The logarithmic gauged linear sigma model: Applications

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joint works (partially in progress) with Q. Chen, S. Guo, Y. Ruan and A. Sauvaget
arXiv: 1906.04345, 1812.11908, 1805.02304, 1709.07392

Example applications

1. Structure of Gromov–Witten theory of $X_5 \subset \mathbb{P}^4$ (finite generation, HAEs, orbifold regularity)

   Chang-Li-Li-Liu-Guo

2. Structure of Gromov–Witten theory of $X_{33} \subset \mathbb{P}^5$
3. LG/CY correspondence for $X_5 \subset \mathbb{P}^4$
4. Conjectures of Oberdieck–Pixton for Gromov–Witten theory of Weierstrass elliptic fibrations w/ X. Wang

in progress

Using log GLSM in a nutshell

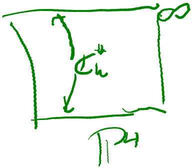
1. Set-up GLSM target $(\mathbb{P}^\circ, W, \mathbb{C}^*_\omega)$ for problem
2. Set-up log GLSM $(\mathbb{P}, W, \mathbb{C}^*_\omega)$ target that recovers $(\mathbb{P}^\circ, W, \mathbb{C}^*_\omega)$.
3. Apply $\mathbb{C}^*_\omega$-localization.
4. Compute, compute, compute!

Target set-up for $X_5 \subset \mathbb{P}^4$

$$(\mathbb{P}^\circ \overset{\text{compactify}}{\subset} \mathbb{P},\ W : \mathbb{P}^\circ \to \mathbb{C},\ \mathbb{C}^*_\omega)$$

$$\mathbb{P}^\circ = \mathcal{O}_{\mathbb{P}^4}(-5) = \mathbb{C}^6 /\!/ \mathbb{C}^* \leftarrow \text{gauge gp}$$

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & -5 \\ x_1 & & & & x_5 & p \end{array}$$



$$\mathbb{P} = \mathbb{P}_{\mathbb{P}^4}(\mathcal{O}(-5) \oplus \mathcal{O})$$

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & -5 \\ & & & & & 1 \quad 1 \end{array}$$

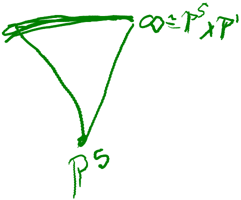
$$W = p\, F_5(\vec{x})$$ pole of order 1 along ∞

Target set-up for $X_{33} \subset \mathbb{P}^5$

$$(\mathbb{P}^\circ \quad \subset \quad \mathbb{P}, \;\; W : \mathbb{P}^\circ \;\to\; \mathbb{C}, \;\; \mathbb{C}^*_\omega)$$

$$\mathbb{P}^\circ = \mathcal{O}_{\mathbb{P}^5}(-3) \oplus \mathcal{O}_{\mathbb{P}^5}(-3) = \mathbb{C}^8 // \mathbb{C}^*$$

$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & -3 & -3 \end{array}$$



$$\mathbb{P} = \mathbb{P}_{\mathbb{P}^5}(\mathcal{O}(-3) \oplus \mathcal{O}(-3) \oplus \mathcal{O})$$

$$\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & -3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{array}$$

$$W = p_1 F_3(\vec{x}) + p_2 G_3(\vec{x})$$

Moduli space and localization formula

$\mathbb{C}_\omega$

Theorem (Chen-J-Ruan) Qile's talk of log maps

There is a compactification $\mathcal{U}$ of the GLSM instanton *moduli space whose reduced virtual cycle $[\mathcal{U}]^{\mathrm{red}}$ recovers the GLSM invariants.*

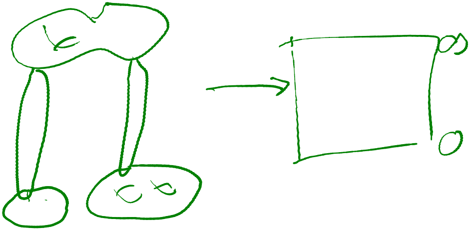
Theorem (Chen-J-Ruan (work in progress))

Localization formula for $[\mathcal{U}]^{\mathrm{red}}$. $= \lim_{\lambda \to 0} \sum_{F} \prod_{Z} \ldots$

fixed loci

graphs

components of C



C

More on localization formula for X_5

$(\mathbb{P}^0, 0, \mathbb{C}^*_w)$
$\S$

- Component $Z \to 0$: twisted Gromov–Witten theory of $\mathcal{O}_{\mathbb{P}^4}(5)$.
    - compute generating series via Givental formalism

- Component $Z \to \infty$: new "effective" invariants BCOV
    - don't know how to compute
    - one for every (g, d) such that $2g - 2 - 5d \geq 0$ (i.e. $\underline{d \leq \frac{2g-2}{5}}$)

$f: Z \to \infty \cong \mathbb{P}^4$, $\deg_{Z}(\omega_Z \otimes f^*\mathcal{O}(-5))$

$\rightsquigarrow$ effective is polynomial in Novikov variable

From localization formula to structure of $\mathrm{GW}(X_5)$

- Prove structure for twisted theory using Givental formalism (study Picard–Fuchs equation, ring of generators, compute S- and R-matrices).
- Prove that structure is preserved under localization graph sum.
- Perform equivariant limit.

Target geometry for LG/CY for X_5

$GW(X_5) \longleftrightarrow LG(X_5)$

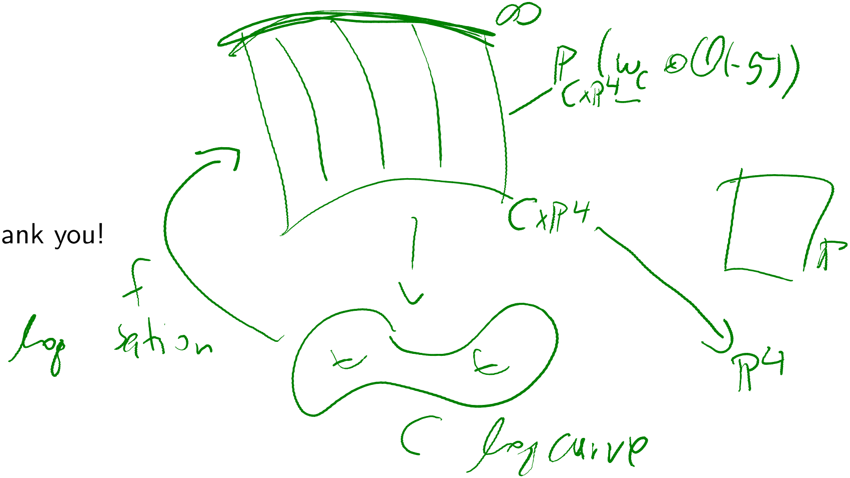
$\mathbb{P}^{\circ}_{LG} = [\mathbb{C}^5/\mathbb{Z}_5] \subset \mathbb{P}^5/\mathbb{Z}_5$

$\begin{pmatrix} 11111-5 \\ 51 \end{pmatrix}^{T} \sim \begin{matrix} 11111-5 \\ 11111 \quad 1 \end{matrix}$

$\infty_{\mathbb{P}_{X_5}} \simeq \infty_{\mathbb{P}_{LG}}$

correspondence of effective invts.

Thank you!


∞
$\mathbb{P}_{C\times\mathbb{P}^4}(\omega_C \otimes \mathcal{O}(-5))$
$C\times\mathbb{P}^4$
$\mathbb{P}^4$
f
log section
C log curve