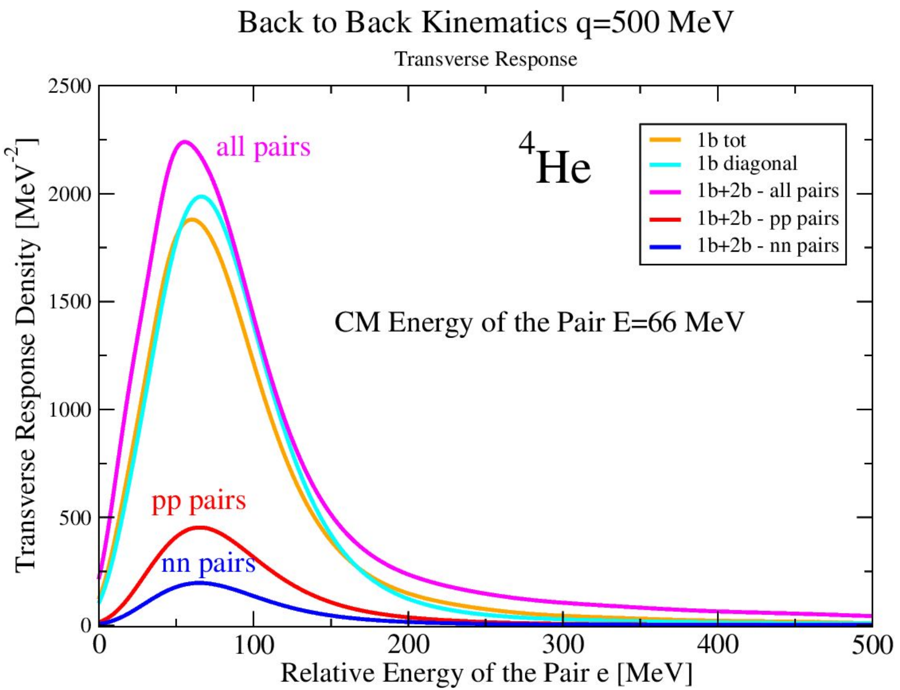


Correlated pairs vs uncorrelated pairs



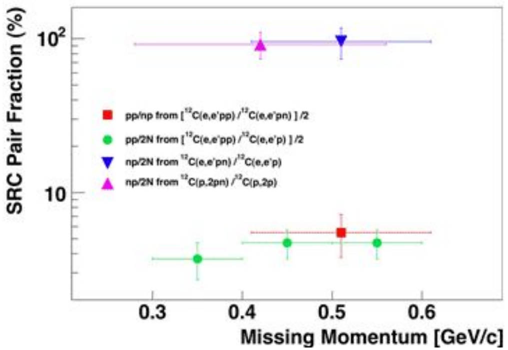
Scattering from uncorrelated vs correlated nucleon pairs

e-^{4}He scattering in the back-to-back kinematic



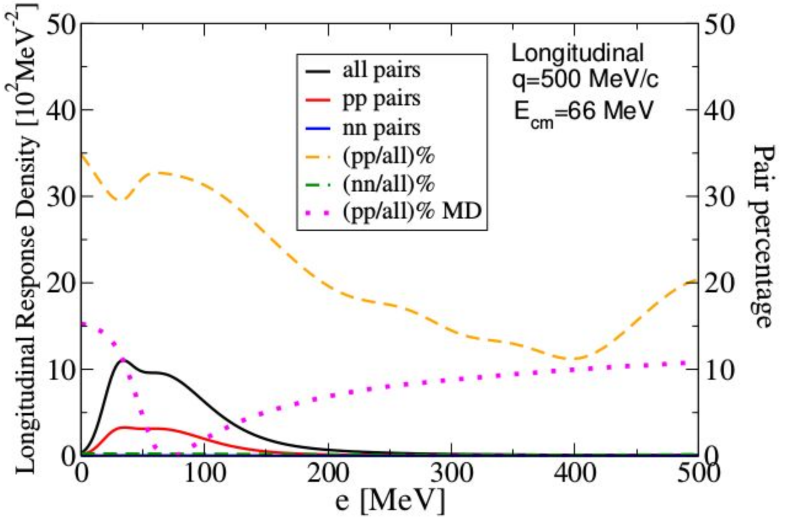
SP *et al.* PRC101(2020)044612

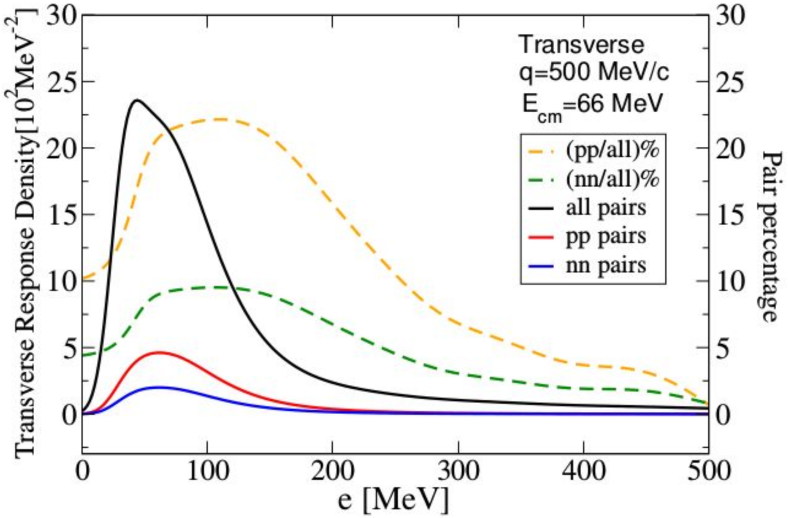
- pp pairs
- nn pairs
- all pairs 1body
- all pairs tot



Subedi et al. Science320(2008)1475

Back to back scattering and particle identity





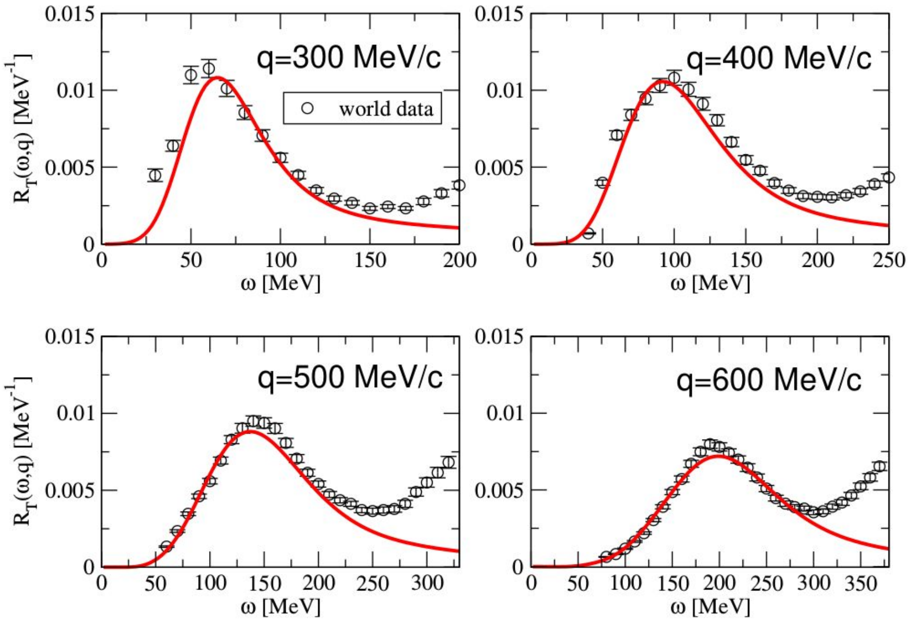
tot

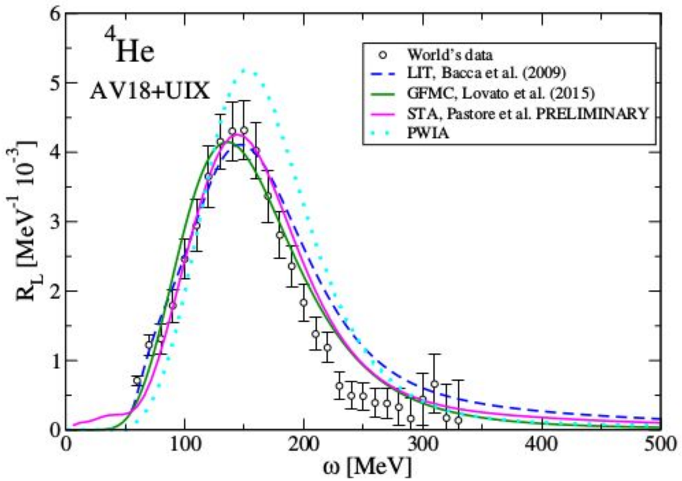
pp nn

pp/all % pp/all % from momentum distributions

nn/all %

Helium-4 comparison with the data



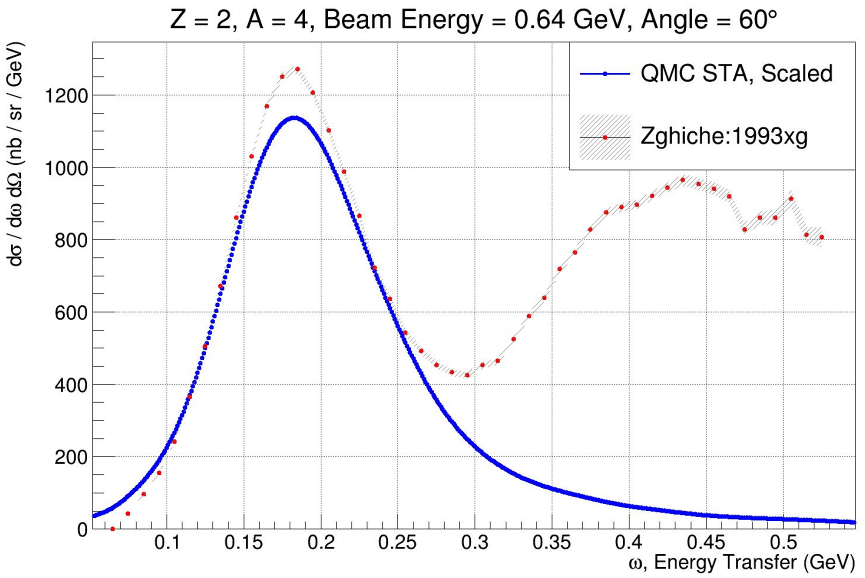


SP *et al.* PRC101(2020)044612

Implementations in GENIE

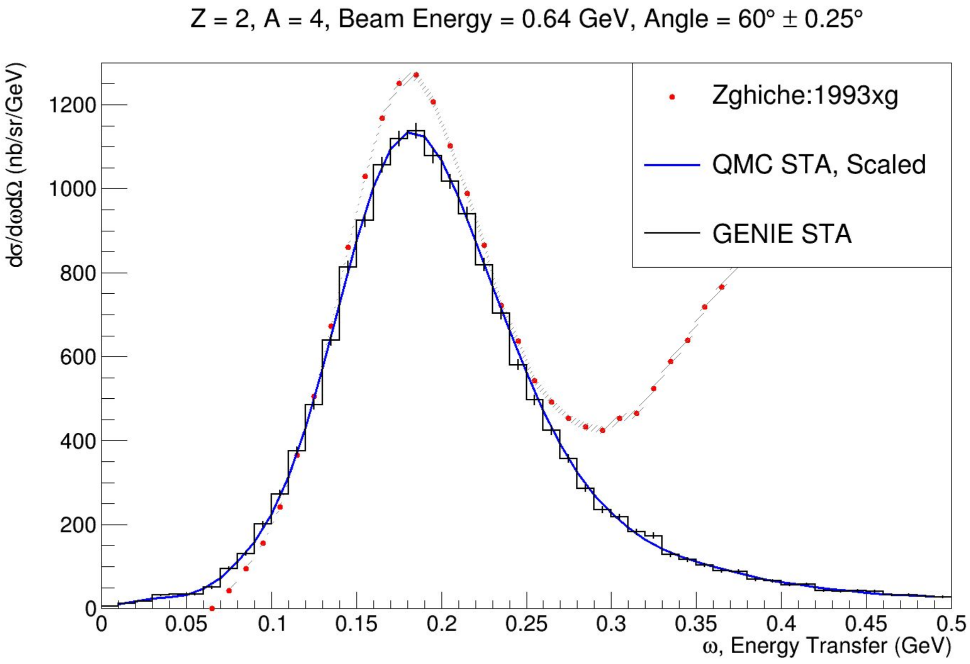
- STA responses used to build the cross sections
- Responses calculated on a finer grid of momentum transfer using scaling functions
- A finer grid in momentum transfer is required to achieve smoother interpolations

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M \left[v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)\right]$$



Barrow, Gardiner, Betancourt *et al.* PRD (2021)

GENIE validation using e-scattering

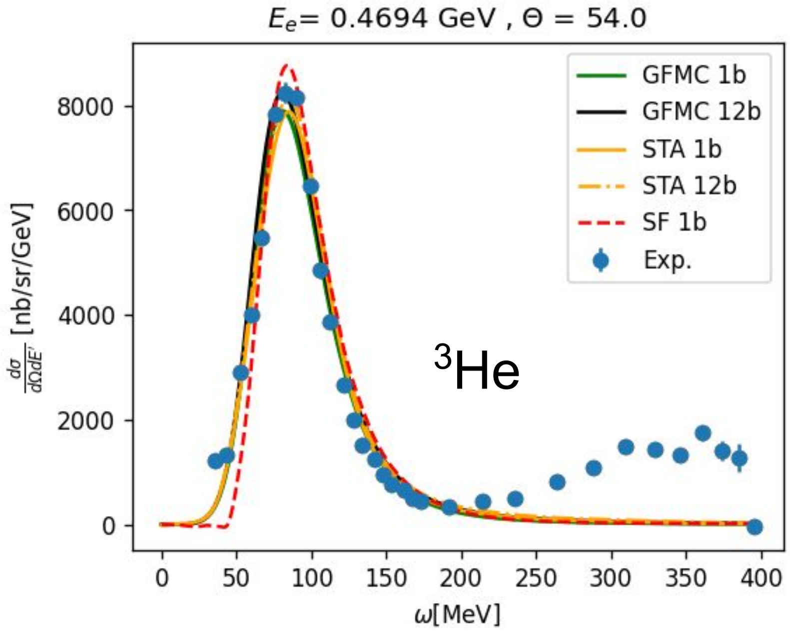


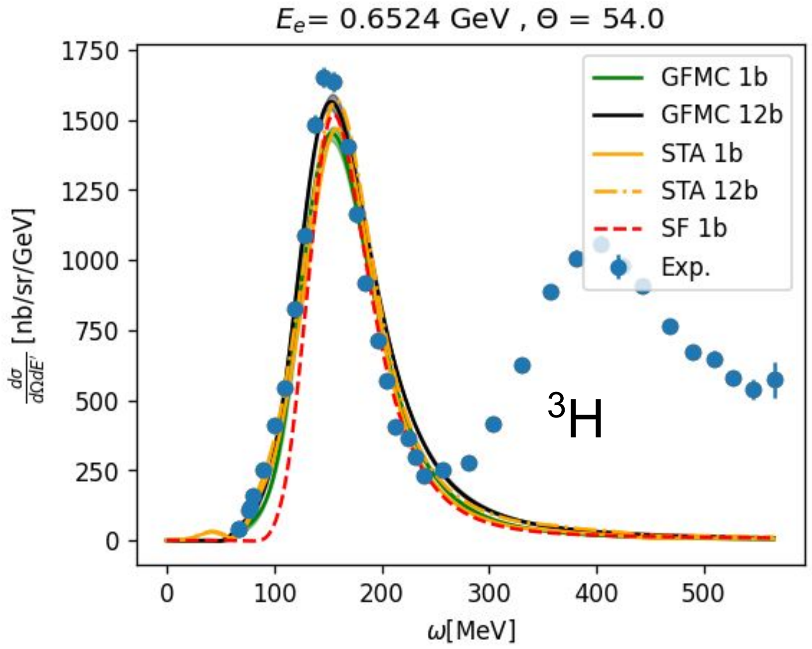
Barrow, Gardiner *et al.* to appear on PRD (2021)

Ongoing work

- Implementation of moment-morphin interpolation techniques
- Implementations of response ***Densities*** in GENIE
- ^{12}C response densities with Lorenzo Andreoli

GFMC SF STA: Benchmark & error estimate

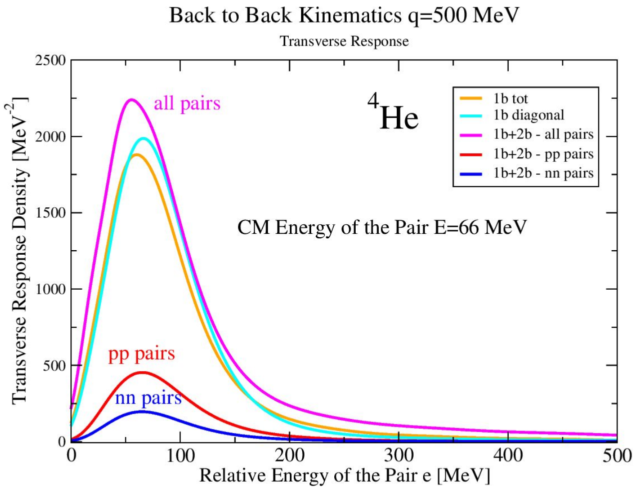


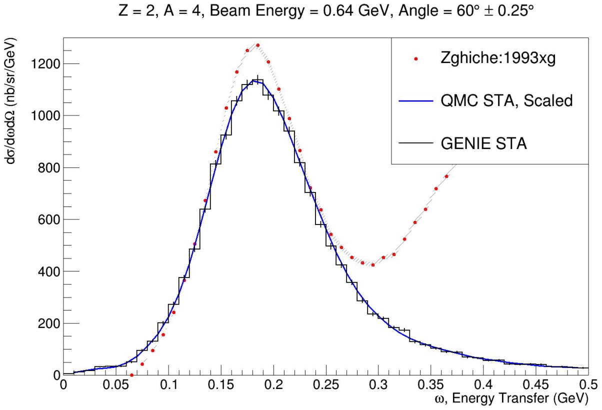


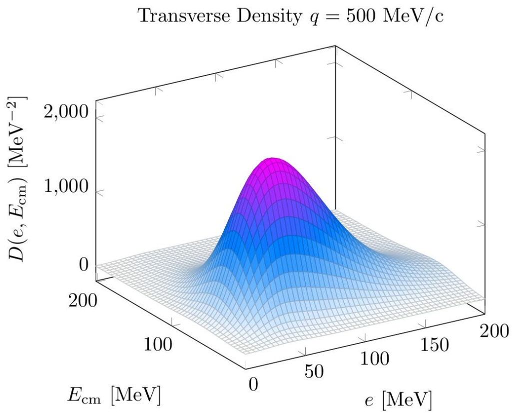
Rocco, Lovato, SP *et al.* ongoing

Summary and Outlook

Ab initio calculations of light nuclei yield a picture of nuclear structure and dynamics where many-body effects play an essential role to explain available data.







Close collaborations between NP, LQCD, Pheno, Hep, Comp, Expt, ...
are required to progress

It's a very exciting time!

Collaborators

WashU: **Andreoli Bub King** Piarulli
LANL: **Baroni** Carlson Cirigliano Gandolfi Hayes Mereghetti
JLab+ODU: Schiavilla
ANL: Lovato Rocco Wiringa
UCSD/UW: Dekens
Pisa U/INFN: Kievsky Marcucci Viviani
Salento U: Girlanda
Huzhou U: Dong Wang

Washington University in St. Louis
Arts & Sciences

Theory Alliance
FACILITY FOR RARE ISOTOPE BEAMS

U.S. DEPARTMENT OF ENERGY
Office of Science

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

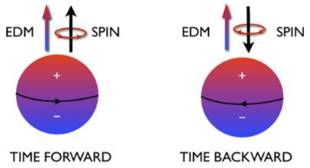
Los Alamos NATIONAL LABORATORY EST. 1943

Argonne NATIONAL LABORATORY

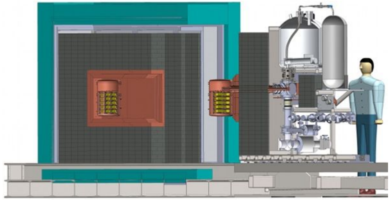
McDONNELL CENTER FOR THE SPACE SCIENCES

Fermilab

Ground States' Electroweak Moments, Form Factors, Radii



Neutrinoless Double Beta Decay, Muon-Capture



Accelerator Neutrino Experiments, Lepton-Nucleus XSecs

(ω,q)~0 MeV

ω~few MeVs
q~0 MeV

ω~few MeVs
q~10^2 MeV

ω~tens of MeVs

ω~10^2 MeV

FRIB

Electromagnetic Decay, Beta Decay, Double Beta Decay & inverse processes

JINA-CEE

Nuclear Rates for Astrophysics

DUNE

Jefferson Lab