M2-Branes, Local G_2 Manifolds and a Colored Quantum Mechanics

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GLSMs – 2020

Overview

Introduction and Motivation

- Compact G_2 manifolds in M-theory engineer 4d models with minimal supersymmetry.

  [Joyce, 1996], [Kovalev, 2003], [Corti, Haskins, Nordström, Pacini, 2015]

  [Acharya, 1998], [Halverson, Morrison, 2015], [Guio, Jockers, Klemm, Yeh, 2017],

  [Braun, Schafer-Nameki, 2017], [Braun, Del Zotto, 2017], [Xu, 2020]

- The gauge theory sector can be isolated by considering non-compact, i.e. local, G_2 manifolds.

  [Acharya, 2000], [Acharya, Witten, 2001], [Witten, 2001], [Atiyah, Witten, 2003],

  [Pantev, Wijnholt, 2009], [Braun, Cizel, H, Schafer-Nameki, 2018],

  [Barbosa, Cvetič, Heckman, Lawrie, Torres, Zoccarato, 2019],

  [Cvetič, Heckman, Rochais, Torres, Zoccarato, 2020]

- F-theory methods relying on Higgs bundles and their spectral covers can be applied to study the physics of local G_2 manifolds. [Beasley, Heckman, Vafa, 2009], [Hayashi, Kawano, Tatar, Watari, 2009], [Marsano, Saulina, Schafer-Nameki, 2009], [Marsano, Saulina, Schafer-Nameki, 2010], [Blumenhagen, Grimm, Jurke, Weigand, 2010], [Donagi, Wijnholt 2011], [Donagi, Wijnholt 2014]
- Supersymmetric sigma models probing the geometries give insight into non-perturbative effects. [Alvarez-Gaumé, Witten, 1981], [Witten, 1982], [Pantev, Wijnholt, 2009], [Braun, Cizel, H, Schafer-Nameki, 2018]

Motivation: Improved understanding of the gauge theory sector in G_2 compactifications

ALE Fibered, Local G_2 Manifolds

Geometric data

$$\begin{aligned} \text{Local } G_2 \text{ Manifold} &: \quad \widetilde{\mathbb{C}^2/\Gamma_{\text{ADE}}} \hookrightarrow X_7 \rightarrow M_3 \\ \text{Fibral 2-Spheres} &: \quad \sigma_I \in H_2(\widetilde{\mathbb{C}^2/\Gamma_{\text{ADE}}}, \mathbb{R}) \\ \text{Hyperkähler Triplet} &: \quad (\omega_1, \omega_2, \omega_3) \in H^2(\widetilde{\mathbb{C}^2/\Gamma_{\text{ADE}}}, \mathbb{R}) \end{aligned}$$

The Higgs field collects the Kähler periods

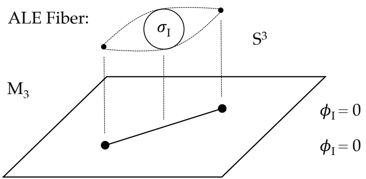
$$\text{Higgs field} : \quad \phi_I = \left(\int_{\sigma_I} \omega_i \right) dx^i \in \Omega^1(M_3)$$

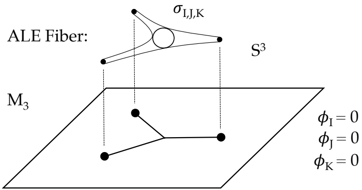
where $I = 1, \ldots, \text{rank}\, \mathfrak{g}_{\text{ADE}}$.

Singularities and Supersymmetric 3-cycles

Singularity Enhancement at $x \in M_3$: $\quad \phi_I(x) = 0 \quad$ (isolated)

The vanishing cycles trace out 3-spheres:





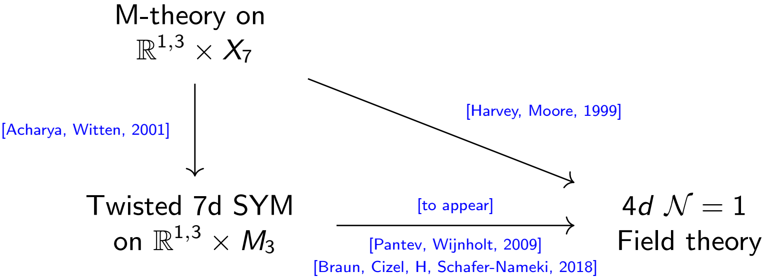
Questions

What do the supersymmetric 3-spheres descend to in an effective 7d field theory description?

What is the global structure of the network of supersymmetric 3-spheres?

How can their contribution to the 4d superpotential be computed from the 7d field theory description?

Previous Work



Effective 7d Physics

M-theory on the local G_2 manifold X_7 gives a

Partially twisted 7d SYM on $\mathbb{R}^{1,3} \times M_3$
with gauge group G_{ADE} .

Complex bosonic 1-form on M_3 : $\varphi = \phi + iA \in \Omega^1(M_3, \mathfrak{g}_{\text{ADE}})$

Supersymmetric vacua are solutions of a Hitchin system:

$$i(F_A)_{ij} + [\phi_i, \phi_j] = 0\,, \qquad (d_A\phi)_{ij} = 0\,, \qquad * d_A * \phi = 0$$

Zero modes along M_3 are determined by

$$H = \frac{1}{2}\left\{ Q, Q^\dagger \right\}, \qquad Q = d + [\varphi, \cdot\,]$$

and counted by the cohomologies

$$H^*_Q(M_3, \mathfrak{g}_{\text{ADE}}) .$$

Alternatively, consider perturbative zero modes

$$\chi_a \in \Omega^*(M_3, \mathfrak{g}_{\text{ADE}}) \qquad \leftrightarrow \qquad \text{Codimension 7 Singularity}$$

The zero modes are recovered from

$$\text{Mass Matrix :} \qquad M_{ab} = \int_{M_3} \langle \chi_b, Q\chi_a \rangle$$

$$\text{Yukawa Couplings :} \qquad Y_{abc} = \int_{M_3} \langle \chi_c, [\chi_a, \chi_b] \rangle .$$

Colored $\mathcal{N} = 2 = (1,1)$ SUSY Quantum Mechanics

The physical Hilbertspace is given by Lie algebra valued forms

$$\mathcal{H}_{\text{phys.}} = \Lambda(M_3, \mathfrak{g}_{\text{ADE}})$$

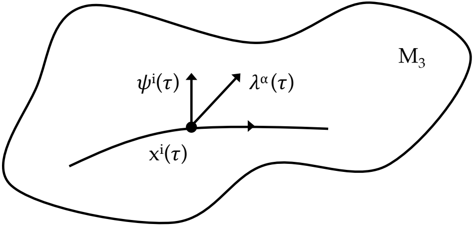
and the supercharge acts on it as the operator

$$Q = d + [\varphi, \cdot\,].$$

The colored SQM is an extension of Witten's SQM [Witten, 1982] by an adjoint bundle on the target space.

The dynamical fields are

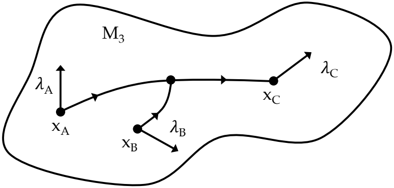
$$\text{Bosonic Coordinates on } M_3 : \quad x^i\,, \quad i = 1,2,3$$
$$\text{Fermions in } x^*(TM_3) : \quad \psi^i\,, \quad i = 1,2,3$$
$$\text{Color fermions in } x^*(\text{ad}\,G_{\text{ADE}}) : \quad \lambda^\alpha\,, \quad \alpha = 1,\ldots,\dim \mathfrak{g}_{\text{ADE}}$$



Perturbative ground states of $H = \frac{1}{2}\left\{Q, Q^\dagger\right\}: \quad (x, \lambda)$

1/2-BPS instantons are solutions to the flow equations

Flow line instanton : $\quad \dot{x}^i - \phi^i_\lambda = \dot{x}^i - ic^\alpha{}_{\beta\gamma}\phi^i_\alpha \bar{\lambda}^\beta \lambda^\gamma = D_\tau \lambda^\alpha = 0\,.$



Colored Instantons $\longleftrightarrow$ Euclidean M2-brane instantons

Abelian Higgs Backgrounds

Abelian solutions to the BPS equations: $A = 0\,, \quad \phi = \mathrm{diag}(\Lambda_K)\,.$

With eigenvalue 1-forms Λ harmonic up to source terms

$$d\Lambda_K = *j_K\,, \qquad *d*\Lambda_K = \rho_K\,,$$

which are supported on codimension ≥ 1 subloci in M_3.

Two classes of solutions distinguished by their spectral cover:

$$\mathcal{C} = \{(x, \Lambda_K(x))\,|\,x \in M_3\} \subset T^*M_3 \quad \rightarrow \quad \begin{cases} \Lambda_K \text{ globally defined} \\ \Lambda_K \text{ connected by branch cuts} \end{cases}$$

Colored SQM for Split Spectral Covers

Globally defined eigenvalues Λ_K :

The Higgs field is valued in the Cartan subalgebra $\phi = \mathrm{diag}(\Lambda_K)$ and the supercharge $Q = d + [\phi, \cdot\,]$ preserves color.

Root α	$\leftrightarrow$	Witten SQM into M_3 with supercharge $Q^{(\alpha)} = d + \alpha^I \phi_I$

$$M_{ab} = \int_{M_3} \langle \chi_b, Q\chi_a \rangle = \sum_{\Gamma_{ab}} (\pm)_{\Gamma_{ab}} \exp\left(- \int_{\Gamma_{ab}} \alpha^I \phi_I \right)$$

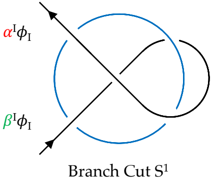
$$Y_{abc} = \int_{M_3} \langle \chi_c, [\chi_a, \chi_b] \rangle = \sum_{\Gamma_{abc}} (\pm)_{\Gamma_{abc}} \exp\left(- \int_{\Gamma_a} \alpha^I \phi_I - \int_{\Gamma_b} \beta^I \phi_I + \int_{\Gamma_c} \gamma^I \phi_I \right)$$

Colored SQM for Non-split Spectral Covers

The eigenvalue 1-forms of the Higgs field are interchanged along a paths linking the branch locus

$$\text{Monodromy Action}: \quad \phi \to g\phi g^{-1}$$
$$\text{Color Mixing}: \quad E^\alpha \to gE^\alpha g^{-1}$$

Monodromy orbit $[\alpha]$ $\leftrightarrow$ Witten SQM into $\mathcal{C}_k$ with supercharge $Q^{([\alpha])} = d + \phi_{[\alpha]}$



4d Symmetries, Matter and Interactions

	Globally defined Λ_K	Locally defined Λ_K
Gauge Symmetry:	Commutant of ϕ	Stabilizer of ϕ
Chiral Multiplets:	$H^1(M_3, \partial_-^{(\alpha)} M_3)$	$H^1_{\text{Nov.}}(\mathcal{C}_k, \phi_{[\alpha]})$
Conj. Chiral Multiplets:	$H^2(M_3, \partial_-^{(\alpha)} M_3)$	$H^2_{\text{Nov.}}(\mathcal{C}_k, \phi_{[\alpha]})$
Light Multiplets:	Instanton Corrected Modes $m = \mathcal{O}(\lvert M_{ab} \rvert)$	
Interactions:	Cup-Product Y	

$$Y = [\,\cdot \wedge , \cdot\,] , \qquad (\chi_a, \chi_b) \;\mapsto\; \sum_c Y_{abc} \chi_c \,.$$

Summary

We introduced a colored SQM and established the correspondence

$$\text{Euclidean M2 Instantons} \quad \leftrightarrow \quad \text{Instantons of Colored SQM}$$

For abelian backgrounds we compute instanton effects giving the

$$\text{Mass Matrix}: \qquad M_{ab} = \int_{M_3} \langle \chi_b, Q\chi_a \rangle$$

$$\text{Yukawa Couplings}: \qquad Y_{abc} = \int_{M_3} \langle \chi_c, [\chi_a, \chi_b] \rangle \,.$$

Mathematical difficulties remain

- Description of Higgs backgrounds with non-split spectral covers allowing for the evaluation of its Novikov groups

Outlook

- Study fluxed solutions (T-branes)
- Probe other Higgs bundle vacua
- Develop the colored SQM

The End

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