# An overview of (0,2) theories

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# Many paths to (0,2) theories

- ⋆ critical heterotic strings with $\mathbb{R}^{1,3}$ N=1 superPoincaré
- ⋆ heterotic stringy geometry for $\mathcal{E} \to X$
- ⋆ surface defects/strings in d=4 N $\geq$ 1 gauge theory
- ⋆ $AdS_3/CFT_2$ and (2,0) compactification
- ⋆ branes at Calabi-Yau singularities
- ⋆ F-theory/heterotic compactification to two dimensions
- ⋆ *last refuge of holomorphy in SUSY QFT*

# Plan

A~~n over~~view of (0,2) theories

1. general properties of (0,2) theories
2. linear sigma models
3. the (0,2) swampland
4. outlook

# (0,2) supercurrents live in an $\mathcal{S}$-multiplet

[Dumitrescu&Seiberg,**1106.0031**]

- $\mathbb{R}^{1,1}$: $x^\mu = (x^0, x^1)$ or light-cone $x^{\pm\pm}$
- $S^\mu_+$, $\overline{S}^\mu_+$ conserved supercurrents $\rightarrow Q_+$, $\overline{Q}_+$
- energy-momentum tensor $T^{\mu\nu} \rightarrow P^\mu$
- weak assumptions $\Longrightarrow$ $T, S, \overline{S} \subset \mathcal{S}$-supermultiplet

$$\{Q_+, \overline{S}_{+\pm\pm}\} = -T_{\pm\pm++} \pm i\partial_{\pm\pm} j_{++} \qquad \{Q_+, S_{+++}\} = 0$$
$$\{Q_+, S_{+--}\} = i\overline{C} \ .$$

- spin 1 operator $j_{++}$ not necessarily conserved
- $\overline{C} \neq 0 \Longrightarrow$ deformed (0,2) supercurrent algebra, a UV property
- $\overline{C}$ can be generated by quantum corrections, e.g. $(0,2)$ $\mathbb{P}^1$ NLSM

# Many (0,2) QFTs have $\mathcal{R}$-multiplet

[Dumitrescu&Seiberg,**1106.0031**]

- $\overline{C} = 0$
- conserved R-current $j^\mu \leftrightarrow j_{\pm\pm} \rightarrow$ conserved charge $R$
- $j^\mu, S^\mu_+, \overline{S}^\mu_+, T \subset \mathcal{R}$-multiplet $\implies$ standard (0,2) SUSY algebra

$$\{Q_+, \overline{Q}_+\} = -4P_{++} \qquad [R, Q_+] = -Q_+ \qquad [R, \overline{Q}_+] = +\overline{Q}_+$$
$$\{Q_+, Q_+\} = 0 \qquad \{\overline{Q}_+, \overline{Q}_+\} = 0$$

- a key property: $\overline{Q}_+^2 = 0$
- we will focus on theories with $\mathcal{R}$-multiplet and their IR limits

# (0,2) SCFTs are the building blocks for (0,2) QFTs

⋆ $T^{\mu}_{\mu} = 0$ on Euclidean world-sheet

$$\mathrm{Vir}_c \oplus \overline{\mathrm{N{=}2\ sVir}}_{\bar{c}} \qquad T(z) \qquad ; \qquad \bar{J}(\bar{z})\ ,\ G^{\pm}(\bar{z})\ ,\ \overline{T}(\bar{z})$$

$$h \qquad\qquad \bar{q} \qquad\qquad \bar{h}$$

⋆ we will discuss unitary and compact SCFTs

⋆ partial characterization

- symmetries, e.g. left-moving Kac-Moody algebra $\supset \mathrm{U}(1)_{\mathrm{L}}$ current $J$
- elliptic genus $Z = \mathrm{Tr}_{RR}(-1)^F q^H \bar{q}^{\overline{H}} y^{J_0}$
- spectrum of marginal operators [depends on moduli!]
- 2 and 3-point functions [super-primary not sufficient!]

# Marginal (0,2) deformations are characterized

- ⋆ SUSY marginal deformations via conformal perturbation theory [Bertolini et al,**1405.4266**], cf N=1 d=4 [Green et al,**1005.3546**]

$$\Delta S = \lambda \int d^2 z \{Q_+, U\} + \text{h.c.}$$

$U$ a chiral primary operator $\quad h = 1, \quad \bar{h} = \bar{q}/2 = 1/2$

- ⋆ if $\bar{q}$ constant, $\bar{h} \geq \bar{q}/2 \implies \lambda$ at worst marginally irrelevant
- ⋆ "lifting" of marginal operators over moduli space:
  - "D-term" $(\mathcal{J}_{\text{KM}}(z), U, \overline{U}) \to$ long multiplet $\implies \lambda$ breaks KM
  - "F-term" $(U_{\bar{q}=1}, F_{\bar{q}=2}) \quad \to$ long multiplet
- ⋆ "Shortening anomalies" can affect moduli space structure [Gomis et al,**1611.03101**]

# RG flows satisfy some simple constraints

- relevant $\Delta S = \lambda \int d^2 z \{Q_+, U\} + \text{h.c.}$ ; $U$ c.p. with $\bar{q} < 1$
- $(c - \bar{c})$ is RG-invariant; $\dot{c} < 0$
- $\bar{c}_{\text{IR}}$ via $c$-extremization [Benini&Bobev,**1211.4030**,**1302.4251**] [IVM,**1603.08935**]
- elliptic genus, computed in UV, constrains IR
- chiral algebra of $\overline{Q}_+$-closed operators

  [Witten,**0504078**],[Tan&Yagi,**08014742**],[Borisov&Kaufmann,**1102.5444**],[Dedushenko,**1501.07589**]
- topological heterotic rings generalize A and B model of (2,2)

  [Adams,Basu&Sethi,**0309226**],[Adams,Distler&Ernebjerg,**0506263**]
- we will see some caveats to these results

# The Chiral algebra

- ⋆ $H_{\overline{Q}_+} \simeq \{$vector space of chiral primary $\bar{h} = \bar{q}/2$ operators$\}$
- ⋆ $H_{\overline{Q}_+}$ graded by holomorphic quantum numbers $h, q, \ldots$ $[T,\ J, \ldots]$
- ⋆ OPE gives $H_{\overline{Q}_+}$ structure of holomorphic CFT:

$$\mathcal{O}_i(z)\mathcal{O}_j(0) \sim \sum_s C_{ij}{}^s(\lambda)\ \ z^{h_s - h_i - h_j}\mathcal{O}_s(0)\ + \overline{Q}_+(\cdots)$$

- ⋆ in principle computable from UV description
- ⋆ caveat $H^{\text{classical}}_{\overline{Q}_+} \neq H^{\text{UV}}_{\overline{Q}_+}$

e.g. [Tan&Yagi,**0801.4782**],[Yagi,**1001.0118**],[McOrist&IVM,**1103.1322**],[Aspinwall&Plesser,**1106.2998**],

[Guo et al,**1501.00987**]

# B/2 rings in theories with $\mathrm{U}(1)_\mathrm{L}$ symmetry

- ⋆ assumptions
  - $\mathrm{U}(1)_\mathrm{L}$ with "level" r : $J(z)J(0) \sim r z^{-2}$
  - $s = h - \bar{h} \in \mathbb{Z}$ (bosonic operator) or $\in \mathbb{Z} + \frac{1}{2}$ (fermionic operator)
  - $q - \bar{q} \in \mathbb{Z}$, $(-1)^F = (-1)^{q-\bar{q}}$
  - a point in moduli space with bound $h \geq q/2$ on spectrum
- ⋆ result [Adams et al, **0506263**]: OPE induces ring structure on

$$H_{B/2} = \{\mathcal{O} \in H_{\overline{Q}_+} \ \mid \ h_\mathcal{O} = q_\mathcal{O}/2\}$$

- ⋆ Sugawara: $h \geq q^2/2r \implies \dim H_{B/2} < \infty$ in compact SCFT
- ⋆ Generalization of (c,c) ring of (2,2) theories
- ⋆ Compute by localization: LG [Melnikov, **0902.3908**], hybrids [Bertolini&Romo, **1801.04100**]

# A/2 rings: generalization of the (a,c) ring of (2,2)

- if $\bar{q} \in \mathbb{Z}$ as well, define A/2 ring:

$$H_{A/2} = \{\mathcal{O} \in H_{\overline{Q}_+} \;\mid\; h_{\mathcal{O}} = -q_{\mathcal{O}}/2\}$$

- Computed by localization in deformations of certain (2,2) theories

[McOrist&IVM,**0810.0012**],[Lu,**1511.09158**],[Closset et al,**1512.08058**]

- worldsheet instanton sums in heterotic compactifications $\mathcal{E} \to X$:

$$(\overline{\mathbf{27}})^3 : \quad H^1(X,\mathcal{E}^*) \times H^1(X,\mathcal{E}^*) \times H^1(X,\mathcal{E}^*) \to \mathbb{C}$$

  - $\mathcal{E} = T_X$ is (2,2) locus
  - ring structure depends on (0,2) moduli in computable manner

- (0,2) mirror symmetry $\implies H_{A/2}(X,\mathcal{E}) \simeq H_{B/2}(X^\circ,\mathcal{E}^\circ)$

[Kreuzer et al,**1001.2104**],[IVM&Plesser,**1003.1303**]

# A-ring of *gapped* theory with $\mathcal{R}$-multiplet

- ⋆ (2,2) reminder: SUSY $\mathbb{CP}^{n-1}$ theory
  - believed to flow to a SUSY TFT: $n$ massive vacua
  - quantum cohomology relation: $\mathcal{I} = \sigma^n - \boldsymbol{q} \implies H_A = \mathbb{C}[\sigma]/\mathcal{I}$
  - GLSM realization: (2,2) U(1) gauge theory with $n$ chiral multiplets
  - $\sigma = \Sigma|_{\theta=0}$ *twisted chiral superfield*
  - $-\log \boldsymbol{q}$ is complexified F-I parameter
- ⋆ vast generalization: quantum cohomology for toric NEF Fano variety $V$

$$H_A(V) = \mathbb{C}[\sigma_1, \cdots, \sigma_k]/\mathcal{I}(\boldsymbol{q})$$

  - write it down [Batyrev,**alg-geom/9310004**]
  - sum instantons in rank $k$ abelian (2,2) GLSM [Morrison&Plesser,**9412236**]
  - Coulomb branch localization [IVM&PLesser,**0507187**], [Closset et al,**1504.06308**]

# A/2 ring of *gapped* theory with $\mathcal{R}$-multiplet

⋆ vast(er) (0,2) generalization: quantum sheaf cohomology for $(V, \mathcal{E})$

$$0 \longrightarrow W^* \otimes \mathcal{O}_V \xrightarrow{E} \oplus_\rho \mathcal{O}_V(D_\rho) \longrightarrow \mathcal{E} \longrightarrow 0$$

- $W = H^2(V, \mathbb{C})$, and $D_\rho$ are toric divisors
- $\mathcal{E} = T_X$ obtained at special value of map $E$
- $H_{A/2} = \mathbb{C}[\sigma_1, \sigma_2, \cdots, \sigma_{\dim W}]/\mathcal{I}(\boldsymbol{q}, E)$

[Adams et al,**0309226**],[Katz&Sharpe,**0406226**],[Guffin&Katz,**0710.2354**],[McOrist&IVM,**0712.3272**],

[Donagi et al,**1110.3751**],[Donagi et al,**1409.4353**]

⋆ rich interplay with Hori-Vafa duality, e.g. [Chen et al,**1705.08472**],

[Gu&Sharpe,**1707.05274**],[Gu et al,**1908.06036**]

⋆ input for A/2 SCFT computations for $X \subset V$

# Plan

1. general properties ✓
2. linear sigma models
3. accidents
4. outlook

# Linear sigma models : UV structure

⋆ two-dimensional gauge theories with (0,2) susy [Witten, 9301042]

- gauginos in chiral fermi multiplets $\Upsilon^i = \upsilon^i_- + \cdots$
- chiral bosonic multiplets $\Phi$ and fermi multiplets $\Gamma^A = \gamma^A_- + \cdots$
- gauge group $G$ and holomorphic potentials $E^A(\Phi)$, $J_A(\Phi)$

$$\overline{\mathcal{D}}\Gamma^A = E^A(\Phi) \ , \qquad L \supset \int d\theta \sum\nolimits_A \Gamma^A J_A(\Phi) + \text{h.c.}$$

- (0,2) SUSY requires $\sum_A E^A J_A = 0$
- "phases" determined by F-I parameters $L \supset \int d\theta \sum_i \Upsilon_i \tau_i + \text{h.c.}$
- geometric phases include $X \subset V_{\text{toric}}$ & $\mathcal{E} \to X$ monad bundle
- abelian (2,2) theories: $E^A = \sum_i Q^A_i \Sigma_i \Phi^A$, $J_A = \frac{\partial W}{\partial \Phi^A}$

# Linear sigma models : (optimistic) IR structure

- ⋆ $U(1)_L \times U(1)_R$ symmetries and anomalies fixed by combinatorics
- ⋆ compactness of field space fixed by combinatorics
- ⋆ chiral algebra computations in principle possible

$\Longrightarrow$

- vast classes of (2,2) and (0,2) SCFTs
- many examples of heterotic geometries: conformal (0,2) NLSMs
- interpolate between disparate descriptions — CY/LG correspondence
- generalize (2,2) dualities
- obtain new ones , e.g. target space duality [Distler&Kachru,**9501111**], [Blumenhagen&Rahn,**1106.4998**],[Anderson&Feng,**1607.04628**]

- ⋆ the art of the linear sigma model : lift IR queries to UV fields
  - simplicity of theories with (2,2) locus: A/2 sub-ring generated by $\sigma_i$

# (0,2) linear sigma model developments

- ⋆ elliptic genus [Gadde&Gukov,**1305.0266**],[Benini et al,**1308.4896**]

  $\Longrightarrow$ probe novel (0,2) SCFTs, e.g. [Gadde et al,**1404.5314**],[Haghighat et al,**1412.3152**],[Franco et al,**1702.02948**],[Kim et al,**1710.06069**]

- ⋆ vanishing theorem for spacetime superpotential [Bertolini&Plesser,**1410.4541**]

  $\Longrightarrow$ $(X, \mathcal{E})$ large-radius geometries safe from worldsheet instantons

- ⋆ torsional linear sigma models: linear sigma models for heterotic geometries with $H$-flux [Adams et al,**0611084,1206.5815**],

  [Quigley et al,**1206.3228,1212.1212**],[Israel&Sarkis,**1606.08982**]

# Plan

1. general properties ✓
2. linear sigma models ✓
3. (0,2) swampland (upcoming work with Bertolini and Plesser)
4. outlook

# (0,2) Landau-Ginzburg theory: optimistic view

- ⋆ (0,2) Landau-Ginzburg theory with $\mathrm{U}(1)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{R}$ symmetry

$$L = \text{free kinetic term} + \int d\theta \sum_A \Gamma^A J_A(\Phi;\alpha) + \text{h.c.}$$

- ⋆ quasi-homogeneity: $J_A(t^q \Phi;\alpha) = t^{-Q_A} J_A(\Phi;\alpha)$
- ⋆ compact field space $\iff \mathbb{J} = \langle J_1, \ldots, J_N \rangle$ zero-dimensional
- ⋆ charges $(q, Q)$ determine family of LG theories
- ⋆ generic $J_A \implies \mathrm{U}(1)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{R}$ symmetry
- ⋆ compact IR SCFT with $(c, \bar{c})$ & elliptic genus determined by $(q, Q)$

# There are accidents in (0,2) RG flows

[Bertolini et al,**1405.4266**]

- ⋆ LG kinetic term irrelevant and slaved to superpotential
- ⋆ $J_A(\Phi;\alpha)$ and $J_A(\Phi;\alpha')$ are IR-equivalent if linked by field redefinition
- ⋆ symmetries of $J_A(\Phi;\alpha)$ depend on $\alpha$

  $\Longrightarrow$ currents in IR can modify chiral algebra: $T \to T + \partial J$

  $\Longrightarrow$ $c$-extremization can lead to $c(\alpha) \neq c(\alpha')$!

  $\Longrightarrow$ UV parameter space stratified according to basin of attraction

  $\Longrightarrow$ naive $c$ may not have realization for *any* $\alpha$

- ⋆ what is the relation between combinatorics and accidents?

# Unicorns and unitarity

- ⋆ If $J_A$ has no terms linear in $\Phi$, then

$$\mathcal{U} = \gamma^1\gamma^2\cdots\gamma^N \in H_{\overline{Q}_+} \quad .$$

- ⋆ $\bar{q}_{\mathcal{U}} = \sum_A (1 + Q_A) \leq \bar{c}/3$
- ⋆ powerful constraint! e.g. bounds on $\#\Phi$ in terms of $c$
- ⋆ what if $\bar{q}_{\mathcal{U}} > \bar{c}/3$?
  - can be an accident: an additional spin 1 current in $H_{\overline{Q}_+}$
  - can be a mystery: no additional spin 1 or spin 2 currents in $H_{\overline{Q}_+}$

# A mysterious bad unicorn

- ⋆ LG with 3 chiral and 3 fermi fields
- ⋆ $\mathbb{J} = \langle \Phi_1\Phi_2 \, , \Phi_2^3 + \Phi_1\Phi_3 \, , \Phi_1^{11} + \Phi_3^2 \rangle$
- ⋆ unique $\mathrm{U(1)_L \times U(1)_R}$
- ⋆ $\mathbb{J}$ most general ideal compatible with charges and compactness
- ⋆ unique $T, J \subset H_{\overline{Q}_+}$, but : $\bar{q}_{\mathcal{U}} > \bar{c}/3$ !
- ⋆ possible resolutions
  - decompactification in IR?
  - ~~SUSY~~ ?
  - ??

# The (0,2) Swampland

- ⋆ Chiral algebra: a powerful tool to uncover IR dynamics of (0,2) GLSM
- ⋆ Question: When is UV $H_{\overline{Q}_+}$ reliable guide to IR physics of a GLSM?
- ⋆ Today's Answer: Sometimes.
- ⋆ (0,2) Swampland program:
  - constrain combinatorics of (0,2) LG or GLSM to give necessary and sufficient conditions for positive answer.
  - describe IR limit even when answer is negative

# Outlook

⋆ impressive results for just two supercharges

⋆ some future directions

- (0,2) swampland!
- can we classify compact (2,2) GLSMs with fixed $c$, $\bar{q} \in \mathbb{Z}$?

  [Aspinwall&Plesser,**1507.00301**]

- torsional sigma models and connections to geometry
- geometric transitions in (0,2) and heterotic stringy geometry
- perspectives on local models: non-compact (0,2) SCFTs

⋆ hope: new sources of (0,2) questions may also yield new (0,2) answers

⋆ a little book : *An introduction to two-dimensional quantum field theory with (0,2) supersymmetry*, **LNP951**, Springer 2019.