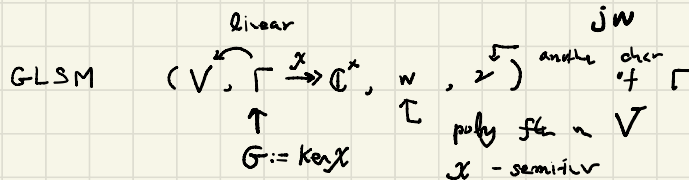


General GLSM Invariants & Coh FTs

David Favero



← some "line" \mathbb{P}^1

$V/G \supset Z(dw)$ is opt.

Want: a Coh FT

- \mathbb{H} a fin. dim graded ex vector sp or a supercomm nondeg pairing $(,)$

$$\left\{ \Omega_{g,r} : \mathbb{H}[\![\xi]\!]^{\otimes r} \xrightarrow{\quad} H^*(\bar{M}_{g,r}) \otimes_{\mathbb{C}} \mathbb{H}[\![\xi]\!] \right\}_{g,r}$$

s.t. some axiom over $\mathbb{C}[\![\xi]\!]$

- gluing axioms
- axioms for unit $1 \in \mathbb{H}$

Then \exists such a Coh FT using virtual fundamental matrix factorization on the moduli of stable LG maps.

previous works

- ① G finite abelian gp case \Rightarrow • FJRW

Fan - Jarvis - Ruan (2013) moduli of curves of W -str & with eqn (analytic)

- Polishchuk - Vaintrob (2016) contr a certain matrix factorization over the above moduli of W -str. (algebraic)

- Kiem - Li (2018) using intersection homology of w/0 and correction localisation

② General case

- Fan - Jarvis - Ruan (2018) defines invariants for narrow sectors with crepant localization
- Convex hybrid model : Ciucan-Fortman-Guéré
- Frenkel - Shrawmker - K (2018) for any sectors! using matrix factorizations

Remarks on the constructions

- moduli of stable LG maps



$$(C, \delta = (\delta_1, \dots, \delta_r); P, \kappa, u)$$

$$\text{LGM}_{g,r,A}$$

$$\kappa: P(\mathcal{O}_C) = P \times C / \Gamma$$

$\omega_C^{\log} = \omega_C(\sum \delta_i)$
 \uparrow
 principal Γ -bundle on C

$$u \in H^0(C, P(V) := P \times V / \Gamma)$$

$$(C, P, u) \xLeftrightarrow[\text{stable}] C \xrightarrow{u^*} [V/\Gamma]$$

$$\Rightarrow \left(\begin{array}{c} \downarrow \\ [V^{\otimes 2}/\Gamma] \end{array} \right)$$

a ex of vector bundles
 $[A \rightarrow B]$

$$R^0 \pi_* P(V)$$

kind

LGM

$$R^0 \pi_* P(V)$$

univ curve

$$\text{Bun} = \{ (C, \delta, P, \kappa) \}$$

transient dist ex

sublocus of A

$$A \xrightarrow{p} B$$

$$B \hookrightarrow d$$

$$Z(\beta) = \text{LGM (upto stability)}$$

$$[LGM]^{und, vr} = \frac{c_{+p}(B) \cap [A]}{= \text{non cpt}}$$

$$\begin{array}{ccc} \emptyset & \xrightarrow{\beta} & p^*B \\ & \searrow & \downarrow \\ LGM & \hookrightarrow & A \\ & \swarrow \text{ev} & \searrow \text{ev} \\ & & ([V^u/G]^{xr}, \boxplus w) \end{array}$$

$\alpha \xrightarrow{w} \text{superstitied}$
 $\alpha \circ \beta = -W \leftarrow$
 $W := \text{ev}^*(\boxplus w)$

$$(\wedge^* p^* B^v, \underbrace{\alpha \wedge + \langle \beta}_{\delta}) = ; K(\alpha, \beta)$$

\uparrow
 $\delta^2 = -W \cdot \text{id}$
 α matrix factorization on $(A, -W)$
 $\text{superstitied } \{ \alpha = 0 \} \cap \underbrace{\{ \beta = 0 \}}_{\text{LGM}}$
 $\swarrow \searrow$
 $\text{LGC} \quad \boxed{\text{cpt}} \quad (\text{FJR})_{2018}$

$$\text{c.f. } c_{+p}(p^*B) = \text{td } p^*B \cdot \text{ch}(\wedge^* p^*B)$$

Define

$$[LGM]^{v,v} = \text{td } p^* B \underbrace{\text{ch}^A(K(\alpha, \beta))}_{\text{LGC}}$$

$$\Rightarrow H_{\text{cpt}}^*(A, (\Omega_A^\bullet, -dW \wedge))$$

$$(V, \tau \xrightarrow{x} \mathbb{C}^x, w, \nu) \quad \begin{array}{ccc} \Gamma & \xrightarrow{\text{phase}} & \mathbb{C}^x \\ \cup & \nearrow \theta & \\ G = \ker \nu & & \end{array}$$

x -inv

$$X = [V^{ss}(\theta)/G]$$

$$(\mathbb{C}^k, \mathbb{C}^{k^2} = \Gamma, w = p \cdot f(x), \nu) \quad \begin{array}{ccc} \cup & & \\ G = \mathbb{C}^x & & \theta = \pm 1 \end{array}$$

~~mmmmmm~~

$$\mathcal{H} := H^*(IX, (\Omega_{IX}^\bullet, dW \wedge))$$

$$IX = X \sqcup X_{g_1} \sqcup X_{g_2} \sqcup \dots$$

$$(\text{if } w|_{X_{g_i}} = 0, \text{ call narrow center})$$