New $2d$ $\mathcal{N} = (0, 2)$ dualities from four dimensions

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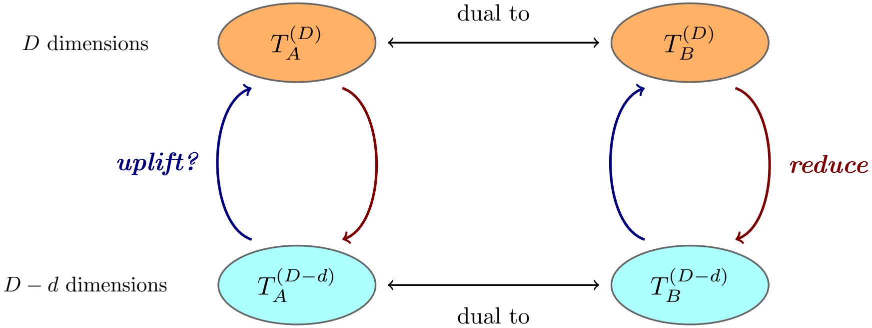
Infra-red dualities in 4, 3 and 2 dimensions can be related by different types of **dimensional reduction** limits.

Why study dimensional reductions of dualities?

- they provide an organizing principle behind the currently known dualities
- they can be used to find **new dualities** from already known ones



It is not always guaranteed that a duality **survives** a dimensional reduction.

Two limits are involved in the process:

- flow to low energy keeping the compactification scale fixed
- dimensional reduction limit keeping the energy scale fixed

The duality is preserved if these two limits commute.

We can use dimensional reductions and uplifts to **conjecture** a new duality, but then this has to be **supported by tests**.

Renewed interest in **$2d$ $\mathcal{N} = (0, 2)$ dualities** after the trialities of [Gadde, Gukov, Putrov '13].

The trialities and many other new dualities have been later derived by compactifications of **$4d$ $\mathcal{N} = 1$ dualities on $\mathbb{S}^2$ with a topological twist**. [Gadde, Razamat, Willett '15]

Several problems to address when studying this type of dimensional reduction:

- truncation to the zero modes
- non-compact target space [Aharony, Razamat, Seiberg, Willett '16; Aharony, Razamat, Willett '17]
- anomalous vs. non-anomalous global symmetries

Define a $4d$ $\mathcal{N}=1$ theory on $\mathbb{R}^2 \times \mathbb{S}^2$ preserving half of the supercharges by turning on a background gauge field for a $U(1)_R^{4d}$ with "properly" quantized flux. [Closset, Shamir '13; Benini, Zaffaroni '15; Honda, Yoshida '15]

We get the direct sum of many $2d$ $\mathcal{N}=(0,2)$ theories on $\mathbb{R}^2$ describing the KK modes. Choosing $U(1)_R^{4d}$ "properly" only the zero modes survive. [Gadde, Razamat, Willett '15]

$U(1)_R^{4d}$ should be such that:

- it is **non-anomalous**
- all chiral fields have **integer** R-charge
- all chiral fields have **non-negative** R-charge

It doesn't have to be the superconformal one! We can mix it with non-abelian global symmetries

$$R = R_0 + \sum_{i=1}^{\mathrm{rk}(F)} q_i R_i, \qquad \text{where} \qquad \prod_{i=1}^{\mathrm{rk}(F)} U(1)_i \subset F$$

The rules of the game: [Kutasov, Lin '13; Gadde, Razamat, Willett '15]

- a $4d$ $\mathcal{N}=1$ chiral of R-charge r gives
  - $r-1$ $2d$ $\mathcal{N}=(0,2)$ Fermis if $r>1$
  - $1-r$ $2d$ $\mathcal{N}=(0,2)$ chirals if $r<1$
  - nothing if $r=1$
- a $4d$ $\mathcal{N}=1$ vector gives a $2d$ $\mathcal{N}=(0,2)$ vector
- the $4d$ $\mathcal{N}=1$ superpotential determines the interactions of the $2d$ matter fields

Comments:

- many choices for $U(1)_R^{4d}$ may be allowed, which lead to different $2d$ dualities
- use the rules only to conjecture a duality, which should be then supported by tests (matching anomalies, elliptic genera, ...)

$4d$ Intriligator–Pouliot duality

Theory $\mathcal{T}_A^{\mathbf{4d}}$: $USp(2N)$ gauge theory with $2N+4$ fundamental chirals and

$$\mathcal{W}_{\mathcal{T}_A^{\mathbf{4d}}} = 0$$

Theory $\mathcal{T}_B^{\mathbf{4d}}$: WZ model of $(N+2)(2N+3)$ chirals M_{ab}, where $a < b = 1, \cdots, 2N+4$, interacting with

$$\mathcal{W}_{\mathcal{T}_B^{\mathbf{4d}}} = \mathrm{Pf}\, M$$

Some properties:

- reduces to Seiberg duality between $SU(2)$ with 6 chirals and WZ model with 15 chirals for $N=1$
- the non-anomalous global symmetry of the two theories is $SU(2N+4)$ (the $U(1)$ part is anomalous)

Transformation properties of the fields under the global symmetry

	$SU(2N+4)$	$U(1)_{R_0}$
Q	$\mathbf{2N+4}$	$\frac{1}{N+2}$
M	$\mathbf{(N+2)(2N+3)}$	$\frac{2}{N+2}$

Consider the subgroup $SU(2N+2) \times SU(2) \times U(1)_s \subset SU(2N+4)$

$$\begin{aligned} \mathbf{2N+4} &\rightarrow (\mathbf{2N+2}, \mathbf{1})^{1} \oplus (\mathbf{1}, \mathbf{2})^{-(N+1)} \\ \mathbf{(N+2)(2N+3)} &\rightarrow (\mathbf{2N+2}, \mathbf{2})^{-N} \oplus (\mathbf{(N+1)(2N+1)}, \mathbf{1})^{2} \oplus (\mathbf{1}, \mathbf{1})^{-2(N+1)} \end{aligned}$$

We allow for a mixing of $U(1)_{R_0}$ with $U(1)_s$ and define $U(1)_R^{4d}$ as

$$R = R_0 + q_s R_s$$

Choosing $R_s = -\frac{1}{N+2}$:

- on the side of theory $\mathcal{T}_A^{\text{4d}}$ we get $2N+2$ chirals
- on the side of theory $\mathcal{T}_B^{\text{4d}}$ we get $(N+1)(2N+1)$ chirals and one Fermi

We get the following $2d$ $\mathcal{N} = (0,2)$ duality [Gadde, Razamat, Willett '15]

Theory $\mathcal{T}_{\mathbf{A}}$: $USp(2N)$ gauge theory with $2N+2$ fundamental chirals and

$$W_{\mathcal{T}_{\mathsf{A}}} = 0$$

Theory $\mathcal{T}_{\mathbf{B}}$: LG model of one Fermi Ψ and $(N+1)(2N+1)$ chirals, where $a < b = 1, \cdots, 2N+2$, interacting with

$$W_{\mathcal{T}_{\mathsf{B}}} = \Psi \,\mathrm{Pf}\, \Phi$$

Global symmetry $SU(2N+2) \times U(1)_s$

	$SU(2N+2)$	$U(1)_s$	$U(1)_{R_0}$
Q	$\mathbf{2N+2}$	1	0
Ψ	$\bullet$	$-2(N+1)$	1
Φ	$\mathbf{(N+1)(2N+1)}$	2	0

New $2d$ duality for $USp(2N)$ with antisymmetric

Consider the $4d$ $\mathcal{N}=1$ duality [Csaki, Skiba, Schmaltz '96]

Theory $\mathcal{T}_A^{\mathbf{4d}}$: $USp(2N)$ gauge theory with one antisymmetric chiral A, six fundamental chirals Q_a and N chiral singlets β_i with superpotential

$$\mathcal{W}_{\mathcal{T}_A^{\mathbf{4d}}} = \sum_{i=1}^{N} \beta_i \mathrm{Tr}_N A^i$$

Theory $\mathcal{T}_B^{\mathbf{4d}}$: WZ model with $15N$ chiral singlets $\mu_{ab;i}$ for $i=1,\cdots,N$, $a<b=1,\cdots,6$ interacting with the cubic superpotential

$$\mathcal{W}_{\mathcal{T}_B^{\mathbf{4d}}} = \sum_{i,j,k=1}^{N} \sum_{a,b,c,d,e,f=1}^{6} \epsilon_{abcdef}\, \mu_{ab;i}\, \mu_{cd;j}\, \mu_{ef;k}\, \delta_{i+j+k,2N+1}$$

Some properties:

- reduces to Seiberg duality between $SU(2)$ with 6 chirals and WZ model with 15 chirals for $N=1$ after integrating out β_1 and A
- the non-anomalous global symmetry of the two theories is $SU(6)\times U(1)_x$

Transformation properties of the fields under the global symmetry

	$SU(6)$	$U(1)_x$	$U(1)_{R_0}$
β_i	$\bullet$	$-i$	2
Q	$\mathbf{6}$	$\frac{1-N}{3}$	$\frac{1}{3}$
A	$\bullet$	1	0
μ_i	$\mathbf{15}$	$i-\frac{2N+1}{3}$	$\frac{2}{3}$

Consider the subgroup $SU(4)\times SU(2)\times U(1)_s \subset SU(6)$

$$
\begin{aligned}
\mathbf{6} &\rightarrow (\mathbf{4},\mathbf{1})^{1}\oplus(\mathbf{1},\mathbf{2})^{-2}\\
\mathbf{15} &\rightarrow (\mathbf{4},\mathbf{2})^{-1}\oplus(\mathbf{6},\mathbf{1})^{2}\oplus(\mathbf{1},\mathbf{1})^{-4}
\end{aligned}
$$

We allow for a mixing of $U(1)_{R_0}$ with $U(1)_s$ with mixing $R_s=-\frac{1}{3}$:

- on the side of theory $\mathcal{T}_A^{4d}$ we get 4 chirals from Q, one antisymmetric chiral from A and N Fermi singlets from β_i
- on the side of theory $\mathcal{T}_B^{4d}$ we get $6N$ chirals and N Fermis

We get the following putative $2d$ $\mathcal{N} = (0,2)$ duality [MS '20]

Theory $\mathcal{T}_{\mathbf{A}}$: $USp(2N)$ gauge theory with one antisymmetric chiral A, four fundamental chirals Q_a and N Fermi singlets β_i with superpotential

$$W_{\mathcal{T}_{\mathbf{A}}} = \sum_{i=1}^{N} \beta_i \mathrm{Tr}_N A^i$$

Theory $\mathcal{T}_{\mathbf{B}}$: LG model with N Fermi fields Ψ_i and $6N$ chiral fields $\Phi_{ab;i}$ for $i = 1, \cdots, N$, $a < b = 1, \cdots, 4$ interacting with cubic superpotential

$$W_{\mathcal{T}_{\mathbf{B}}} = \sum_{i,j,k=1}^{N} \sum_{a,b,c,d=1}^{4} \epsilon_{abcd} \, \Psi_i \, \Phi_{ab;j} \, \Phi_{cd;k} \, \delta_{i+j+k,2N+1}$$

Some properties:

- reduces to the duality between $SU(2)$ with 4 chirals and LG model with 6 chirals and one Fermi for $N = 1$ after integrating out β_1 and A [Gadde, Razamat, Willett '15; Gukov, Dedushenko '17]
- the non-anomalous global symmetry of the two theories is $SU(4) \times U(1)_s \times U(1)_x$

The 4d duality for $USp(2N)$ with antisymmetric can be derived iterating the IP duality in the confining case $N_f = N_c + 2$. [Rains '03]

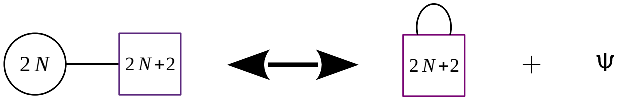
There exists a 3d $\mathcal{N} = 2$ version of the duality relating $U(N)$ with one adjoint and one fundamental flavor to a WZ model of 3N chirals with cubic superpotential. [Benvenuti '18]

Also the 3d duality can be derived iterating Aharony duality [Aharony '97] and a variant with monopole superpotential [Benini, Benvenuti, Pasquetti '17] in the confining case. [Pasquetti, MS '19]

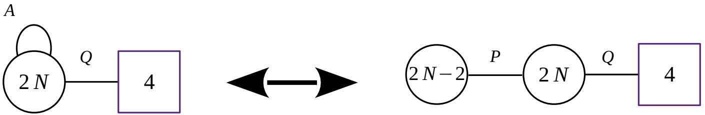
The 3d derivation is the uplift of the one for the DOZZ evaluation formula of the 3-point function in Liouville CFT in the free field formalism. [Fateev, Litvinov '07]

We can complete this picture and provide a **derivation of the 2d duality**!

Recall $2d$ IP duality



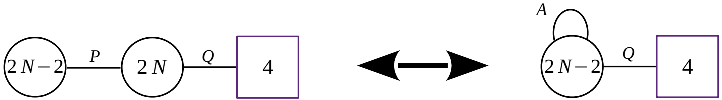
Use it to find an auxiliary dual frame



$\mathcal{W} = \sum_{i=1}^{N} \beta_i \mathrm{Tr}_N A^i$

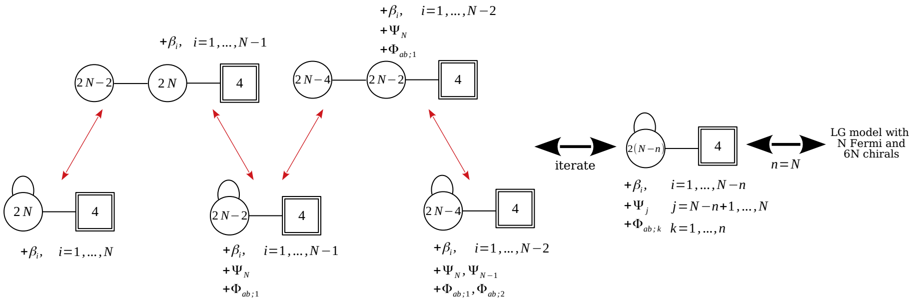
$\mathcal{W} = \sum_{i=1}^{N-1} \beta_i \mathrm{Tr}_N \left(\mathrm{Tr}_{N-1} P\, P\right)^i$

Apply IP on the original $USp(2N)$ node



$\mathcal{W} = \sum_{i=1}^{N-1} \beta_i \mathrm{Tr}_N \left(\mathrm{Tr}_{N-1} P\, P\right)^i$

$\mathcal{W} = \sum_{i=1}^{N-1} \beta_i \mathrm{Tr}_{N-1} A^i +$
$+\epsilon_{abcd} \Psi_N \Phi_{ab;1} \mathrm{Tr}_{N-1}(Q_c A^{N-2} Q_d)$



$+\beta_i, \quad i=1,\dots,N-1$
$2N-2$
$2N$
4
$+\beta_i, \quad i=1,\dots,N-2$
$+\Psi_N$
$+\Phi_{ab;1}$
$2N-4$
$2N-2$
4
$2N$
4
$+\beta_i, \quad i=1,\dots,N$
$2N-2$
4
$+\beta_i, \quad i=1,\dots,N-1$
$+\Psi_N$
$+\Phi_{ab;1}$
$2N-4$
4
$+\beta_i, \quad i=1,\dots,N-2$
$+\Psi_N, \Psi_{N-1}$
$+\Phi_{ab;1}, \Phi_{ab;2}$
iterate
$2(N-n)$
4
$+\beta_i, \quad i=1,\dots,N-n$
$+\Psi_j \quad j=N-n+1,\dots,N$
$+\Phi_{ab;k} \quad k=1,\dots,n$
$n=N$
LG model with
N Fermi and
6N chirals

Some open questions

- clarify some of the fundamental issues
- are there other $4d$ dualities that can be ***reduced*** to $2d$?
- reduction of $4d$ theories with IR global symmetry enhancement
- relation to $2d$ $\mathcal{N} = (0, 2)$ preserving boundary conditions for $3d$ $\mathcal{N} = 2$ theories [Dimofte, Gaiotto, Paquette '17]