m-dimers: A unifying perspective on worldvolume gauge theories on D-branes

arXiv:1811.07016 (JHEP03(2019)053)
arXiv:1904.07954 (JHEP11(2019)104)
arXiv:2004.13765

Azeem Ul Hasan

Dipartimento di Matematica e Fisica Ennio De Giorgi,
Università del Salento & I. N. F. N. - sezione di Lecce,
Via Arnesano 1, I-73100 Lecce, Italy

Outline

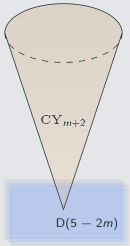
- Introduction and Motivation
- Graded Quivers
- m-Dimers
- Products
- Topological B-model
- Conclusions

Introduction

Gauge Theories and Calabi-Yau: A Recap

- D-branes probing a singularity is a powerful setup to study both Physics and Geometry.
- Supersymmetry requires that the probed singularity is a Calabi-Yau.
- The probed geometry appears as the moduli space of vacua for the corresponding theory.
- This setup realizes the theories with minimal SUSY in even dimensions starting from 6 to 0.

D-Branes probing CY



m	0	1	2	3
CY	CY_2	CY_3	CY_4	CY_5
SUSY	6d $\mathcal{N}=(1,0)$	4d $\mathcal{N}=1$	2d $\mathcal{N}=(0,2)$	0d $\mathcal{N}=1$

A Unified Description

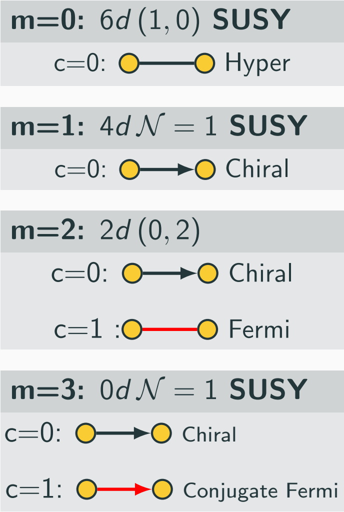
- Minimally supersymmetric theories in different dimensions have different structures but there are also remarkable connections.
- The number of multiplets and types of interactions increases in a uniform manner as we go down in spacetime dimension.
- There are “dualities” relating these theories. The order of these dualities increases by 1 at each step.
- These are indications that these theories fit in a family.
- Describing this family requires a new language that captures the similarities and organizes the differences.

Graded Quivers

m-Graded Quivers

The fields in the D$(5-2m)$-brane worldvolume theory realize an m-graded quiver.

- We represent $U(N)$ gauge groups as nodes.
- Bifundamental or adjoint fields are represented as arrows.
- To distinguish between different types of fields we assign each arrow a "degree" c $(0 \leq c \leq m)$.
- For every field $i \xrightarrow{(c)} j$ there is also the conjugate field $j \xrightarrow{(m-c)} i$



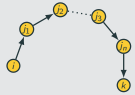
Superpotentials (Interactions)

- A cycle C is a path that begins and end at the same node subject to identification

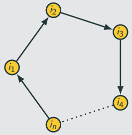
$$A_{ij}^{(c)} B_{ji}^{(d)} \sim (-1)^{cd} B_{ji}^{(d)} A_{ij}^{(c)}$$

- Superpotential W is a linear combination of cycles of degree $m-1$.

Path



Cycle



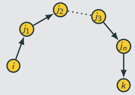
Superpotentials (Interactions)

- A cycle C is a path that begins and end at the same node subject to identification

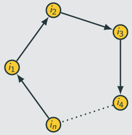
$$A_{ij}^{(c)} B_{ji}^{(d)} \sim (-1)^{cd} B_{ji}^{(d)} A_{ij}^{(c)}$$

- Superpotential W is a linear combination of cycles of degree $m-1$.

Path



Cycle



The Degree Constraint

- **m=0**: No Superpotential
- **m=1**: $W = W(X)$
- **m=2**: $W = \Lambda_a J_a(X) + \bar{\Lambda}_a E_a(X)$
- **m =3**: $W = \Lambda_a J_a(X) + \bar{\Lambda}_a \bar{\Lambda}_b H_{ab}(X)$

Constraints due to SUSY: Kontsevich Bracket

- Kontsevich bracket is defined to be

$$\{f, g\} = \sum_{\Phi} \frac{\partial f}{\partial \Phi} \frac{\partial g}{\partial \bar{\Phi}} - (-1)^{s(\Phi)} \frac{\partial f}{\partial \bar{\Phi}} \frac{\partial g}{\partial \Phi}$$

- Superpotential is required to have vanishing Kontsevich bracket with itself.

$$\{W, W\} = 0$$

Constraints due to SUSY: Kontsevich Bracket

- Kontsevich bracket is defined to be

$$\{f, g\} = \sum_{\Phi} \frac{\partial f}{\partial \Phi} \frac{\partial g}{\partial \bar{\Phi}} - (-1)^{s(\Phi)} \frac{\partial f}{\partial \bar{\Phi}} \frac{\partial g}{\partial \Phi}$$

- Superpotential is required to have vanishing Kontsevich bracket with itself.

$$\{W, W\} = 0$$

Constraint for small m

- **m=0**: No superpotential
- **m=1**: Trivial
- **m=2**: $\sum_a \operatorname{tr} E_a J_a = 0$
- **m =3**: $\sum_b H_{ab} J_b = 0$

Ranks and Anomaly Cancellation

- The ranks of gauge groups satisfy the anomaly cancellation equations.

$$(1 + (-1)^m)N_i = \sum_j \sum_{c=0}^{m} (-1)^{m-c} n_{ji}^{(c)} N_j$$

  Where $n_{ji}^{(c)}$ is the number of arrows of degree c going from node j to i.

- We will restrict to the case where all nodes have the same rank.

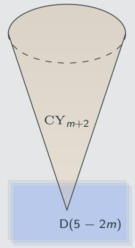
m-Dimers: Connecting Geometry and QFT

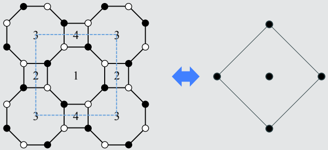
The Problem:

We want a dictionary between:

- The worldvolume QFT described as an m-graded quiver with superpotential.
- The probed Calabi-Yau singularity described by its realization as a GLSM or equivalently as a convex polygon in $\mathbb{Z}^{m+1}$ i.e the toric diagram.

D-Branes probing CY

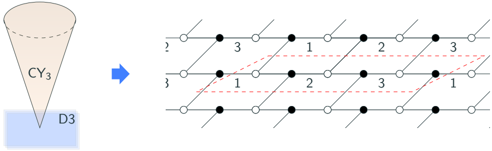




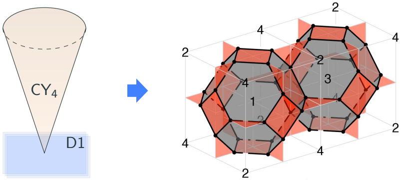
Brane Tilings and Brane Brick Models

The brane configuration we are chasing is T-dual to another one that involves D-branes suspended from NS5-branes. This configuration directly encodes the field theory.

D3-Branes and CY3 [Franco, Hanany et al (2006)]



D1-Branes and CY4 [Franco, Lee et al (2015)]



An m-Dimer is a tessellation of $\mathbb{T}^{m+1}$ that connects a CY_{m+2} with the corresponding SUSY field theory.

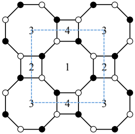
m-dimer	**Gauge Theory**
Codimension-0 face (brick)	Gauge group
Codimension-1 face of degree c between bricks i and j	Degree c field in the bifundamental representation of nodes i and j (adjoint for $i = j$)
Codimension-2 face	A monomial in the superpotential

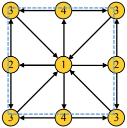
They generalize brane tilings and brane brick models and can be defined for any m.

Periodic Quivers

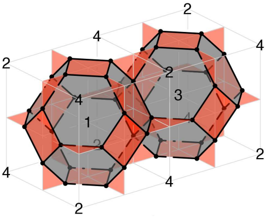
The dual to the dimer model is a quiver embedded on $\mathbb{T}^{m+1}$ called a periodic quiver.

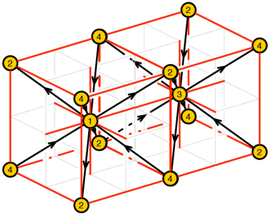
m=1: F_0





m=2: $\mathbb{C}^4/\mathbb{Z}_4$



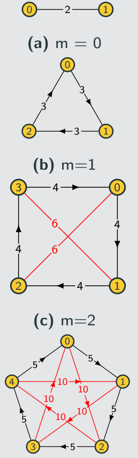


An Infinite Family: $\mathbb{C}^{m+2}/\mathbb{Z}_{m+2}$ [Closset, Franco, Guo, AH (2018)]

- Has a global $SU(m+2)$ symmetry.
- Arrows in quiver fall in representations of this $SU(m+2)$.
- There are $m+2$ nodes.
- Between i and $i+k$ there is an arrow $\Phi^{(k-1;k)}_{i,i+k}$. It has degree $k-1$ and transforms in the representation with k antisymmetric indices.
- Potential is

$$W = \sum_{i+j+k<m+2} \Phi^{(j-1;j)}_{i,i+j} \Phi^{(k-1;k)}_{i+j,i+j+k} \bar{\Phi}^{(m+1-j-k;m+2-j-k)}_{i+j+k,i}$$

Quivers



(a) m = 0

(b) m=1

(c) m=2

(d) m=3

Perfect Matchings

- A perfect matching p of an m-dimer is a collection of fields satisfying two simple properties
  - For every field Φ, either $\Phi \in p$ or $\bar{\Phi} \in p$ but not both.
  - p contains exactly one field from each term in the superpotential W.
- A perfect matching is determined entirely by chiral fields in it.
- Due to special structure of dimer models, they can be mapped to GLSM fields parameterizing the moduli space of the corresponding theory.
- Their intersections of fields in p with the fundamental cycles of the torus, determine the position of corresponding GLSM in the toric diagram.

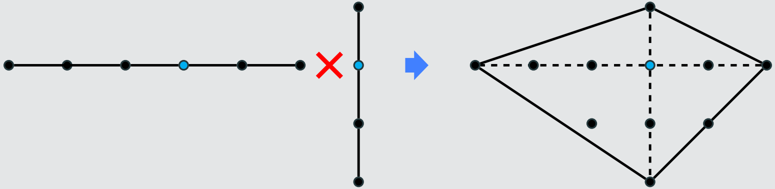
Product of Dimer Models

Franco, AH (2020)

The Algorithm

- Products is an algorithm that takes as inputs
  - An m-dimer P with a perfect matching p of P.
  - An n-dimer Q with a perfect matching q of Q.
- The output is an $n+m+1$-dimer $P_p \times Q_q$.
- This allows us to construct complicated geometries starting from simple ones.

An example of product connecting geometries

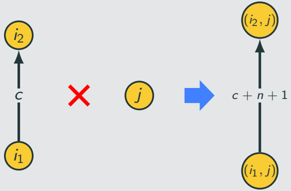


Periodic Quivers

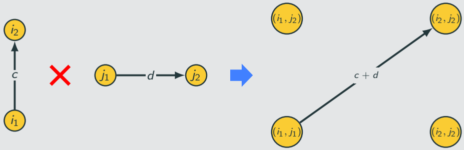
Gauge Group × Gauge Group

Gauge Group × Field in q

Field in p × Gauge Group



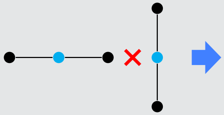
Field in p × Field in q



An example: F_0

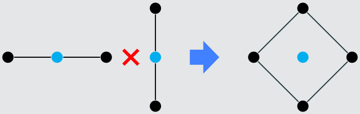
An example: F_0

Toric Diagram



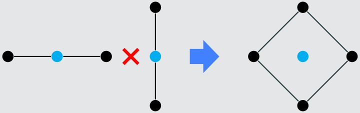
An example: F_0

Toric Diagram

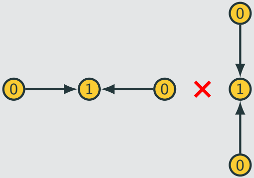


An example: F_0

Toric Diagram

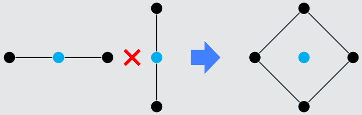


Quivers

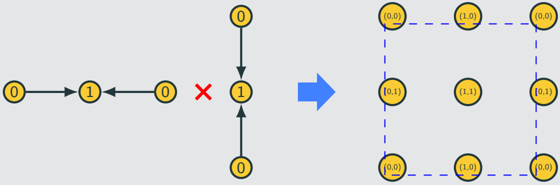


An example: F_0

Toric Diagram

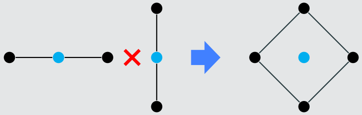


Quivers

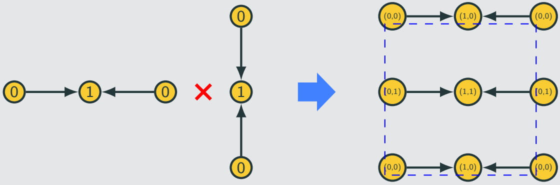


An example: F_0

Toric Diagram

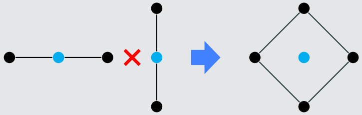


Quivers

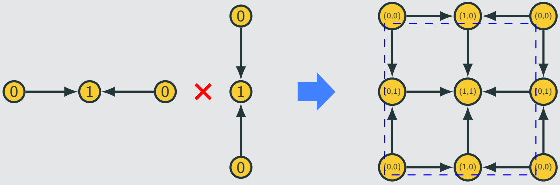


An example: F_0

Toric Diagram

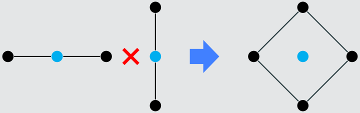


Quivers

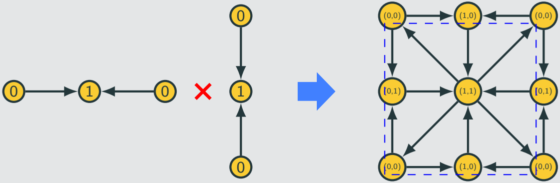


An example: F_0

Toric Diagram

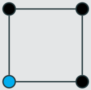


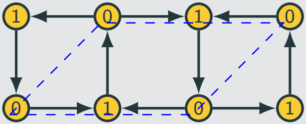
Quivers



Conifold × Conifold

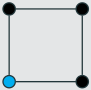
$4d\ \mathcal{N} = 1$: Conifold

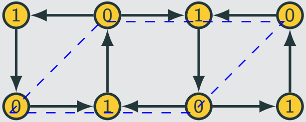




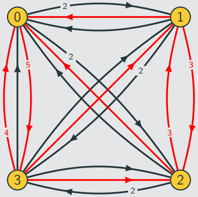
Conifold $\times$ Conifold

$4d\,\mathcal{N}=1$: Conifold





$0d\,\mathcal{N}=1$: Conifold $\times$ Conifold



- 13 chiral and 19 fermi fields.
- 56 quartic terms descending from superpotentials of parent conifolds.
- 32 additional cubic and 36 additional quintic terms.

Connected Sums?

- Products can be regarded as an algorithm for constructing an graph embedded on $X \times Y$ given graphs embedded on X and Y.
- Simplest example: Quiver blocks are graphs on $\mathbb{T}^{m+1} \times I$

$$\mathcal{Q}_p^{(m+1)} = Q_p^{(m)} \times$$

- Periodic quivers can be formed by gluing a number of quiver block for a given quiver Q but possibly different perfect matching p along their boundaries.
- Maybe such operations are part of a large class of "surgeries" on dimer models.

Topological B-model

Aspinwall, Katz (2004); Closset, Franco, Guo, AH (2018)

Topological B-model

- Topological B-model on a Calabi Yau $m+2$-fold X_{m+2} can be defined for any $m \geq 2$.
- It is described by an m-graded quiver.
- Topological B-model can be formalized in terms of an exceptional collection $\mathcal{E}_i$ of sheafs on a resolution of X_{m+2}.
- The dictionary is as follows

B-Model	Graded Quiver	Sheaf Theory
Brane	Node i	Sheaf $\mathcal{E}_i$
Open String	$i \xrightarrow{(c)} j$	Generator of $\mathrm{Ext}^{c+1}(\mathcal{E}_i, \mathcal{E}_j)$
Correlation function	Potential	Products m_k
"Supersymmetry"	$\{\mathrm{W},\mathrm{W}\} = 0$	A_∞ relations on m_k

Conclusions and Outlook

Conclusions

- m-graded quivers for provide us a language for unified description of minimally supersymmetric theories across dimensions.
- This unified description helps us chart the space of theses theories and reveals some interesting relationships between theories in different dimensions.
- m-dimers facilitate the connections between Calabi-Yau $m+2$-folds and the corresponding SUSY theory.
- For all m they can be realized as the configurations of B-branes in topological string theory.