Genie

Tuning effort

Marco Roda
mroda@liverpool.ac.uk

on behalf of GENIE collaboration

University of Liverpool

15 March 2021
New Directions in Neutrino-Nucleus Scattering

Neutrino MC generators: our vision

- Connect neutrino fluxes and observables
    - event topologies and kinematics
- *Good* generators
    - optimal coverage of physics processes
    - Uncertainty validation
    - Tune the *physics* models
- Specific requirements for *experiments*
    - Shareable configurations
    - Data agreement
    - ⇒ Simple models can be perfectly acceptable

We don't believe in a *perfect theory* approach

- There are always things that need to be derived from measurements

⇒ Dealing with errors is unavoidable

⇒ Errors are part of the analysis procedures

- See reweight approach

GENIE - www.genie-mc.org

Core GENIE mission - from GENIE by-law

Framework "... provide a state-of-the-art neutrino MC generator for the world experimental neutrino community ..."

Universality "... simulate all processes for all neutrino species and nuclear targets, from MeV to PeV energy scales ..."

Global fit "... perform global fits to neutrino, charged-lepton and hadron scattering data and provide global neutrino interaction model tunes ..."

- Well established generator
  - main generator for LAr experiments at Fermilab
- Modelling effort
  - Discussed by Steven
- Tuning effort
  - Deal with quantities that cannot be predicted
  - Control transition regions - from a model to another
- Other tools: Reweight package, flux drivers, geometry drivers, etc.

GENIE v3.00.06 (latest version) - v3.02.00 (soon to be released)

- Version 3 release introduced new concepts
- "Comprehensive Model Configurations"
  - Self-consistent collections of primary process models
  - Unique string identifier
    - `--tune G18_02a_00_000`
- Some of these models have been tuned
  - Identified by datasets and parameters
  - Unique string identifier
    - `--tune G18_02a_02_11b`

UNIVERSAL NEUTRINO GENERATOR & GLOBAL FIT

Tuning procedure

1. "Brute force" scan the observables to be used for tuning (bins)
   - Decoupling prediction generation and minimisation

2. The bin's behaviour is summarised by response function
   - Polynomial functions of desired order
   - Including all the correlation terms up to the order of the polynomial
   - Fitted against the brute force points

Parameterisation - P dimensional parameter space, M order

$$O^i(\boldsymbol{\theta}) = \alpha_0^i + \sum_{n=1}^{P} \beta_n^i \theta_n + \sum_{n \leq m} \gamma_{nm}^i \theta_n \theta_m + \ldots + \sum_{n_1 \leq \ldots \leq n_M} \xi_{n_1 \ldots n_M}^i \prod_{\ell=1}^{M} \theta_{n_\ell} \quad (1)$$

More complex scenarios can be imagined

3. The minimisation is performed using the response functions

- The response functions are extracted using `Professor` [1, 2]
- The minimisation code is developed internally and it's based on Minuit
  - Takes into account datasets correlations
  - Can add nuisance parameters

Pros and Cons

Pros and Cons

- All parameters can be tuned
  - Not only reweight-able

- It does not work on an event-by-event basis

Pros and Cons

- All parameters can be tuned
  - Not only reweight-able
- Re-usability
  - Response functions can be stored

- It does not work on an event-by-event basis
- Changes in the analysis might require a re-run of the full chain
  - New bins
  - changes in the parameter ranges

Pros and Cons

- All parameters can be tuned
    - Not only reweight-able
- Re-usability
    - Response functions can be stored
- Reweight modules are not necessary

- It does not work on an event-by-event basis
- Changes in the analysis might require a re-run of the full chain
    - New bins
    - changes in the parameter ranges
- Incompatible with reweight

Pros and Cons

- All parameters can be tuned
  - Not only reweight-able
- Re-usability
  - Response functions can be stored
- Reweight modules are not necessary

- It does not work on an event-by-event basis
- Changes in the analysis might require a re-run of the full chain
  - New bins
  - changes in the parameter ranges
- Incompatible with reweight

Some details

- Proved to be successful up ~ 20 parameters
  - Limit due to disk/CPU $\Rightarrow$ It can be overcome
- 10 parameters, 5 order polynomial $\Rightarrow$ about 4.5 k scan points
- 20 parameters, 5 order polynomial $\Rightarrow$ about 23 k scan points

Response functions and reweight

- Response functions and reweight are separated (GENIE v3)
  - No reweighting for the tuned parameters
    - Unless a dedicated rweighting tool is already in place
  - We aim to find the best parameter values $\Rightarrow$ best data description
  - We understand the release of tune results can be frustrating

$\Rightarrow$ We encourage experiments to develop their own analyses
- Based on their own response functions
- In this way they will be able to use our statistical results as their priors

Response functions and reweight

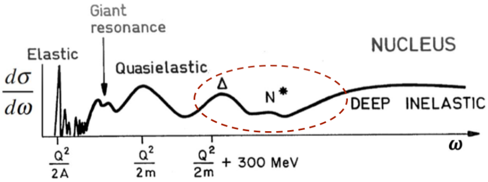
- Response functions and reweight are separated (GENIE v3)
  - No reweighting for the tuned parameters
    - Unless a dedicated rweighting tool is already in place
  - We aim to find the best parameter values ⇒ best data description
  - We understand the release of tune results can be frustrating

⇒ We encourage experiments to develop their own analyses
  - Based on their own response functions
  - In this way they will be able to use our statistical results as their priors

- Reweight updated to include response function concepts (GENIE v4)
  - Response functions could be created in every parameter space
    - ⇒ Including each event generator parameter space
  - Reweight will be configurable with response functions produced by users
    - Overcome the intrinsic limitation of “writing a dedicated module“
  - Still the reweight limitation will apply
    - Hard to reweight things like binding energies, masses, thresholds, etc
    - ⇒ Experiments will produce their own response functions for their analysis bins
    - Optimised for their flux, detector composition, etc.

Tuning details

Resonant to Deep Inelastic Scattering transition



$$\frac{d^2\sigma^{inel}}{dQ^2dW} = \begin{cases} \frac{d^2\sigma^{RES}}{dQ^2dW} + \frac{d^2\sigma^{SIS}}{dQ^2dW} & \text{for } W < W_{cut} \\ \frac{d^2\sigma^{DIS}}{dQ^2dW} & \text{for } W \geq W_{cut} \end{cases}$$

$$\frac{d^2\sigma^{SIS}}{dQ^2dW} = \frac{d^2\tilde{\sigma}^{DIS}}{dQ^2dW}\Theta(W_{cut} - W)\sum_m f_m(Q^2, W)$$

$$f_m(Q^2, W) = R_m \cdot P_m^{had}(Q^2, W)$$

Parameter	GENIE parameter name	*Default* value	Min value	Max value	Prior
W_{cut} (GeV/c^2)	`Wcut`	1.7	1.5	2.3	
M_A^{QE} (GeV/c^2)	`QEL-Ma`	0.999	0.75	1.10	1.014 ± 0.014 [3]
M_A^{RES} (GeV/c^2)	`RES-Ma`	1.12	0.8	1.3	1.12 ± 0.03 [4]
$R_{\nu p}^{CC1\pi}$	`DIS-HMultWgt-vp-CC-m2`	0.10	0.0	0.4	
$R_{\nu p}^{CC2\pi}$	`DIS-HMultWgt-vp-CC-m3`	1.00	0.0	2.0	
$R_{\nu n}^{CC1\pi}$	`DIS-HMultWgt-vn-CC-m2`	0.30	0.0	0.35	
$R_{\nu n}^{CC2\pi}$	`DIS-HMultWgt-vn-CC-m3`	1.00	0.8	3.0	
S_{RES}	`RES-CC-XSecScale`	1.0	0.6	1.2	
S_{DIS}	`DIS-CC-XSecScale`	1.032	0.9	1.15	1 ± 0.05

Observables and data

- Bubble chambers data
  - Muon (anti)neutrinos CC interactions on hydrogen and deuterium targets
    - ANL 12FT, BNL 7FT, FNAL 15FT and BEBC
- Integrated cross sections as a function of the neutrino energy
  - ν_μ and $\bar{\nu}_\mu$ CC inclusive scattering [5–32]
  - ν_μ and $\bar{\nu}_\mu$ CC quasi-elastic scattering [5, 15, 29, 33–41]
  - ν_μ and $\bar{\nu}_\mu$ CC single-pion production [34, 42–50]
    - $\nu_\mu + n \to \mu^- + n + \pi^+$ and $\nu_\mu + p \to \mu^- + p + \pi^+$
    - $\nu_\mu + n \to \mu^- + p + \pi^0$
    - $\bar{\nu}_\mu + p \to \mu^+ + p + \pi^-$ and $\bar{\nu}_\mu + n \to \mu^+ + n + \pi^-$
  - ν_μ CC two-pion production [51]
    - $\nu_\mu + p \to \mu^- + n + 2\pi^+$ and $\nu_\mu + p \to \mu^- + n + \pi^+ + \pi^-$
    - $\nu_\mu + p \to \mu^- + p + \pi^+ + \pi^0$
- Not all datasets are used:
  - Only bins with $E_\nu > 0.5\ GeV$
  - Only latest available version from each experiment
- Our predictions take into account experimental cuts
- Correlations between data from the same experiments

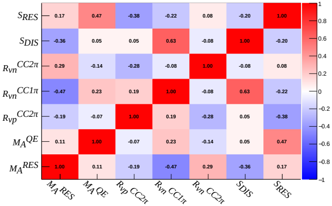
  ⇒ Taken into account with experiment related nuisance parameters

Best fit parameters

- Two fits: one for each CMC
  - Both behaving similarly

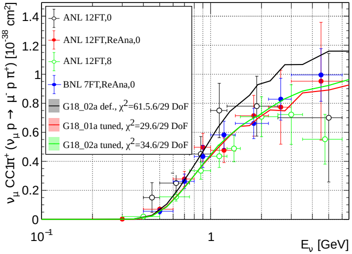
  ⇒ confidence in the procedure
- Results as expected from previous iterations
  - RES interaction suppressed $\sim 15\,\%$
  - 1π very suppressed
    - at the limit of the region
  - 2π enhanced
- Reasonable goodness of fit
  - These numbers take into account the nuisance parameters

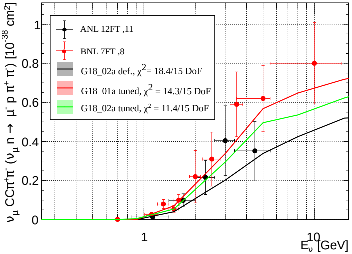
Parameter	G18_01a(/b)	G18_02a(/b)
W_{cut}	1.94	1.81
M_A^{QE}	1.00 ± 0.01	1.00 ± 0.013
M_A^{RES}	1.09 ± 0.02	1.09 ± 0.014
$R_{\nu p}^{CC1\pi}$	0.06 ± 0.03	0.008
$R_{\nu p}^{CC2\pi}$	1.1 ± 0.2	0.94 ± 0.075
$R_{\nu n}^{CC1\pi}$	0.14 ± 0.03	0.03 ± 0.010
$R_{\nu n}^{CC2\pi}$	2.8 ± 0.4	2.3 ± 0.12
S_{RES}	0.89 ± 0.04	0.84 ± 0.028
S_{DIS}	1.03 ± 0.02	1.06 ± 0.01
χ^2/157 DoF	1.84	1.64



Correlations for G18_02a(/b)

Best fit predictions

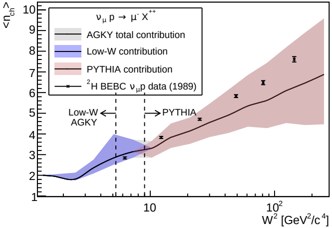




- The agreement improves for every observable
  - True also for inclusive datasets despite observed tensions with exclusive channels
- The predictions of the two CMC after tuning are in strong agreement

Hadronisation modeling

- Another aspect of the inelastic modeling
  - Independent to cross section modeling
  - Focusing on $\mathcal{P}(n|W)$ instead of $\mathcal{P}(W)$
  - Empirically
    - $\langle n_{\text{ch}}\rangle(W) = \alpha_{\text{ch}} + \beta_{\text{ch}} \ln\left(\frac{W^2}{\text{GeV}^2/c^4}\right)$
    - $\langle n_{\text{neut}}\rangle = \frac{1}{2}\langle n_{\text{ch}}\rangle$
- Two different models inside GENIE
  - Pythia - for high W
    - based on Lund fragmentation model
  - Low W AGKY
    - Empirical distribution based on Lévy distribution
    - $\langle n\rangle P\left(\frac{n}{\langle n\rangle}\middle| c\right) = 2e^{-c} c^{c\frac{n}{\langle n\rangle}+1}\left[\Gamma\left(c\frac{n}{\langle n\rangle}+1\right)\right]^{-1}$
  - Joint linearly
    - from $W^{\text{tr}}_{\text{min}} = 2.3$ GeV to $W^{\text{tr}}_{\text{max}} = 3.0$ GeV



Default GENIE v3 low-W AGKY parameters

Parameter	$\nu_\mu p$	$\nu_\mu n$	$\bar{\nu}_\mu p$	$\bar{\nu}_\mu n$
α_{ch}	0.40	-0.20	0.02	0.80
β_{ch}	1.42	1.42	1.28	0.95
c	7.93	5.22	5.22	7.93

Default GENIE v3 PYTHIA parameters

Parameter	Name in PYTHIA	Value
$P_{s\bar{s}}$	PARJ(2)	0.30
$\langle p^2_\perp \rangle$ [GeV^2]	PARJ(21)	0.44
E_{CutOff} [GeV]	PARJ(33)	0.20
Lund a	PARJ(41)	0.30
Lund b [GeV^{-2}]	PARJ(42)	0.58

Datasets and previous attempts

- Data from FNAL 15 ft and BEBC
- Data in the form of $\langle n_{ch} \rangle$ vs W
  - from muon (anti)neutrinos on H, ^2H, heavier targets
    - We only used hydrogen and deuterium data
  - there are other observables but not used this time
    - data sensitive to c parameters
    - to take into account those observables priors have been used on E_{CutOff} and $\langle p_\perp^2 \rangle$
  - Data samples divided by the nucleon hit in the interaction
    - Selection based on the topology
- Every experiment tried to make their own estimation of the linear coefficients
  - Not enough for Pythia contribution
  - Not satisfactory as a global fit
    - Tensions between hydrogen and deuterium data

	$\nu_\mu + p \to \mu^- X^{++}$		
FNAL 15 ft (1976)	H	FNAL 15 ft,0	[52]
BEBC (1983)	H	BEBC,0	[53]
BEBC (1990)	H	BEBC,1	[54]
BEBC (1992)	H	BEBC,2	[55]
FNAL 15 ft (1983)	^{2}H	FNAL 15 ft,1	[56]
BEBC (1989)	^{2}H	BEBC,3	[57]
	$\nu_\mu + n \to \mu^- X^{+}$		
FNAL 15 ft (1983)	^{2}H	FNAL 15 ft,2	[56]
BEBC (1984)	^{2}H	BEBC,4	[58]
BEBC (1989)	^{2}H	BEBC,5	[57]
	$\bar{\nu}_\mu + p \to \mu^+ X^{0}$		
FNAL 15 ft (1981)	H	FNAL 15 ft,3	[59]
BEBC (1983)	H	BEBC,6	[53]
BEBC (1990)	H	BEBC,7	[54]
BEBC (1992)	H	BEBC,8	[55]
BEBC (1982)	^{2}H	BEBC,9	[60]
BEBC (1989)	^{2}H	BEBC,10	[57]
	$\bar{\nu}_\mu + n \to \mu^+ X^{-}$		
BEBC (1982)	^{2}H	BEBC,11	[60]
BEBC (1989)	^{2}H	BEBC,12	[57]

Best fit parameters

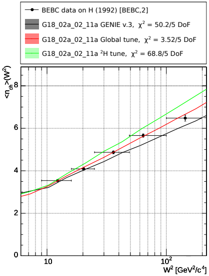
Parameter	GENIE parameter name	Nominal value	Allowed range	Global Fit	^{2}H Fit
		Low-W empirical model			
$\alpha_{\nu p}$	`KNO-Alpha-vp`	0.40	$[-1.0, 2.0]$	1.1 ± 0.3	1.2 ± 0.4
$\alpha_{\nu n}$	`KNO-Alpha-vn`	-0.20	$[-1.0, 2.0]$	$1.75^{+0.14}_{-0.11}$	-0.58 ± 0.07
$\alpha_{\bar{\nu} p}$	`KNO-Alpha-vbp`	0.02	$[-1.0, 2.0]$	$1.32^{+0.16}_{-0.14}$	1.9 ± 0.08
$\alpha_{\bar{\nu} n}$	`KNO-Alpha-vbn`	0.80	$[-1.0, 2.0]$	1.11 ± 0.09	1.07 ± 0.3
$\beta_{\nu p}$	`KNO-Beta-vp`	1.42	$[0.0, 2.5]$	0.79 ± 0.15	0.9 ± 0.3
$\beta_{\nu n}$	`KNO-Beta-vn`	1.42	$[0.0, 2.5]$	0.5 ± 0.1	1.9 ± 0.3
$\beta_{\bar{\nu} p}$	`KNO-Beta-vbp`	1.28	$[0.0, 2.5]$	0.8 ± 0.1	0.3 ± 0.1
$\beta_{\bar{\nu} n}$	`KNO-Beta-vbn`	0.95	$[0.0, 2.5]$	$0.88^{+0.09}_{-0.08}$	0.9 ± 0.2
		PYTHIA			
$P_{s\bar{s}}$	`PYTHIA-SSBarSuppression`	0.30	$[0.0, 1.0]$	0.27 ± 0.04	0.29 ± 0.05
$P_{\langle p_\perp^2 \rangle}$ [GeV$^2/c^2$]	`PYTHIA-GaussianPt2`	0.44	$[0.1, 0.7]$	0.43 ± 0.05	0.43 ± 0.04
E_{CutOff} [GeV]	`PYTHIA-RemainingEnergyCutoff`	0.20	$[0.0, 1.0]$	0.30 ± 0.04	0.24 ± 0.05
Lund a	`PYTHIA-Lunda`	0.30	$[0.0, 2.0]$	1.53 ± 0.13	1.85 ± 0.15
Lund b [GeV$/c^2$]	`PYTHIA-Lundb`	0.58	$[0.0, 1.5]$	1.16 ± 0.09	1.0 ± 0.2
			$\chi^2 =$	87.9/62 DoF	29.5/32 DoF

- Generally good goodness of fit
- Mostly symmetrical χ^2 profiles $\Rightarrow$ Gaussian behaviour
- A few strong (anti)correlation observed: due to transition region

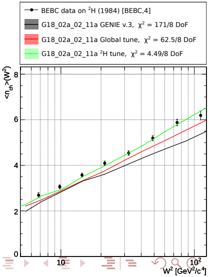
Best fit predictions

- The data prefers a higher multiplicity at high W
  - Low-W is bound by energy conservation
- In general better agreement with data
  - Very strong tension identified
    - Global fit p-value $4 \cdot 10^{-12}$
    - Deuterium fit p-value 0.94
  - Previously observed [61]
    - Not completely understood
- The problem gets complicated when other observables are included
  - Already tension of these data vs others
    - e.g. π^0 multiplicity, ratio of multiplicity variance over average multiplicity
  - Interference with the free nucleon tune
    - A proper tuning procedure should go in inverse order
    - Possible systematic errors in the reconstruction of W

$\nu_\mu + p \to \mu + X^0$



$\nu_\mu + n \to \mu + X^+$



Summary

- Overview of the tuning effort within GENIE
  - Procedures
  - Expected plan
- Satisfactory free nucleon tune
  - Better understood data
  - Analyses from different groups have been going on for a long time
  - Paper ready to submission
- Hadronisation tune is new
  - Data is more tricky
  - First time a global tune of this scale has been attempted
  - Much more to be learned
  - Paper in internal review

Backup

Collaboration

Luis Alvarez-Ruso [9], **Costas Andreopoulos** [5,7], **Adi Ashkenazi** [4], **Christopher Barry** [5], **Steve Dennis** [5], **Steve Dytman** [6], **Hugh Gallagher** [8], **Steven Gardiner** [3], **Walter Giele** [3], **Robert Hatcher** [3], **Or Hen** [4], **Libo Jiang** [6], **Rhiannon Jones** [5], **Igor Kakorin** [2], **Konstantin Kuzmin** [2], **Anselmo Meregaglia** [1], **Donna Naples** [6], **Vadim Naumov** [2], **Afroditi Papadopoulou** [4], **Gabriel Perdue** [3], **Marco Roda** [5], **Vladyslav Syrotenko** [8], **Jeremy Wolcott** [8], **Júlia Tena Vidal** [5], **Julia Yarba** [3]

1. **CENBG, Université de Bordeaux, CNRS/IN2P3**
   33175 Gradignan, France
2. **Joint Institute for Nuclear Research (JINR)**
   Dubna, Moscow region, 141980, Russia
3. **Fermi National Accelerator Laboratory**
   Batavia, Illinois 60510, USA
4. **Massachusetts Institute of Technology (MIT)** Dept. of Physics
   Cambridge, MA 02139, USA
5. **University of Liverpool**, Dept. of Physics
   Liverpool L69 7ZE, UK
6. **University of Pittsburgh**, Dept. of Physics and Astronomy
   Pittsburgh PA 15260, USA
7. **UK Research and Innovation**, **Science and Technology Facilities Council**
   Rutherford Appleton Laboratory, Particle Physics Dept.
   Harwell Oxford Campus, Oxfordshire OX11 0QX, UK
8. **Tufts University**, Dept. of Physics and Astronomy
   Medford MA 02155, USA
9. **University of Valencia**
   Valencia, Spain

Roles of generators in oscillation physics

- Compare data and models
  - Reliability and validity region
  - $\Rightarrow$ You cannot study oscillations without fully understood models
- Compare dataset against dataset
  - Data quality and data sources are increasing $\Rightarrow$ tensions
  - $\Rightarrow$ joint analyses
  - $\Rightarrow$ comparing results from different experiments
- Global fits
  - A generator is the ideal place for global fits
    - Controls the model implementation
  - Finding the best parameters
  - Cross Section priors based on data
- Feedback for experiments
  - Drive the format of cross section releases
  - Hint toward key measurements

Bibliography I

P Abreu et al. “Tuning and Test of Fragmentation Models Based on Identified Particles and Precision Event Shape Data”. In: *Z. Phys. C* 73.CERN-PPE-96-120 (1996), pp. 11–60. DOI: 10.1007/s002880050295. URL: http://cds.cern.ch/record/309559.

Andy Buckley et al. “Systematic event generator tuning for the LHC”. In: *Eur. Phys. J. C* 65 (2010), pp. 331–357. DOI: 10.1140/epjc/s10052-009-1196-7. arXiv: 0907.2973 [hep-ph].

A. Bodek et al. “Vector and axial nucleon form factors: A duality constrained parameterization”. In: *Eur. Phys. J.* C53 (2008), pp. 349–354. DOI: 10.1140/epjc/s10052-007-0491-4. arXiv: 0708.1946 [hep-ex].

Konstantin S. Kuzmin, Vladimir V. Lyubushkin, and Vadim A. Naumov. “Axial masses in quasielastic neutrino scattering and single-pion neutrinoproduction on nucleons and nuclei”. In: *Acta Phys. Polon.* B37 (2006), pp. 2337–2348. arXiv: hep-ph/0606184 [hep-ph].

Bibliography II

- S. J. Barish et al. "Study of Neutrino Interactions in Hydrogen and Deuterium. 1. Description of the Experiment and Study of the Reaction $\nu + d \to \mu^- + p + p_s$". In: *Phys. Rev.* D16 (1977), pp. 3103–3121. DOI: 10.1103/PhysRevD.16.3103.
- P. C. Bosetti et al. "Total Cross Sections for Charged-Current Neutrino and anti-neutrino Interactions in BEBC in the Energy Range 20–200 GeV". In: *Phys. Lett.* B70 (1977), pp. 273–277. DOI: 10.1016/0370-2693(77)90537-8.
- P. Bosetti et al. "Total cross-sections for ν_μ and $\bar{\nu}_\mu$ charged-current interactions between 20 and 200 GeV". In: *Phys. Lett.* B110 (1982), pp. 167–172. DOI: 10.1016/0370-2693(82)91028-0.
- C. Baltay et al. "Cross-sectons and scaling variable distributions of neutral and charged current neutrino nucleon interactions from a low-energy narrow band beam". In: *Phys. Rev. Lett.* 44 (1980), pp. 916–919. DOI: 10.1103/PhysRevLett.44.916.

Bibliography III

- William Glenn Seligman. "A Next-to-Leading Order QCD Analysis of Neutrino–Iron Structure Functions at the Tevatron". PhD thesis. Nevis Labs, Columbia U., 1997. DOI: 10.2172/1421736. URL: http://lss.fnal.gov/cgi-bin/find_paper.pl?thesis-1997-21.
- M. Jonker et al. "Experimental Study of Neutral-Current and Charged-Current Neutrino Cross Sections". In: *Phys. Lett.* B99 (1981). [Erratum-ibid. B 100, 520 (1981)], pp. 265–270. DOI: 10.1016/0370-2693(81)91123-0.
- T. Kitagaki et al. "Charged-current exclusive pion production in $\nu_\mu n \to \mu^- p$ interactions". In: *Phys. Rev.* D34 (1986), pp. 2554–2565. DOI: 10.1103/PhysRevD.34.2554.
- T. Eichten et al. "Measurement of the neutrino-nucleon antineutrino-nucleon total cross sections". In: *Phys. Lett.* B46 (1973), pp. 274–280. DOI: 10.1016/0370-2693(73)90702-8.
- J. G. Morfin et al. "Total cross-sections and nucleon structure functions in the Gargamelle SPS neutrino/antineutrino experiment". In: *Phys. Lett.* B104 (1981), pp. 235–238. DOI: 10.1016/0370-2693(81)90598-0.

Bibliography IV

A. S. Vovenko. “Total cross section measurements for ν_μ, $\bar{\nu}_\mu$ interactions in 3–30 GeV energy range with IHEP-JINR detector and future plans”. In: *Nucl. Phys. B (Proc. Suppl.)* 112 (2002), pp. 116–123. DOI: 10.1016/S0920-5632(02)01770-X.

V Lyubushkin et al. “A Study of quasi-elastic muon neutrino and antineutrino scattering in the NOMAD experiment”. In: *Eur. Phys. J.* C63 (2009), pp. 355–381. DOI: 10.1140/epjc/s10052-009-1113-0. arXiv: 0812.4543 [hep-ex].

P. Adamson et al. “Neutrino and Antineutrino Inclusive Charged-current Cross Section Measurements with the MINOS Near Detector”. In: *Phys. Rev.* D81 (2010), p. 072002. DOI: 10.1103/PhysRevD.81.072002. arXiv: 0910.2201 [hep-ex].

S. J. Barish et al. “Study of Neutrino Interactions in Hydrogen and Deuterium: Inelastic Charged Current Reactions”. In: *Phys. Rev.* D19 (1979), pp. 2521–2542. DOI: 10.1103/PhysRevD.19.2521.

Bibliography V

D. C. Colley et al. "Cross-sections for charged current ν and $\bar{\nu}$ interactions in the energy range 10 to 50 GeV". In: *Z. Phys.* C2 (1979), pp. 187–224. DOI: 10.1007/BF01474659.

Michael Andrew Parker et al. "A comparison of charged current cross sections and structure functions for neutrino and anti-neutrinos beams on Hydrogen and Neon". In: *Nucl. Phys.* B232 (1984), pp. 1–20. DOI: 10.1016/0550-3213(84)90358-4.

N. J. Baker et al. "Total cross-sections for $\nu_\mu n$ and $\nu_\mu p$ charged current interactions in the 7-foot bubble chamber". In: *Phys. Rev.* D25 (1982), pp. 617–623. DOI: 10.1103/PhysRevD.25.617.

D. MacFarlane et al. "Nucleon Structure Functions from High-Energy Neutrino Interactions with Iron and QCD Results". In: *Z. Phys.* C26 (1984), pp. 1–12. DOI: 10.1007/BF01572534.

J. V. Allaby et al. "Total cross sections of charged-current neutrino and antineutrino interactions on isoscalar nuclei". In: *Z. Phys.* C38 (1988), pp. 403–410. DOI: 10.1007/BF01584388.

Bibliography VI

- N. J. Baker et al. “Measurement of the ν_μ charged-current cross section”. In: *Phys. Rev. Lett.* 51 (1983), pp. 735–738. DOI: 10.1103/PhysRevLett.51.735.
- S. Ciampolillo et al. “Total cross-section for neutrino charged current interactions at 3 and 9 GeV”. In: *Phys. Lett.* B84 (1979), pp. 281–284. DOI: 10.1016/0370-2693(79)90303-4.
- A. E. Asratian et al. “Total antineutrino-nucleon charged current cross section in the energy range 10–50 GeV”. In: *Phys. Lett.* B137 (1984), pp. 122–124. DOI: 10.1016/0370-2693(84)91118-3.
- V. B. Anikeev et al. “Total cross-section measurements for ν_μ, $\bar{\nu}_\mu$ interactions in 3–30 GeV energy range with IHEP-JINR neutrino detector”. In: *Z. Phys.* C70 (1996), pp. 39–46. DOI: 10.1007/s002880050078.
- D. S. Baranov et al. “Measurements of the $\nu_\mu N$ total cross section at 2–30 GeV in SKAT neutrino experiment”. In: *Phys. Lett.* B81 (1979), pp. 255–257. DOI: 10.1016/0370-2693(79)90536-7.

Bibliography VII

Y. Nakajima et al. “Measurement of inclusive charged current interactions on carbon in a few-GeV neutrino beam”. In: *Phys. Rev.* D83 (2011), p. 012005. DOI: 10.1103/PhysRevD.83.012005. arXiv: 1011.2131 [hep-ex].

G. Fanourakis et al. “Study of low-energy antineutrino interactions on protons”. In: *Phys. Rev.* D21 (1980), pp. 562–568. DOI: 10.1103/PhysRevD.21.562.

G. N. Taylor et al. “$\overline{\nu}_{\mu}$-Nucleon Charged Current Total Cross Section for 5–250 GeV”. In: *Phys. Rev. Lett.* 51 (1983), pp. 739–742. DOI: 10.1103/PhysRevLett.51.739.

O. Erriquez et al. “Antineutrino-nucleon total cross section and ratio of antineutrino cross section on neutrons and protons”. In: *Phys. Lett.* B80 (1979), pp. 309–313. DOI: 10.1016/0370-2693(79)90224-7.

A. E. Asratian et al. “Charged current neutrino interactions below 30 GeV”. In: *Phys. Lett.* B76 (1978), pp. 239–242. DOI: 10.1016/0370-2693(78)90286-1.

Bibliography VIII

W. A. Mann et al. “Study of the reaction $\nu + n \to \mu^- + p$”. In: *Phys. Rev. Lett.* 31 (1973), pp. 844–847. DOI: 10.1103/PhysRevLett.31.844.

D. Allasia et al. “Investigation of exclusive channels in $\nu/\bar{\nu}$-deuteron charged current interactions”. In: *Nucl. Phys.* B343 (1990), pp. 285–309. DOI: 10.1016/0550-3213(90)90472-P.

T. Kitagaki et al. “High-Energy Quasielastic $\nu_\mu n \to \mu^- p$ scattering in Deuterium”. In: *Phys. Rev.* D28 (1983), pp. 436–442. DOI: 10.1103/PhysRevD.28.436.

S. V. Belikov et al. “Quasielastic $\nu_\mu n$ scattering at 3–30 GeV energy”. In: *Yad. Fiz.* 35 (1982), pp. 59–63.

J. Brunner et al. “Quasielastic nucleon and hyperon production by neutrons and antineutrinos with energies below 30 GeV”. In: *Z. Phys.* C45 (1990), pp. 551–555. DOI: 10.1007/BF01556267.

N. J. Baker et al. “Quasielastic Neutrino Scattering: A Measurement of the Weak Nucleon Axial Vector Form-Factor”. In: *Phys. Rev.* D23 (1981), pp. 2499–2505. DOI: 10.1103/PhysRevD.23.2499.

Bibliography IX

- S. Bonetti et al. “Study of Quasi-elastic Reactions of ν and $\bar{\nu}$ in Gargamelle”. In: *Nuovo Cim.* A38 (1977), pp. 260–270. DOI: 10.1007/BF02730023.
- S. V. Belikov et al. “Restraints on parameters of oscillations of muon neutrinos from quasielastic scattering data.”. In: *Yad. Fiz.* 41 (1985), pp. 919–924.
- N. Armenise et al. “Charged current elastic antineutrino interactions in propane”. In: *Nucl. Phys.* B152 (1979), pp. 365–375. DOI: 10.1016/0550-3213(79)90087-7.
- J. Campbell et al. “Study of the reaction $\nu p \to \mu^- \pi^+ p$”. In: *Phys. Rev. Lett.* 30 (1973), pp. 335–339. DOI: 10.1103/PhysRevLett.30.335.
- G. M. Radecky et al. “Study of single-pion production by weak charged currents in low-energy μd interactions”. In: *Phys. Rev.* D25 (1982). [Erratum-ibid. D 26, 3297 (1982)], pp. 1161–1173. DOI: 10.1103/PhysRevD.25.1161.

Bibliography X

- Callum Wilkinson et al. "Reanalysis of bubble chamber measurements of muon-neutrino induced single pion production". In: *Phys. Rev.* D90.11 (2014), p. 112017. DOI: 10.1103/PhysRevD.90.112017. arXiv: 1411.4482 [hep-ex].
- W. Lerche et al. "Experimental Study of the Reaction $\nu p \to \mu^- \pi^+ p$". In: *Phys. Lett.* 78B (1978), pp. 510–514. DOI: 10.1016/0370-2693(78)90499-9.
- P. Allen et al. "Single π^+ production in charged current neutrino-hydrogen interactions". In: *Nucl. Phys.* B176 (1980), pp. 269–284. DOI: 10.1016/0550-3213(80)90450-2.
- J. Bell et al. "A study of the reaction $\nu p \to \mu^- \Delta^{++}$ at high-energies and comparisons with theory". In: *Phys. Rev. Lett.* 41 (1978), pp. 1012–1015. DOI: 10.1103/PhysRevLett.41.1012.
- P. Allen et al. "A study of single meson production in neutrino and antineutrinos charged-current interactions on protons". In: *Nucl. Phys.* B264 (1986), pp. 221–242. DOI: 10.1016/0550-3213(86)90480-3.

Bibliography XI

- S. J. Barish et al. “Study of the reaction $\bar{\nu}_\mu p \to \mu^+ p \pi^-$”. In: *Phys. Lett.* B91 (1980), pp. 161–164. DOI: 10.1016/0370-2693(80)90684-X.
- T. Kitagaki et al. “Neutrino flux and total charged current cross sections in high-energy neutrino-deuterium interactions”. In: *Phys. Rev. Lett.* 49 (1982), pp. 98–101. DOI: 10.1103/PhysRevLett.49.98.
- D. Day et al. “Study of νd Charged current two pion production in the threshold region”. In: *Phys. Rev.* D28 (1983), pp. 2714–2720. DOI: 10.1103/PhysRevD.28.2714.
- J. W. Chapman et al. “Multiplicity distributions in high-energy neutrino interactions”. In: *Phys. Rev. Lett.* 36 (1976), pp. 124–126. DOI: 10.1103/PhysRevLett.36.124.
- H. Grässler et al. “Multiplicities of secondary hadrons produced in νp and $\bar{\nu} p$ charged current interactions”. In: *Nucl. Phys. B* 223.2 (1983), pp. 269–295. ISSN: 0550-3213. DOI: https://doi.org/10.1016/0550-3213(83)90057-3. URL: http://www.sciencedirect.com/science/article/pii/0550321383900573.

Bibliography XII

- G. T. Jones et al. "W^2 and Q^2 dependence of charged hadron and pion multiplicities in νp and $\overline{\nu} p$ charged current interactions". In: *Z. Phys. C* 46 (1990), pp. 25–34. DOI: 10.1007/BF02440830.

- G. T. Jones et al. "Multiplicity distributions of charged hadrons in νp and $\overline{\nu} p$ charged current interactions". In: *Z. Phys. C* 54 (1992), pp. 45–54. DOI: 10.1007/BF01881707.

- Daria Zieminska et al. "Charged particle multiplicity distributions in νn and νp charged current interactions". In: *Phys. Rev. D* 27 (1983), pp. 47–57. DOI: 10.1103/PhysRevD.27.47.

- B. Jongejans et al. "Multiplicity distributions of charged hadrons produced in (anti)neutrino–deuterium charged- and neutral-current interactions". In: *Nuovo Cim. A* 101 (1989), pp. 435–453. DOI: 10.1007/BF02789427.

- D. Allasia et al. "Fragmentation in neutrino and antineutrino charged current interactions on proton and neutron". In: *Z. Phys. C* 24 (1984), pp. 119–131. DOI: 10.1007/BF01571716.

Bibliography XIII

M. Derrick et al. "Multiplicity distributions in $\overline{\nu}_\mu p$ interactions". In: *Phys. Rev. D* 25 (1982), pp. 624–633. DOI: 10.1103/PhysRevD.25.624.

S. Barlag et al. "Charged hadron multiplicities in high energy $\overline{\nu}_\mu n$ and $\overline{\nu}_\mu p$ interactions". In: *Z. Phys. C* 11 (1982). [Erratum: *ibid.* **14**, 281 (1982)], pp. 283–292. DOI: 10.1007/BF01578279.

Konstantin S. Kuzmin and Vadim A. Naumov. "Mean charged multiplicities in charged-current neutrino scattering on hydrogen and deuterium". In: *Phys. Rev. C* 88 (2013), p. 065501. DOI: 10.1103/PhysRevC.88.065501. arXiv: 1311.4047 [hep-ph].