GLSMs, localisation, and the stringy Kähler moduli space

[with M. Romo, E. Scheidegger: arXiv:2003.00182[hep-th]] [with D. Erkinger: arXiv:2008.03089[hep-th]]

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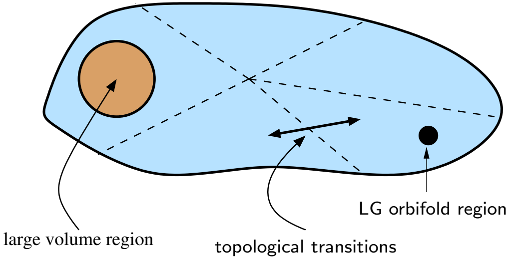
Outline

Stringy Kähler moduli space $\mathcal{M}_K$

- This talk: Calabi-Yau threefolds in a type II setting (+ branes)
- $\mathcal{M}_K$ is divided into chambers



  - Quantum corrections due to worldsheet instantons.
  - One can cross the chamber boundaries.
  - Categories associated to chambers (conjecturally) equivalent.
- Study $\mathcal{M}_K$ in limiting regions that are not large volume.

$\mathcal{M}_K$ via GLSMs

- The $2D$ $(2,2)$ gauged linear sigma model (GLSM) provides a common UV description of the CFTs in $\mathcal{M}_K$. [Witten 93]
- Phases: different low-energy (IR) configurations depending on the values of the FI-theta parameters:

$$t = \zeta - i\theta \quad \leftrightarrow \quad t \in \mathcal{M}_K \quad \leftrightarrow \quad \text{chambers}$$

- Use the GLSM to map out $\mathcal{M}_K$.
- SUSY localisation computes (quantum) exact expressions.
  - Can be evaluated in any phase.
  - t-dependence $\leftrightarrow$ worldsheet instanton corrections.
  - Compute quantum corrections directly in the GLSM.

Beyond geometry

- Geometric phases are well-understood and we know how to do computations.
  - Toric geometry, topological strings, mirror symmetry, enumerative invariants, etc.
- The structures we know in geometry should also be visible in non-geometric settings. Why?
  - The worldsheet CFT of the phases does not care whether there is a geometry.
  - Whatever is computed in non-geometric phases using localisation has the same UV origin as the geometric results.
- If we evaluate localisation results in phases of the GLSM, how do we interpret the result in the IR theory?

Universal structures in phases of GLSMs

- Claim: When evaluating the GLSM partition functions in different types of phases the (exact) result has the same structure in every phase.
- To see this, we need structures that are available beyond geometric settings, e.g.
  - Worldsheet: CFT, topological gravity, tt^*-geometry
  - FJRW theory [Witten 92][Fan-Jarvis-Ruan 07]
  - Givental's mirror construction [Givental 96-03]
  - Categories
- Ingredients that appear in every phase:
  - State space and pairing
  - I/J-function
  - Gamma class [Hosono 00][Iritani 07][Katzarkov-Kontsevich-Pantev 08]
  - D-brane data

Results

- Hemisphere partition function: [Romo-Scheidegger-JK 20]

$$Z_{D^2}^{phases}(\mathcal{B}) = \langle \mathrm{ch}(\mathcal{B}), \Gamma \cdot I \rangle$$

  - Works in geometry and Landau-Ginzburg orbifold phases.
- Sphere partition function: [Erkinger-JK 20]

$$Z_{S^2}^{phases} = \langle \bar{I}, I \rangle, \qquad \langle \bar{I}| = (-1)^{\mathrm{Gr}} \frac{\Gamma}{\Gamma^*} \bar{I}$$

  - Works in hybrid phases that are LG orbifolds fibered over some base manifold.

GLSM data

- G ... a compact Lie group (gauge group)
- V ... space of chiral fields $\phi_i \in V$
- $\rho_V : G \to GL(V)$ ... faithful complex representation
  - CY condition: $G \to SL(V)$
- $R : U(1)_V \to GL(V)$ ... vector R-symmetry
  - R_i ... R-charges
- $T \subset G$ ... maximal torus
  - Lie algebras: $\mathfrak{g} = Lie(G)$, $\mathfrak{t} = Lie(T)$
  - $Q_i^a \in \mathfrak{t}_{\mathbb{C}}^*$ ... gauge charges of chiral fields

GLSM Data (ctd.)

- $t^a \in \mathfrak{g}_{\mathbb{C}}^*$ . . . FI-theta parameters
  - $t^a = \zeta^a - i\theta^a$ $\quad$ ζ^a : real, $\quad$ θ^a : 2π-periodic
  - $t^a \leftrightarrow$ Kähler moduli of the CY
- $\sigma_a \in \mathfrak{g}_{\mathbb{C}}$ . . . scalar components of the vector multiplet
- $W \in \mathrm{Sym}\, V^*$ . . . superpotential
  - G-invariant
  - R-charge 2
  - non-zero for compact CYs

D-branes in GLSMs

- D-branes (B-type) in the GLSM are G-invariant matrix factorisations of the GLSM potential with R-charge 1

  [Herbst-Hori-Page 08][Honda-Okuda,Hori-Romo 13]

- Data:
  - $\mathrm{Sym}\, V^*$-module (Chan-Paton space): $M = M^0 \oplus M^1$
  - Matrix Factorisation: $Q \in End^1(M)$ with

$$Q^2 = W \cdot \mathrm{id}_M$$

  - G-action: $\rho : G \to GL(M)$ with

$$\rho(g)^{-1} Q(g\phi)\rho(g) = Q(\phi) \qquad g \in G$$

  - R-action: $r_* : u(1)_V \to gl(M)$ with

$$\lambda^{\mathbf{r}_*} Q(\lambda^R \phi)\lambda^{-\mathbf{r}_*} = \lambda Q(\phi) \qquad \lambda \in U(1)_V$$

Sphere partition function Z_{S^2}

- Sphere partition function

[Benini-Cremonesi 12][Doroud-Gomis-LeFloch-Lee 12]

$$Z_{S^2} = C \sum_{m\in\mathbb{Z}^{\mathrm{rk}G}} \int_\gamma d^{\mathrm{rk}G}\sigma \prod_{\alpha>0}\left(\frac{\alpha(m)^2}{4}+\alpha(\sigma)^2\right)$$
$$\cdot \prod_{j=1}^{\dim V} \frac{\Gamma\left(iQ_j(\sigma)-\frac{Q_j(m)}{2}+\frac{R_j}{2}\right)}{\Gamma\left(1-iQ_j(\sigma)+\frac{Q_j(m)}{2}-\frac{R_j}{2}\right)} e^{-4\pi i\zeta(\sigma)-i\theta(m)}$$

  - $\alpha > 0$ positive roots
  - γ ...integration contour (s.t. integral is convergent)
- Z_{S^2} computes the exact Kähler potential on $\mathcal{M}_K$.

[Jockers-Kumar-Lapan-Morrison-Romo 12][Gomis-Lee 12]

[Gerchkovitz-Gomis-Komargodski 14][Gomis-Hsin-Komargodski-Schwimmer-Seiberg-Theisen 15]

Hemisphere partition function

- Hemisphere partition function: [Sugishita-Terashima][Honda-Okuda][Hori-Romo 13]

$$Z_{D^2}(\mathcal{B}) = C \int_\gamma d^{\mathrm{rk}_G}\sigma \prod_{\alpha>0} \alpha(\sigma) \sinh(\pi\alpha(\sigma)) \cdot \prod_{j=1}^{\dim V} \Gamma\left(iQ_j(\sigma) + \frac{R_j}{2}\right) e^{it(\sigma)} f_{\mathcal{B}}(\sigma)$$

- Brane factor

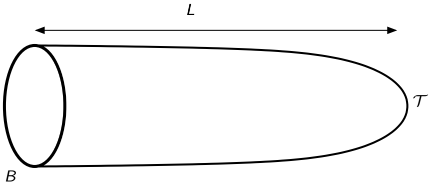
$$f_{\mathcal{B}}(\sigma) = \mathrm{tr}_M \left(e^{i\pi \mathbf{r}_*} e^{2\pi\rho(\sigma)}\right)$$

- Z_{D^2} computes the exact D-brane central charge.

D-brane central charge – worldsheet perspective

- The central charge connects A/B-branes with $(c,c)/(a,c)$-operators: [Ooguri-Oz-Yin 96][Hori-Iqbal-Vafa 00]

$$Z(B) = \langle B|\mathbf{0}\rangle$$



- Here we consider B-branes $\mathcal{B}$ and (a,c)-operators.
  - Note: "B-branes in the A-model"

Z_{D^2} in LG orbifold phases

- We want to collect evidence that [Romo-Scheidegger-JK 20]

$$Z_{D^2}^{phases}(\mathcal{B}) = \langle \mathrm{ch}(\mathcal{B}), \Gamma \cdot I \rangle.$$

  - $\langle \cdot, \cdot \rangle$ ... pairing (state space)
  - Γ ... Gamma class
  - I ... I-function
  - $\mathrm{ch}(\mathcal{B})$ ... Chern character
- Focus on Landau-Ginzburg orbifold phases.
- Plan:
  - Define the objects on the right-hand side for LG orbifolds.
  - Show that the results match with Z_{D^2} and FJRW theory.

LG data

- Landau-Ginzburg orbifold: $(W, G, \overline{\rho}_m, U(1)_{L/R})$

[Vafa 89][Intriligator-Vafa 90]

  - $W(x_i)$ ... superpotential
  - G ... orbifold group (assume $\mathbb{Z}_{d_1} \times \mathbb{Z}_{d_2} \times \ldots$)
  - $\overline{\rho}_m$ ... matter representation
  - $U(1)_{L/R}$ ... left/right R-symmetry
- State space
  - (c, c), (a, c), ...- chiral rings $\leftrightarrow$ γ-twisted sectors ($\gamma \in G$) with basis elements ϕ_γ
  - Restriction to one-dimensional ("narrow") sectors ($\delta \in G$)
  - (Topological) pairing on the (a, c)-ring:

$$\langle \phi_\delta, \phi_{\delta'} \rangle = \frac{1}{|G|} \delta_{\delta, \delta'^{-1}}$$

  - Note: Spectral flow required to map (c, c) to (a, c).

LG branes

- LG orbifold B-brane: $\overline{\mathcal{B}} = (\overline{M}, \overline{Q}, \overline{\rho}_\gamma, \overline{r}_*)$

[Kapustin-Li 02][Brunner-Herbst-Lerche-Scheuner 03][Walcher 04]

  - $\overline{M}$... Chan-Paton space
  - $\overline{Q}$... matrix factorisation of W
  - $\overline{\rho}_\gamma, \overline{r}_*$... representation of G and $u(1)_V$ on the boundary
- Chern character:

$$\mathrm{ch}(\overline{\mathcal{B}})_\gamma = \frac{1}{n_\gamma!}\mathrm{Res}_{W_\gamma}\left(\Phi_\gamma \cdot \mathrm{Tr}_{\overline{M}}\left[\overline{\rho}_\gamma(\partial\overline{Q}_\gamma)^{\wedge n_\gamma}\right]\right)$$

  - Φ_γ ...state(s) in the γ-twisted sector $\mathcal{H}_\gamma$ (to be precise, this is the (c,c)-ring)
  - n_γ ...dimFix_γ
  - $W_\gamma = W|_{\mathrm{Fix}_\gamma}$

Gamma class

- Taking into account deformations away from the LG point, one can define a $h \times (N+h)$ matrix q.
  - N ... number of chirals x_i.
  - h ... number of marginal deformations (narrow sectors).
- Gamma class:

$$\Gamma\phi_\delta = \Gamma_\delta \phi_\delta \qquad \Gamma_\delta = \prod_{j=1}^{N} \Gamma\left(1 - \left\langle \sum_{a=1}^{h} k_a q_{a,h+j} \right\rangle\right).$$

  - $k_a \in \mathbb{Z}^h_{\geq 0}$: each gets associated to a sector δ.
  - $\langle x \rangle = x - \lfloor x \rfloor$

I-function

- I-function

$$I_{LG}(u) = \sum_{\delta \in G} I_\delta(u) \phi_\delta$$

$$I_\delta(u) = - \sum_{\substack{k_1,\dots,k_h \geq 0 \\ k_i' = \delta_i \mod d_i}} \frac{u^k}{\prod_{a=1}^h \Gamma(k_a+1)} \times \prod_{j=1}^N \frac{(-1)^{\langle -\sum_{a=1}^h k_a q_{a,h+j} + q_j \rangle} \Gamma(\langle \sum_{a=1}^h k_a q_{a,h+j} - q_j \rangle)}{\Gamma(1 + \sum_{a=1}^h k_a q_{a,h+j} - q_j)}$$

  - u ... local coordinates/deformation parameters
  - q_j ... left R-charges of the x_j

J-function

- One obtains the J-function from the I-function through a transformation to flat coordinates.
    - These correspond to the deformation parameters of the marginal deformations in the worldsheet CFT.
- Select components of the I_{δ_a} $(a = 1, \dots, h)$ of the I-function associated to the subspace of (narrow) marginal deformations $\leftrightarrow$ (a, c) states with $(q, \overline{q}) = (-1, 1)$ and the unique component I_0 with $(q, \overline{q}) = (0, 0)$.
- The flat coordinates and the J-function are

$$t_a(u) = \frac{I_{\delta_a}(u)}{I_0(u)} \qquad J(t) = \frac{I_{LG}(u(t))}{I_0(u(t))}$$

    - This coincides with the mirror map.

Consistency checks

- The LG quantities consistent with FJRW theory.

  [Chiodo-Ruan 08][Chiodo-Iritani-Ruan 12]

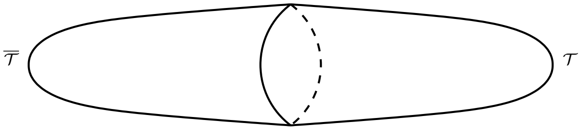
  - This gives an independent check for our results.
  - Our results generalise results from FJRW theory: more moduli, more general G.
- The hemisphere partition function of the GLSM reproduces the results from the central charge formula when evaluated at LG points.
  - GLSM gauge group broken to orbifold group G
  - Matrix q is related to the gauge charges of the GLSM fields
  - GLSM branes: matrix factorisations of the GLSM superpotential reduce to LG matrix factorisations

  [Herbst-Hori-Page 08][Clarke-Guffin 10]
- Tested in examples with up to four Kähler moduli, including computation of FJRW invariants.

Kähler potential – worldsheet perspective

- The Kähler potential of $\mathcal{M}_K$ is [Cecotti-Vafa '91]

$$e^{-K(t,\bar{t})} = \langle \overline{0} | 0 \rangle$$



- $\langle \overline{0} |$ and $| 0 \rangle$ are related by CPT conjugation.

Hybrids

- Consider a GLSM phase that is a Landau-Ginzburg orbifold with orbifold group G fibered over some base manifold B.
- This includes:
  - Calabi-Yau complete intersections in toric ambient spaces
  - Landau-Ginzburg orbifolds
- The sphere partition function evaluates to [Erkinger-JK 20]

$$Z_{S^2}^{phases}(t) = \sum_{\delta \in G} \int_B (-1)^{\mathrm{Gr}} \frac{\Gamma_\delta(H)}{\Gamma_\delta^*(H)} |I_\delta(t,H)|^2 = \langle \bar{I}, I \rangle$$

  - $\delta \in G$ . . . narrow sectors
  - Basis $H \in H^2(B)$
  - Gr . . . grading operator on the state space

Z_{S^2} in phases

- Landau-Ginzburg phases:
  - B is a point (no H)
  - Definitions of I/J, Gr, pairing, and the Gamma class coincide with definitions above
- Geometric phases:
  - B is a complete intersection CY X, G is trivial
  - The pairing is $\langle \alpha, \beta \rangle = \int_X \alpha^\vee \wedge \beta$, $\alpha, \beta \in H^{even}(X)$.
  - I/J, Gr, and Gamma class coincide with results in the literature
- Results suggest a definition [Iritani 07][Halverson-Jockers-Lapan-Morrison 13]

$$\langle \bar{I} | = (-1)^{\mathrm{Gr}} \frac{\Gamma}{\Gamma^*} I(\bar{t}).$$

Consistency checks

- Tested for GLSMs associated to 14 one-parameter complete intersections in toric ambient phases.
  - The small radius phases are Landau-Ginzburg, hybrids, and pseudo-hybrids. [Aspinwall-Plesser 09]
  - The structure is observed in all phases, even pseudo-hybrids, where we have a sum of terms.
  - I-functions and Gamma class in hybrid phases match with FJRW theory. [Clader 13]
- Two-parameter CY hypersurface with geometric, LG, and hybrid phase.
  - New conjectural results for the I-function and the Gamma class in multi-parameter hybrid models.

Summary

- We conjectured universal expressions for the hemisphere and sphere partition functions for phases of (abelian) GLSMs.
- Evidence that this works for geometric, Landau-Ginzburg and hybrid phases.
- Results match with mathematical results from FJRW theory and mirror symmetry, where available.
- Tested for lots of examples.

Open Questions

- Gamma class from the worldsheet perspective?
- Non-abelian GLSMs.
- Broad sectors.
- Hybrids, in particular with branes and enumerative invariants.
- More localisation results.
- Mathematical proofs.