GLSM Conference – August 18, 2020

3d Twists & Algebraic Structures

Based on work/interaction w/ K. Costello, D. Gaiotto, N. Paquette, M. Bullimore, J. Hilburn, P. Yoo, A. Ballin, W. Niu, N. Garner

Focus: 3d $\mathcal{N}=2$ & $\mathcal{N}=4$ theories in flat Euclidean space $\mathbb{R}^3$

Twist: choice of Q s.t. $Q^2=0$

3d $\mathcal{N}=2$: unique (up to iso) $\qquad \mathbb{R}^3 \simeq \mathbb{C}_z \times \mathbb{R}_t$

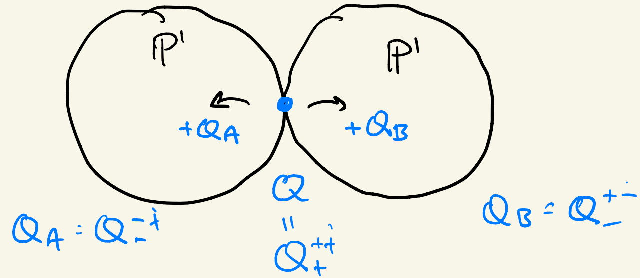
$\mathrm{Im}\,[Q,-]$ contains $\partial_{\bar z}$, ∂_t

8 Q's $\quad$ 3d $\mathcal{N}=4$: generic nilp Q is fully topological: $\mathrm{Im}\,[Q,-]$ contains ∂_z, $\partial_{\bar z}$, ∂_t

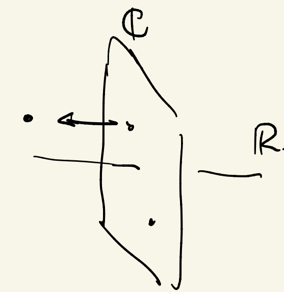
$SU(2)_H \times SU(2)_C$

$SU(2)_E$

$$\underbrace{\overbrace{Q_+^{+\dot{+}}}^{\text{hol-top}} + \varepsilon\, Q_-^{-\dot{+}}}_{\text{top A-twist}} \quad \text{or} \quad \underbrace{Q_+^{+\dot{+}} + \varepsilon\, Q_-^{+\dot{-}}}_{\text{top B-twist}}$$



In each twist, get a vec space of local ops $\mathcal{A} = H^\bullet_Q(\mathrm{Ops})$



HT : $\mathcal{A}$ is a (-1 shifted) Poisson vertex algebra (commutative)

$\mathcal{N}=4$: have $\mathcal{A}$

also $\mathcal{A}_A$, $\mathcal{A}_B$ (-2 shifted) Poisson algebras

E.g. $\mathcal{A}_A = H^\bullet_{Q+Q_A}(\mathrm{Ops})$

$$\underset{\text{hope/expect}}{\simeq} H^\bullet_{Q_A}(H^\bullet_Q(\mathrm{Ops})) = H^\bullet_{Q_A}(\mathcal{A})$$

<u>Example</u> 3d $\mathcal{N}=2$ free chiral

$$\mathcal{A} = \mathbb{C}\langle \underset{\text{bos}}{\overset{1}{\phi(x)}}, \underset{\text{ferm}}{\overset{-1}{\psi(x)}} \rangle \qquad \text{flavor}$$

3d $\mathcal{N}=4$ free hyper

$$\mathcal{A} = \mathbb{C}\langle \overset{1}{\phi(x)}, \overset{-1}{\tilde{\phi}(x)}, \overset{-1}{\psi(x)}, \overset{1}{\tilde{\psi}(x)} \rangle \qquad \text{(huge)}$$

Deformations:

$$Q_B \psi = \partial \tilde{\phi} \qquad\qquad Q_A \phi = \tilde{\psi}$$

$$Q_B \tilde{\psi} = - \partial \phi \qquad\qquad Q_A \tilde{\phi} = - \psi$$

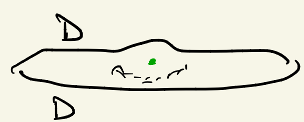
hol^c symp^c form on target

$\Rightarrow$

(tiny) $\mathcal{A}_B \simeq \mathbb{C}[\phi, \tilde{\phi}]$ $\qquad\qquad$ $\mathcal{A}_A \simeq \mathbb{C}$

$= \text{functions on } T^*\mathbb{C}$ $\qquad\qquad$ $\simeq H^\bullet_{dR}(T^*\mathbb{C})$

State-op correspondence: schematically $\mathcal{A} = H_Q^\bullet(\text{Hilb space on } D \cup_{D^\circ} D)$



free chiral: state-op corresp says that

$$\mathcal{A} = H^{\bullet}_{\bar\partial}\Big(\underbrace{\mathbb{C}[[z]] \times_{\mathbb{C}((z))} \mathbb{C}[[z]]}_{\substack{\phi(z) \qquad \phi'(z) \\ \text{st } \phi(z) = \phi'(z) \text{ for } z \neq 0}} \Big)$$

$= H^{\bullet}_{\bar\partial}(\mathbb{C}[[z]])$ namely $\rightsquigarrow \mathbb{C}\langle \phi(z) \rangle$

claim: correct this by using a derived intersection

ie. imposing $\phi(z) = \phi'(z)$

by introducing fermions st $Q(\ ) = \phi - \phi'$

Corrected version: $\mathcal{A} = \mathbb{C}\langle \phi(z), \psi(z) \rangle$

$$H^{\bullet}_{\bar\partial}(N^{\bullet}[1]\, \mathbb{C}[[z]])$$

A-twist $H^{\bullet}_{dR}(N^{\bullet}[1]\, \mathbb{C}^2[[z]]) = \mathbb{C}$ vs B-twist $H^{\bullet}_{\bar\partial}(\mathbb{C}^2)$

$= \mathcal{A}_A \qquad\qquad = \mathcal{A}_B$

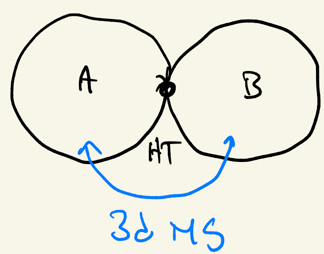
Gauge theory: $\mathcal{A}$ is not (fully) known

3d $\mathcal{N}=2$ $\quad G, V \quad$ expect $\mathcal{A} = \underbrace{H^{\bullet}_{\bar\partial}\big(\text{holomorphic } G\text{-bundles } \& \text{ hol}^c \text{ sections of } V\text{-bundle on } D \cup_{D^\times} D}_{\text{derived alg. geom}} \,,\ \mathcal{L}^{\text{cs level}}\big)$

3d $\mathcal{N}=4$

$$\mathcal{A}_A = \text{BFN construction of Coulomb branches}$$
$$= H_{dR}^{\bullet}(\text{same})$$
$$\mathcal{A}_B = H^{\bullet}_{\bar\partial}(T^*V /\!/\!/ G)$$

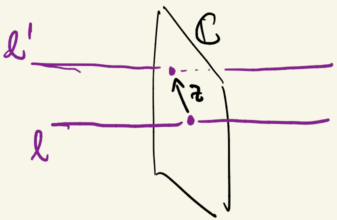
Recall: for 3d $\mathcal{N}=4$



$$\begin{tikzcd} & \mathcal{A} \arrow[loop above, "\text{3d MS}"] \arrow[dl] \arrow[dr] & \\ \mathcal{A}_A \arrow[rr, bend right, leftrightarrow, "\text{3d MS}"'] & & \mathcal{A}_B \end{tikzcd}$$

Line ops

Form a category (in any twist)



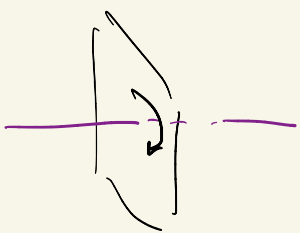
$\mathrm{Hom}(\ell,\ell') = H^\bullet_Q(\text{local ops at junction})$

- factorization cat $\mathcal{C}$ in HT twist

  $\ell \otimes_z \ell'$

  $\mathbb{1}$ identity line (trivial line)

  $A = \mathrm{End}^\bullet_{\mathcal{C}}(\mathbb{1})$



- $\mathcal{C}$ is graded by rotations

  $\rightsquigarrow K(\mathcal{C}) =$ Quantum K-thy showing up in 3d $\mathcal{N}=2$ on S^1

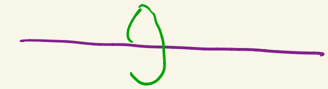
In 3d $\mathcal{N}=4$

$$\mathcal{C} \swarrow \quad \searrow$$

$$\underbrace{\mathcal{C}_A \qquad\qquad \mathcal{C}_B}_{\text{braided } \otimes \text{ categories}}$$

Baby example free chiral 3d $\mathcal{N}=2$

$$\mathcal{C} = \mathrm{Coh}(\underset{\substack{\uparrow \\ \text{target}}}{\mathbb{C}}(\!(\underset{\substack{\uparrow \\ \text{loops}}}{z})\!))$$



= bdy conds for 2d B-model on loop space of 3d target

Contains: $\mathbb{1} = \mathcal{O}_{\mathbb{C}[\![z]\!]}$

and vortex lines $V_n = \mathcal{O}_{z^n\mathbb{C}[\![z]\!]}$ $n \in \mathbb{Z}$

$$\mathrm{End}^\bullet(\mathbb{1}) = \text{functions on } \mathbb{C}[\![z]\!]$$

& deformations of $\mathbb{C}[\![z]\!]$ into $\mathbb{C}(\!(z)\!)$ (fermionic)

$$= \mathbb{C}\langle \phi(z), \psi(z)\rangle = \mathcal{A}$$

3d $\mathcal{N}=4$ free hyper

$$\mathcal{C} = \mathrm{Coh}(T^*\mathbb{C}(\!(z)\!))$$

$\swarrow$ $\searrow$

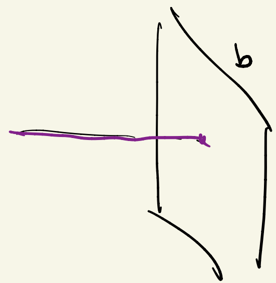
$$\mathcal{C}_A = \text{D-mod}(\mathbb{C}(\!(z)\!)) \qquad \mathcal{C}_B = \mathrm{Coh}(T^*\mathbb{C})$$

Rozansky-Witten

In 3d $\mathcal{N}=4$, $\mathcal{C}_A$ & $\mathcal{C}_B$ cats are now mostly understood

$\mathcal{C}$ is not!

Bdy conds



$\rightsquigarrow$ bdy chiral algebras $A[b]$ $\leftarrow$ interesting OPE's

(modules for A)

$$\mathcal{C} \xrightarrow{F_b} A[b]\text{-mod}$$

Discussion

3d $\mathcal{N}=4$ A or B twist

(0,4) b.c. $\rightsquigarrow$ bdy VOA

(2,2) b.c. $\rightsquigarrow$ bdy algebra hyper

$\mathbb{C}\langle \phi(z), \tilde{\phi}(z), \psi, \tilde{\psi} \rangle$

bulk $\mathbb{C}$

bdy $\boxed{\phi(z)\,\tilde{\phi}(0) \sim \frac{1}{z}}$

$(0,4)$ b.c. NN $T\mathbb{C} \subseteq T^*\mathbb{C}$ $\quad \mathbb{C}\langle \phi(z), \tilde{\phi}(z)\rangle \xrightarrow{A}$

DD $pt \subseteq T^*\mathbb{C}$ $\quad \mathbb{C}\langle \psi(z), \tilde{\psi}(z)\rangle \xrightarrow{B}$

bulk $\mathbb{C}[\phi, \tilde{\phi}]$

bdy $\psi(z)\,\tilde{\psi}(0) \sim \frac{1}{z^2}$

$(2,2)$ ND

DN

$\mathbb{C}\langle \phi(z), \tilde{\psi}(z)\rangle$ ∂ $\xrightarrow{A}$ $\downarrow B$

kills all bdy modes

$C_A = D\text{-mod}(\mathbb{C}((z))) \simeq \boxed{\beta\text{-}\gamma\text{-mod}}$

$\phi_n \quad n \in \mathbb{Z}$

$\tilde{\phi}_{-n-1} := \frac{\partial}{\partial \phi_n}$ $\quad [\phi_n, \tilde{\phi}_m] = \delta_{m+n+1,0}$